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Studies on a standby repairable system with two types of failure

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ABSTRACT. In this paper, we study the reliability analysis of a repairable system with two types of failure in which switching failures and reboot delay are considered. Let units in this system be cold standby, and failure rate and repair rate of [type1, type2] components be exponentially distributed. The expressions of reliability characteristics – such as the system reliability and the mean time to system failure $MTTF$ – are derived. We use several cases to graphically analyze the effect of various system parameters on the system reliability and $MTTF$. We also perform a sensitivity analysis of the reliability characteristics with changes in specific values of the system's parameters.

Key Words: mean time to system failure, reboot delay, reliability, sensitivity analysis, switching failures

1. INTRODUCTION

'Standby' is a technique that has been widely applied to improve system reliability in system design. In general, there are mainly three types of standby redundancy: hot standby, cold standby, and warm standby. A hot standby component has the same failure rate as the active component; while a cold standby component has a zero failure rate.

Ke et al (2011) studied the reliability measures of a repairable system with warm standby switching failures and reboot delay. Jain et al (2004) studied the degraded model with warm standbys and two repairmen. Wang et al (2006) compared four different system configurations with warm standby components and standby switching failures. Hsu et al (2011) examined an availability system with reboot delay, standby switching failures and an unreliable repair facility, which consists of two active components and one warm standby.

Recently El-Damcese and Shama studied Reliability measures of a degradable system with standby switching failures and reboot delay in (2013) and reliability and availability

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analysis of a standby repairable system with two types of failure in (2014).
There are three objectives of this paper: 1) to develop the explicit expressions reliability function and mean time to failure using Laplace transform techniques, 2) to perform a parametric investigation which presents numerical results to analyze the effects of the various system parameters on the system reliability, 3) to perform a sensitivity analysis in the system reliability and the $MTTF$ along with changes in specific values of the system parameters.

1.1 Notations

M : the number of operating units in the initial state
W : the number of cold standby units in the initial state
R_1 : the number of repairman in the first service line
R_2 : the number of repairman in the second service line
λ_1 : the failure rate of type1
λ_2 : the failure rate of type2
μ_1 : the repair rate of type1
μ_2 : the repair rate of type2
$\mu_{1,n}$: mean repair rate when there are n failed units of type1
$\mu_{2,n}$: mean repair rate when there are n failed units of type2
$P_{i,j,x,y}(t)$: probability that there are i, j failed units of type1and of type2 respectively in standby units and x, y failed units of type 1and of type 2 respectively in operating units in the system at time t where $x, y = 0,1$.
$i, j = 0,1,2,...,W \ ,0 \le i + j \le W,$
s : Laplace transform variable
$P^*_{i,j,x,y}(s)$: Laplace transform of $P_{i,j,x,y}(t)$
Y : time to failure of the system
$R_Y(t)$: reliability function of the system
$MTTF$: mean time to failure

1.2 Problem description

We consider a machining system consisting of M identical units operating simultaneously in parallel, W cold standby units, R_1 repairmen in the first service line that repairs failed units of type1 and R_2 repairmen in the second service line that repairs failed units of type2. The assumptions of the model are described as follows: We suppose that the failure rates of type1 and type2 occur independently of the states of other units and follow exponential distributions with λ_1, λ_2 , respectively. It is assumed that there is a significant failure probability g in the switching process, and the delay times of reboot are assumed to be exponentially distributed with parameter β.

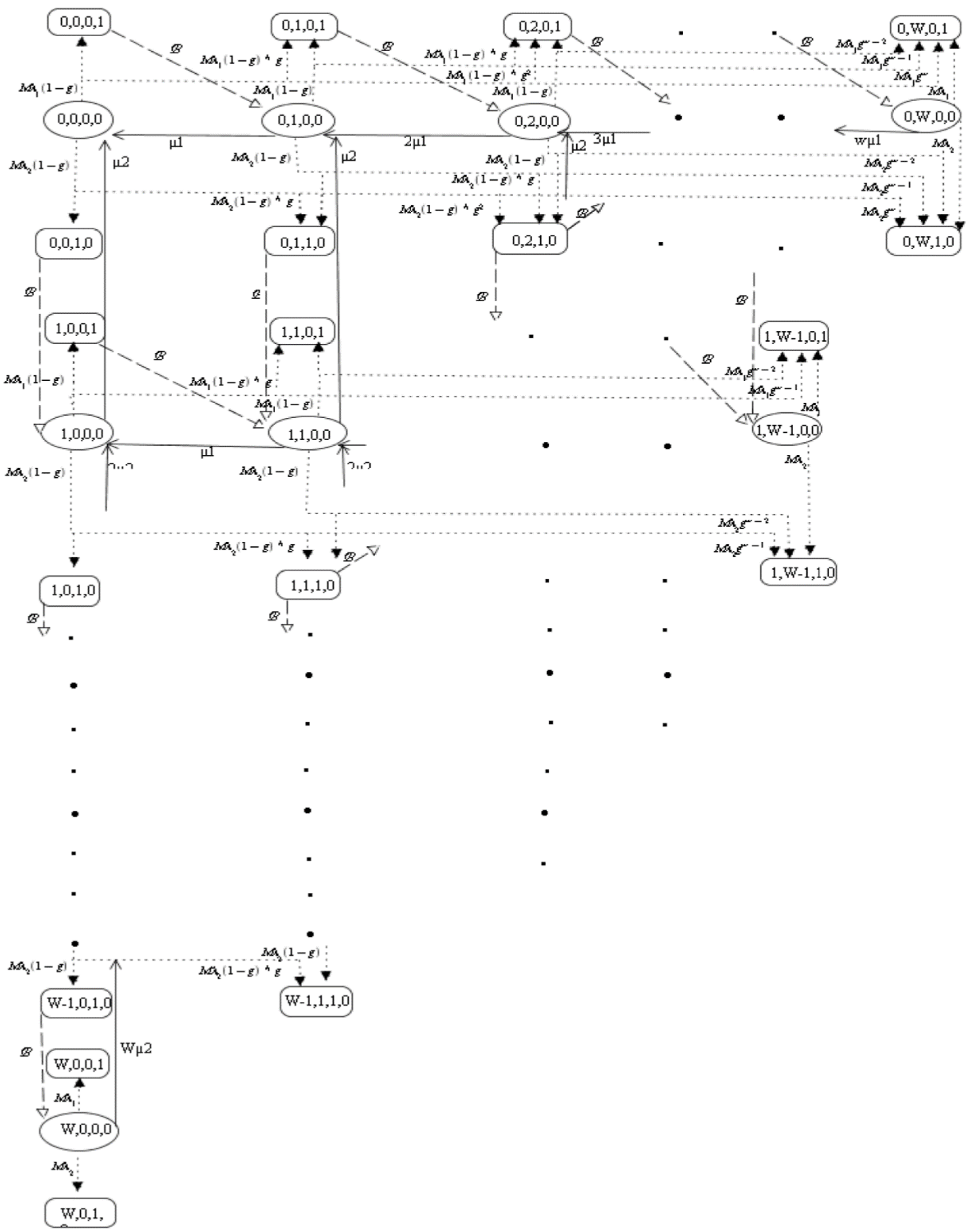

Figure 1. State-transition-rate diagram

An operating unit is replaced by a cold standby if, and only if, it failed [type1 or type2], and one cold standby unit is available; and then it is immediately sent to the appropriate service line where it is repaired with time-to-repair which is exponentially distributed with parameter μ_1 or μ_2 according to the type of failure. Suppose that the switching time from failure to repair, or from repair to standby state, is instantaneous. We also assume that there is always the failure possibility g during the switching process from standby state to operating state and these units are failed of type1. After the switching, reboot delay takes place with mean time 1/β for a standby unit which is exponentially distributed. In the system, we assume that no other event can take place during a reboot.
Let the secession of failure times and the secession of repair times are independently distributed random variables. Moreover, we assume that the secession of failure times and repair times are independently distributed random variables. Failed [type1 and/or type2] units are delivered to the repairmen, forming a single waiting line and are repaired in the order of their breakdowns; in other word, according to the first-come, first-served discipline. Suppose that the repairmen in the two service lines can repair only one failed unit at a time and the repair is independent of the failure of the units. Once a unit is repaired, it is as good as new. System reliability is investigated according to the assumptions that the system is safe when all M -operating units are working.

2. AVAILABILITY AND RELIABILITY ANALYSIS OF THE SYSTEM

At time $t = 0$ the system start operation with no failed units. The reliability function under the exponential failure time and exponential repair time distributions can be developed through the birth–death process. Let Y be the random variable representing the time to failure of the system.
The mean repair rate $\mu_{1,n}$ is given by:

$$\mu_{1,n} = \begin{cases} n\mu_1 & , \quad if\ 1 \le \mathrm{n} \le \min(\mathrm{R}_1, \mathrm{W}) \\ R_1\mu_1 & , \quad if\ \mathrm{R}_1 \le \mathrm{n} \le \mathrm{W} \\ 0 & \quad otherwise \end{cases}$$

The mean repair rate $\mu_{2,n}$ is given by:

$$\mu_{2,n} = \begin{cases} n\mu_2 & , \quad if\ 1 \le \mathrm{n} \le \min(\mathrm{R}_2, \mathrm{W}) \\ R_2\mu_2 & , \quad if\ \mathrm{R}_2 \le \mathrm{n} \le \mathrm{W} \\ 0 & \quad otherwise \end{cases}$$

The Laplace transforms of $P_{i,j}(t)$ are defined by:

$$P^*_{i,j,x,y}(s) = \int_0^\infty e^{-st} P_{i,j,x,y}(t)\,dt, \quad i,j = 0,1,2,\ldots,W \ ,0 \le i + j \le W \ ,x,y = 0,1.$$

State –transition-rate diagram can be obtained in Fig.1, and it leads to the following Laplace transform expressions for $P_{i,j,x,y}{}^*(s)$:

$$\left(s+M\lambda_1+M\lambda_2\right)P^*_{0,0,0,0}(s)-\mu_{1,1}P^*_{0,1,0,0}(s)-\mu_{2,1}P^*_{1,0,0,0}(s)=1$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{1,n}\right)P^*_{0,n,0,0}(s)-\mu_{1,n+1}P^*_{0,n+1,0,0}(s)-\mu_{2,1}P^*_{1,n,0,0}(s)-\beta P^*_{0,n-1,0,1}(s)=0,1\le n\le W-1$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{1,W}\right)P^*_{0,W,0,0}(s)-\beta P^*_{0,W-1,0,1}(s)=0$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{2,u+1}+\mu_{1,n}\right)P^*_{u+1,n,0,0}(s)-\beta P^*_{u+1,n-1,0,1}(s)$$
$$-\mu_{1,n+1}P^*_{u+1,n+1,0,0}(s)-\mu_{2,u+2}P^*_{u+2,n,0,0}(s)-\beta P^*_{u,n,1,0}(s)=0,0\le u\le W-3,1\le n\le W-u-2$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{2,n}+\mu_{1,W-n}\right)P^*_{n,W-n,0,0}(s)-\beta P^*_{n,W-n-1,0,1}(s)-\beta P^*_{n-1,W-n,1,0}(s)=0,1\le n\le W-1$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{2,W}\right)P^*_{W,0,0,0}(s)-\beta P^*_{W-1,0,1,0}(s)=0$$

$$\left(s+M\lambda_1+M\lambda_2+\mu_{2,u}\right)P^*_{u,0,0,0}(s)-\mu_{1,1}P^*_{u,1,0,0}(s)-\beta P^*_{u-1,0,1,0}(s)=0,\quad 0\le u\le W-1$$

$$\left(s\right)P^*_{W,0,0,1}(s)-M\lambda_1P^*_{W,0,0,0}(s)=0$$

$$\left(s\right)P^*_{W,0,1,0}(s)-M\lambda_2P^*_{W,0,0,0}(s)=0$$

$$\left(s+\beta\right)P^*_{0,n,0,1}(s)-\sum_{k=0}^{n}M\lambda_1(1-g)g^kP^*_{0,n-k,0,0}(s)=0,\ 0\le n\le W-1$$

$$\left(s\right)P^*_{0,W,0,1}(s)-\sum_{k=0}^{W}M\lambda_1g^{W-k}P^*_{0,k,0,0}(s)=0$$

$$\left(s+\beta\right)P^*_{0,n,1,0}(s)-\sum_{k=0}^{n}M\lambda_2(1-g)g^kP^*_{0,n-k,0,0}(s)=0,0\le n\le W-1$$

$$\left(s\right)P^*_{0,W,1,0}(s)-\sum_{k=0}^{W}M\lambda_2g^{W-k}P^*_{0,k,0,0}(s)=0$$

$$\left(s+\beta\right)P^*_{u,n,0,1}(s)-\sum_{k=0}^{n}M\lambda_1(1-g)g^kP^*_{u,n-k,0,0}(s)=0,0\le n\le W-u-1,1\le u\le W-1$$

$$\left(s+\beta\right)P^*_{u,n,1,0}(s)-\sum_{k=0}^{n}M\lambda_2(1-g)g^kP^*_{u,n-k,0,0}(s)=0,0\le n\le W-u-1,1\le u\le W-1$$

$$\left(s\right)P^*_{n,W-n,0,1}(s)-\sum_{k=0}^{W-n}M\lambda_1 g^{W-n-k}P^*_{n,k,0,0}(s)=0,1\le n\le W-1$$

$$\left(s\right)P^*_{n,W-n,1,0}(s)-\sum_{k=0}^{W-n}M\lambda_2 g^{W-n-k}P^*_{n,k,0,0}(s)=0,1\le n\le W-1$$

By solving the above equations and taking inverse Laplace transforms, we obtain the reliability function as follows:

$$R_Y(t)=L^{-1}\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P^*_{i,j,x,y}(s)\right)=\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P_{i,j,x,y}(t)\right),i,j=0,1,2,...,W. \quad (1)$$

The mean time to system failure $MTTF$ can be obtained from the following relation.

$$MTTF=\lim_{s\to 0}R^*_Y(s)=\lim_{s\to 0}\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P^*_{i,j,,x,y}(s)\right)=\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P^*_{i,j,x,y}(0)\right) \quad (2)$$

We first perform a sensitivity analysis for changes in the $R_Y(t)$ resulting from changes in system parameters $\lambda_1,\lambda_2,\mu_1,\mu_2,g$ and β .By differentiating equation (1) with respect to λ_1 we obtain,

$$\frac{\partial R_Y(t)}{\partial\lambda_1}=\frac{\partial}{\partial\lambda_1}\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P_{i,j,x,y}(t)\right)=\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}\frac{\partial}{\partial\lambda_1}P_{i,j,x,y}(t)\right) \quad (3)$$

We can use the same procedure to get $\frac{\partial R_Y(t)}{\partial\lambda_1},\frac{\partial R_Y(t)}{\partial\lambda_2},\frac{\partial R_Y(t)}{\partial\mu_1},\frac{\partial R_Y(t)}{\partial\mu_2},\frac{\partial R_Y(t)}{\partial g},\frac{\partial R_Y(t)}{\partial\beta}$.

By differentiating equation (2) with respect to g , β we can perform a sensitivity analysis for changes in $MTTF$ with respected their parameters.

$$\frac{\partial MTTF}{\partial g}=\frac{\partial}{\partial g}\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}P^*{}_{i,j,x,y}(0)\right)=\left(\sum_{\substack{i+j=0\\x,y\neq 1}}^{W}\frac{\partial}{\partial g}P^*{}_{i,j,x,y}(0)\right) \quad (4)$$

We use the same procedure to get $\frac{\partial MTTF}{\partial\beta}$.

3. NUMERICAL RESULTS

In this section, we use MAPLE computer program to provide the numerical results of the effects of various parameters on system reliability and $MTTF$. We choose $\lambda_1=0.0007$, $\lambda_2=0.001$ and fix $\mu_1=0.1,\mu_2=0.03$ .The following cases are analyzed graphically to study the effect of various parameters on system reliability $R_Y(t)$.

Case 1: Fix $M = 2, R_1 = 1, R_2 = 1, \beta = 0.24, g = 0.01$ and choose W =1, 2, 3.
Case 2: Fix $M = 2, W = 2, R_1 = 1, R_2 = 1, \beta = 0.2$ and choose g=0.0,0.2,0.4,0.6
Case 3: Fix $M = 2, W = 2, R_1 = 1, R_2 = 1, g = 0.01$ and choose $\beta = 0.2, 0.3, 0.4$.
Case 4: Fix $M = 1, W = 3, R_2 = 1, \beta = 0.24, g = 0.01$ and choose $R_1 = 1,2,3$
Case 5: Fix $M = 1, W = 3, R_1 = 1, \beta = 0.24, g = 0.01$ and choose $R_2 = 1,2,3$.

We find that system reliability $R_Y(t)$ increases by increasing the standby units W from Figure 2. Figure3 shows that $R_Y(t)$ increases as switching failure probability g decreases. It is also noticed that $R_Y(t)$ rarely changes for β from the Fig.4. It appears from Figure 5 and Figure 6 that $R_Y(t)$ doesn't change with increasing the number of repairmen R_2 or R_1.

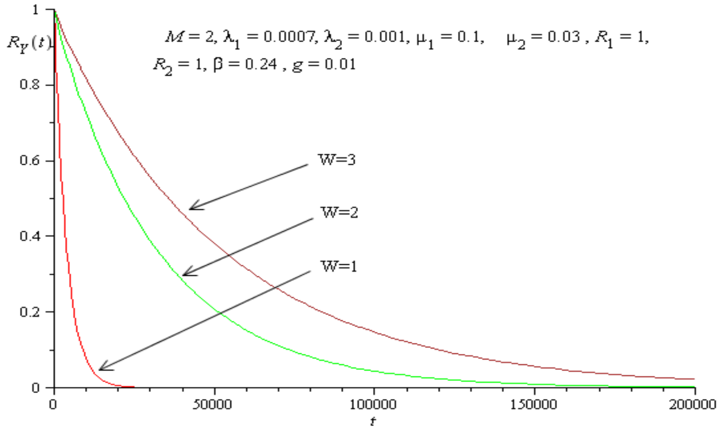

Figure 2. System reliability for different numbers of standby units

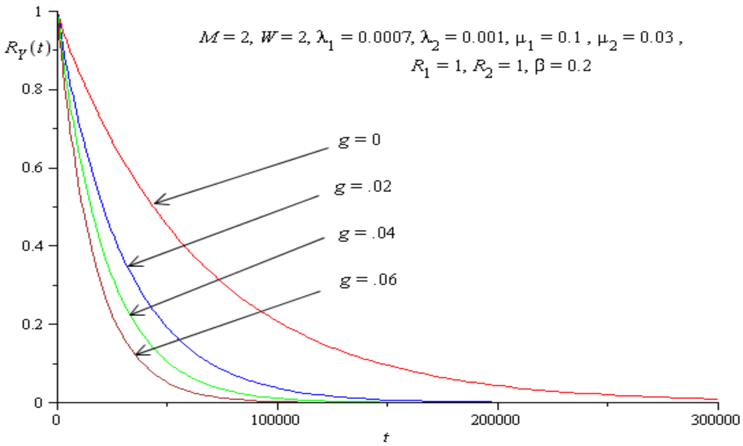

Figure 3. System reliability for different switching failure probabilities

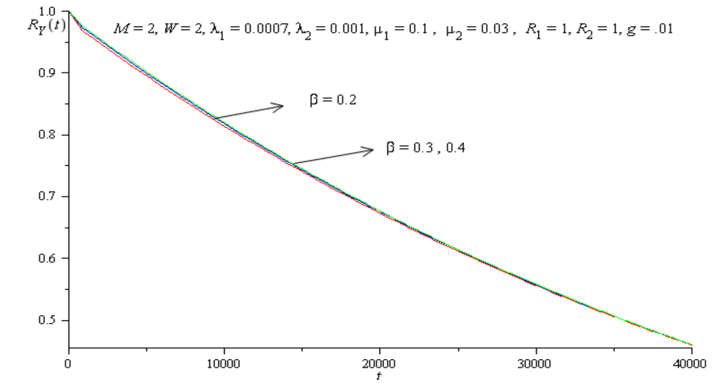

Figure 4. System reliability for different reboot delay rates

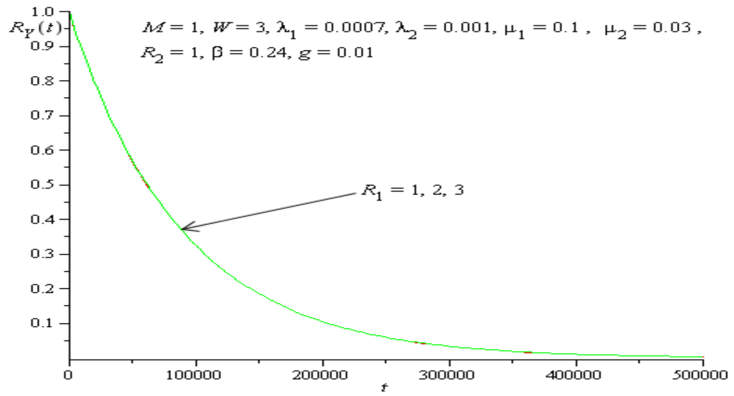

Figure 5. System reliability for different numbers of repairmen in first service line

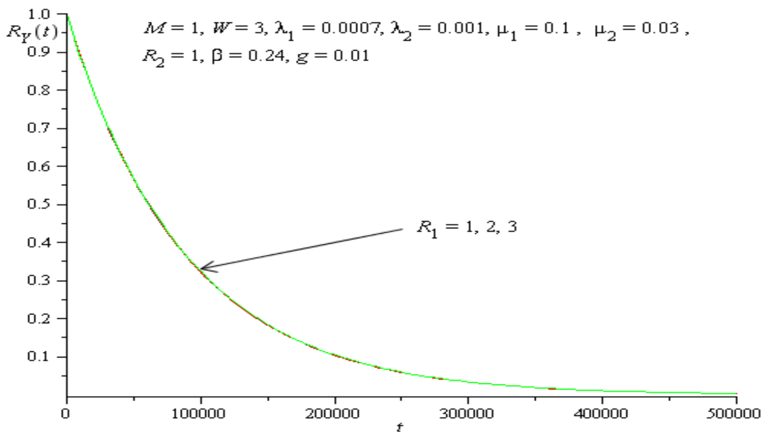

Figure 6. System reliability for different numbers of repairmen in second service line

Next, we study the cross effect of various parameters on $MTTF$. As presented in Figure 7 and Figure 8, we find that the effect of g on the $MTTF$ becomes more significant when λ_2 is smaller than λ_1. Figure 9, Figure 10 show that the effect of g on the $MTTF$ becomes more significant when R_2 is larger than R_1. In Figure 11 we observe that the effect of g on the $MTTF$ becomes more significant as M decreases.

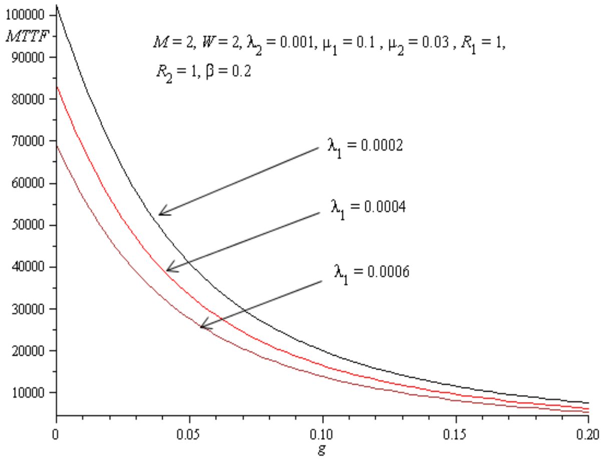

Figure 7. $MTTF$ with changes in λ_1 and g

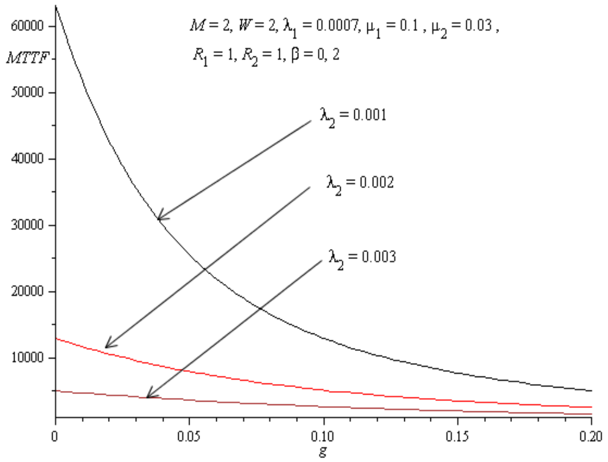

Figure 8. $MTTF$ with changes in λ_2 and g

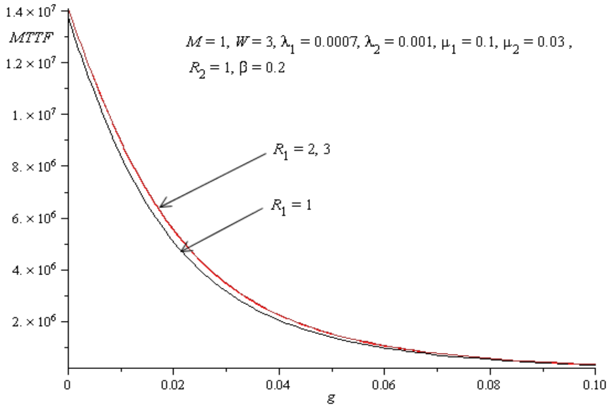

Figure 9. *MTTF* with changes in R_1 and g

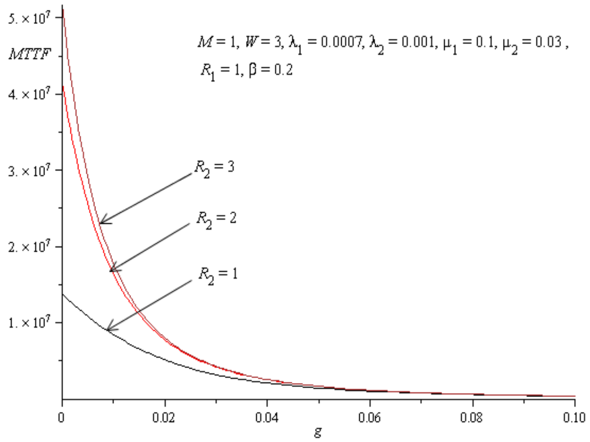

Figure 10. *MTTF* with changes in R_2 and g

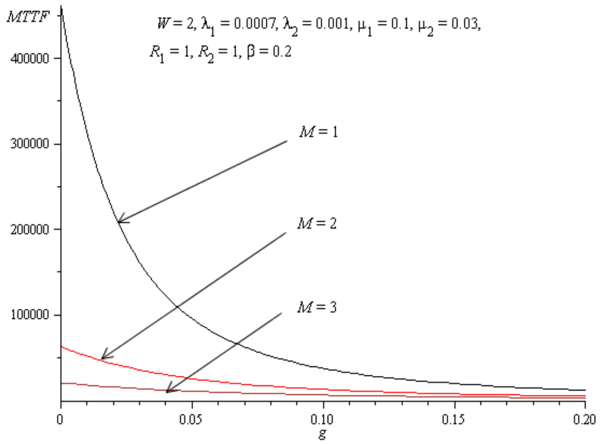

Figure 11. *MTTF* with changes in M and g

Finally we perform sensitivity analysis of system reliability $R_Y(t)$ with respect to system parameters $\lambda_1, \lambda_2, \mu_1$ and μ_2 . In Figure 12, we can easily observe that the order of magnitude of the effect is $(\lambda_2 > \lambda_1 > \mu_2 > g)$ and the sensitivities of μ_1 and β on the $R_Y(t)$ are almost equal to zero. Table 1 shows that the gross sensitivity of various values of g on the $MTTF$ decreases rapidly from -1,015,041 to -500,072as g increases from 0.01 to 0.04 . In Figure 13 we can easily observe that the sensitivities of various values of β on the $R_Y(t)$ which reverse the sign from positive to negative nearly at the same certain time (t_0 =3900) for three cases, this means faster reboot increases the system reliability in the interval of $t \geq t_0$, but decreases the system reliability after that time. In Table 2, we find that the total effect of various reboot rates on the $MTTF$ are all equal to zeros.

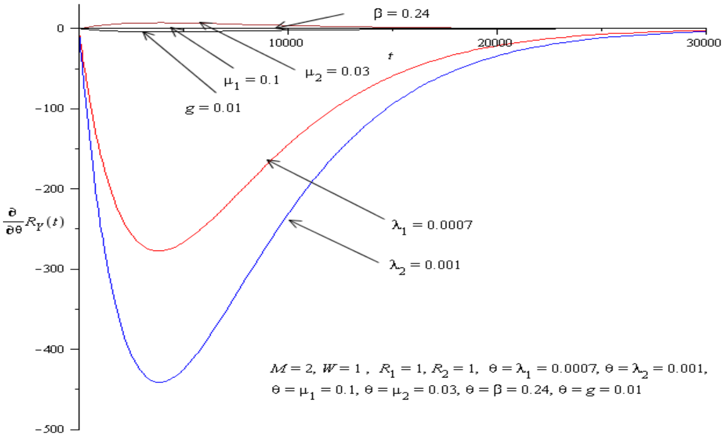

Figure 12. Sensitivity of system reliability with respect to system parameters

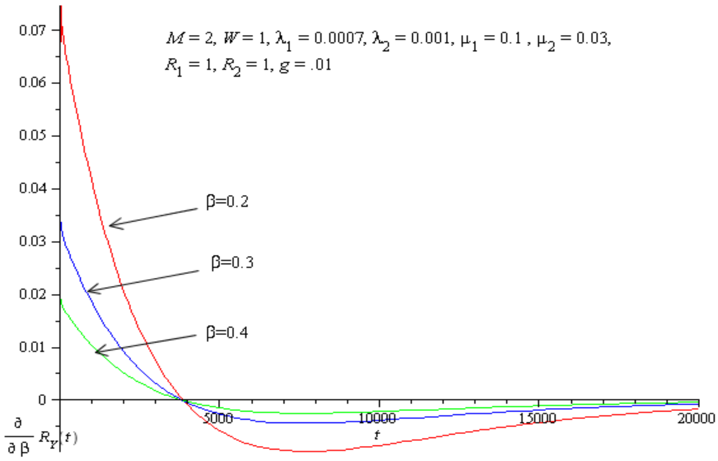

Figure 13. Sensitivity of system reliability with respect to β

Table 1. Sensitivity analysis for $MTTF$ with respect to g

g	$\frac{\partial MTTF}{\partial g}$
0.01	-1,015,041
0.02	-799,735
0.03	-630,476
0.04	-500,072

Table 2. Sensitivity analysis for $MTTF$ with respect to β

g	$\frac{\partial MTTF}{\partial \beta}$
0.02	0
0.04	0
0.04	0
0.06	0
0.08	0

4. CONCLUSIONS

In this paper, a mathematical model was constructed for a repairable system with standby switching failures and reboot delay. Reliability and mean time to failure in addition to sensitivity analysis for the system reliability were obtained and the results were shown graphically by the aid of MAPLE program. Sensitivity analysis of the system parameters on the system reliability and $MTTF$ are also studied. Results indicate that the increasing the number of cold standby units W can greatly improve the system performance and the reboot delay parameter β only affects the reliability but not $MTTF$ of the system.

REFERENCES

El-Damcese, M. A. and Shama, M. S. (2013). Reliability measures of a degradable system with standby switching failures and reboot delay, *Applied Mathematics in Engineering, Management and Technology*, **1**, 1–16.

El-Damcese, M. A. and Shama, M. S. (2014) Reliability and availability analysis of a repairable system with two types of failure, *Jokull Journal,* **64**, 75–83.

Hsu, Y. L., Ke, J. C. and Liu, T. H. (2011). Standby system with general repair, reboot delay, switching failure and unreliable repair facility A statistical standpoint, *Mathematics and Computers in Simulation*, **81**, 2400–2413.

Jain M, Rakhee and Singh, M. (2004). Bilevel control of degraded machining system with warm standbys, setup and vacation, *Applied Mathematical Modeling*, **28**, 1015–1026.

Ke, J. B., Chen, J. W. and Wang, K. H. (2011). Reliability Measures of a Repairable System with Standby Switching Failures and Reboot Delay, *Quality Technology & Quantitative Management*, **8**, 15-26.

Wang, K. H, Dong, W. L. and Ke, J. B. (2006). Comparison of reliability and the availability between four systems with warm standby components and standby switching failures, *Applied Mathematics and Computation*, **183**, 1310–1322.