Optimal User Density and Power Allocation for Device-to-Device Communication Underlaying Cellular Networks

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Abstract

This paper analyzes the optimal user density and power allocation for Device-to-Device (D2D) communication underlaying cellular networks on multiple bands with the target of maximizing the D2D transmission capacity. The entire network is modeled by Poisson point process (PPP) which based on stochastic geometry. Then in order to ensure the outage probabilities of both cellular and D2D communication, a sum capacity optimization problem for D2D system on multiple bands is proposed. Using convex optimization, the optimal D2D density is obtained in closed-form when the D2D transmission power is determined. Next the optimal D2D transmission power is obtained in closed-form when the D2D density is fixed. Based on the former two conclusions, an iterative algorithm for the optimal D2D density and power allocation on multiple bands is proposed. Finally, the simulation results not only demonstrate the D2D performance, density and power on each band are constrained by cellular communication as well as the interference of the entire system, but also verifies the superiority of the proposed algorithm over sorting-based and removal algorithms.

Keywords: Device-to-Device communication, user density, power allocation, stochastic geometry, convex optimization

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1. Introduction

Nowadays due to the ongoing development of wireless communication technology, the issue of spectrum shortage has become more and more serious [1]. Device-to-device (D2D) communication is one technology that may solve this problem, D2D communication can effectively enhance spectral efficiency by providing a direct link between user terminals in an underlay way with cellular networks [2][3]. D2D communication has many benefits, such as improving system capacity, reducing user power consumption, and enhancing the instantaneous data rate, D2D has drawn much attention in recent years [4]-[6].

Transmission capacity is an important indicator of a spectrum sharing system such as a cellular system where D2D users coexisted. H. Min et al. [7] proposed an interference management strategy to enhance the capacity of cellular and D2D system. Other researchers [8] studied network capacity with outage constraints. In order to maximize the transmission capacity of the system, a robust distributed solution was adopted for relay-assisted D2D communication [9]. The authors in [10] and [11] considered the achievable transmission capacity of secondary users in a system that had the outage probability constraints. An optimum resource allocation method was discussed in [12], where D2D can improve capacity under different resource sharing modes.

Some previous works analyzed methods of enhancing the performance of D2D communication. Several of these works [13]-[16] focused on modeling cellular networks underlaid with D2D communication and discussed outage probability and interference in the system. Using stochastic geometry, authors in [17] studied the distribution of transmission power and signal-to-interference-plus-noise ratio (SINR) in D2D networks. In order to minimize the average interference power level and solve the problem of interference constraints, an improved joint subcarrier and bit allocation scheme, introduced with the cooperation of primary users, was proposed in [18]. In [19], a mode selection mechanism was introduced to improve the reliability of D2D communication in different interference environment. Moreover, several resource sharing mechanisms were investigated in [20]-[22] to enhance the D2D successful transmission probability, energy efficiency and spectrum efficiency.

Previous studies considered resource allocation or interference management of a D2D system. However, user density and power allocation of D2D communication are also very important. In [23], we studied the optimization of D2D density and power with constraints on multiple bands. For this paper, the system outage probabilities and the D2D achievable transmission capacity are obtained based on stochastic geometry, then, three aspects of a D2D communication underlaying cellular network are explored:

1) The conditions that whether the sum D2D density and power constraints should consider in the system are discussed;

2) Using optimization, the optimal D2D densities with or without the sum user density constraint are derived in closed-form when D2D transmission power is fixed;

3) When D2D density is determined, the optimal D2D transmission powers are obtained in closed-form with or without the sum power constraint.

Combing the former conclusions, an iterative algorithm of optimal D2D density and power on multiple bands is proposed. Numerical results demonstrate that the D2D performance, density and power boundary are constrained by cellular communication as well as the interference of the entire system. In addition, the optimal values that are confined by sum density and power constraints are determined, and the results verify the superiority of the proposed algorithm over sorting-based and removal algorithms [24].

The rest of this paper is organized as follows: Section 2 describes the system model. Section 3 presents the system outage probabilities and the definition of D2D achievable transmission capacity on multiple bands. In Section 4, optimal D2D density and transmission power are derived, and an iterative algorithm to determine the maximum D2D achievable transmission capacity is proposed. Simulation results are shown in Section 5. Finally our conclusions are summarized in Section 6.

2. Scenario Description and System Model

2.1 Scenario Description

The basic scenario contains cellular system (S_1) and D2D system (S_0), as shown in **Fig. 1**. Cellular networks are deployed on N multiple independent bands, the bandwidths are denoted as W_i, $i=1,2,...,N$, respectively. D2D transmission shares the uplink (UL) frequency resources of the cellular system. Each D2D user is allowed to use multiple bands to transmit data at the same time

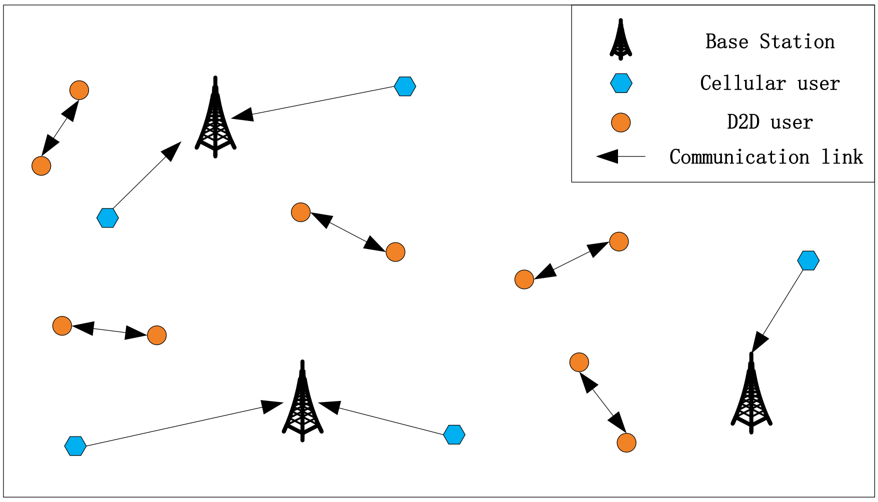


Fig. 1. System model of D2D communication underlaying cellular networks

On each band, cellular UL frequency spectrum is divided into K frequency-flat sub-channels by using Orthogonal Frequency Division Multiplexing (OFDM) technology. The full set of the sub-channels can be used by D2D communication as an underlay sharing with the cellular networks. In the cellular and D2D networks, a transmitter modulates signals by using frequency-hopping spread spectrum [8] and the signals randomly hop over all sub-channels on each band assigned to the affiliated network.

2.2 System Model

Based on stochastic geometry theory, following assumptions are made:

Assumption 1. The transmitters of D2D system form a Poisson Point Process (PPP) on the two dimensional plane M, which is denoted as Π_0 with the density $\lambda_{0,i}$ on band i,

$i = 1, 2, ..., N$. The transmission powers of D2D transmitters are denoted as $P_{0,i}$, $(i = 1, 2, ..., N)$ on each band.

Assumption 2. The cellular system forms stationary PPPs on each band which are denoted as Π_1^i with the density $\lambda_{1,i}$, $(i = 1, 2, ..., N)$ on M. The transmission powers of cellular users are denoted as $P_{1,i}$, $(i = 1, 2, ..., N)$ on each band, respectively.

Assumption 3. According to Palm theory [25], a typical receiver of system S_j, $j \in \{0,1\}$ is assumed to be located in the origin, which does not influence statistics of the PPP.

2.3 Channel Models

Path loss and Rayleigh fading are considered as the propagation channel model, which can be formed as:

$$P_{rx} = \delta P_{tx} |D|^{-\alpha} \tag{1}$$

where P_{tx} and P_{rx} represent the transmitter and receiver power respectively, α is the path loss exponent, $|D|$ is the distance between the transmitter and the receiver. δ stands for Rayleigh fading coefficient, which has an independent exponential distribution with unit mean for every communication link in the system.

In the spectrum sharing environment, the receiver suffers from the interference generated by transmitters in both cellular and D2D system. So δ_{jk} and X_{jk} can be defined respectively as Rayleigh fading coefficient and the distance from the origin to the node k, $(k \in \Pi_j)$ of system S_j, $j \in \{0,1\}$ on each band.

3. Achievable Transmission Capacity of D2D System on Multi-bands

3.1 The Outage Probability on One Single Band

The interference received at a typical receiver is generated by both cellular and D2D systems occupying the specific band, the SINR (Signal to Interference plus Noise Ratio) of system S_n, (n is 0 or 1) on the i th band at the receiver is:

$$SINR_{n,i} = \frac{P_{n,i}\delta_{n0,i}R_{n0,i}^{-\alpha}}{\sum_{j\in\{0,1\}}\sum_{(X_{jk},\delta_{jk})\in\Pi_j} P_{j,i}\delta_{jk}|X_{jk}|^{-\alpha} + N_0} \tag{2}$$

where $\delta_{n0,i}$ and $R_{n0,i}$ are the Rayleigh fading and the distance from the desired transmitter to the typical receiver of system S_n on the i th band respectively. N_0 is the thermal noise. Because the spectrum sharing of D2D communication is the main consideration, which means cellular and D2D hybrid system is interference limited, the thermal noise is negligible. Then SINR is replaced by SIR (Signal to Interference Ratio) as follows:

$$SIR_{n,i} = \frac{\delta_{n0,i}R_{n0,i}^{-\alpha}}{I_{n,i,0} + I_{n,i,1}} \tag{3}$$

where $I_{n,i,0}=\sum_{(X_{0k},\delta_{0k})\in\Pi_0}\left(\frac{P_{0,i}}{P_{n,i}}\right)\delta_{0k}\left|X_{0k}\right|^{-\alpha}$, $I_{n,i,1}=\sum_{(X_{1k},\delta_{1k})\in\Pi_1}\left(\frac{P_{1,i}}{P_{n,i}}\right)\delta_{1k}\left|X_{1k}\right|^{-\alpha}$. Set $T_{n,i}$ as the threshold of SIR on i th band, following lemma shows the outage probability of a typical receiver:

Lemma 1. The outage probability of a typical receiver of system S_n, (n is 0 or 1) on the i th band $(i=1,2,...,N)$ satisfies:

$$\Pr\left(SIR_{n,i}\le T_{n,i}\right)=1-\exp\left\{-\varsigma_{n,i}\sum_{j\in\{0,1\}}\lambda_{j,i}\left(\frac{P_{j,i}}{P_{n,i}}\right)^{\frac{2}{\alpha}}\right\} \tag{4}$$

where $\Pr(\bullet)$ represent the probability, $\varsigma_{n,i}=\left[\pi\Gamma\left(1+\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)\right]T_{n,i}^{\frac{2}{\alpha}}R_{n0,i}^{2}$.

Proof: See Appendix A.

Based on **Lemma 1**, the successful transmission probability of a typical receiver of system S_n, (n is 0 or 1) on the i th band $(i=1,2,...,N)$ can be expressed as:

$$\Pr_{n,i}^{suc}\left(\lambda_{n,i},\lambda_{j,i}\right)=1-\Pr\left(SIR_{n,i}\le T_{n,i}\right)=\Pr\left(SIR_{n,i}\ge T_{n,i}\right) \tag{5}$$

where $\lambda_{n,i}$ is the node density of system S_n on the i th band.

3.2 Achievable Transmission Capacity of D2D System over Multi-bands

The achievable transmission capacity of D2D system is defined as D2D density multiplies the successful transmission probability [10]. According to equation (5), following definition is given:

Definition 1. The achievable transmission capacity of D2D communication underlaying cellular networks on multi-bands is defined as follows:

$$f\left(\lambda_{0,i},P_{0,i}\right)=\sum_{i=1}^{N}\omega_i\lambda_{0,i}e^{-\varsigma_{0,i}\left[\lambda_{0,i}+\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}\right]} \tag{6}$$

where $\omega_i=\frac{W_i}{\sum_{m=1}^{N}W_m}$, W_i is the bandwidth of the i th band, $\sum_{m=1}^{N}W_m$ is the whole bandwidth, $P_{0,i}$ is the D2D power and $\lambda_{0,i}$ is the density of D2D pairs on the i th band.

D2D communication can reuse up to N bands in an underlay way to the cellular network, and the power and density of D2D pairs should meet outage threshold of cellular transmission and D2D transmission, so we have the following constraints:

$$1-e^{-\varsigma_{0,i}\left[\lambda_{0,i}+\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}\right]}\le\theta_0 \tag{7}$$

$$1-e^{-\varsigma_{1,i}\left[\lambda_{1,i}+\left(\frac{P_{0,i}}{P_{1,i}}\right)^{\frac{2}{\alpha}}\lambda_{0,i}\right]}\le\theta_1 \tag{8}$$

$$0\le\lambda_{0,i}\le\lambda_{\max,i},\left(i=1,2,...,N\right) \tag{9}$$

$$0\le P_{0,i}\le P_{\max,i},\left(i=1,2,...,N\right) \tag{10}$$

where θ_0 is the maximum outage probability for D2D pairs setting up link on a single band, and θ_1 is the outage probability threshold for cellular users on their working band. $\lambda_{\max,i}$ and $P_{\max,i}$ are the maximum density and power of D2D system on each band respectively.

4. Optimal Achievable Transmission Capacity of D2D System on Multi-bands

In this section, achievable transmission capacity of D2D system on multi-bands is analyzed under the constraints of D2D density and D2D transmission power respectively. After getting the optimal D2D density and D2D transmission power in closed-form, an iterative algorithm is proposed in order to get the maximum D2D transmission capacity.

4.1 Optimal Achievable Transmission Capacity of D2D System on Multi-bands with the Constraint of D2D density

First the optimization with the constraint of D2D density is analyzed, notice that here the D2D transmission power can be seen as fixed when we analyzing D2D density on each band, from inequalities (7) and (8), we have:

$$\lambda_{0,i} \le \frac{-1}{\varsigma_{0,i}} \ln\left(1-\theta_0\right) - \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i} \tag{11}$$

$$\lambda_{0,i} \le \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \left(\frac{-1}{\varsigma_{1,i}} \ln\left(1-\theta_1\right) - \lambda_{1,i}\right) \tag{12}$$

Make $\lambda_{0,i,\sup_1} = \frac{-1}{\varsigma_{0,i}} \ln\left(1-\theta_0\right) - \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i}$ and $\lambda_{0,i,\sup_2} = \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \left(\frac{-1}{\varsigma_{1,i}} \ln\left(1-\theta_1\right) - \lambda_{1,i}\right)$, from constraint (9), the upper limit of the D2D density in a single band is $\lambda_{0,i,\sup} = \min\left\{\lambda_{0,i,\sup_1}, \lambda_{0,i,\sup_2}, \lambda_{\max,i}\right\}$, $\left(i=1,2,\ldots,N\right)$.

Denote density of the whole D2D system as λ_0, so $\sum_{i=1}^{N} \lambda_{0,i} = \lambda_0$ and $\lambda_0 \le \sum_{i=1}^{N} \lambda_{0,i,\sup}$ should be satisfied, otherwise when $\lambda_0 > \sum_{i=1}^{N} \lambda_{0,i,\sup}$, only $0 \le \lambda_{0,i} \le \lambda_{0,i,\sup}$, $\left(i=1,2,\ldots,N\right)$ can be satisfied by controlling the activation rate of D2D users on each band. So the optimization of D2D density on each band should be discussed in two aspects:

1) When the density of the whole D2D system $\lambda_0 > \sum_{i=1}^{N} \lambda_{0,i,\sup}$, we have:

$$\begin{aligned} \max \quad & f\left(\lambda_{0,i}, P_{0,i}\right) = \sum_{i=1}^{N} \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\left[\lambda_{0,i} + \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i}\right]} \\ s.t. \quad & 0 \le \lambda_{0,i} \le \lambda_{0,i,\sup}, \left(i=1,2,\ldots,N\right) \end{aligned} \tag{13}$$

Take the partial derivate of $f\left(\lambda_{0,i}, P_{0,i}\right)$ with respect to $\lambda_{0,i}$:

$$\frac{\partial f\left(\lambda_{0,i},P_{0,i}\right)}{\partial\lambda_{0,i}}=\left(1-\varsigma_{0,i}\lambda_{0,i}\right)\omega_i e^{-\varsigma_{0,i}\left[\lambda_{0,i}+\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}\right]} \tag{14}$$

Make $\frac{\partial f\left(\lambda_{0,i},P_{0,i}\right)}{\partial\lambda_{0,i}}=0$, $\lambda_{0,i}=\frac{1}{\varsigma_{0,i}}$ is obtained, so the optimal density of D2D pairs on i th band $\lambda_{0,i,opt1}^{*}$ is:

$$\lambda_{0,i,opt1}^{*}=\begin{cases}\lambda_{0,i,\sup} & \lambda_{0,i,\sup}<\frac{1}{\varsigma_{0,i}}\\ \frac{1}{\varsigma_{0,i}} & \lambda_{0,i,\sup}\geq\frac{1}{\varsigma_{0,i}}\end{cases}\left(i=1,2,...,N\right) \tag{15}$$

2) When the density of the whole D2D system $\lambda_0\leq\sum_{i=1}^{N}\lambda_{0,i,\sup}$, we have:

$$\begin{aligned}&\max && f\left(\lambda_{0,i},P_{0,i}\right)=\sum_{i=1}^{N}\omega_i\lambda_{0,i}e^{-\varsigma_{0,i}\left[\lambda_{0,i}+\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}\right]}\\ &s.t. && 0\leq\lambda_{0,i}\leq\lambda_{0,i,\sup},\left(i=1,2,...,N\right)\\ &&& \sum_{i=1}^{N}\lambda_{0,i}=\lambda_0\end{aligned} \tag{16}$$

The following theorem presents the optimal D2D density:

Theorem 1. Under given values of D2D transmission power $P_{0,i}$, $(i=1,2,...,N)$ the optimal D2D density $\lambda_{0,i,opt2}^{*}$ in the i th band is:

$$\lambda_{0,i,opt2}^{*}=\begin{cases}\lambda_{0,i,\sup} & , \quad 0\leq\psi<\left(1-\varsigma_{0,i}\lambda_{0,i,\sup}\right)e^{-\varsigma_{0,i}\lambda_{0,i,\sup}}\\ \frac{1}{\varsigma_{0,i}}\left(1-\sqrt{\rho}\right) & , \quad \left(1-\varsigma_{0,i}\lambda_{0,i,\sup}\right)e^{-\varsigma_{0,i}\lambda_{0,i,\sup}}\leq\rho<1\\ 0 & , \quad 1\leq\rho\end{cases} \tag{17}$$

where $\rho=\frac{v}{A_i}$, $A_i=\omega_i e^{-\varsigma_{0,i}\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}}$, $\left(i=1,2,...,N\right)$. v is a Lagrange multiplier coefficient which is readily determined with the condition $\sum_{i=0}^{N}\lambda_{0,i,opt2}^{*}=\lambda_0$.

Proof: See Appendix B.

4.2 Optimal Achievable Transmission Capacity of D2D System on Multi-bands with the Constraint of D2D Transmission Power

Next the optimization with the constraint of D2D transmission power is analyzed, notice that here the D2D density can be seen as fixed when we analyzing D2D transmission power on each band, from inequalities (11) and (12), we have:

$$P_{0,i}\geq P_{1,i}\left[\frac{-\ln\left(1-\theta_0\right)}{\lambda_{1,i}\varsigma_{0,i}}-\frac{\lambda_{0,i}}{\lambda_{1,i}}\right]^{\frac{-\alpha}{2}} \tag{18}$$

$$P_{0,i} \le P_{1,i}\left[\frac{-\ln\left(1-\theta_1\right)}{\lambda_{0,i}\varsigma_{1,i}} - \frac{\lambda_{1,i}}{\lambda_{0,i}}\right]^{\frac{\alpha}{2}} \tag{19}$$

Let $P_{0,i,\inf_1} = P_{1,i}\left[\frac{-\ln\left(1-\theta_0\right)}{\lambda_{1,i}\varsigma_{0,i}} - \frac{\lambda_{0,i}}{\lambda_{1,i}}\right]^{\frac{-\alpha}{2}}$ and $P_{0,i,\sup_1} = P_{1,i}\left[\frac{-\ln\left(1-\theta_1\right)}{\lambda_{0,i}\varsigma_{1,i}} - \frac{\lambda_{1,i}}{\lambda_{0,i}}\right]^{\frac{\alpha}{2}}$, from constraint (10), the lower and upper limit of D2D transmission power in a single band are $P_{0,i,\inf} = \max\left\{0, P_{0,i,\inf_1}\right\}$ and $P_{0,i,\sup} = \min\left\{P_{\max,i}, P_{0,i,\sup_1}\right\}$, $i = 1,2,...,N$ respectively.

The constraint of D2D transmission power is denoted by P_0, so $P_0 \le \sum_{i=1}^{N} P_{0,i,\sup}$ should be satisfied, otherwise when $P_0 > \sum_{i=1}^{N} P_{0,i,\sup}$, only $P_{0,i,\inf} \le P_{0,i} \le P_{0,i,\sup}$, $\left(i = 1,2,...,N\right)$ can be satisfied by controlling the activation rate of D2D users on each band. So the optimization of D2D transmission power on each band should be discussed in two aspects:

1) When the constraint of D2D transmission power $P_0 > \sum_{i=1}^{N} P_{0,i,\sup}$, we have:

$$\begin{aligned} \max \quad & f\left(\lambda_{0,i}, P_{0,i}\right) = \sum_{i=1}^{N} \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\left[\lambda_{0,i} + \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i}\right]} \\ s.t. \quad & P_{0,i,\inf} \le P_{0,i} \le P_{0,i,\sup}, \left(i = 1,2,...,N\right) \end{aligned} \tag{20}$$

It is obvious that when $P_{0,i} = P_{0,i,\sup}$, $f\left(\lambda_{0,i}, P_{0,i}\right)$ can get the maximum value in the definition domain of $P_{0,i}$, so the optimal value of D2D density is:

$$P^{*}_{0,i,opt1} = P_{0,i,\sup}, \left(i = 1,2,...,N\right) \tag{21}$$

However, notice that the maximum value is obtained when $P_{0,i,\inf} \le P_{0,i,\sup}$, $\left(i = 1,2,...,N\right)$. While $P_{0,i,\inf} > P_{0,i,\sup}$ on the i th band, following inequality holds:

$$P_{1,i}\left[\frac{-\ln\left(1-\theta_0\right)}{\lambda_{1,i}\varsigma_{0,i}} - \frac{\lambda_{0,i}}{\lambda_{1,i}}\right]^{\frac{-\alpha}{2}} > P_{1,i}\left[\frac{-\ln\left(1-\theta_1\right)}{\lambda_{0,i}\varsigma_{1,i}} - \frac{\lambda_{1,i}}{\lambda_{0,i}}\right]^{\frac{\alpha}{2}} \tag{22}$$

Let $\xi_{0,i} = \frac{-\ln\left(1-\theta_0\right)}{\varsigma_{0,i}}$ and $\xi_{1,i} = \frac{-\ln\left(1-\theta_1\right)}{\varsigma_{1,i}}$, then reshape (26), we have:

$$\left(\frac{\xi_{0,i}}{\lambda_{1,i}} - \frac{\lambda_{0,i}}{\lambda_{1,i}}\right)\left(\frac{\xi_{1,i}}{\lambda_{0,i}} - \frac{\lambda_{1,i}}{\lambda_{0,i}}\right) < 1 \tag{23}$$

Remark 1. With the growing of D2D density $\lambda_{0,i}$ on the i th band, the interference is becoming more and more serious. Once D2D density is big enough which makes inequality (23) established, both cellular and D2D communication cannot be ensured at the same time as long as D2D transmits signals on this band. The only way of D2D is to abandon choosing the band, so $P_{0,i}$ is zero under this condition. For the analysis below, $P_{0,i,\inf} \le P_{0,i,\sup}$ is also established while D2D has to forbidden transmitting on the band when $P_{0,i,\inf} > P_{0,i,\sup}$.

2) When the constraints of D2D transmission power $P_0 < \sum_{i=1}^{N} P_{0,i,\sup}$, we have

$$\begin{aligned} \max \quad & f\left(\lambda_{0,i}, P_{0,i}\right) = \sum_{i=1}^{N} \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\left[\lambda_{0,i} + \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i}\right]} \\ s.t. \quad & P_{0,i,\inf} \le P_{0,i} \le P_{0,i,\sup}, \left(i = 1,2,...,N\right) \\ & \sum_{i=1}^{N} P_{0,i} = P_0 \end{aligned} \tag{24}$$

Here the outage probability constraint of cellular communication is θ_1 , and it is defined as a very small value to ensure the reliability of cellular transmission.

Denote $B_i = \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\lambda_{0,i}}$, $D_i = \varsigma_{0,i} P_{1,i}^{\frac{2}{\alpha}} \lambda_{1,i}$, $(i = 1,2,...,N)$. Following lemma and theorem are obtained:

Lemma 2. When the cellular outage probability threshold $\theta_1 \in \left(0, 1 - e^{-\lambda_{1,i}\varsigma_{1,i}}\right)$, the negative function of D2D transmission capacity $-f\left(\lambda_{0,i}, P_{0,i}\right) = -\sum_{i=1}^{N} \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\left[\lambda_{0,i} + \left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}} \lambda_{1,i}\right]}$ is convex in the D2D transmission power definition domain $\left[P_{0,i,\inf}, P_{0,i,\sup}\right]$.

Proof: See Appendix C.

Then following theorem demonstrates the optimal D2D transmission power in each band:

Theorem 2. Under given values of D2D density on each band $\lambda_{0,i}$, $(i = 1,2,...,N)$, the optimal D2D transmission power on the i th $\left(i = 1,2,...,N\right)$ band $P_{0,i,opt2}^{*}$ is:

$$P_{0,i,opt2}^{*} = \begin{cases} P_{0,i,\sup} & , \quad u \le h_{0,i,\min} \\ P_{0,i,solution}^{*} & , \quad h_{0,i,\min} < u \le h_{0,i,\max} \\ P_{0,i,\inf} & , \quad h_{0,i,\max} < u \end{cases} \tag{25}$$

where for each band, $\left[h_{0,i,\min}, h_{0,i,\max}\right]$ is the range of the function $h\left(P_{0,i}\right) = \frac{2B_i D_i}{\alpha} e^{-D_i P_{0,i}^{\frac{-2}{\alpha}}} P_{0,i}^{-\left(1+\frac{2}{\alpha}\right)}$, and $P_{0,i,solution}^{*}$ is the solution of $u - h\left(P_{0,i}\right) = 0$. While u is a Lagrange multiplier coefficient which is readily determined with the condition $\sum_{i=1}^{N} P_{0,i} = P_0$.

Proof: See Appendix D.

4.3 Iterative Algorithm of D2D Density and Power for Maximum Achievable Transmission Capacity of D2D System

Based on the analysis before, the target of D2D capacity is a convex function of D2D density/power when the other parameter (D2D power/density) is fixed. Here an iterative algorithm of D2D density and power is proposed, i.e., D2D density and power are kept adjusting to maximum achievable transmission capacity of D2D system until the capacity is stable. The detail of the algorithm is described in **Algorithm 1**.

Algorithm 1. Iterative Algorithm of D2D Density and Power for Maximum Achievable Transmission Capacity of D2D System*

Initialization:

Initialize N, ω_i, θ_0, θ_1, λ_0, P_0, $\lambda_{1,i}$, $P_{1,i}$, $\varsigma_{1,i}$, $\varsigma_{0,i}$, $\lambda_{0,i}=0$, $P_{0,i}=\frac{P_0}{N}$, $\lambda_{0,i,\sup}$, $P_{0,i,\inf}$, $P_{0,i,\sup}$, $(i=1,2,...,N)$, $C_k=0$, $k=0$, $flag=0$.

Iteration:

```
while ΔC ≥ ε do
    k ← k+1;
    if flag = 0 then
        if λ_0 > Σ_{i=1}^{N} λ_{0,i,sup} then
            Update D2D density λ_{0,i}, (i = 1,2,...,N) on each band according to equation (15);
        else
            Update D2D density λ_{0,i}, (i = 1,2,...,N) on each band according to Theroem 1.
        end if
        Update λ_{0,i,sup}, (i = 1,2,...,N);
    end if
    if flag = 1 then
        if P_0 > Σ_{i=1}^{N} P_{0,i,sup} then
            D2D density P_{0,i} = P_{0,i,inf}, (i = 1,2,...,N) on each band;
        else
            Update D2D power P_{0,i}, (i = 1,2,...,N) on each band according to Theorem 2;
        end if
        Update P_{0,i,inf}, P_{0,i,sup}, (i = 1,2,...,N);
    end if
    Calculate achievable transmission capacity of D2D system C_k = Σ_{i=1}^{N} C_i;
    ΔC = |C_{k+1} − C_k| / C_k;
    flag = flag ⊕ 1;
end while
```

Output:

Optimal parameters of D2D density and power $\left(\lambda^*_{0,i,opt}, P^*_{0,i,opt}\right)$ which satisfy $\left(\lambda^*_{0,i,opt}, P^*_{0,i,opt}\right)=\arg\max\{C_k\},(i=1,2,...,N)$.

*Note: 1. C_k and C_i are the achievable transmission capacity of D2D system in k th iteration and i th band respectively.
2. $flag$ means the flag bit in every iteration which has only the value 0 or 1.
3. ε is a pre-defined threshold of ΔC.
4. The operational symbol '$\oplus$' means logical XOR operation.

Remark 2. In Algorithm 1, the first 'if-end' statement block (from line 3 to line 10) optimizes the D2D density when D2D transmission power is fixed, the second 'if-end' statement block (from line 11 to line 18) optimizes D2D transmission power when the D2D density is fixed. In essence, updating D2D density and power allocation are two ways to adjust the interference to the cellular system. These two parameters of a D2D system have a mutual coupling relationship, i.e., the optimal value of one parameter (density or power) can be completely decided by the other. The iteration continues until the capacity is stable, and finally leads to the optimal D2D density and power which not only make the maximum D2D transmission capacity but also cause the interference to the cellular system within the tolerance range of cellular users.

5. Simulation Results and Discussions

In this section, the performance of D2D communication underlaying cellular networks is evaluated. The outage probability and achievable transmission capacity of the D2D system on a single band are analyzed first. Next the optimal D2D achievable transmission capacity on multiple bands is investigated for three cases, which include five bands with different bandwidth ratios. Finally, optimal D2D density and power on different bands are discussed and the sum optimal D2D achievable transmission capacity is compared under three algorithms in order to make the results more insightful.

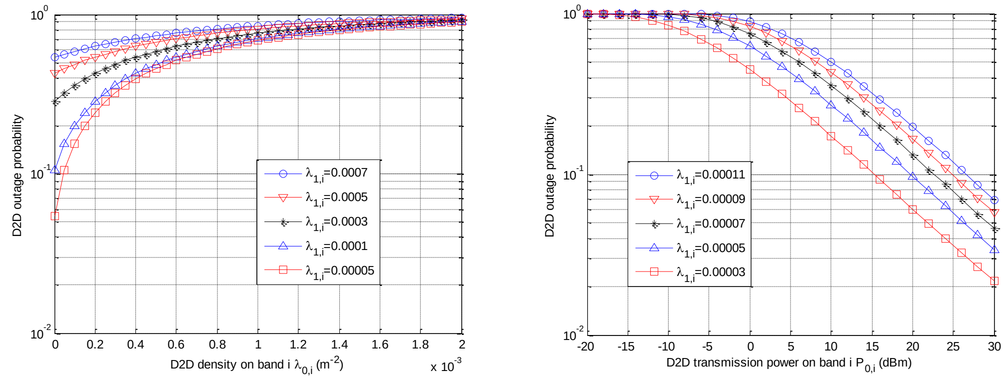
5.1 Simulation Analysis of D2D Outage Probability and Achievable Transmission Capacity on One Single Band

The basic parameters in the simulation on one single band are listed in **Table 1**. The band is assumed with a bandwidth normalized to 1. Here i means the serial number of this band. Consider that the D2D users usually have a closed distance, and the D2D communication should also ensure the cellular communication, so the D2D transmission power and the link distance are defined as smaller than the cellular transmission in our simulation.

Table 1. Basic parameters in the simulation on one single band (Unless Otherwise Noted)

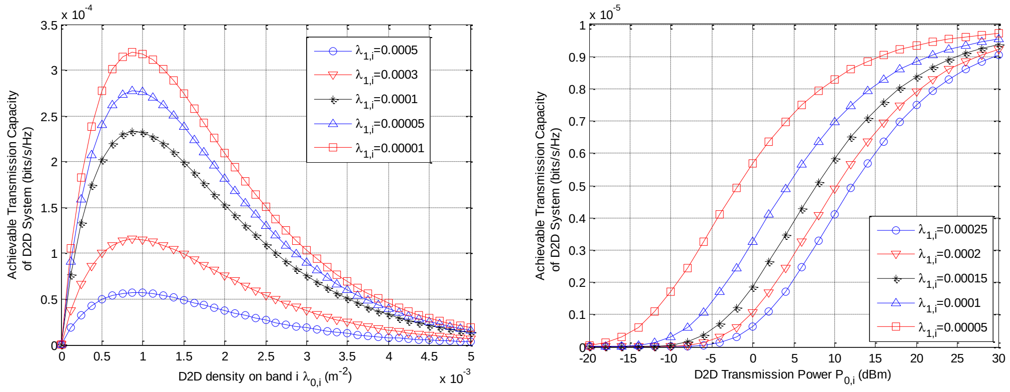
Parameter	Physical Mean	Default Value
$\lambda_{0,i}$, $\lambda_{1,i}$	D2D/Cellular user density	0.0001m^{-2} / 0.0001m^{-2}
$P_{0,i}$, $P_{1,i}$	D2D/Cellular transmission power	15dBm / 25dBm
$R_{00,i}$, $R_{10,i}$	D2D/Cellular average link distance	15m / 50m
α	Path loss coefficient	4
θ_0 , θ_1	Maximum D2D/Cellular outage probability	0.1
$T_{0,i}$, $T_{1,i}$	D2D/Cellular threshold of SIR	0dB / 0dB

Fig. 2(a) illustrates the relationship between D2D outage probability and D2D density on a single band. D2D outage probability is rising as D2D density increasing because of higher D2D density causing more serious interference to the D2D system itself. Furthermore, **Fig. 2(a)** shows the D2D outage probability under different cellular user density on a same band. With the increasing cellular user density, D2D system suffers more interference from cellular communication, so the D2D outage probability is bigger when cellular user density is higher. In **Fig. 2(b)**, the relationship between D2D outage probability and D2D transmission power on a single band is showed. From **Fig. 2(b)**, D2D outage probability is reducing as the D2D transmission power increasing because high D2D transmission power can bring the improvement of SIR of D2D communication under the same environment. In addition, **Fig. 2(b)** indicates when cellular user density is increasing, D2D outage probability is increasing. High cellular density can cause more interference from cellular system to D2D system. Compare with the cellular density from $7\times10^{-5}m^{-2}$ to $9\times10^{-5}m^{-2}$, this influence is more obvious from $3\times10^{-5}m^{-2}$ to $5\times10^{-5}m^{-2}$ because high D2D transmission power enhances the anti-interference of D2D system.



(a) **(b)**

Fig. 2. (a). D2D outage probability vs. D2D density on band i
(b). D2D outage probability vs. D2D transmission power on band i



(a) **(b)**

Fig. 3. (a). Achievable transmission capacity of D2D system vs. D2D density on band i
(b). Achievable transmission capacity of D2D system vs. D2D transmission power on band i

Fig. 3(a) shows the relationship between D2D achievable transmission capacity and D2D density on a single band. First, when D2D density is low, the achievable transmission capacity is increasing as D2D density increasing which is due to the increasing of D2D density can bring the improvement of D2D system performance. Second, when D2D density is high and continues to increase, the interference among each D2D pairs becomes large and causes harmful interference to the D2D system, so the D2D achievable transmission capacity begins to reduce. Furthermore, with the increasing cellular user density, the decrease of D2D achievable transmission can be observed due to the increment of harmful interference from the cellular system. **Fig. 3(b)** demonstrates the change of D2D achievable transmission capacity with D2D transmission power on a single band. It can be seen that the D2D achievable transmission capacity increases with D2D transmission power because higher D2D transmission power can improve the SIR of D2D system. In addition, the figure shows that in an environment with low cellular user density, D2D system suffers low interference from cellular system, so it can get a high achievable transmission capacity.

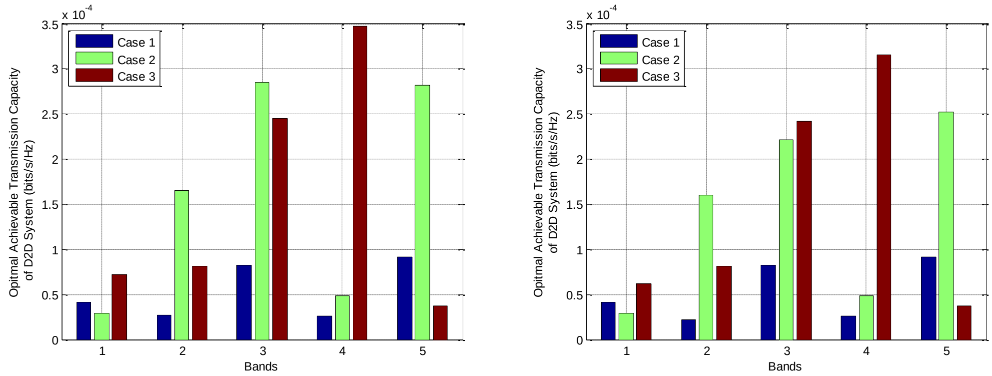
5.2 Simulation Analysis of Optimal D2D Achievable Transmission Capacity on Multiple Bands

Next the simulation results of D2D communication underlaying cellular networks on multiple bands are discussed. The whole system band is divided into five bands with different bandwidth ratios in three cases. Generally, different with the cellular users in the networks, the kinds of D2D users are various (e.g. smartphone, tablet PC, etc.), in addition, the D2D may reuse different cellular users' spectrum according to different networks (e.g. the network with user number very dense), so three kinds of simulation case is designed: Case 1 has the smallest transmission power on the whole bands averagely, Case 2 has the longest cellular average link distance over the whole bands, while in Case 3, the cellular user density is the biggest. The key parameters are listed in **Table 2**.

Table 2. Key parameters of the simulation on multiple bands

	Parameters as [Band1 Band2 Band3 Band4 Band5]		
Parameter	**Case 1**	**Case 2**	**Case 3**
System bandwidth ratio	[1:1:2:2:1]	[0.5:1:1.5:2.5:1.5]	[0.5:1:3:2:0.5]
Cellular transmission powers (dBm)	[10,10,10,20,20]	[20,25,15,10,20]	[10,30,15,30,20]
Cellular user density ($10^{-5}\cdot m^{-2}$)	[1,1,1,1,3]	[1,3,2,5,1]	[10,30,50,20,30]
Cellular average link distances (m)	[50,60,50,80,50]	[70,30,10,50,20]	[20,10,15,10,20]
D2D Transmission power	$\leq$ 20dBm on each band		
D2D density	$\leq 3\times10^{-3}\cdot m^{-2}$ on each band		
Cellular average link distances	15m		
Path loss coefficient	4		
Maximum Outage probability	0.1		
Threshold of SIR	0dB		

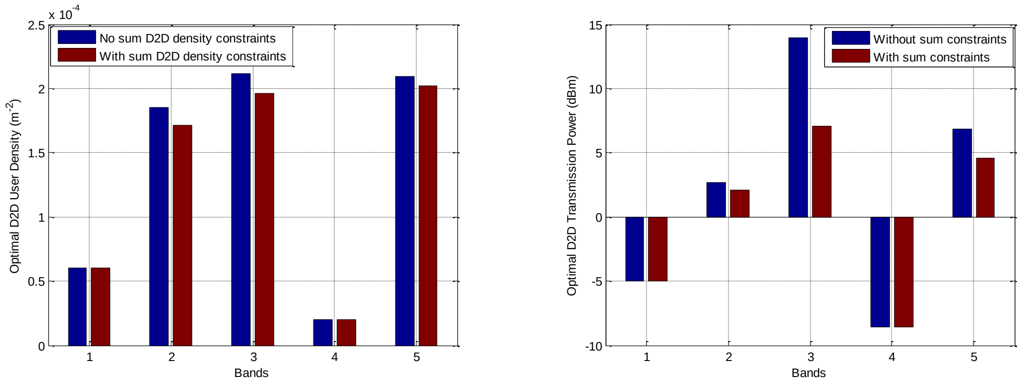
Fig. 4(a) illustrates the optimal achievable transmission capacity of D2D system on five bands without the constraints of sum D2D user density and sum D2D transmission power. From the figure, the optimal D2D achievable transmission capacity is low in Case 1 for the long cellular average link distances which make the cellular system cannot bear much interference from D2D system. While in Band 3 and 5 in Case 2, the cellular average distance is short, so D2D can reach a high optimal value. Compare with Case 2, the optimal D2D achievable transmission capacity on Band 1, 2 and 5 in Case 3 is small because of the constraints of high cellular user density. While in Band 3 and 4 in Case 3, due to the short cellular link distances and the wide bandwidths, the high achievable transmission capacities can be observed, also a bigger gain on Band 4 is get because of the high cellular transmission which can resist more D2D interference. While **Fig. 4(b)** shows the optimal D2D achievable transmission capacities on five bands with the constraints of sum D2D density λ_0 $6.5\times10^{-3}m^{-2}$ and sum D2D transmission power P_0 10dBm. And the reduction of values on Band 2, 3, 4 in Case 2 and Band 1, 3, 4 in Case 3 can be observed because of the sum constraints which lead to a decline of D2D density and transmission power on these bands. In addition, compare **Fig. 4 (a)** and **(b)**, the optimal D2D achievable transmission capacity has a small decline when the sum constraints of D2D density and transmission power are added. This means with the sum constraints, D2D system is confined with not only the constraints of power and density on each band, but also with the maximum transmission power of each user device and the user number of the whole D2D system, which take less effect on the optimal D2D achievable transmission capacity on each band.



(a) **(b)**

Fig. 4. Optimal D2D achievable transmission capacity on each band
(a)/(b). Without/With the constraints of sum D2D transmission powers and D2D densities

Further, **Fig. 5(a)** shows the optimal D2D density of Case 2 with and without the sum D2D density constraint respectively. On band 2, 3 and 4, D2D densities are decreased from high values to the lower ones because of the sum constraint of D2D density. Also, in **Fig. 5(b)**, the reduction of high D2D power on these bands can be seen which is due to the constraint of the sum D2D transmission power. This also verifies the decrease of optimal D2D achievable transmission capacity on those bands of Case 2 in **Fig. 4**.

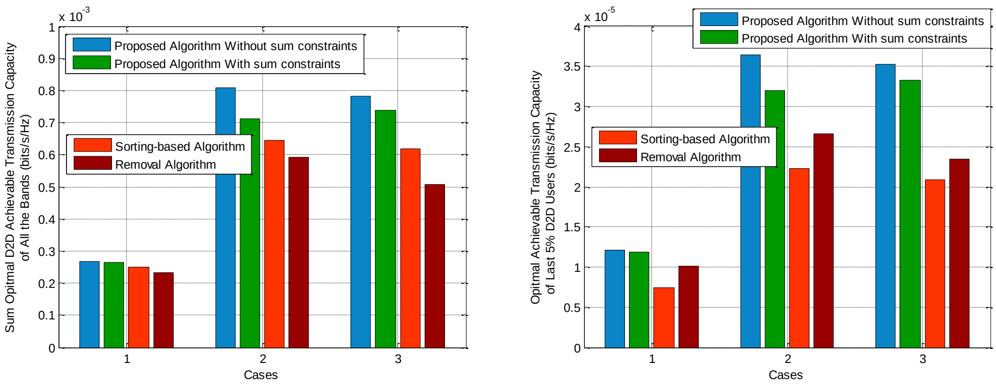


(a) **(b)**

Fig. 5. (a). Optimal D2D user density of Case 2 on each band
(b). Optimal D2D transmission power of Case 2 on each band

Finally, the propsed algorithm is compared with the sorting-based algorithm and the removal algorithm. In sorting-based algorithm, D2D users access the cellular spectrum according to the interference to the base station, i.e., the D2D user which casues the smallest interference accesses the spectrum first, then the other D2D users access the spectrum according to the interference order until the cellular communcation cannot bear any more interference. In the removal algorithm, first a power control of D2D users is executed, then the D2D links which cannot satisfy the D2D outage probability are removed in this algorithm, and power control and removeal process are repeatly executed until all the D2D users can ensure the quality of the cellular communication. **Fig. 6(a)** shows that even with the consitaint of sum

D2D density and transmission power, the proposed algorithm results a better value over the other two algorithms. This due to the proposed algorithm is not only consider the optimization of transmission power, but also consider the distribution of the users in the networks. All the D2D users can choose a more appropriate BS to reuse the cellular spectrum. This phenomenon is also proved in **Fig. 6(b)**, which shows the optimal achievable transmsision capacity of last 5% D2D users, from the figure, it can be seen that the sorting-based algorithm declines much, which means the main contribution of the transmission capacity is from those D2D users with high communication quality in the system. While consider a more reasonable user density in the system, the proposed algorithm also reveals a better fairness in the whole network.



(a) **(b)**

Fig. 6. (a). Sum optimal D2D achievable transmission capacity of each case
(b). Optimal achievable transmission capacity of Last 5% D2D Users of each case

6. Conclusion

In this paper, an optimal density and power allocation method for D2D communication underlaying cellular networks on multiple bands was studied. System outage probabilities and the definition of D2D achievable transmission capacity were obtained using networks modeled with stochastic geometry. The optimal D2D densities and powers were derived in closed-form under the constraints of both cellular and D2D constraints. An iterative algorithm of D2D density and power was proposed with the target of maximizing D2D achievable transmission capacity. The simulation showed that D2D outage probability, achievable transmission capacity, density and power on each band are constrained by cellular communication as well as the interference of the system. The optimal values on multiple bands are reduced when sum constraints are added. Finally, the results verified the superiority of the proposed algorithm over the sorting-based algorithm and the removal algorithm.

Appendix A: Proof of Lemma 1

From equation (3), the outage probability satisfies:

$$\begin{aligned}\Pr\left(SIR_{n,i} \le T_{n,i}\right) &= 1-\Pr\left(SIR_{n,i} \ge T_{n,i}\right) = 1-\Pr\left(\delta_{n0,i} \ge T_{n,i} R_{n0,i}^{\alpha}\left(I_{n,i,0}+I_{n,i,1}\right)\right) \\ &= 1-\int_0^{\infty} e^{-sT_{n,i}R_{n0,i}^{\alpha}} d\left[\Pr\left(I_{n,i,0}+I_{n,i,1} \le s\right)\right] = 1-\psi_{I_{n,i,0}}\left(T_{n,i}R_{n0,i}^{\alpha}\right)\psi_{I_{n,i,1}}\left(T_{n,i}R_{n0,i}^{\alpha}\right)\end{aligned} \quad (26)$$

where $\psi_{I_{n,i,0}}(\bullet)$ and $\psi_{I_{n,i,1}}(\bullet)$ are Laplace transformation of $I_{n,i,0}$ and $I_{n,i,1}$ respectively. Because the analysis is based on the two dimensional plane and $\delta_{n0,i}$ satisfies independent exponential distribution, we have [26]:

$$\psi_{I_{n,i,0}}(s)=\exp\left\{-\lambda_{0,i}\pi\left(sP_{0,i}/P_{n,i}\right)^{2/\alpha}\Gamma(1+2/\alpha)\Gamma(1-2/\alpha)\right\} \tag{27}$$

Here $\Gamma(\bullet)$ is the gamma function with the form $\Gamma(z)=\int_0^\infty e^{-t}t^{z-1}dt$. Similarly,

$$\psi_{I_{n,i,1}}(s)=\exp\left\{-\lambda_{1,i}\pi\left(sP_{1,i}/P_{n,i}\right)^{2/\alpha}\Gamma(1+2/\alpha)\Gamma(1-2/\alpha)\right\} \tag{28}$$

Substitute (27) and (28) back into (26), the result is:

$$\begin{aligned}&\Pr\left(SIR_{n,i}\le T_{n,i}\right)=1-\psi_{I_{n,i,0}}\left(T_{n,i}R_{n0,i}^{\alpha}\right)\psi_{I_{n,i,1}}\left(T_{n,i}R_{n0,i}^{\alpha}\right)\\&=1-\exp\left\{-\pi\Gamma(1+2/\alpha)\Gamma(1-2/\alpha)T_{n,i}^{2/\alpha}R_{n0,i}^{2}\sum_{j\in\{0,1\}}\lambda_{j,i}\left(P_{j,i}/P_{n,i}\right)^{2/\alpha}\right\}\end{aligned} \tag{29}$$

Denote $\varsigma_{n,i}=\pi\Gamma(1+2/\alpha)\Gamma(1-2/\alpha)T_{n,i}^{2/\alpha}R_{n0,i}^{2}$, equation (4) is obtained. ■

Appendix B: Proof of Theorem 1

Make $\lambda_{0,i}=x_i$, change the optimization problem (16) into the standard form as:

$$\begin{aligned}\min\quad & -f(x_i)=-\sum_{i=1}^{N}A_i x_i e^{-\varsigma_{0,i}x_i}\\ s.t.\quad & x_i\ge 0\\ & x_i-\lambda_{0,i,\sup}\le 0,(i=1,2,...,N)\\ & \sum_{i=1}^{N}x_i=\lambda_0\end{aligned} \tag{30}$$

The second derivative of target function with respect to $\lambda_{0,i}$ satisfies:

$$-f''(x_i)=A_i\varsigma_{0,i}\left(2-\varsigma_{0,i}x_i\right)e^{-\varsigma_{0,i}x_i} \tag{31}$$

In practical cellular and D2D hybrid networks, the outage probability constraints of cellular and D2D communication θ_0 and θ_1 are both very small values, which leads to $\varsigma_{0,i}x_i<2$, so $-f''(x_i)>0$, $\left(0\le x_i\le\lambda_{0,i,\sup}\right)$ which makes the optimization problem a convex problem, define the symbol of optimal x_i as x_i^*, then we have Lagrange function as follows:

$$L\left(x_i^*,k_i,l_i,v\right)=-\sum_{i=1}^{N}A_i x_i^* e^{-\varsigma_{0,i}x_i^*}-\sum_{i=1}^{N}k_i x_i^*+\sum_{i=1}^{N}l_i\left(x_i^*-\lambda_{0,i,\sup}\right)+v\left(\sum_{i=1}^{N}x_i^*-\lambda_0\right) \tag{32}$$

According to the KKT condition, we get following algebras:

1) $x_i^*\ge 0$; 2) $k_i\ge 0$; 3) $l_i\ge 0$; 4) $x_i^*-\lambda_{0,i,\sup}\le 0$; 5) $k_i x_i^*=0$; 6) $l_i\left(x_i^*-\lambda_{0,i,\sup}\right)=0$; 7) $-A_i\left(1-\varsigma_{0,i}x_i^*\right)e^{-\varsigma_{0,i}x_i^*}-k_i+l_i+v=0$; 8) $\sum_{i=1}^{N}x_i^*=\lambda_0$.

From 7) we have $k_i=l_i+v-A_i\left(1-\varsigma_{0,i}x_i^*\right)e^{-\varsigma_{0,i}x_i^*}$, then from 6) we have $l_i x_i^*=l_i\lambda_{0,i,\sup}$, take the two equations above into 5) and transform:

$$\left[v-A_i\left(1-\varsigma_{0,i}x_i^*\right)e^{-\varsigma_{0,i}x_i^*}\right]x_i^*+l_i\lambda_{0,i,\sup}=0 \tag{33}$$

Combine with 1) to 5), we can know:

If $v\ge A_i$, $v-A_i\left(1-\varsigma_{0,i}x_i^*\right)e^{-\varsigma_{0,i}x_i^*}>0$, so $x_i^*=0$, $l_i=0$.

If $v<A_i$, following results are obtained:

If $v\ge A_i\left(1-\varsigma_{0,i}\lambda_{0,i,\sup}^*\right)e^{-\varsigma_{0,i}\lambda_{0,i,\sup}^*}$, so $l_i=0$. According to (33), we have

$$v-A_i\left(1-\varsigma_{0,i}x_i^*\right)e^{-\varsigma_{0,i}x_i^*}=0 \tag{34}$$

Because the equivalent infinite of $e^{-\varsigma_{0,i}x_i^*}$ is $\left(1+\varsigma_{0,i}x_i^*\right)$, we get $x_i^*=\left(1/\varsigma_{0,i}\right)\left(1-\sqrt{v/A_i}\right)$. Otherwise, we have $x_i^*=\lambda_{0,i,\sup}$. So from above, results in equation (17) are obtained. ■

Appendix C: Proof of Lemma 2

For the target function in (24), we get:

$$-\omega_i\lambda_{0,i}e^{-\varsigma_{0,i}\left[\lambda_{0,i}+\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}\right]}=-\omega_i\lambda_{0,i}e^{-\varsigma_{0,i}\lambda_{0,i}}\cdot e^{-\varsigma_{0,i}\left(\frac{P_{1,i}}{P_{0,i}}\right)^{\frac{2}{\alpha}}\lambda_{1,i}} \tag{35}$$

Then the equation above can be transformed into $-f\left(\lambda_{0,i},P_{0,i}\right)=-B_ie^{-D_iP_{0,i}^{\frac{-2}{\alpha}}}$. Calculate its second partial derivative with respective to $P_{0,i}$. We get:

$$-f''\left(\lambda_{0,i},P_{0,i}\right)=\frac{2B_iD_ie^{-D_iP_{0,i}^{\frac{-2}{\alpha}}}P_{0,i}^{\frac{-2(\alpha+2)}{\alpha}}}{\alpha^2}\left[-2D_i+(\alpha+2)P_{0,i}^{\frac{2}{\alpha}}\right] \tag{36}$$

The domain of $P_{0,i}$ is $P_{0,i,\inf}\le P_{0,i}\le P_{0,i,\sup}$, and it is obvious that $-2D_i+(\alpha+2)P_{0,i}^{2/\alpha}$ is monotonous increasing with $P_{0,i}$, so if the lower limit of $P_{0,i}$ makes the second partial derivative greater than zero, all the values of $P_{0,i}$ in the domain make so. Because $P_{0,i,\inf}=\left\{0,P_{1,i}\left[-\ln\left(1-\theta_0\right)/\lambda_{1,i}\varsigma_{0,i}-\lambda_{0,i}/\lambda_{1,i}\right]^{-\alpha/2}\right\}$, we have:

1) When $P_{0,i,\inf}=0$, it is obvious that $-f''\left(\lambda_{0,i},P_{0,i}\right)\ge 0$.

2) When $P_{0,i,\inf}>0$, $-2D_i+(\alpha+2)P_{0,i,\inf}^{2/\alpha}=-\varsigma_{0,i}P_{1,i}^{2/\alpha}\lambda_{1,i}\left\{2+(\alpha+2)/\left[\ln\left(1-\theta_0\right)+\lambda_{0,i}\varsigma_{1,i}\right]\right\}$, then from $P_{0,i,\inf}>0$, we know $P_{0,i,\inf}=P_{1,i}\left[-\ln\left(1-\theta_0\right)/\lambda_{1,i}\varsigma_{0,i}-\lambda_{0,i}/\lambda_{1,i}\right]^{-\alpha/2}>0$. So we have:

$$P_{1,i}\left(1/\lambda_{1,i}\right)^{-\alpha/2}\left(1/\varsigma_{0,i}\right)^{-\alpha/2}\left[-\ln\left(1-\theta_0\right)-\varsigma_{0,i}\lambda_{0,i}\right]^{-\alpha/2}>0 \tag{37}$$

So $\ln\left(1-\theta_0\right)+\varsigma_{0,i}\lambda_{0,i}<0$. And the threshold of D2D outage probability θ_0 is a very tiny value which ensures D2D communication in practice. Here we can make $0<\theta_0<1-e^{-\left(1+\varsigma_{0,i}\lambda_{0,i}\right)}$, so we get $-1<\ln\left(1-\theta_0\right)+\varsigma_{0,i}\lambda_{0,i}<0$, then $\left\{2+(\alpha+2)/\left[\ln\left(1-\theta_0\right)+\varsigma_{0,i}\lambda_{0,i}\right]\right\}<0$, and we have

$$-2D_i+(\alpha+2)P_{0,i,\inf}^{2/\alpha}>0 \tag{38}$$

Thus we know $-f''\left(\lambda_{0,i},P_{0,i}\right)>0$ when $P_{0,i}=P_{0,i,\inf}$. While $-2D_i+(\alpha+2)P_{0,i}^{2/\alpha}$ is monotonous increasing with $P_{0,i}$, so $-f\left(\lambda_{0,i},P_{0,i}\right)$ is convex when $P_{0,i}\in\left[P_{0,i,\inf},P_{0,i,\sup}\right]$. ■

Appendix D: Proof of Theorem 2

Make $P_{0,i} = p_i$, change the optimization problem (24) into the standard form as:

$$\begin{aligned} \min \quad & -f(p_i) = -\sum_{i=1}^{N} \omega_i \lambda_{0,i} e^{-\varsigma_{0,i}\left[\lambda_{0,i} + (P_{1,i}/P_{0,i})^{2/\alpha}\lambda_{1,i}\right]} \\ s.t. \quad & p_i - P_{0,i,\inf} \geq 0, i = 1,2,\ldots,N \\ & p_i - P_{0,i,\sup} \leq 0, i = 1,2,\ldots,N \\ & \sum_{i=1}^{N} p_i = P_0 \end{aligned} \tag{39}$$

From **Lemma 2**, we know the target function is convex. Define the symbol of optimal p_i as p_i^*, construct Lagrange function as follows:

$$L(p_i^*, s_i, t_i, u) = -\sum_{i=1}^{N} B_i e^{-D_i p_i^{*-2/\alpha}} - \sum_{i=1}^{N} s_i (p_i^* - P_{0,i,\inf}) + \sum_{i=1}^{N} t_i (p_i^* - P_{0,i,\sup}) + u\left(\sum_{i=1}^{N} p_i^* - P_0\right) \tag{40}$$

From KKT condition, we get:

1) $s_i \geq 0$; 2) $t_i \geq 0$; 3) $p_i^* - P_{0,i,\inf} \geq 0$; 4) $p_i^* - P_{0,i,\sup} \leq 0$; 5) $s_i(p_i^* - P_{0,i,\inf}) = 0$; 6) $t_i(p_i^* - P_{0,i,\sup}) = 0$; 7) $-(2B_iD_i/\alpha)e^{-D_i p_i^{*-2/\alpha}} p_i^{*-(1+2/\alpha)} - s_i + t_i + u = 0$; 8) $\sum_{i=1}^{N} p_i^* = P_0$.

Transform 7) into $s_i = -(2B_iD_i/\alpha)e^{-D_i p_i^{*-2/\alpha}} p_i^{*-(1+2/\alpha)} + t_i + u$, then from 6), we get $t_i p_i^* = t_i P_{0,i,\sup}$. Substitute the two equations above into 5), we get:

$$\left[u - (2B_iD_i/\alpha)e^{-D_i p_i^{*-2/\alpha}} p_i^{*-(1+2/\alpha)}\right](p_i^* - P_{0,i,\inf}) + t_i(P_{0,i,\sup} - P_{0,i,\inf}) = 0 \tag{41}$$

If $P_{0,i,\sup} \leq P_{0,i,\inf}$, we get $p_i^* = 0$, otherwise make $h(p_i^*) = (2B_iD_i/\alpha)e^{-D_i p_i^{*-2/\alpha}} p_i^{*-(1+2/\alpha)}$, then it is a continuous function with its definition domain a compact closed set. So define its range $[h_{0,i,\min}, h_{0,i,\max}]$, we have:

1) When $u \leq h_{0,i,\min}$, we have $p_i^* = P_{0,i,\inf}$, $t_i = 0$.

2) When $h_{0,i,\min} < u \leq h_{0,i,\max}$, $t_i = 0$, and $u - (2B_iD_i/\alpha)e^{-D_i p_i^{*-2/\alpha}} p_i^{*-(1+2/\alpha)} = 0$. And the equivalent infinite of $e^{-D_i p_i^{*-2/\alpha}}$ is $(1 - D_i p_i^{*-2/\alpha})$, and substitute it into the equation before, we have $u - (2B_iD_i)/\alpha(1 - D_i p_i^{*-2/\alpha}) p_i^{*-(1+2/\alpha)} = 0$. When all the parameters are fixed, the numerical solution of p_i^* can be obtained, and we define it as $P_{0,i,solution}^*$.

3) When $u > h_{0,i,\max}$, we have $p_i^* = P_{0,i,\sup}$ which satisfies $u - \frac{2B_iD_i}{\alpha} e^{-D_i P_{0,i,\sup}^{\frac{-2}{\alpha}}} P_{0,i,\sup}^{-\left(1+\frac{2}{\alpha}\right)} + t_i = 0$.

Substitute the results above into 8), i.e., $\sum_{i=1}^{N} p_i^* = P_0$. We can get the numerical solution of u, so we can obtain the specific value of p_i^* on each band. ■

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