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부석사 무량수전 배흘림 목재 기둥의 좌굴강도

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Buckling Strength of Wooden Column with Entasis at the Muryangsugeon in Buseoksa-Temple

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Abstract: In this paper we present the result of investigations pertaining to the buckling strength of Zelkova Serrata (Elm-like) tree column with entasis at the Muryangsujeon in Buseoksa-Temple, Korea. Wooden columns with entasis had been used in the construction of ancient architectural buildings in Korea. It was not known why did they design columns with entasis of the buildings. It is just presumed that the reason may be the compensation of optical illusion, aesthetics, and/or structural safety. The question is not answered even today and it may not be possible to answer clearly and easily. In the paper, the buckling analyses are conducted on both of the wooden column with entasis and the prismatic wooden column by the successive approximations technique and the finite element methods, respectively. The results of analyses are compared and discussed.

Key Words: Buckling analysis, Wooden column with entasis, Muryangsujeon, Successive approximations technique, Finite element method

1. INTRODUCTION

1.1 Historical Review

The word “Entasis”, which is originated from Greek word, means the application of a convex curve to a surface for aesthetic purposes (Wikipedia/Entasis 2014). Definition of entasis is a slight convexity especially in the shaft of a column (Webster/Entasis 2014). This type of columns with entasis was used in an architecture early from Egyptian pyramids, but it continues to use in Greek column designs (i.e., Doric-order temples), to use in Roman temples, and to use even in Chinese Song Dynasty (1103 A.D. Yingzao Fashi) (Wikipedia/Etasis 2014). Entasis is also found in Inca walls and double-jamb doorways to counteract the optical illusion (Wikipedia/Entasis 2014). Even in Korea and Japan, this type of columns with entasis was used in the ancient architectural buildings, for example, Muryangsugeon (the Hall of Eternal Life) at the Buseoksa-temple, in which the word “Buseok sa-temple” means the “temple of the floating rock”, in Korea and the columns at Horyu-ji in Japan (Wikipedia/Entasis 2014).

Design of column with entasis in the architectural

주요어: 좌굴해석, 배흘림목재기둥, 무량수전, 연속근사법, 유한요소법

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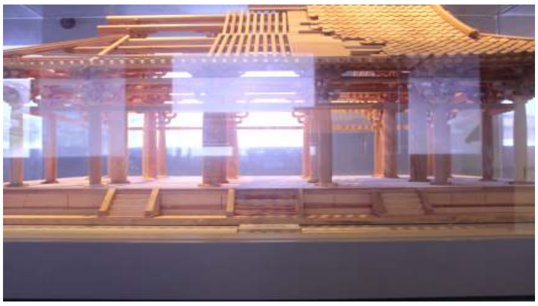
buildings still continues even nowadays for various purposes with various shapes and materials such as merely decorations in addition to the compensation of optical illusion, aesthetics, and structural safety. Variety of such design illustrations are available not only in domestic but also in international Internet Web Site (Google/Muryangsugeon 2015; Google/Entasis Architecture 2015; Cheyenne 2014).

Muryangsugeon at the Buseoksa-temple is the National Treasure No. 18 in Korea. Muryangsugeon is the main building of the Buseoksa-temple. According to the reference in the Museum at the Buseoksa-temple, after the victory of war between Silla (Korea) and Tang (China), Uisang (a great Buddhist priest) builds the Buseoksa-temple in 676 A.D. (King Munmu's 16th year) under orders from King Munmu (661-681 A.D.). This building is famous for its architectural appearance including column style and bracket sets in addition to its beauty and its famous folk story (i.e., The Tale of Seonmyo). Fig. 1 shows the Muryangsugeon at the Buseoksa-temple.

(a) Side View (1)

(b) Side View (2)

(c) Front View



(d) Scale-Down Model

Fig. 1 Muryangsugeon, Buseoksa-Temple

1.2 Muryangsujeon

Muryangsugeon is the National Treasure No. 18 as shown in Fig. 1, as mentioned. The material for the column is made of wood "Zelkova Serrata tree (Neuti-namu in Korean) which is one of Elm-like tree species". Zelkova tree is one of the hard wood species. Most of broad-leaf tree (a latifoliate tree) is categorized in the hard woods while most of a needle-leaf tree (a coniferous tree) belongs to the soft woods, in general.

Wood is one of the typical natural composite materials and it may be classified as an orthotropic material, macroscopically. More closely it is similar to the transversely isotropic material. Among the man-made composites, pultruded fiber reinforced polymer matrix composite is one of the typical examples of transversely isotropic material.

The mechanical properties with respect to the direction in the wood are different significantly. The mechanical properties of wood are different because of various conditions such as the moisture content (it is usually measured at 12% moisture content), the temperature at measurement, direction and location of test samples taken, etc. (Kiaei 2011). The mechanical properties of Zelkova tree is not measured in this study. Therefore, we found in the literature as the mechanical properties of Zelkova tree (Elm-like tree) such as $E_l = 10.5\,GPa$, $E_t = 1.25\,GPa$, and $E_r = 1.90\,GPa$ for the modulus of elasticity (E) and for the Poisson's ratios (ν) $\nu_{lt} = \nu_{rt} = 0.6$ and $\nu_{lr} = 0.4$, respectively. In the properties, the letter used in the subscripts is defined as l for longitudinal, r for radial, and t for tangential direction, respectively (Kim 2012).

1.3 Objective

Why did they design columns with entasis in the ancient architectural buildings? To find the answers many researches were conducted throughout the world. Recently, Thompson et al. (2007) studied on "The origins of entasis: illusion, aesthetics or engineering?" In the paper, they conducted a series of psychological experiments to find such visual illusion exists on both in parallel sided columns (prismatic columns) and in tapering columns (columns with convexity in the middle). They found little evidence to support any illusion-compensation theory. In addition, they also explored the possibility that entasis was employed for purely aesthetic reasons, but they did not find the results supporting aesthetic reasons. Finally, they presented some evidences supporting the engineering hypothesis (i.e., the buckling strength of column with entasis is higher than that of the parallel sided column). They argued that the reason of entasis is purely engineering. However, they did not present any calculations on the buckling strength but just cited several research results conducted by others. In fact, because they are psychologists, it is natural that they could not provide evidences supporting engineering hypothesis.

In order to find the evidence supporting the engineering hypothesis, theoretical calculations on the buckling strength of column with entasis may need to be conducted.

2. COLUMN WITH ENTASIS AT THE MURYANGSUJEON IN BUSEOKSA-TEMPLE

2.1 Mechanical Properties of Column Material

Column material of the Muryangsugeon is known to be Zelkova Seratta tree. The mechanical properties of Zelkova Serrata tree is not measured in this study. Kim (2012) presented the special article on the mechanical properties of wood as a material for the wooden bridge construction. In fact, wood is, typically, an anisotropic heterogeneous material and its hygrothermal effects are also significant. According to the literature presented by Kim (2012), we assumed the mechanical properties: $E_l = 10.5\,GPa$, $E_t = 1.25\,GPa$, and $E_r = 1.90\,GPa$ for the modulus of elasticity (E) and for the Poisson's ratios (ν) $\nu_{lt} = \nu_{rt} = 0.6$ and $\nu_{lr} = 0.4$, where the subscripts are defined as l for longitudinal, r for radial, and t for tangential direction, respectively.

For the sake of simplicity of our discussion relating to the effect of entasis on the buckling strength of the column, we assumed the wood as an isotropic material with the modification of mechanical properties as:

$$E_{tr} = (E_t + E_r)/2 \qquad (\because E_t \approx E_r),$$

$E_{wood} = (E_{tr} \times E_l)^{1/2}$, and with a similar reason, $\nu_{lt} = \nu_{rt}$, $\nu_{wood} = (\nu_{lt} \times \nu_{lr})^{1/2}$. Finally, the modulus of elasticity of Zelkova wood $E_{wood} = 4.067\,GPa$ and Poisson's ratio of Zelkova wood $\nu_{wood} = 0.490$ are decided, respectively.

According to the research conducted by Nam et al. (2004), if we use approximated mechanical properties by using geometric mean of the modulus of elasticity and Poisson's ratio in the structural analysis of orthotropic plate it results in close agreement between the results of orthotropic and approximated isotropic plate analyses. In other words, the structural member composed of an orthotropic material can be analyzed as the structural member composed of an isotropic material in which the orthotropic elastic constants are modified into the isotropic elastic constants by using geometric mean (Wikipedia/Geometric Mean 2014).

In addition, in the investigation of the effect of entasis on the column buckling strength, the differences either orthotropic or isotropic in the mechanical property of material on the column buckling analysis may not hinder the comparison of results as long as the same material is used in the analysis.

2.2 Dimension and Modeling for the Buckling Analysis

For the buckling analysis it is necessary to determine the dimension of column member to be analyzed. Dimensions of eight columns measured at the site by other researchers are analyzed. Dimensions of the columns are different each other and the location of the maximum convexity of the column is also different.

Therefore, based on the results of analysis, the dimension of wooden column with entasis is decided, for the sake of simplicity, as given in Table 1. As can be seen in the table, the maximum convexity is in the middle of the column height, and hence, the column is symmetric with respect to its mid-height.

For the buckling analysis, the column is modeled by dividing into ten segments and each segment is assumed to be prismatic, and hence, each segment has uniform cross-sectional area (i.e., same diameter and same moment of inertia). In Table 1, ten evenly spaced segments are numbered from node 0 to node 10, and the diameter and corresponding moment of inertia at each node of circular cross-section of the column are presented.

Table 1. Dimension of Wooden Column

Node No.	Diameter D (mm)	Moment of Inertia I (mm^4)
0	414.065	$1.00I_0$
1	452.940	$1.43I_0$
2	471.000	$1.67I_0$
3	483.345	$1.86I_0$
4	490.230	$1.96I_0$
5	492.250	$2.00I_0$
6	490.230	$1.96I_0$
7	483.345	$1.86I_0$
8	471.000	$1.67I_0$
9	452.940	$1.43I_0$
10	414.065	$1.00I_0$

Remark: $I_0=144.293\times10^7$ mm^4

3. BUCKLING ANALYSIS OF COLUMN WITH ENTASIS

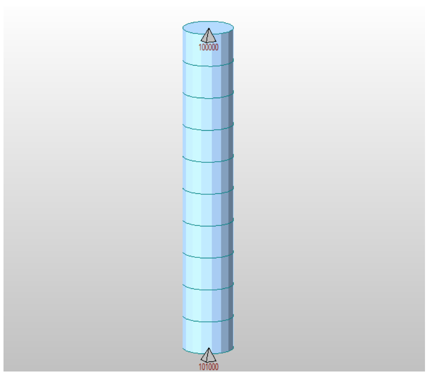
3.1 Successive approximations technique

The method of successive approximations is usually used to determine buckling loads in cases where the exact solution is not known or very complicated. It is known that the application of the method to buckling problems is due to Engesser (Timoshenko and Gere 1961).

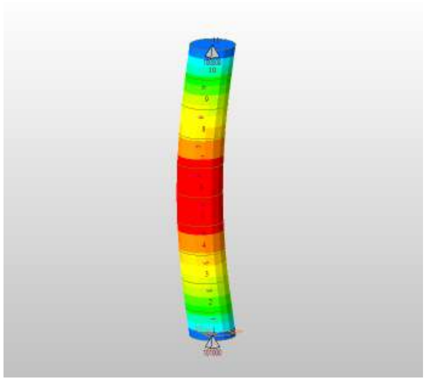
Prior to continue buckling analysis by the method, the accuracy of the result needs to be verified. Buckling analyses on the simply supported prismatic column with ten evenly spaced segments are conducted by the Euler buckling analysis ($P_{cr}=\pi^2 EI/l^2$) and the successive approximations technique, respectively, and the results are compared. It was found that the buckling load by the successive approximations technique is 99.984% of the Euler buckling load. Therefore, it can be said that the result obtained by the method of successive approximations yields virtually the same as exact one. Detail calculation procedure with the buckling load for the column with entasis is given in Table 2.

3.2 Numerical Analysis by the Finite Element Methods (ANSYS®, MIDAS®, GTSTRUDL®)

In addition to the buckling analysis by the method of successive approximations, the buckling analyses by the finite element method are also conducted. In the finite element analyses, commercial softwares ANSYS®, MIDAS®, and GTSTRUDL® are utilized. For the modeling of the columns with and without entasis, column dimension given in Table 1 is used. For the buckling analysis of parallel sided column (i.e., prismatic column), average diameter of the column cross-section is used. The buckling analysis on the prismatic column with the same mechanical properties of the material is conducted for comparison purposes and to investigate the effect of entasis on the column buckling strength. The results are shown in Fig. 2 to Fig. 4 and the buckling loads are also given in Table 3 to Table 5, respectively.

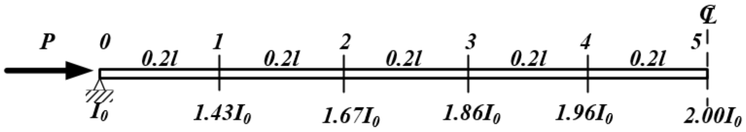


(a) Prismatic Column Model



(b) Buckled Mode Shape of Prismatic Column

Table 2. Successive Approximations Analysis



Station Number	0	1	2	3	4	5	Common Factor
Degree	0°	18°	36°	54°	72°	90°	
y_1	0	31.000	59.000	81.000	95.000	100.000	$\delta_1/100$
M_1/EI	0	21.678	35.329	43.548	48.469	50.000	$P\delta_1/100EI_0$
R_1		4.202	6.975	8.655	9.637	9.949	$P\delta_1 l/100EI_0$
Average Slope 1		34.444	30.242	23.267	14.612	4.975	$P\delta_1 l/100EI_0$
y_2	0	6.889	12.937	17.590	20.512	21.507	$P\delta_1 l^2/100EI_0$
y_1/y_2		4.500	4.561	4.605	4.631	4.650	EI_0/Pl^2
y_2	0	32.031	60.153	81.787	95.374	100.000	$\delta_2/100$
M_2/EI	0	22.399	36.020	43.972	48.660	50.000	$P\delta_2/100EI_0$
R_2		4.334	7.110	8.740	9.676	9.955	$P\delta_2 l/100EI_0$
Average Slope 2		34.838	30.504	23.394	14.654	4.978	$P\delta_2 l/100EI_0$
y_3	0	6.968	13.069	17.748	20.679	21.675	$P\delta_2 l^2/100EI_0$
y_2/y_3		4.597	4.603	4.608	4.612	4.614	EI_0/Pl^2
y_3	0	32.148	60.295	81.882	95.405	100.000	$\delta_3/100$
M_3/EI	0	22.481	36.105	44.023	48.676	50.000	$P\delta_3/100EI_0$
R_3		4.349	7.126	8.750	9.680	9.956	$P\delta_3 l/100EI_0$
Average Slope 3		34.883	30.534	23.408	14.658	4.978	$P\delta_3 l/100EI_0$
y_4	0	6.977	13.084	17.766	20.698	21.694	$P\delta_3 l^2/100EI_0$
y_3/y_4		4.608	4.608	4.609	4.609	4.610	EI_0/Pl^2
y_4	0	32.161	60.312	81.894	95.409	100.000	$\delta_4/100$
M_4/EI	0	22.490	36.115	44.029	48.678	50.000	$P\delta_4/100EI_0$
R_4		4.350	7.128	8.751	9.680	9.956	$P\delta_4 l/100EI_0$
Average Slope 4		34.887	30.537	23.409	14.658	4.978	$P\delta_4 l/100EI_0$
y_5	0	6.977	13.084	17.766	20.698	21.694	$P\delta_4 l^2/100EI_0$
y_4/y_5		4.610	4.610	4.610	4.610	4.610	EI_0/Pl^2

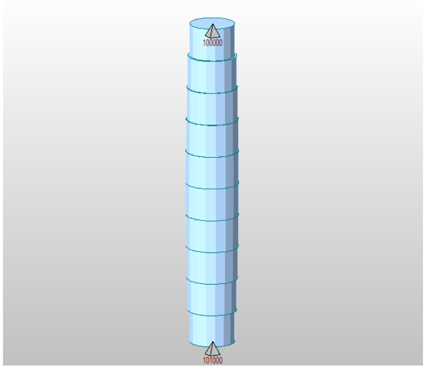
E (GPa)	I_0 (mm^4)	l (mm)	P_{cr} (MN)
4.067	144.293×10^7	1500.000	12.024

Remark:

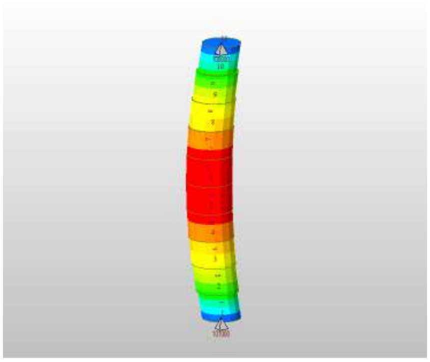
① $P_{cr(average)} = \frac{\pi^2 EI_{average}}{L^2} = \frac{\pi^2(4067N/mm^2)(234.043\times10^7 mm^4)}{3000^2 mm^2} = 10.438MN$ (where L=3000 mm)

② $P_{cr} = 4.610\frac{EI_0}{l^2} = 4.610\times\frac{(4067N/mm^2)(144.293\times10^7 mm^4)}{1500^2 mm^2} = 12.024MN$ (where l = L/2 = 1500 mm)

②/①=1.152

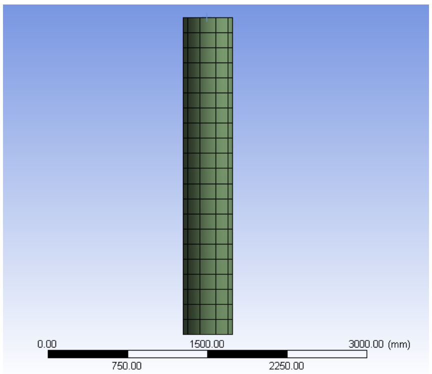


(c) Column with Entasis Model

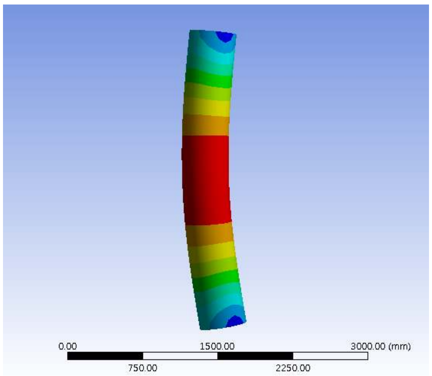


(d) Buckled Mode Shape of Column with Entasis

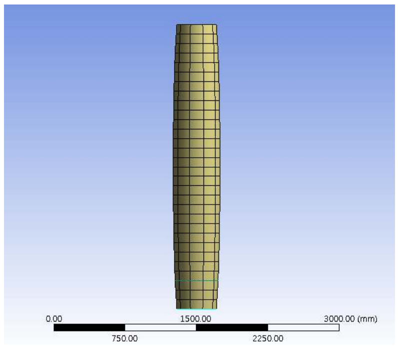
Fig. 2 Result of Buckling Analysis by MIDAS®



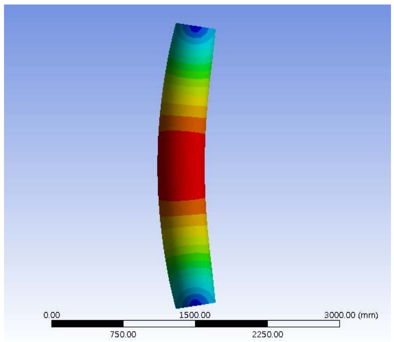
(a) Prismatic Column Model



(b) Buckled Mode Shape of Prismatic Column

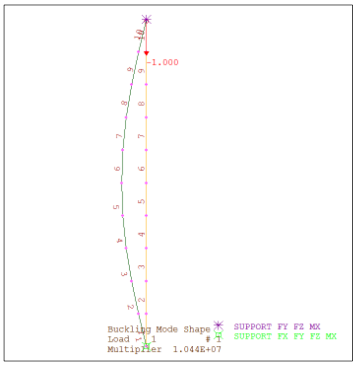


(c) Column with Entasis Model

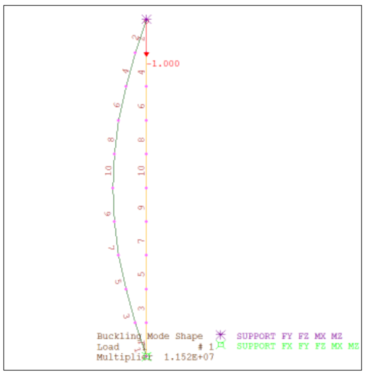


(d) Buckled Mode Shape of Column with Entasis

Fig. 3 Result of Buckling Analysis by ANSYS®



(a) Buckled Mode Shape of Prismatic Column



(b) Buckled Mode Shape of Column with Entasis

Fig. 4 Result of Buckling Analysis by GTSTRUDL®

Table 3. Buckling Load by MIDAS®

Buckling Load	① Prismatic Column with Diameter 467.284mm	② Column with Entasis	②/①
P_{cr} (MN)	10.040	10.960	1.092

Table 4. Buckling Load by ANSYS®

Buckling Load	① Prismatic Column with Diameter 467.284mm	② Column with Entasis	②/①
P_{cr} (MN)	10.170	11.610	1.142

Table 5. Buckling Load by GTSTRUDL®

Buckling Load	① Prismatic Column with Diameter 467.284mm	② Column with Entasis	②/①
P_{cr} (MN)	10.438	11.518	1.103

4. COMPARISON OF ANALYSIS RESULTS

4.1 Comparison of Results

As mentioned above, the buckling analyses on the columns with and without entasis are performed by the method of successive approximations and the numerical methods. For the numerical analysis, ANSYS®, MIDAS®, and GTSTRUDL®, which are commercial finite element analysis softwares, are used.

In the analysis, the dimension of columns with entasis at the Muryangsugeon is modified for the sake of simplicity as given in Table 1. In addition, the mechanical properties of material of the Zelkova Serrata tree used in the column are also modified from orthotropic to approximated isotropic by geometric mean of the material properties. The result obtained in the study is summarized as given in Table 6. Even in Table 3 to Table 5, the effect of entasis on the column buckling strength is given, respectively.

Table 6. Comparison of Results

Analysis / Buckling Load	Successive Approximations	Numerical Analysis		
		MIDAS®	ANSYS®	GTSTRUDL®
P_{cr} (MN)	12.024	10.960	11.610	11.518

4.2 Discussion

As can be seen in Table 2 to Table 5, the buckling load of column with entasis is larger than the buckling load of parallel sided column, as expected. The buckling loads obtained by the method of successive approximations, MIDAS®, ANSYS®, and GTSTRUDL® are larger about 15.2%, 9.2%, 14.2%, and 10.3%, respectively.

According to Thompson et al. (2007), they reported, by citing Keller's work, that the buckling load of column with entasis is 61.2% larger than that of the prismatic column. They also reported that the work presented by Cox was about 33% larger buckling load.

During conducting present study details of Keller's and Cox's works including the informations about the material properties, geometrical conditions of columns, analysis conditions, etc. are not available. Therefore, direct comparison with other's work is meaningless.

If we concluded the discussion for the effect of entasis on the column buckling load (hence strength), it can say that the buckling strength of column with entasis is 15%, approximately, larger than that of parallel sided column based on the result obtained by the method of successive approximations which is the most accurate and almost exact one.

About 15% larger buckling load may not so strong evidence to support for the engineering hypothesis in consideration to the tolerance of measurement in the structural engineering. This result is somewhat controversial with the conclusion of Thomson et al. (2007).

5. CONCLUSION

In order to answer to the question, "why did they design column with entasis in the ancient architectural buildings in Korea?", we have examined the effect on the buckling strength of column with and without entasis. In the conclusion of the work by Thomson et al. (2007), the compensation of optical illusion and aesthetics have no strong evidence to support hypotheses. In our study it was found that the structural safety by studying the buckling strength of columns with and without entasis does not support strongly the engineering hypothesis. Therefore it is concluded that they might design the columns with entasis in the ancient architectural buildings in Korea for the compensation of an optical illusion, aesthetics, and structural safety, combined all together, not for the individual purpose.

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References

ANSYS® (2013), ANSYS Workbench 14.5.

Cheyenne, PVC Columns (2014), Available from: http://www.cheyennecompany.com, Accessed Aug. 24.

Google, Muryangsugeon (2015), Available from: http://www.google.co.kr, Accessed Jan. 17.

Google, Entasis Architecture (2015), Available from: http://www.google.co.kr, Accessed Jan. 17.

GTSTRUDL® (2013), GTSTRUDL Ver. SE13.

Kiaei, M. (2011), "Variation in the Wood Physical and Mechanical Properties of Zelkova Carpinifolia Trees along Longitudinal Direction," Middle-East Journal of Scientific Research, Vol. 9, No. 2, pp. 279-284.

Kim, K.-M. (2012), "Understanding of Wood as a Structural Member for Timber Bridge," Special Article, KSCE Magazine, Vol. 60, No. 9, pp. 10-20.

MIDAS® (2012), MIDAS Civil 2012+ Academic.

Nam, J.-H., Son, K.-W., and Yoon, S.-J. (2004), "Flexural Analysis of Radiata Pine Plywood Plate for the Concrete Form by the Laminate Plate Theory," Mokchae Konghak, Vol. 32, No. 4, pp. 36-45.

Timoshenko, S. P. and Gere, J. M. (1961), Theory of Elastic Stability, Second Edition, McGraw-Hill, Inc., New York, USA.

Thomson, P., Papadopoulou, G., and Vassiliou, E. (2007), "The Origins of Entasis: Illusion, Aesthetics or Engineering?," Spatial Vision, Vol. 20, No. 6, pp. 531-543.

Wikipedia, Entasis (2014), Available from: http://en.wikipedia.org, Accessed Aug. 24.

Webster, Entasis (2014), Available from http://www.merriam-webster.com, Accessed Aug. 24.

Wikipedia, Geometric Mean (2014), Available from: http://en.wikipedia.org, Accessed Sep. 5.