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Introduction of Conventional Stress Analysis Method based on Kowalski's Formula at Internal Thread Undercut

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Abstract : The purpose of this paper is to provide the method of conventional stress analysis for a pressurized hydraulic cylinder with internal thread undercut. In the case of hydraulic cylinder with thread undercut, several loads and stresses occur during operating. So, the thread undercut can be most critical section has to be considered in detail for strength check. In this paper, the conventional stress analysis method at internal thread undercut of hydraulic cylinder based on Kowalski's formula is introduced. The method is verified by comparing to FEM analysis results using ANSYS.

Key Words : Thread Undercut, Kowalski's formula, Hydraulic Cylinder, Stress analysis, FEM analysis

1. Introduction

The pressurized cylinder is one of the most important parts of hydraulic equipment. And most cylinders contain thread undercut that gets several stresses during operation. Prior to the advent of the finite element technique of structural analysis, stresses at the thread undercut of many mechanical parts were calculated using Kowalski's formula. Even with the increasing popularity of computer and finite element programs, the method, mainly because of its simplicity in application, continues to be widely used in practice [1]. So this paper attempts to introduce the method of stress analysis at internal thread undercut of hydraulic cylinder based on Kowalski's formula with other methods. The following stress analysis examples (requirements and dimensions) are referenced from stress analysis report of hydraulic fuses for B737 airplane which are developing in aerospace R&D center of Hanwha with Boeing Company.

2. Static stress analysis at thread undercut of cylinder

Figure 1 is an example of the stress analysis method at thread undercut which is a part of the hydraulic fuse assembly. In the case of hydraulic fuse, thread undercut is exposed to hydraulic pressure and gets external load by pressurized outlet port. The loads acting on the cylinder are calculated from free-body diagram as below. Opposite balancing arrows for internal pressure were omitted from the diagram for simplicity. In this paper, the preload due to

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fastening torque is not considered for calculation simplicity.

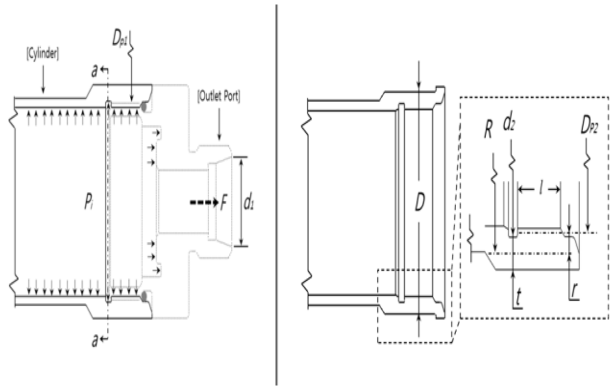

Table 1. Fig. 1 Free body diagram and major dimensions of hydraulic cylinder for stress analysis

Major dimensions so as to calculate the external load due to pressurized outlet port are as below.

(a) Maximum thread pitch diameter of outlet port

: $D_{P1} = 1.2584\ in$

(b) Minmum inner diameter of outlet port

: $d_1 = 0.6010\ in$

Where, thread pitch diameter is for 1.3125-12 UNJ 3A thread as per the AS8879 specification [3]. Required proof pressure is determined by multiplying operating pressure 3,000 psi by 1.5

$$\text{(c) Proof pressure : } P_i = 3{,}000 \times 1.5 = 4{,}500 \text{ psi}$$

External load acting on a cylinder by pressurized outlet port is as below equation (d).

$$\text{(d) External load by outlet port : } F = \frac{P_i \pi (D_{p1}^2 - d_1^2)}{4} = 4{,}320.211 \text{ lbf}$$

Major Dimensions so as to calculate stresses at thread undercut of cylinder are defined as below.

$$\text{(e) Minimum outer diameter of cylinder : } D = 1.5600 \text{ in}$$

$$\text{(f) Maximum inner diameter of cylinder : } d_2 = 1.3390 \text{ in}$$

$$\text{(g) Thread engagement length : } l = 0.3690 \text{ in}$$

$$\text{(h) Thickness of thread undercut : } t = \frac{D - d_2}{2} = 0.1105 \text{ in}$$

$$\text{(i) Maximum thread pitch diameter of cylinder : } D_{P2} = 1.2640 \text{ in}$$

Where, thread pitch diameter is for 1.3125-12 UNJ 3B thread as per the AS8879 specification [3].

$$\text{(j) Mean radius at undercut : } R = \frac{D + d_2}{4} = 0.7248 \text{ in}$$

$$\text{(k) Moment arm : } r = R - \frac{D_{p2}}{2} = 0.0928 \text{ in}$$

One thing to note is that there is an error in the published copy of Ref [2]. The formula r = (D - D_p) / 4 should be r = R - D_p / 2. The former places the moment arm at the center of the thread annulus and the latter places it at the center of the thread relief annulus. From the above definition, stresses are calculated as below. Four stresses (Tensile, Bending, Hoop and Radial) occur at thread undercut due to internal pressure P_i and tensile load F. Minimum cross sectional area at undercut (section a-a in figure 1) is shown in equation (l).

$$\text{(l) Cross section area : } A = \frac{\pi (D^2 - d_2^2)}{4} = 0.5032 \text{ in}^2$$

Tensile stress by external load F is shown in equation (m).

$$\text{(m) Tesnile stress : } f_t = \frac{F}{A} = 8585.68 \text{ psi}$$

Moment and bending stress based on Kowalski's formula are shown in equation (n) and (o) [2].

$$\text{(n) Bending moment per unit circumference : } M_E = \frac{F \times r}{2\pi R} = 87.99 \text{ lbf}$$

$$\text{(o) Bending stress : } f_b = \frac{6M_E}{t^2}\alpha = 12688.35 \text{ psi (Where, } \alpha = 0.2934)$$

$$\text{(Where, } \alpha = \frac{1}{4\beta l}[1 + 4e^{-\beta l}\sin\beta l - e^{-2\beta l}(\sin 2\beta l + \cos 2\beta l)] \text{ and } \beta = \sqrt[4]{\frac{3(1-\mu)}{R^2 t^2}})$$

(Where, poisson's ratio υ=0.27 for 15-5PH (AMS 5659) material properties)

Hoop stress for thick walled (d_2 / 2t <10) pressure vessel is shown in equation (p) [4].

$$\text{(p) Maximum hoop stress at internal surface : } f_h = P_i \frac{D^2 + d_2^2}{D^2 - d_2^2} = 29686.23 \text{ psi}$$

Radial stress for thick walled (d_2 / 2t <10) pressure vessel is shown in equation (q) [4].

$$\text{(q) Maximum radial stress at internal surface : } f_r = -P_i = -4{,}500 \text{ psi}$$

Equivalent stress in tensile, bending, hoop and radial stresses is shown in equation (r).

(r) Eq. stress : f_{eq}

$$= \sqrt{\frac{((f_t + f_b) - f_h)^2 + (f_h - f_r)^2 + (f_r - (f_t + f_b))^2}{2}}$$

$= 30{,}852.58$ psi

3. Validation of conventional stress analysis from FEM analysis.

FEM analysis was performed using ANSYS WB R15.0 so as to validate suitability of the conventional stress analysis method. Same model with figure 1 regarding dimensions was applied to FEM analysis. Figure 3 shows 3D geometry and mesh model for FEM analysis. FEM model consists of hexahedron mesh.

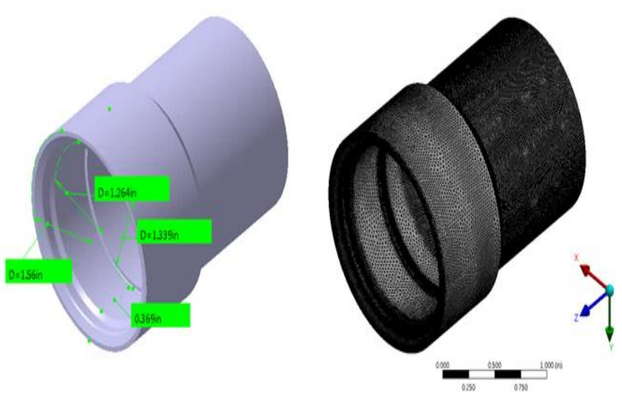

Table 2. Fig. 3 3D geometry and mesh model for FEM analysis

Force reaction for checking accuracy of load applied to undercut of cylinder and the boundary conditions are shown in figure 4.

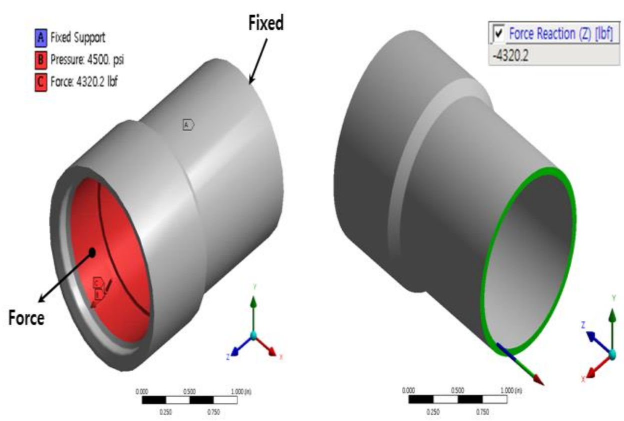

Table 3. Fig. 4 Boundary conditions and force reaction

Figure 5 shows FEM stress analysis results. The center points of thread undercut were picked to show FEM validation results regarding mean stress to avoid stress concentration. The error rate between conventional analysis and FEM analysis results is as below.

(s) Stress from conventional Method : f_{CON}

$= 30{,}853$ psi

(t) Stress from finit element method : f_{FEM}

$\approx 32{,}752$ psi $\sim 32{,}875$ psi

(u) Error rate between (s) and (t) : Error rate

$$= \frac{|f_{CON} - f_{FEM}|}{f_{FEM}} \times 100$$

$= 5.8\ \% \sim 6.2\%$

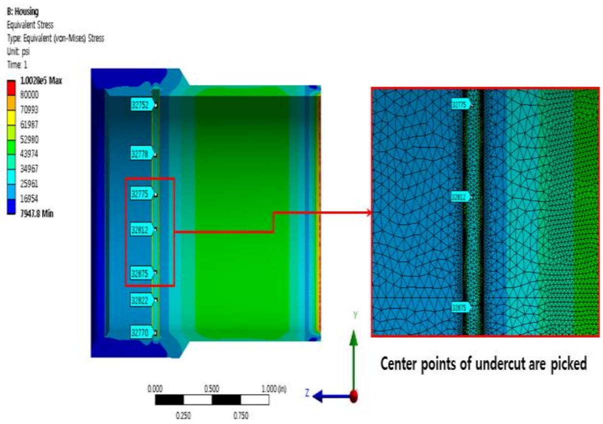

Table 4. Fig. 5 FEM von-Mises stress analysis results

4. Conclusion

Conventional stress analysis method based on Kowalski's formula at thread undercut which is pressurized by hydraulic fluid is examined in this paper. And the result of conventional analyzing stress is validated by comparing with results from an FEM using ANSYS WB R15.0. The FEM is found to provide more conservative results than the conventional method but the conventional analysis method can be trustworthy according to the above small error rate results (5.8% ~ 6.2%). As shown in previous pages, the method based on Kowalski's formula is simple in application and the method is expected to be used in practice.

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