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Time- and Frequency-Domain Optimization of Sparse Multisine Coefficients for Nonlinear Amplifier Characterization

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Abstract

For the testing of nonlinear power amplifiers, this paper suggests an approach to design optimized multisine signals that could be substituted for the original modulated signal. In the design of multisines, complex coefficients should be determined to mimic the target signal as much as possible, but very few methods have been adopted as general solutions to the coefficients. Furthermore, no solid method for the phase of coefficients has been proven to show the best resemblance to the original. Therefore, in order to determine the phase of multisine coefficients, a time-domain nonlinear optimization method is suggested. A frequency-domain-method based on the spectral response of the target signal is also suggested for the magnitude of the coefficients. For the verification, multisine signals are designed to emulate the LTE downlink signal of 10 MHz bandwidth and are used to test a nonlinear amplifier at 1.9 GHz. The suggested phase-optimized multisine had a lower normalized error by 0.163 dB when N = 100, and the measurement results showed that the suggested multisine achieved more accurate adjacent-channel leakage ratio (ACLR) estimation by as much as 12 dB compared to that of the conventional iterative method.

Key Words: Multisine, Nonlinear, Optimization.

I. INTRODUCTION

Multisine signals are widely used in the development and analysis of advanced communication systems because the transmission channel can be accurately identified with the magnitude and phase information of each tone. There have been efforts to replace the modulated signal used in the field with more generic signals because of the cost of equipment and the comparability of the test results. Of the options, multisine signals are the most widely used because of their mathematical clarity, ease of design, and versatility in identifying behavioral models of nonlinear systems [1, 2].

Many design methods have been introduced thus far, mostly based on the discrete Fourier transform (DFT) and the iterative sorting of samples [3, 4]. The brief algorithm of the most common iterative method is as follows [4]:

1. Calculate the probability density function (pdf) of the desired target signal or a noisy signal.
2. Synthesize a multisine with the desired magnitude.
3. Sort both signals by the instantaneous magnitude in descending order.
4. Substitute the magnitudes of the multisine samples with those of the target signal.
5. Restore the multisine in the original time instances of the

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multisine, creating a new multisine with the pdf of the target signal.

6. Calculate the DFT of the multisine and restore the originnal magnitude, retaining the phase of each tone.
7. Repeat steps 3 to 6 until an acceptable convergence criterion is met.

Although this iterative sorting method (hereafter the iterative method) accounts for the time-domain distribution of the signal, it is mainly concerned with the probability distribution of the amplitude; thus, the method does not guarantee the best overall resemblance to the original target signal. In this regard, the authors suggested designing a multisine for the best time-domain resemblance [5]. However, as the number of tones of the multisine decreases, the error in the time domain accumulates onto the existing tones of the multisine, resulting in a significant discrepancy in the frequency domain. In this paper, we therefore suggest a hybrid process to determine the best set of multisine coefficients of sparse multisine tones, in which the magnitude is determined in the frequency domain and the phase is optimized in the time domain. The method of determining the optimal phase of each multisine was mathematically proven rather than by using the numerical method. In Section II, the extraction of the multisine magnitude is discussed. In Section III, the mathematical optimization for the phase is discussed, based on the Wiener-Hopf theory. In Sections IV and V, an experimental comparison between multisines synthesized using the suggested method and the conventional sorting method is discussed, and finally, the conclusion is presented in Section VI.

II. Magnitude Determination of Multisine Signals

A complex multisine signal $x_c(t)$ is defined by the sum of the sinusoids with the amplitude and phase of each tone as follows:

$$x_c(t) = \sum_{k=0}^{N-1} A_k e^{j(\omega_k t + \phi_k)} , \tag{1}$$

where A_k and ϕ_k are the magnitude and phase of the k^{th}-tone, respectively, ω_k is the tone frequency, and N is the number of tones within the bandwidth (BW).

The fundamental idea in determining the magnitude of each tone is to set the magnitude based on the spectral shape of the original signal, for which the Fourier transform is suggested. However, the magnitudes of the Fourier coefficients may show severe fluctuations because of the random nature of the original signal. Thus, it is clear that in order to use this multisine for accurate measurement, post-processing to mitigate the fluctuation within the signal bandwidth is necessary. Other than directly applying the DFT coefficient, the alternative method of using a constant magnitude and leveling off the coefficient to make its power equal to that of the target signal has been suggested [4, 6]. However, this method fails to emulate target signals such as 802.11b or Bluetooth, of which the spectral density is not constant within its channel bandwidth.

In this paper, the signal bandwidth is divided into spectral bins, in which the magnitude of the multisine has a constant value so that its integrated power meets the average power of the target signal within the bin. With this method, the accumulated spectral error can be spread over the bins of the sparsely defined multisine frequency, and thus the synthesized multisine can follow the spectral shape of the original modulated signal.

The magnitude of the k^{th} multisine is calculated so that the following condition is met:

$$P_{avg,n} = \left(\frac{m}{2} \frac{A_n^2}{50} \right), \tag{2}$$

where $P_{avg,n}$ and A_n are the average power and magnitude of the target signal within the n^{th} bin, respectively, and m is the number of tones within the bin.

Thus, the k^{th}-tone magnitude A_k is determined by the n^{th}-bin coefficient, as follows:

$$A_k = A_n = \sqrt{\frac{100 P_{avg,n}}{m}} , \quad m(n-1)+1 \le k < (mn+1) . \tag{3}$$

III. Phase Determination of Multisine Signals

In terms of the phase of multisines, very few general methods are widely used because the distribution of the phase significantly affects the characteristics of the multisine. Therefore, the phase should be designed in consideration of its application. In the linearity measurement of nonlinear devices, signal properties such as sample-to-sample correlation, probability distribution, and the spectral response all affect the measurement result. Fig. 1 shows the variation of $N = 20$ multisines in the time domain when the phase is given in various ways, such as evenly distributed, all aligned, taken from Fourier coefficients, and time domain optimized. In this sense, the iterative method is used for the phase of a multisine by iteratively sorting samples in regard to the pdf of the target signal and by rearranging them according to the original time instances [4]. However, as mentioned above, since this method does not involve any mathematical optimization, it does not guarantee the best sample-to-sample correlation, which has a major effect on the linearity testing results.

Therefore, the mathematical optimization of ϕ_k is suggested by the authors so that the synthesized multisine shows the best resemblance to the target waveform with the given number of tones.

Because the synthesized multisine should resemble the target

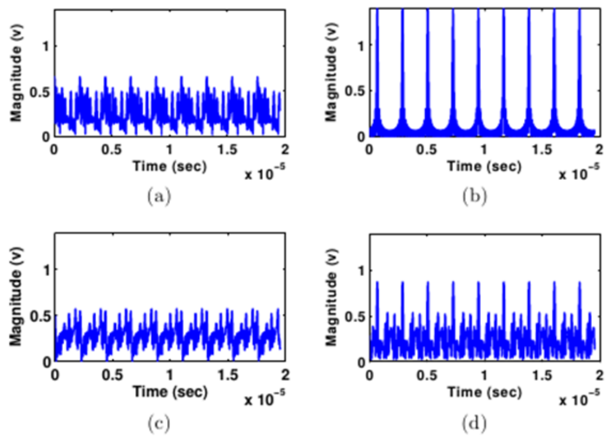

Fig. 1. Magnitude variation of N = 20 multisines emulating an LTE downlink signal. The phases of multisines are (a) evenly distributed, (b) all aligned, (c) taken from Fourier coefficients, and (d) time domain optimized, the process of which is explained in Section III.

signal that is statistically random in most cases, the cost function J_t for the optimization is defined by the expectation of the time-domain error $e_t(t)$:

$$J_t = E\left[|e_t(t)|^2\right] = E\left[|x(t) - x_c(t)|^2\right], \tag{4}$$

where $x(t)$ is the target signal.

Thus, the goal of the optimization is to minimize the cost function with ϕ_k when the magnitude A_k is predetermined by the power spectral density (PSD) of the target signal. Therefore, the optimal phase of the k^{th} tone is determined by the one-dimensional gradient of Eq. (4) [7]:

$$\begin{aligned}\nabla J_t &= \left\{\frac{\partial J_t}{\partial \phi_k}\right\}_{A_k = const.} \\ &= \frac{\partial}{\partial \phi_k}\left(E\left[e_t(t)e_t^*(t)\right]\right) \\ &= E\left[\frac{\partial e_t(t)}{\partial \phi_k}e_t^*(t) + e_t(t)\frac{\partial e_t^*(t)}{\partial \phi_k}\right].\end{aligned} \tag{5}$$

In this equation, $\partial e_t(t)/\partial \phi_k$ is expressed as follows:

$$\begin{aligned}\frac{\partial e_t(t)}{\partial \phi_k} &= \frac{\partial}{\partial \phi_k}\left(x(t) - \sum_{k=0}^{N-1} A_k e^{j(\omega_k t + \phi_k)}\right) \\ &= -jA_k e^{j\phi_k} e^{j\omega_k t}.\end{aligned} \tag{6}$$

Therefore, Eq. (5) is rewritten using the magnitude and phase of the tone.

$$\nabla J_t = E\left[-jA_k e^{j(\omega_k t + \phi_k)} e_t^*(t) + jA_k e^{-j(\omega_k t + \phi_k)} e_t(t)\right]. \tag{7}$$

The gradient of the cost function is redefined so that Eq. (7) has a real value:

$$\nabla J_t' \equiv \frac{\partial J_t}{\partial \phi_k} = E\left[\varsigma_k - \varsigma_k^*\right] = 2jE\left[imag(\varsigma_k)\right], \tag{8}$$

where ς_k is defined as $A_k e^{j(\omega_k t + \phi_k)} e^*(t)$, and $imag(\cdot)$ is the function used to take the imaginary part of a complex number.

In order to obtain the minimal J_t' using phase optimization, the condition of the zero gradient and its equivalent condition are determined as follows:

$$\nabla J_t' = 0, \text{ thus } 2jE\left[imag(\varsigma_k)\right] = imag\left(E[\varsigma_k]\right) = 0. \tag{9}$$

Eq. (9) is expanded by the statistical properties:

$$\begin{aligned}&imag\left(E[\varsigma_k]\right) \\ &= imag\left(E\left[A_k e^{j(\omega_k t + \phi_k)}\left(x^*(t) - \sum_{i=0}^{N-1} A_i e^{-j(\omega_i t + \phi_i)}\right)\right]\right) \\ &= imag\left(\begin{aligned}&E\left[A_k e^{j\phi_k} e^{j\omega_k t} x^*(t)\right] \\ &- E\left[A_k e^{j\phi_k} e^{j\omega_k t}\sum_{i=0}^{N-1} A_i e^{-j\phi_i} e^{-j\omega_i t}\right]\end{aligned}\right) = 0.\end{aligned} \tag{10}$$

Eq. (10) has the sum of the cross-correlation between the k^{th}-tone and the synthesized multisine, so it can be simplified as in the following equation because of the orthogonality of the sinusoidal function as follows:

$$\begin{aligned}&imag\left(E[\varsigma_k]\right) \\ &= imag\left(A_k e^{j\phi_k} E\left[e^{j\omega_k t} x^*(t)\right]\right) \\ &\quad - imag\left(\sum_{i=0}^{N-1} corr\left(A_i e^{j(\omega_i t + \phi_i)}, A_k e^{-j(\omega_k t + \phi_k)}\right)\right) \\ &= imag\left(A_k e^{j\phi_k} E\left[e^{j\omega_k t} x^*(t)\right] - A_k^2(0 + \cdots + 1 + \cdots + 0)\right),\end{aligned} \tag{11}$$

where $corr(\cdot)$ represents the cross-correlation of the given functions.

Thus, the condition for the minimum error is found by Eq. (11), and its solution is equivalent to that of the following equation:

$$imag\left(E\left[e^{j\omega_k t} x^*(t)\right] e^{j\phi_k} - A_k\right) = 0. \tag{12}$$

Because A_k is real, Eq. (12) is equivalent to the following equation:

$$angle\left(E\left[e^{j\omega_k t} x^*(t)\right] e^{j\phi_k}\right) = \pm n\pi. \tag{13}$$

Therefore, the optimal ϕ_k is finally found as follows:

$$\phi_k = -angle\left(E\left[e^{j\omega_k t} x^*(t)\right]\right) \pm n\pi. \tag{14}$$

As a result, it is found that when A_k is given, the optimal phase of each multisine is determined by the correlation of $x(t)$ and $e^{-j\omega_k t}$ at the corresponding frequency. This could also be expressed in a discrete form:

$$\phi_k = angle\left(\frac{1}{L}\sum_{n=0}^{L-1} x(nT_s) e^{-j\omega_k(nT_s)}\right) \pm p\pi. \tag{15}$$

where L is the sample length of the target signal, p is an integer, and T_s is the sampling interval.

In addition, in order to measure the preciseness of the designed multisine, a normalized average error is defined as a figure of merit, as follows:

$$err_{norm} = \frac{\frac{1}{L}\sum_{k=0}^{L-1}\left|e_t\left(kT_s\right)\right|^2}{P_{avg}}. \quad (16)$$

where T_s, L, and P_{avg} are the sampling period, sample length, and the average power of the target signal, respectively.

IV. SIMULATION AND EXPERIMENTAL VALIDATION

In this section, multisine signals from different methods are synthesized to emulate the Long-Term Evolution (LTE) downlink signal of 10-MHz bandwidth and are compared using the linearity test on a commercial amplifier at 1.9 GHz. The target LTE signal has a peak-to-average power ratio (PAPR) of 10.5 dB, a sample length of 520,000, and a sampling frequency of 128.8 Msps. The linearity of the amplifier was evaluated by the adjacent-channel leakage ratio (ACLR) at an offset of 8.6 MHz from the center frequency, where the third-order intermodulation (IMD3) dominates. The first multisine was synthesized using the conventional iterative method with various numbers of tones. The suggested method with the phase-optimized (PO) multisine was synthesized by the aforementioned nonlinear phase optimization, with the same *N* and *BW*. Next, the resemblance of the iterative multisine to the original signal was evaluated over the number of iterations; Fig. 2 shows the difference in the pdfs between the LTE signal and the iterative multisine when $N = 50$. From the figure, the number of iterations is decided as 50 because the accuracy saturates from the 30th iteration. Next, the resemblance of multisines to the original signal was evaluated using the conventional iterative method and the suggested PO method; Fig. 3 shows the pdfs between the LTE signal and multisines when $N = 20$. In the figure, it is clearly shown that the PO method and the iterative method have pdfs that are very similar to that of the target signal because of the optimization process. However, from the

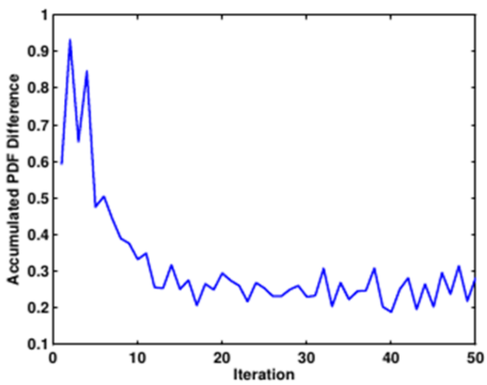

Fig. 2. Convergence of the probability density function (pdf) error of the iterative method ($N = 50$) over iterations.

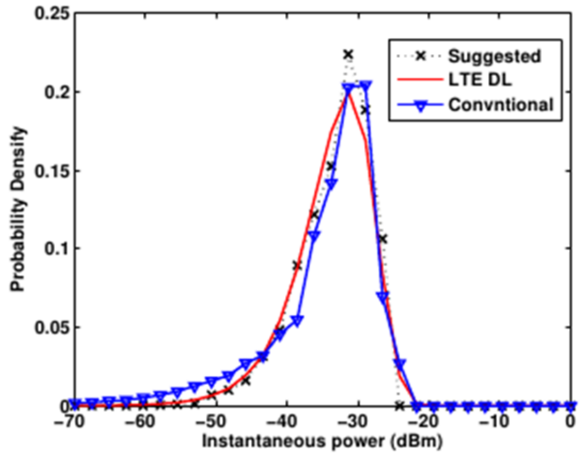

Fig. 3. Comparison of probability density functions (pdfs) between multisines ($N = 50$) and the LTE downlink (DL) signal.

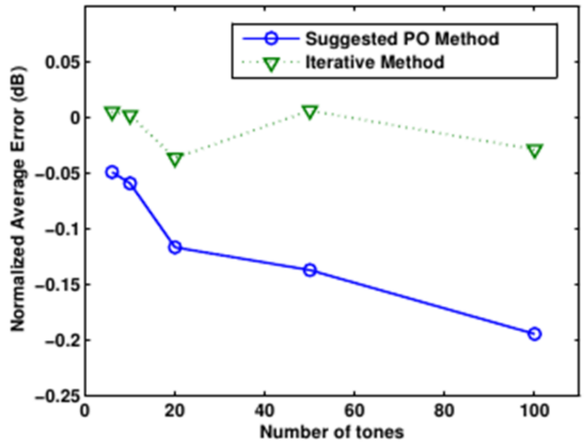

Fig. 4. Comparison of the normalized average error between multisines from the iterative and phase-optimized (PO) methods when $P_{avg} = -30$ dBm.

perspective of the normalized average error in the time domain, which also determines the accuracy of the multisine, the suggested PO method has better results than the iterative method. Fig. 4 compares the results of the different methods for various numbers of tones with $P_{avg} = -30$ dBm.

V. COMPARISON OF ACLR RESULTS

Finally, the designed multisines are used for the linearity testing of a commercial PA and are compared by the accuracy of the ACLR estimation. Fig. 5 shows the spectral responses of the original signal and the multisines of $N = 20$, in which the outlines of the spectral regrowth of the PO multisine show a marginally closer shape to the original signal's ACLR response. Fig. 6 shows the results of the ACLR test with synthesized signals, where the ACLR is calculated by the ratio of the IMD3 frequency (PSD_{fIMD3}) to the power at the center frequency (PSD_{fc}) over the resolution bandwidth (*RBW*):

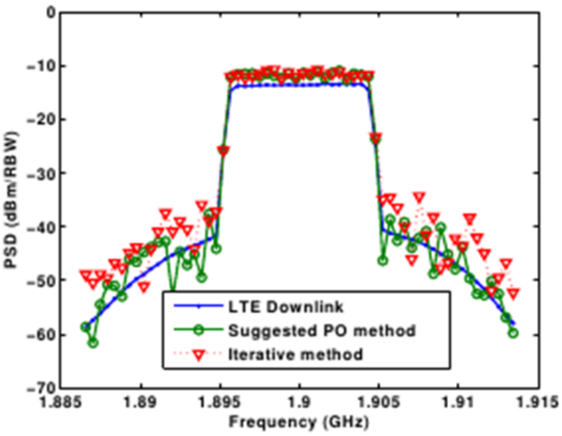

Fig. 5. Spectral regrowth of multisines of $N = 20$ and the LTE downlink signal. PSD = power spectral density, PO = phase-optimized.

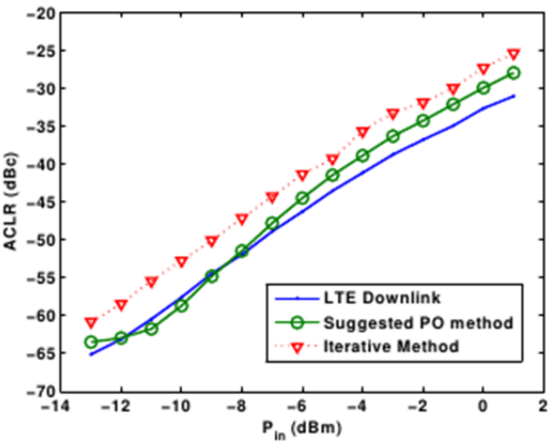

Fig. 6. Comparison of adjacent-channel leakage ratio (ACLR) estimation between N = 50 multisines from the iterative and phase-optimized (PO) methods.

$$ACLR_{dBc} = 10\log\left(\frac{\int_{RBW} PSD(f_c)df}{\int_{RBW} PSD(f_{IMD3})df}\right). \tag{17}$$

From the figure, a remarkable accuracy in the ACLR is observed with the PO multisine over a wide range of input power, whereas the iterative multisine showed relatively inaccurate ACLR results.

As seen in the measurement results, due to the optimized time-domain correlation, it is shown that the suggested PO multisine can estimate ACLR results more accurately than the conventional iterative method.

VI. Conclusion

In this paper, we have suggested a multisine design method that is based on time- and frequency-domain signal processing in which the phase is calculated from the time-domain optimization and the magnitude is determined by the frequency-domain spectral responses. Based on the time-domain optimization, the condition for the optimal phase of each tone is determined so that the multisine can accurately estimate the ACLR. In addition, the magnitude of the multisine is found to follow the spectral characteristic of the original signal. To verify the performance of the suggested method, the method used to synthesize the 10 MHz LTE downlink signal was compared with the conventional iterative method by using the ACLR test results using a commercial amplifier at 1.9 GHz. The measurement results showed that the multisine from the suggested method achieved a lower normalized error by 0.163 dB when N = 100, and its ACLR results outperform the iterative method by up to 12 dB in various numbers of tones.

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