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Integrating Math and Music: Teaching Ideas

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Mathematical creativity, an important goal in mathematics education, can be promoted through an integrated learning environment where students explore mathematics with other subject areas such as science, technology, engineering and art. Establishing such learning environments is not a trivial task. Therefore, this creates a need for the development of instructional resources promoting meaningful integration. This paper focuses on integration of the fields of mathematics and music. Beginning with some of the historical discoveries and views of the connections between mathematics and music, this paper attends to several musical concepts correlating to middle school mathematical content and then provides ideas for teaching.

Keywords: integrating, integrated lesson, music
MESC Classification: C40
MSC2010Classification: 97C40

I. INTRODUCTION

Mathematical creativity is an important learning goal in mathematics education and creative problem solving is regarded as a core competency in handling complex problems that arise in modern society (Wilson, 2005). Solving complex problems often requires utilizing a variety of topics. Interdisciplinary environments, for example, through an integration of science, technology, engineering and art, can provide a meaningful context for students to learn and develop mathematical creativity while learning important ideas in each discipline being used. Establishing such learning environments is not a trivial task: we cannot expect that teachers will develop interdisciplinary lessons with little support.

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This creates a need for the development of instructional resources promoting integration and interdisciplinarity. However, a report by Ju, Moon & Song(2012) reveals that studies examining the integration of school subject areas focus on kindergarten and elementary education and there is little research studying integration, mathematics and music in particular, at the secondary level.

Mathematics and music have commonalities and connections. However it is not often that these two disciplines are integrated for the purpose of education. From elementary to collegiate education, there are content areas within mathematics curriculum into which musical ideas can be integrated. These musical ideas can provide a context for students to explore mathematics, which has received an increasing attention in the mathematics education community as a way to help ensure and improve retention and understanding. This type of interdisciplinary approach can allow students to enjoy and comprehend certain mathematical concepts while developing complex reasoning skills as a result of learning through pragmatic encounters (Edelson & Johnson, 2003).

The integration of the fields of mathematics and music that is occurring is becoming incredibly widespread. Research materials (eg, Harkleroad, 2006) have been developed as a result of courses taught or lectures given on the topic of math and music which encourage their use for such potential. Grants are being awarded to those educators willing to allow the mathematics in their classroom to be enhanced with musical content (NCTM, 2008). Resources including websites are being developed as a way for educators to share the ways they incorporate music in their classroom (Chandler, 2007). Performances are emerging where the focus is on music and mathematics (Hinds, 2006), while activities are being developed for the classroom incorporating mathematics into dance and music (Schaffer et al., 2001). From mathematical concepts being sung in a familiar tune to full courses teaching in-depth connections to mathematical dance, educators recognize the educational potential of music in the classroom setting.

The purpose of this paper is to provide educators with resources from which to draw upon for use in their lessons and classrooms.

While it is exciting to hear or witness how educators are using music in their classrooms, a certain level of musical content is needed to make the integration of music and mathematics educational. Simply using music in a math class does not equate to utilizing the connections between music and mathematics as a context for students to learn mathematics. Although there are many creative songs used in classrooms that are great ways to make material more interesting or to aid in retention, often times these have nothing to do with the mathematics being taught. Such implementation is a very superficial level of integration of mathematics and music. The research surrounding whether a musical background can improve your math skills may also be incentive for teachers to bring music in the classroom. But this only refers to playing a musical instrument or listening to music, not learning mathematical ideas alongside musical content (or vice versa) (Zhan, 2002).

The connections between mathematics and music are wide-ranging in terms of the level of understanding and knowledge of each discipline needed in order to comprehend the implications of their interactions. A routinely known link exists between note values and fractions (Houser, 2002). Some material surrounding the links between these fields need only algebra and trigonometry as prior knowledge, while other associations require more complex mathematics such as calculus and number theory (Madden, 1999). Although there are many more connections than the ones presented in this paper, these connections have a focus on K–12 mathematical topics which correlates to relatively simple musical content.

Beginning with the historical discoveries and views of the connections between mathematics and music, this paper will focus on several musical concepts that relate directly to mathematical content typically presented in the middle school mathematics curriculum and then provide ideas for teaching.

II. A BRIEF HISTORY OF CONNECTIONS BETWEEN MATHEMATICS AND MUSIC

Early mathematicians and philosophers believed that music was originally a 'hidden arithmetic exercise of the soul' (Dyck, 1960, p. 91); 'If music were only a random shower of sounds, it would bear as little relation to' mathematics 'as the clatter of plates and knives at dinner' (Saminsky, 1957, p. 41).

The associations of mathematics and music have been known and developed for millennia which imply a deeper connection perhaps to such philosophical standards of beauty and truth. The interactions between mathematics and music are not simply flimsy findings, but in fact resilient and effective for each field; Not only can mathematical decisions result in new musical works, musical composition can bring to life mathematical questions (Haack, 1998). In their own unique way, both music and mathematics fuse the intellectual and the aesthetic beautifully but unfortunately, just as musicians are not aware of the aesthetic features of mathematics, mathematicians often do not realize the intellectual characteristics of music (Harkleroad, 2006). Both disciplines communicate their own abstract content through their unique historical systems of symbolic notation which have remained modern due to their ability to adapt with new ideas and discoveries (Fauvel et al., 2003). Both are also dependent on their notation because when we think mathematically it is in the language of mathematical symbols while thinking musically, the framework and notations of music are different (Andrews, 2006).

While there are many scholars of the past who have contributed their discoveries to the development of the connections between mathematics and music, Pythagoras was the first to fundamentally realize the connection between mathematics and music. Filling an urn with water, Pythagoras hit the urn with a hammer in order to produce a note. After some investigations into the notes produced based on the amount of water in the urn, he discovered that is he removed half of the water and hit the urn again, the note had gone up by an octave compared to the first note. He continued with this and every time he removed more water from the urn to keep it at one third full then one quarter full, the notes sounded in harmony with the initial note produced. Any other amount of water in the urn formed notes that sounded dissonant with the initial note. Pythagoras had realized there was harmony associated with the fractions 1, 1/2, 1/3, 1/4, etc. (DuSautoy, 2003). To the Pythagoreans, the followers of Pythagoras and his discoveries, music was in her ently mathematical; They believed that musical harmony was dependent on mathematics, hence the existence also of a mathematica lharmony (Dyck, 1960).

It was through this alignment of music with mathematics that it was then connected with arithmetic and geometry. Traditionally, these disciplines (music, arithmetic, geometry, plus astronomy) were a part of the quadrivium of the curriculum in liberal arts from the Middle Ages and earlier (Dyck, 1960). Music was instructed as a science at the university level then assessed as an art through composition and even as recently as the 19^{th} century music still had significant academic status. Mathematical sciences are seen as separate from the arts in the eyes of scientific historians and especially since music is now viewed as strictly an art as opposed to a science (Fauvel et al., 2003), the historical past of the connection between mathematics and music is scarce.

Just as music has intrigued mathematicians for centuries, musicians are drawn towards using mathematics as a part of their compositional efforts. There is research concerning the mathematical aspects of compositional methods and analysis has been done on specific works regarding their mathematical patterns (Fauvel et al., 2003). The primary structures of music - rhythm and pitch- are two strong connections, but others include symmetry found in compositions, the appearance of Fibonacci numbers, the golden mean, and even more recently fractals in compositions also as the mathematical foundation for tuning systems (Haack, 1998).

III. MIDDLE SCHOOL MATH AND MUSIC

1.Musical Space and Geometry

1) Connections

Musical space is simply the space in which music takes place. Initially, one may think that musical space is simply the staff lines where music is systematically represented as a part of a score. However, the two musical components of pitch and time create a 2-dimensional space, a plane, of musical space(Figure 1). Musical space is a bounded region as 'a piece of music lasts a finite amount of time and uses only a finite range of pitches' (Fauvel et al., 2003, p. 98). Musical space can clearly still be represented musically on the staff lines in a musical score, but it can also be represented mathematically on a Cartesian coordinate plane.

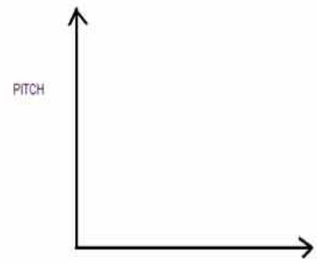


Figure 1. A pitch-time musical space

Graphically, the two components of pitch and time can be represented as the coordinate and the abscissa, respectively (Haack, 1998, p. 72). In order to be more accurate with terminology, it may be better to refer to pitch as frequency and time as duration when using the terms in relation to a mathematical Cartesian coordinate plane. A pitch refers to the frequency, or the measure of vibrations per second (Loy, 2007), usually measured in Hertz (Hz).The time, or duration, refers to the note symbols which indicate the length of each pitch.So, each note has its own frequency and duration indicated on the musical score and these two elements of each note can be represented in the coordinate plane. Whether by making a table of values to plot, or simply plotting directly from the musical composition, each note can now be seen in a mathematical way as opposed to simply in a musical way. Using the first two measures of Andrew Lloyd Webber's "The Music of the Night" as our music, it can be seen below (Figure 2) in two varying formats: musical staff and coordinate plane (Johnson, 2001).Whether by making a table of values to plot, or simply plotting directly from the musical composition, each note can now be seen in a mathematical way as opposed to simply in a musical way. Using the first two measures of Andrew Lloyd Webber's "The Music of the Night" as our music, it can be seen below (Figure 2) in two varying formats: musical staff and coordinate plane (Johnson, 2001).

These two representations are equivalent; it simply depends on how one is looking at the music, as a mathematician or as a musician.

Musical space, as a part of the musical staff and as a coordinate plane, can be transformed just as other geometric objects. Certain transformations are isometries when applied to typical musical spaces, as the distances between notes remain rigid; these are translation, rotation, and reflection (Fauvel et al., 2003). Scalar transformations can also be done on the musical staff, but these distances do not remain rigid, as only the intervals are affected (Madden, 1999). Each geometric transformation performed on a musical image, or space, has a corresponding musical transformation.

A translation moves an image horizontally or vertically. Done vertically, the geometric translation can be compared to transposition in music, which means the repetition of an interval or motive on a different initial pitch. Done horizontally, a translation changes the timing of the piece. Think of the round 'Row, Row, Row Your Boat.' One person begins while the other waits to begin the tune once a line or two have been sung.

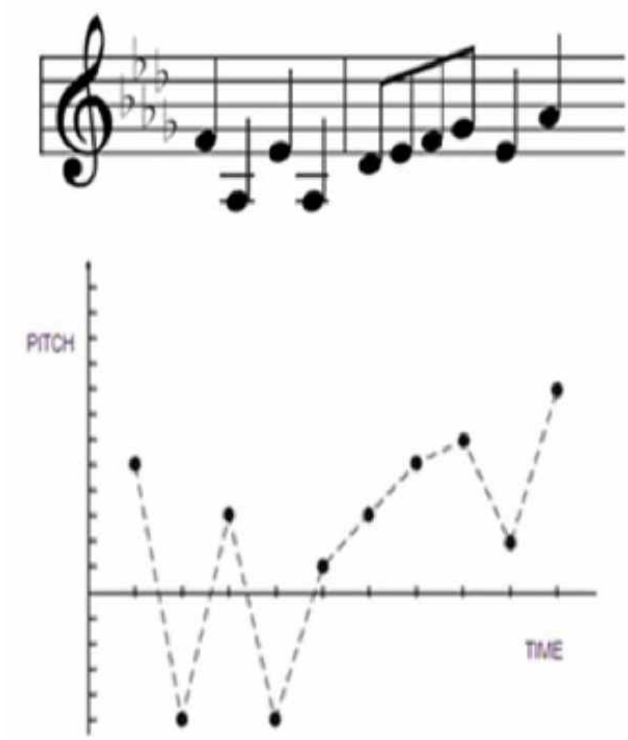


Figure 2. The musical staff and a pitch-time graph of a portion of 'The Music of the Night'

A rotation turns, or rotates, an image around an axis. If rotated 90° clockwise, musically this refers to a melody being changed into a chord, or vice versa. This 90° done counterclockwise is a simultaneity, or verticalization. Done clockwise, the 90° rotation is known as arpeggiation, or horizontalization.

A reflection turns an image over, whether up, down, left, or right; this is done over a line of reflection and the position of the line of reflection dictates the direction of the reflection. In music, a vertical reflection is referred to as inversion while a horizontal reflection is called retrograde (Madden, 1999). Figure 3 is a portion of a Bach fugue score with different transformations. The first measure of the lowest staff is the melody and is used in two different manners. The last measure in the middle staff is the melody inverted; a rise in pitch in this measure corresponds to a fall in pitch in the melody. The notes contained in the top staff are also the melody inverted, however, the time is changed to a scale of two, making it last twice as long as the melody or middle sections (Santu, 1998).

Symmetry can also be addressed musically and geometrically using these transformations. Some musical motifs have implemented within them symmetry through certain transformations, meaning that a motif is repeated but with a transformation performed on it in order to make it different. Most musical compositions do not contain such repeated symmetric motifs, but those that do are interesting to investigate. A reflection over a vertical line as symmetry musically refers to a reversal of pitch only. Symmetry of a horizontal reflection means a musical motif in which only time has been reversed. And finally those with symmetry of a 180° rotation would be a change in pitch and time, but these are rare to find in musical motifs (Fauvel et al., 2003).

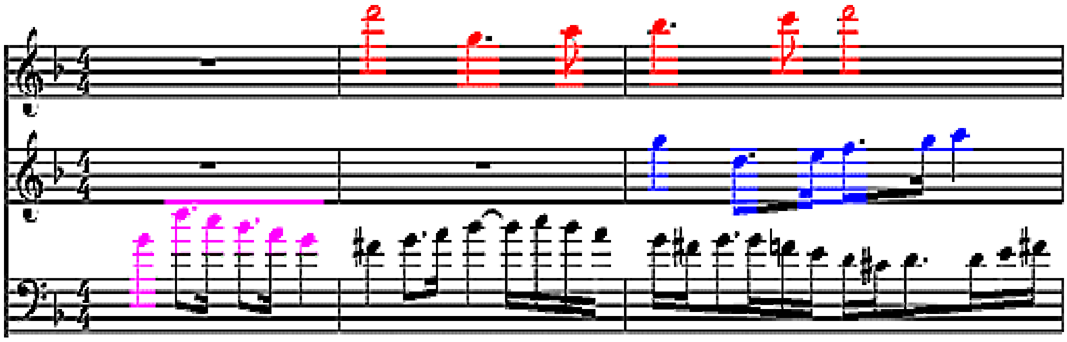


*Figure*3. Bach fugue score

2) Teaching Ideas

The relationship between geometry and musical space elicits many ideas for incorporation within the mathematics classroom. When introducing coordinate geometry, use a musical score as the information for plotted points. This can push the application of coordinates and their space. Once using music to plot on a coordinate grid, comparing certain musical styles can be a way for students to take a critical look at how they differ. Bringing in music samples to listen to, not just the music score itself, will give students a view of the graph they have in front of them in a more realistic way. Even trying to distinguish plotted graphs by their varying audio sounds can give opportunity for reasoning skills to be developed; making real world decisions using mathematics knowledge encourages students' interest in what math is capable of.

3) Musical Space & Geometry Lesson Plan

(1) Title: Space for Music

(2) Topic: Functions and Graphs-Coordinate plane and ordered pairs

(3) Prior Knowledge: Basic music notes

(4) Learning Goals:

- Ability to plot within a coordinate graph
- Relate musical staff to coordinate plane

(5) Materials: Musical Scales and Coordinate Graphs Worksheet

(6) Teaching and Learning Activities:

- Motivator: Draw or display the musical staff for the class to see. Use groups to have students discuss what they know about the image. Make a list of the class brainstorming to refer to later.

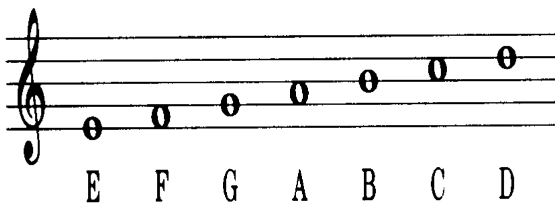
Figure 4. Musical staff

- Introduction: Discuss with the class how notes are placed within a musical staff: The vertical placement on the lines is based on the pitch of the notes, while the horizontal placement relates to when it is supposed to be played or sung, that is the timing of the note. Work through plotting notes from a musical staff to the coordinate plane with the class.
- Development: Pass out the Musical Scales and Coordinate Graphs (cf. Figure5) worksheet to groups. Students will have different sets of notes to plot. Be sure each student at a table has a different set of notes. Allow students time to work on their own, then the groups will discuss the differences and similarities of their sets of notes and the way they are plotted.
- Closure: Display a longer piece of music to the class. Have students plot this musical piece on their own drawn coordinate graph. This can be turned in as a way to check in on their understanding of this connection and how to plot on a graph.
- Extensions, if time: As this lesson focuses on notes being the same length, or note value, have students decide how to plot a piece of music when the notes are of varying lengths.

Musical Scales and Coordinate Graphs Worksheet

Below is a scale showing seven notes and a chart of two sets of notes. Place these notes on the two musical scales provided then plot them on the coordinate graphs.

First Set	A	C	D	B	G	E
Second Set	B	G	E	C	A	F



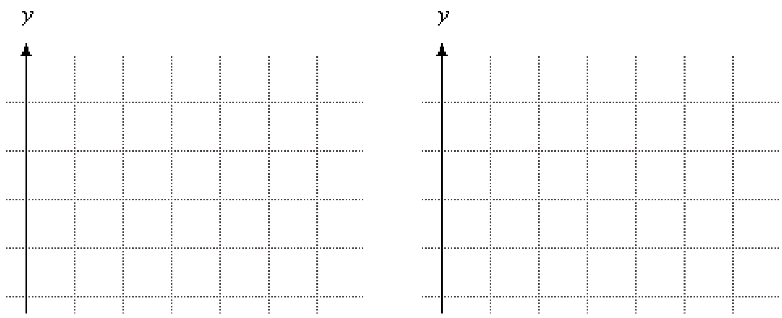


*Figure*5. Musical Scales and Coordinate Graphs Worksheet

2. Frequency and Statistics

1) Connections

Statistical measures can be used to compare musical styles. Certain statistical measures are able to describe a musical piece in more detail than can be seen on the musical staff. In terms of the pitches of the piece, range, midpoint, mean, variance and standard deviation can all be applied to find out more about a musical composition.

The range is the difference between the largest and smallest values; the range of pitches present in a composition can be helpful to know musically. The midpoint is simply one-half the range, and this can reveal where the range lies on the staff in relation to this midpoint pitch. The mean is a useful average found by adding together all the frequencies and dividing this sum by the number of pitches. The variance is a measure of the dispersion of the pitches and the standard deviation is simply a standard unit of dispersion used, so these two statistical measures inform how near or far from the mean pitch the other ones lie (Madden, 1999).

2) Teaching Ideas

Statistics can be developed through any set of data, but the integration of music can create a more meaningful experience for students. Allow students to listen to three to five short pieces, or parts of pieces, of music and analyze them statistically. Drawing conclusions about the music based on these statistics develops reasoning skills using mathematics. Encouraging students to analyze the music they prefer to listen to can have them discover whether there are statistical consistencies in their preferred genre of music.

3) Frequency and Statistics Lesson Plan

(1) Title: Musical Statistics
(2) Topic: Probability and Statistic: Frequency tables and graphs
(3) Prior Knowledge: Reading Music
(4) Learning Goals:
- Calculate statistics including range, midpoint
- Make decisions based on statistics

Materials: Many pieces of music, chart of pitch frequencies

(5) Teaching and Learning Activities:
- Motivator: Play a piece of music and discuss the characteristics of the piece such as the pitch of the song, the range of pitches, etc. Ask students to then compare that piece to another piece which is different in many ways; pitch, loudness, expanse of notes, etc.
- Introduction: Present the ideas of range and midpoint to the class. Discuss the frequency of the notes in music so the students know what they are working with when they are using numbers for notes (such as 220 Hz for middle A). Do a few examples with the class showing how to calculate the range and midpoint of the music selection.
- Development: Allow small groups of students to pick out 3-5 pieces of music to statistically analyze. Have them record the highest, lowest, and middle pitch (use chart of pitch frequency) as well as the range.

- Closure: Play a musical selection then have students choose between three options of range; why is one choice more reasonable based on what they heard than the others. Brainstorm why this statistical information about the music selections could be useful to singers or instrumentalists, or others in the music industry.
- Extensions: Students can pick out some of their favorite music to analyze and see if there is a consistency in the statistics of the type of music they like.

3. Musical Proportions and the Golden Section

1) Connections

The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 377, 610, 987, 1597, …) is a set of numbers in which each new term is the sum of the two values immediately preceding it. For example, 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8. This series was invented by a man by the name of Leonard Pisano Fibonacci as an answer to a problem he posed involving rabbits in his book Liber Abaci (Loy, 2007). There are various organic examples of the Fibonacci numbers such as flower pedals, plant branches, and pine cone scales; the number present in each of these examples is a Fibonacci number (Madden, 1999). This is only one interesting thing about the Fibonacci sequence of numbers. There are many others but in terms of musical relevancy, looking at the ratios of the successive terms reveals an intriguing number.

The ratios of the Fibonacci terms

$$\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\frac{13}{8},\ldots$$

in decimal notation provide a view of the convergence of these ratios: 1,2, 1.5, 1.67, 1.6, 1.625, 1.619, 1.617, 1.618, …The ratios of adjacent Fibonacci numbers converge to the value referred to as phi: ϕ= 1.618. The reciprocal of phi, 0.618, has come to be known as the golden section.

While this number ϕ, and the golden section also appear in natural designs, they have appeared, both deliberately and not, in musical compositions. It is common to find musical patterns change near ratios of phi. The golden section can be found in musical composition from motifs to full pieces and especially in the works of Beethoven, Mozart, and Webern. For example(Figure 6), the first movement of Mozart's Sonata No. 1 in C Major is 100 measures divided into two parts. The exposition section is 38 measures while the development and recapitulation section is 62 measures. The ratio of these two sections, 38/62 (0.613), is as close to the golden section (0.618) as can be achieved with only 100 measures (May, 1998).

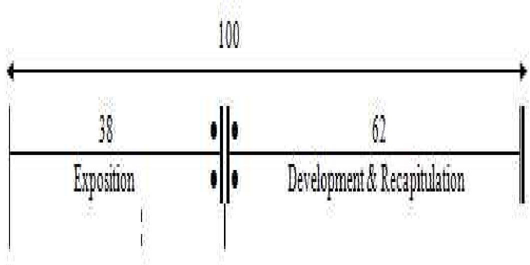


Figure 6.Measures in the Mozart's Sonata No. 1 in C Major

Whether or not these composers deliberately used the golden ratio to dictate their compositions is still unclear, but there are some pieces in which the golden mean is used so frequently that it is to be assumed that it was used knowingly (Loy, 2007).

As a result of the way in which the musical analysis was done, the principle of the golden ratio in music has been doubted. At times, certain parts of pieces were ignored in order to make the ratio work out or other times such a large error range was allowed which led to the presence of the golden section. The needed level of precision was not set up during these analyses to ensure the accurate conclusion as to the use or non use of the golden mean in musical compositions. Now with computer models, a higher level of accuracy is being used by analysts and we can accept that a few composers did decisively engage the golden section in their compositions because their numbers come out exact (Madden, 1999).

2) Teaching Ideas

Objects in nature encompassing Fibonacci numbers are typically the first way to show its existence beyond the theoretical. The proportioning in musical composition using the Fibonacci numbers and the golden section can be an added resource to use in the classroom

3) Musical Proportions and the Golden Section Project Based Assignment

(1) Topic: Number and Operations- Real numbers and radicals
(2) Task: In groups, allow students to investigate for themselves the existence of the golden section within musical compositions. Either gather music selections for them to choose from or give them an opportunity to use the library and internet to find their own pieces. Music departments may even have an extensive collection of older classical pieces. Students should find three to five musical pieces, or parts of pieces, and analyze them with a critical eye towards the golden section. Look at sections/ measures but also push beyond this to look at the notes and lengths of notes used

within the piece of music.

(3) Assessment: Student groups will present their findings to the class. Have them compare their own findings to those of the other groups; Did they use the same musical selections but have different conclusion or was their selection of the types of musical pieces different than others which created such diverse results?

4. Intervals and Ratios

1) Connections

While frequency gives us the ability to discuss relationships between sounds, when trying to compare two notes, it is not the frequency which is important but the ratio of the two frequencies that are imperative (Fauvel et al., 2003). An interval is the difference in pitch between two notes. It is our ears' perception to intervals that creates the basis for melody and harmony. If a note's frequency is f, then a note with frequency $2f$ is one octave higher. The note with frequency 2^2f is two octaves higher than the original note. It is the octave which is the foundation of any musical scale and to which our ears are the most attune as they are considered to be musical equivalent (Loy, 2007, p. 14).

On the piano the octave sequence, in Hertz frequencies, of the note A are 27.5, 55, 110, 220, 440, 880, 1760, 3520.

Each octave up doubles the frequency while each octave down halves the frequency. All of these frequencies are powers of two as the ratios of each octave to any fundamental note (here it was A, but it could be applied to any note) are

2:1, 4:1, 8:1, 16:1, 32:1, 64:1, 128:1, 256:1

or expressed as powers of 2,

2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, & 2^8 (Madden, 1999).

An interval of the frequencies of two notes is similar to making a ratio of these frequencies. Just as in mathematics, ratios are preferred to be represented in lowest terms, or with the lowest common denominator, in order to reduce the complexity of the numbers involved. In the musical sense, there is no need to see the actual frequencies which are involved in an interval; it is the interval itself which is important.

While there are many different intervals in music, there are some specific ones that are more pleasing to the ear, or consonant. These would be those intervals that are ratios of low integers; the simpler the ratio, the more consonant the two notes (Fauvel et al., 2003). Those ratios with higher integers create more complex ratios and thus more dissonant sound, which is rough to the ears (Benson, 2007). The simplest ratio is 1:1, or unison, and the next simplest is one octave up, discussed above, with the ratio 2:1 (Fauvel et al., 2003). An octave down has the ratio of 1:2. These few may be considered trivial as they are the basic ratios which are the basis for the development of scales, but that's exactly why they are important. These simple ratios continue and reveal the harmonic series (Capleton, 2006). The intervals with the simplest ratios of the harmonic series are actually called perfect (Rogers, 2004), including the perfect fifth (3:2) and perfect fourth (4:3).

2) Teaching Ideas

Exponential growth can be investigated in the math classroom by looking at octaves of notes and their corresponding pitches. Using a piano or keyboard in the classroom to give students a chance to hear harmonious sounds and how their foundation in ratios provides such pleasure to the ears. Musical pieces can also be investigated by their intervals; Using coordinate geometry to plot the interval ratios within a piece of music gives another way for students to see music mathematical. Compare the music created based on their graph of interval ratios.

3) Intervals & Ratios Lesson Plan

(1) Title: Sounding off on Ratios
(2) Topic: Functions- Functions and graphs
(3) Prior Knowledge: Ratios/Fractions
(4) Learning Goals:

- Develop an understanding of consonance and dissonance
- Knowledge of musical intervals and their relationship to mathematical ratios

(4) Materials: Keyboard, chart of pitch frequencies
(5) Teaching and Learning Activities:

- Motivator: Using the keyboard, play some intervals for students to listen to which both sound pleasing to the ears and ones which do not.
- Introduction: Ask if the students if they know much about intervals. Brainstorm why they think some sound dissonant while others sound nice when played. Is it the distance between the two notes or frequencies? Could it be the ratio of the two frequencies?
- Development: Students will record the notes they hear (stated before the interval) and whether they feel it is *consonant* or *dissonant*. In order to get enough examples for students to work with, be sure to cover the basic intervals including octave, unison, perfect fifth, perfect fourth, major third, etc., as well as dissonant ratios.
- Given a chart of note frequencies, students will calculate the ratios of the presented intervals. Working with their peers at their tables, students will try and deduce what made some intervals sound pleasant and why others do not.
- Closure: After the groups have finished discussing their ideas, have each group share what they noticed when calculating these intervals into ratios. Develop as a class the reason behind consonance and dissonance based on intervals.
- Extensions: Develop an understanding that these ratios are harmonic numbers, or ratios created from the harmonic series.

IV. IMPLICATIONS FOR THE MATHEMATICS CLASSROOM

Literature associated with mathematics and music provides a wealth of knowledge regarding the connections between these two fields. The extensive examples, complex range of interactions, as well as how each field has allowed for deeper investigation into the other gives a more thorough understanding of the organic relationship between math and music. However, the literature stops short of educational uses. When looked at with an educational perspective, this wealth of knowledge has the potential to further contextual educational opportunities for students. Music in the math classroom means more creative options for instruction and teaching strategies. A deeper awareness of mathematical realities and an appreciation for musical complexity can be achieved by integrating the connections between math and music.

These several connections discussed between the fields of mathematics and music are ones which allow the "average" student or individual to appreciate their elegant intertwinement. All of the mathematical topics covered are ones that are a part of the middle school mathematics curriculum while the musical content may not be previous knowledge for the students. This could be an opportunity to allow students to learn from one another by pairing or grouping those students with some background of musical concepts with those new to the ideas. Integrating this related content in the classroom could also give some students a chance to recognize a new context for their mathematical knowledge which in turn could motivate their success. However these connections between mathematics and music are implemented in the classroom setting, it is just the beginning of a great new way for students to learn mathematics in context.

Integrating music into the math classroom can be complicated, especially if a more meaningful incorporation is desired. Keep in mind that not all students will have the same level of musical background which can create complications for integration but all students will have heard music previously. This can be a jumping off point of music and math collaboration in the classroom. Giving those students who do have a more sophisticated knowledge of musical ideas an opportunity to assist their peers who do not can develop their self-esteem and sense of leadership. In order to make music content relevant in a mathematics classroom, the teacher must take the time to thoroughly understand the musical content as it relates to the math content being covered. This may take extra time depending on previous exposure to musical theory. Remember that students will not only be learning math, but increasing their awareness and understanding of music as well.

As mentioned in the introduction, both music and mathematics fuse the intellectual and the aesthetic beautifully, but musicians are seldom aware of the aesthetic features of mathematics while mathematicians often do not realize the intellectual characteristics of music. Integrating these two fields educationally can provide a wider base of understanding between these two fields

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