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학습과 망각에 대한 작업자들의 이질성 정도가 시스템 생산성에 미치는 영향

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The Effect of Worker Heterogeneity in Learning and Forgetting on System Productivity

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■ Abstract ■

Incorporation of individual learning and forgetting behaviors within worker-task assignment models produces a mixed integer nonlinear program (MINLP) problem, which is difficult to solve as a NP hard due to its nonlinearity in the objective function. Previous studies commonly assume homogeneity among workers in workforce scheduling that takes account of learning and forgetting characteristics. This paper expands previous researches by considering heterogeneous individual learning/forgetting, and investigates the impact of worker heterogeneity in initial expertise, steady-state productivity, learning and forgetting on system performance to assist manager's decision-making in worker-task assignments without tackling complex MINLP models. In order to understand the performance implications of workforce heterogeneity, this paper examines analytically how heterogeneity in each of the four parameters of the exponential learning and forgetting (L/F) model affects system performance in three cases : consecutive assignments with no break, *n* breaks of s-length each, and total b break-periods occurred over *T* periods. The study presents the direction of change in worker performance under different assignment schedules as the variance in initial expertise, steady-state productivity, learning or forgetting increases. Thus, it implies whether having more heterogenous workforce in terms of each of four parameters in the L/F model is desired or not in different schedules from the perspective of system productivity measurement.

Keywords : Decision-Making, Worker Heterogeneity, Learning, Forgetting, Productivity, Multi-Skilled Worker, Flexibility

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1. Introduction

Intensified global competition has led to shorter product life cycles while providing high-quality products and services, forcing organizations to seek improvements in the efficiency and developments in the flexibility of the processes. As the processes need to make appropriate adjustments as quickly as possible in response to rapidly changing environments, workforce agility becomes an important part of the current production systems, wherein workers are facing the challenges that they should frequently learn and master new tasks. Prior studies suggest that cross-training increases workforce flexibility [5, 6, 11, 23], so it is essential to consider worker learning and forgetting from the productivity perspective when hiring workers and assigning them to tasks [2, 3, 22].

Workers learn and forget based on how often and when they perform tasks. Learning and forgetting characteristics have been studied in various fields including psychology (e.g. [10]) and engineering (e.g. [24]) over the past decades. Individual learning and forgetting (L/F) models consist of various individual-based parameters such as previous expertise, steady-state productivity rate, and learning and forgetting rates.

Previous studies commonly assume homogeneity among workers when learning and forgetting are incorporated; however, Shafer et al. [19] and Buzacott [1] claim the importance of heterogeneity of workers when measuring productivity. They suggest that the heterogeneity in the workforce is critical in a scheduling decision of workers to tasks. Shafer et al. [19] also show that consideration of the central tendency not the variation among workers would lead to underestimation of the overall productivity of a system, in which workers perform tasks independently. Buzacott [1] examines the impact of worker differences on production system output. Nembhard and Shafer [17] investigate the magnitude of the effects of heterogeneity in each of the four parameters of Hyperbolic Recency Learning and Forgetting (L/F) model on productivity both analytically and using simulation, which is based on empirical data employed in the study by Shafer et al. [19]. Nembhard and Shafer [17] measure the impact of workforce heterogeneity in parallel systems. Kim and Nembhard [8] study optimal cross-training levels with heterogeneous learning and forgetting in parallel systems. Neidigh and Harrison [12] investigate the impact of various production rates caused by different learning rates on lot sizing in a multi-product single-machine system. Kim and Nembhard [9] present an association rule mining based framework for cross-trained worker assignments in parallel systems to assist managers with robust real-time assignment decisions in presence of heterogeneous learning and forgetting behaviors.

Scheduling a group of workers to a set of parallel jobs can be modeled as a mixed integer program (MIP) assignment problem when the objective is to maximize production if the performance of each worker and task pair is constant. However, the existence of learning and forgetting effects leads to dynamic changes of productivity of an individual worker to a specific task through time, and the productivity of each worker and task pair could be different with a heterogeneous workforce even if everything is the same. Consideration of worker learning and forgetting means the additional complexity of non-linear, perhaps non-deterministic, productivity of each worker in the assignment problem. Thus, incorporation of individual learning and forgetting behaviors within worker-task assignment models produces a mixed integer nonlinear program (MINLP) problem, which is difficult to solve as a NP hard due to its nonlinearity in the objective function. Many of learning and forgetting models are not intended for use within a math programming context due to their complexity [14]. This study aims to analytically investigate the impact of worker heterogeneity in learning and forgetting on system performance without solving the MINLP problems when the exponential learning and forgetting model is adapted.

The remainder of this paper is organized as follows. Section 2 presents the exponential learning and forgetting model. Section 3 is devoted to a presentation of analytical approaches to evaluate the effects of worker heterogeneity on system throughput. And Section 4 concludes with discussion and future research.

2. Exponential Learning and Forgetting Model

Teyarachakulet et al. [20] propose some basic properties of forgetting functions. They suggest that forgetting is an incremental function of the length and the number of the interruptions. Nembhard and Osothsilp [15] present an empirical comparison of several learning and forgetting models. Several papers describe how the data for L/F models were collected in real assembly lines [e.g., 15, 19]. This study employs the exponential learning-forgetting (L/F) model proposed by Thomas and Nembhard [21]. Although detailed explanation of the model is illustrated in Appendix A of Kim and Nembhard [8], this section briefly describes the model one more time for better explanation of the analytical approaches the next section discusses.

2.1 Parameters

The exponential L/F model consists of four parameters, l_{ij}, K_{ij}, L_{ij}, and F_{ij}.

l_{ij} : The initial expertise rate for worker i on task j

K_{ij} : The steady-state productivity for worker i on task j

L_{ij} : The learning rate for worker i on task j

F_{ij} : The forgetting rate for worker i on task j

where

i : workers available to perform the tasks, $i = 1, 2, \cdots, I$

j : tasks (or jobs or work stations) available to be operated, $j = 1, 2, \cdots, J$

t : time periods, $t = 1, 2, \cdots, T$

2.2 Variable

x_{ijt} : The binary variable indicating worker i is assigned to task j in time t

2.3 Model

As presented in equation (1), Q_{ijt} represents a measure of the productivity rate corresponding to worker i on task j during time period t.

Q_{ijt} = the productivity rate for worker i to tast j during time period t

$$= l_{ij} + K_{ij}\left[1-\exp\left(-\frac{1}{L_{ij}}\sum_{k=1}^{t} x_{ijk}\right)\right]$$

$$\exp\left[\frac{1}{F_{ij}}\left(\sum_{k=1}^{t} x_{ijk} - t + \widetilde{t_{ij}}\right)\right] \quad (1)$$

In the equation, $\tilde{t}_{ij}$ indicates the number of periods unassigned before an initial assignment occurred for worker i on task j, and the binary variable x_{ijt} is a decision variable representing worker-task assignment. The productivity function Q_{ijt} is nonlinear and non-convex, and it can be simplified as equation (2) as follows :

$$Q(t) = l + K\left[1 - \exp\left(-\frac{1}{L}\sum_{k=1}^{t} x_k\right)\right] \exp\left[\frac{1}{F}\left(\sum_{k=1}^{t} x_k - t + \tilde{t}\right)\right] \tag{2}$$

As suggested in (2), the exponential L/F model has one weakness that it does count the cumulative number of periods for learning and forgetting from time 0 to t to determine productivity at time t not when learning and forgetting have occurred throughout t in the concept of intermittent learning and forgetting, also called as the learn-forget-relearn cycle. The model is developed, in which, forgetting is modeled similarly to the learning curve that it has a convex shape and is a function of the break length and the performance time. However, it has a comparatively simple mathematical form to represent individual L/F characteristics effectively compared to a number of other more elaborated learning and forgetting models in the literature which have more complex forms such as the S-Shaped model [7] or the hyperbolic recency L/F model [13, 18, 16]. The S-shaped model developed by Globerson and Levin [7] has the S-shaped forgetting function to better capture the forgetting phenomenon after the break, but it is more complex than the exponential model. The hyperbolic recency model which was introduced by Nembhard and Uzumeri [13] performs consistently well to capture individual L/F behaviors in terms of several model performance criteria including efficiency, stability and balance [15], but it is also too complex to be examined analytically, so it probably be used in research based on simulation not analytical approaches. Thus, this paper employs the exponential L/F model to evaluate the impact of the level of heterogeneity within workforce on productivity in terms of the direction of changes rather than the degree of changes. This study analytically examines how the variance of each parameter of the model affects productivity for three management cases such as "case 1 : consecutive assignments with no break", "case 2 : n breaks with s-length each", and "case 3 : Total b break-periods occurred over T periods". Because the exponential model has a shortcoming stated above, the case 2 could be collapsed to the case 3 as the special case where each of n breaks has the same length, s. However, this paper differentiates these cases by intention to guide a potential work that two cases would be different with models considering timing of the break.

3. Analytical Approaches to Evaluate the Effects of Worker Heterogeneity on Performance

3.1 Case 1 : Consecutive Assignments with No Break

In this case, there is no break occurred over the time periods of T. A worker has been consecutively assigned to the specific task and forgetting has not been occurred as shown in (3) :

$$Q(t) = l + K\left[1 - \exp\left(-\frac{1}{L}t\right)\right] \tag{3}$$

The production output (PO; i.e., the system throughput) over the time horizon of $T(t=0 \rightarrow T)$ is derived by integrating (3) as shown in (4) :

$$PO(T) = \int_0^T Q(t)dt = \int_0^T \left\{ l + K\left[1-\exp\left(-\frac{1}{L}t\right)\right]\right\}dt$$
$$= (l+K)T + K \cdot L\left[\exp\left(-\frac{T}{L}\right)-1\right] \quad (4)$$

The effects of heterogeneity (i.e., variance) in each of the four parameters of the exponential L/F model on production $PO(t)$ are examined in the following subsections.

3.1.1 Initial Expertise l

Since $PO(t)$ is linear with respect to individual's initial expertise l, by Jensen's inequality we obtain $\overline{PO(T|l)} = PO(T|\bar{l})$ which represents that the average production through time T with given l, $\overline{PO(T|l)}$, is equal to production through time T when the mean initial expertise is given, $PO(T|\bar{l})$. Therefore, increasing the variance in l while the other parameters are being constant does not affect production.

3.1.2 Steady-state Productivity K

Since $PO(T)$ is linear with respect to individual's steady-state productivity rate K, applying Jensen's inequality results in $\overline{PO(T|K)} = PO(T|\overline{K})$, which suggests that the average production through time T with given K, $\overline{PO(T|K)}$, is the same as production through time T with given the mean steady-state productivity rate ($\overline{K}$), $PO(T|\overline{K})$. Therefore, increasing the variance in K while the other parameters are maintained as constant does not have any impact on production.

3.1.3 Learning Rate L

Taking the second partial derivative of equation (4) with respect to individual's learning rate L results in (5) :

$$\frac{\partial^2 PO(T)}{\partial L^2} = \frac{KT^2}{L^3} exp\left(-\frac{T}{L}\right) \quad (5)$$

Since L cannot be nonpositive in the exponential L/F model, equation (5) is positive for T and K, where both are also positive (Nembhard [13] observes some negative learning with a hyperbolic recency L/F model). Thus, $PO(T)$ is concave up (convex) with respect to L. By Jensen's inequality, we get the following result : $\overline{PO(T|L)} \geq PO(T|\overline{L})$, which indicates increasing the variance in the learning rate (L) while the other parameters are being constant is expected to have the tendency to *increase* the production output. Thus, having more heterogeneous workforce in terms of learning would increase the system productivity.

3.2 Case 2 : n Breaks with s-Length Each

In this case, it assumes that n breaks have been occurred until time t and each break lasts for the same duration of s periods where $n > 0$, $s > 0$ and $t - ns \geq 0$. Once the break occurs, forgetting also takes place on individual expertise so it lowers worker's productivity level. Thus, the numbers of time periods for learning and forgetting for time t are equal to $\Sigma_{k=1}^{t} x_k = t - ns$ and ns, respectively. Here, $t-ns$ becomes zero if the worker has not performed any task at all, so learning does not occur for t. $Q(t)$ in equation (2) can be represented as (6) :

$$Q(t)_{n,s} = l + K\left[1-\exp\left(-\frac{1}{L}(t-ns)\right)\right]$$
$$\exp\left[\frac{1}{F}(-ns+\tilde{t})\right] \quad (6)$$

where $0 \le \tilde{t} \le t - \Sigma_{k=1}^{t} x_k = ns$. Thus, production through time T with n breaks of s-length each, $PO(T)_{n,s}$, is computed by integrating (6) as shown in (7) :

$$\begin{aligned} PO(T)_{n,s} &= \int_0^T Q(t)_{n,s} dt \\ &= \int_0^T \left\{ l + K\left[1 - \exp\left(-\frac{1}{L}(t-ns)\right)\right] \right. \\ &\quad \left. \exp\left[\frac{1}{F}(-ns+\tilde{t})\right] \right\} dt \\ &= lT + KL \exp\left(\frac{ns-T}{L} + \frac{\tilde{t}-ns}{F}\right) \\ &\quad \left[1 - \exp\left(\frac{T}{L}\right)\right] + KT \exp\left(\frac{\tilde{t}-ns}{F}\right) \end{aligned} \tag{7}$$

The effects of heterogeneity of each parameter on production over time t, $PO(T)_{n,s}$, are assessed in the following subsections.

3.2.1 Initial Expertise l

Since $PO(T)_{n,s}$ is linear with respect to individual's initial expertise l, applying Jensen's inequality leads to $\overline{PO(T|l)}_{n,s} = PO(T|\bar{l})_{n,s}$, which indicates that $\overline{PO(T|l)}_{n,s}$, the average production through time T with given l, is the same as $PO(T|\bar{l})_{n,s}$, production through time T with given the mean initial expertise $\bar{l}$. This relationship suggests that increasing the variance in individual's initial expertise while the other parameters are remained constant does not have any effect on the production level.

3.2.2 Steady-state Productivity K

Since $PO(T)_{n,s}$ is linear with respect to individual's steady-state productivity rate K, applying Jensen's inequality results in $\overline{PO(T|K)}_{n,s} = PO(T|\overline{K})_{n,s}$, which means that the mean production through time T with given K, $\overline{PO(T|K)}_{n,s}$, is equal to production through time T with given the mean steady-state productivity, $PO(T|\overline{K})_{n,s}$. Therefore, increasing the variance in the steady-state productivity while the other parameters are being constant does not affect production.

3.2.3 Learning Rate L

Taking the second partial derivative of (7) with respect to individual's learning rate L leads to (9) :

$$\begin{aligned} \frac{\partial^2 PO(T)_{n,s}}{\partial L^2} &= KT\left(\frac{T-2ns}{L^3}\right)\exp\left(\frac{ns}{L} + \frac{\tilde{t}-ns}{F}\right) \\ &\quad + K\left(\frac{(T-ns)^2}{L^3}\right)\left[1-\exp\left(\frac{T}{L}\right)\right] \\ &\quad \exp\left(\frac{ns-T}{L} + \frac{\tilde{t}-ns}{F}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} &= \frac{K}{L^3}\exp\left(\frac{ns}{L} + \frac{\tilde{t}-ns}{F}\right) \\ &\quad \left[(T-ns)^2 \exp\left(-\frac{T}{L}\right) - (ns)^2\right] \end{aligned} \tag{9}$$

Since $KT > 0$, $\exp\left(\frac{ns}{L} + \frac{\tilde{t}-ns}{F}\right) > 0 \left(\frac{(t-ns)^2}{L^3}\right) \ge 0$, $\left[-1+\exp\left(\frac{T}{L}\right)\right] > 0$, and $\exp\left(\frac{ns-T}{L} + \frac{\tilde{t}-ns}{F}\right) > 0$ for $\forall\, T > 0, K > 0, L > 0$, (8) becomes negative for all $T, K, L > 0$ if $T < 2ns$ (i.e., T is shorter than two breaks). Thus, $PO(T)_{n,s}$ is concave down (concave) with respect to L. The following relationship is obtained through Jensen's inequality : $\overline{PO(T|L)}_{n,s} \le PO(T|\overline{L})_{n,s}$, which indicates that increasing the variance in individual's learning rate while the other parameters are being constant is expected to have the tendency to *decrease* production when T is shorter than two breaks. However, if

$\exp\left(-\frac{T}{L}\right) > \left(\frac{ns}{T-ns}\right)^2$, i.e., $T < -2L\ln\frac{ns}{T-ns}$, equation (9) becomes positive for $T, K, L>0$ and $T \neq ns$. It turns out that $PO(T)_{ns}$ is concave up (convex) with respect to L. By Jensen's inequality, we get $\overline{PO(T|L)}_{ns} \geq PO(T|\overline{L})_{ns}$, suggesting that increasing the variance in the learning rate while the other parameters are constant is expected to have the tendency to *increase* production. Thus, having more heterogeneous workforce in terms of learning would increase system productivity.

3.2.4 Forgetting Rate F

After taking the second partial derivative of (7) with regard to individual's forgetting rate F, equation (10) is obtained as follows :

$$\begin{aligned}\frac{\partial^2 PO(T)_{ns}}{\partial F^2} &= \frac{KT}{F^3}\left(\frac{(\tilde{t}-ns)^2}{F}+2(\tilde{t}-ns)\right)\exp\left(\frac{\tilde{t}-ns}{F}\right)\\ &\quad -\frac{KL}{F^3}\left(\frac{(\tilde{t}-ns)^2}{F}+2(\tilde{t}-ns)\right)\\ &\quad \left(-1+\exp\left(\frac{T}{L}\right)\right)\exp\left(\frac{ns-T}{L}+\frac{\tilde{t}-ns}{F}\right)\\ &= \frac{K}{F^3}\left(\frac{(\tilde{t}-ns)^2}{F}+2(\tilde{t}-ns)\right)\exp\left(\frac{\tilde{t}-ns}{F}\right)\\ &\quad \left[L\left(1-\exp\left(\frac{T}{L}\right)\right)\exp\left(\frac{ns-T}{L}\right)+T\right]\\ &= \frac{K}{F^3}\left(\frac{(\tilde{t}-ns)^2}{F}+2(\tilde{t}-ns)\right)\exp\left(\frac{\tilde{t}-ns}{F}\right)\\ &\quad \left[L\cdot\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T\right] \qquad (10)\end{aligned}$$

Since $\frac{KT}{F^3}>0$, $\frac{KL}{F^3}>0$, $\frac{K}{F^3}>0$, $\exp\left(\frac{\tilde{t}-ns}{F}\right)>0$, $KL>0$, $\left[-1+\exp\left(\frac{T}{L}\right)\right]>0$, and $\exp\left(\frac{ns-T}{L}+\frac{\tilde{t}-ns}{F}\right)>0$ for $\forall T>0, K>0, L>0, F>0$, equation (10) is positive in the cases of A and B while the equation for the cases of C and D is negative :

- Case A : $\left(\frac{(\tilde{t}-ns)^2}{F}-2(\tilde{t}-ns)\right)$ is positive, i.e., $\tilde{t}-ns>2F$, and $L\cdot\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is positive, i.e., $\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)>-\frac{T}{L}$

- Case B : $\left(\frac{(\tilde{t}-ns)^2}{F}-2(\tilde{t}-ns)\right)$ is negative, i.e. $\tilde{t}-ns>2F$, and $L\cdot\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is negative, i.e., $\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)<-\frac{T}{L}$

- Case C : $\left(\frac{(\tilde{t}-ns)^2}{F}-2(\tilde{t}-ns)\right)$ is positive, i.e., $\tilde{t}-ns>2F$, and $L\cdot\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is negative, i.e., $\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)<-\frac{T}{L}$

- Case D : $\left(\frac{(\tilde{t}-ns)^2}{F}-2(\tilde{t}-ns)\right)$ is negative, i.e., $\tilde{t}-ns>2F$, and $L\cdot\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is positive, i.e., $\exp\left(\frac{ns}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)>-\frac{T}{L}$

Therefore, if the second derivative of $PO(T)_{ns}$ is positive, it is concave up (convex) with respect to F. By Jensen's inequality, we get the relationship of $\overline{PO(T|F)}_{ns} \geq PO(T|\overline{F})_{ns}$, which means that increasing the variance in F, while the other parameters are constant, is expected to have the tendency to *increase* overall production. However, if equation (10) is negative, $PO(T)_{ns}$ is concave down (concave) with respect to F. Thus, applying Jensen's inequality leads to ship of $\overline{PO(T|F)}_{ns} \leq PO(T|\overline{F})_{ns}$, which indicates that increasing the variance in the forgetting rate while the other parameters are being constant is expected to have the tendency to *decrease* overall production. Therefore, having more heterogeneous workforce in terms of forgetting would reduce system productivity.

3.3 Case 3 : Total b Break-Periods Occurred Over T Periods

When the total b periods for forgetting are assumed over time t with $t \geq b$ and $b \geq 0$, the total number of learning periods becomes $\Sigma_{k=1}^{t} x_k = t-b$. Here, $t-b$ is zero if the worker has not been assigned to any task for t so only forgetting not learning occurs throughout the time. Thus, $Q(t)$ in (2) can be represented as (11) as follows :

$$Q(t)_b = l + K\left[1-\exp\left(-\frac{1}{L}(t-b)\right)\right]\exp\left[\frac{1}{F}(-b)\right] \quad (11)$$

The total output throughout T, $PO(T)_b$, is derived to (13) through (12) by integrating (11) as shown below :

$$PO(T)_b = \int_0^T Q(t)_b dt$$
$$= \int_0^T \left\{ l + K\left[1-\exp\left(-\frac{1}{L}(t-b)\right)\right] \exp\left[\frac{1}{F}(-b)\right]\right\} dt \quad (12)$$
$$= lT - KL\exp\left(\frac{b-T}{L} - \frac{b}{F}\right)\left[-1+\exp\left(\frac{T}{L}\right)\right] + KT\exp\left(-\frac{b}{F}\right) \quad (13)$$

The effects of heterogeneity of each parameter on the total output over time t, $PO(t)_b$, are explored in the following subsections :

3.3.1 Initial Expertise l

Since $PO(t)_b$ is linear with respect to individual's initial expertise l, the following relationship is observed through Jensen's inequality : $\overline{PO(T|l)}_b = PO(T|\bar{l})_b$ where the mean production through T periods with given l, $\overline{PO(T|l)}_b$ is equal to production through T with given the mean initial expertise $\bar{l}$, $PO(T|\bar{l})_b$. Therefore, increasing the variance in the initial expertise (l) while the other parameters are being constant does not have any effect on the overall production level.

3.3.2 Steady-State Productivity K

Since $PO(T)_b$ is linear with respect to the steady-state productivity rate K, applying Jensen's inequality brings about $\overline{PO(T|K)}_b = PO(T|\overline{K})_b$, which indicates that the mean production through T periods with given K, $\overline{PO(T)}_b$, is equal to the production output through T given the mean steady-state productivity rate $\overline{K}$, $PO(T|\overline{K})_b$. Therefore, increasing the variance in the steady-state productivity rate while the other parameters are being constant does not have any impact on overall production.

3.3.3 Learning Rate L

After taking the second partial derivative of (13) with regard to individual's learning rate L, the following equation is observed :

$$\frac{\partial^2 PO(T)_b}{\partial L^2} = KT\left(\frac{T-2b}{L^3}\right)\exp\left(\frac{b}{L}-\frac{b}{F}\right)K\left(\frac{(T-b)^2}{L^3}\right)\left[-1+\exp\left(\frac{T}{L}\right)\right]\exp\left(\frac{b-T}{L}-\frac{b}{F}\right)$$
$$= \frac{K}{L^3}\exp\left(\frac{b}{L}-\frac{b}{F}\right)\left[(T-b)^2\exp\left(-\frac{T}{L}\right)-b^2\right] \quad (14)$$

Since $KT>0$, $\exp\left(\frac{b}{L}-\frac{b}{F}\right)>0$, $\left(\frac{(T-b)^2}{L^3}\right) \geq 0$, $\left[-1+\exp\left(\frac{T}{L}\right)\right]>0$, and $\exp\left(\frac{b-T}{L}-\frac{b}{F}\right)>0$ for $\forall T$

> 0, $K > 0$, $L > 0$, (14) is negative for all T, K, $L > 0$ if $T < 2b$. In other words, T is shorter than two times of break-periods. Hence, $PO(T)_b$ is concave down (concave) with respect to L. Through Jensen's inequality, $\overline{PO(T|L)}_b \le PO(T|\overline{L})_b$ is obtained, suggesting that increasing the variance in the learning rate (L) while the other parameters are being the same is expected to have the tendency to decrease overall production when T is shorter than two times of break-periods b. However, if $\exp\left(-\frac{T}{L}\right) > \left(\frac{b}{T-b}\right)^2$ (i.e., $T \leftarrow 2L \ln \frac{b}{T-b}$), equation (14) becomes positive for all T, K, $L > 0$ and $T \neq b$. Therefore, $PO(T)_b$ is concave down (concave) with respect to L. Applying Jensen's inequality results in $\overline{PO(T|L)}_b \ge PO(T|\overline{L})_b$, which implies that increasing the variance in L while the other parameters are constant is expected to have the tendency to increase overall production. Thus, having more heterogeneous workforce in terms of learning is desired to improve the system performance.

3.3.4 Forgetting Rate F

Taking the second partial derivative of (13) with respect to the forgetting rate F, it observes that

$$
\begin{aligned}
\frac{\partial^2 PO(T)_b}{\partial F^2} &= \frac{KT}{F^3}\left(\frac{b^2}{F}-2b\right)\exp\left(-\frac{b}{F}\right) - \frac{KL}{F^3}\left(\frac{b^2}{F}-2b\right) \\
&\quad \left(-1+\exp\left(\frac{T}{L}\right)\right)\exp\left(\frac{b-T}{L}-\frac{b}{F}\right) \\
&= \frac{K}{F^3}\left(\frac{b^2}{F}-2b\right)\exp\left(-\frac{b}{F}\right) \\
&\quad \left[L\left(1-\exp\left(\frac{T}{L}\right)\right)\exp\left(\frac{b-T}{L}\right)+T\right] \\
&= \frac{K}{F^3}\left(\frac{b^2}{F}-2b\right)\exp\left(-\frac{b}{F}\right) \\
&\quad \left[L\cdot\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T\right] \qquad (15)
\end{aligned}
$$

It is clear from this expression that $\frac{KT}{F^3} > 0$, $\frac{K}{F^3} > 0$, $\exp\left(-\frac{b}{F}\right) > 0$, $KL > 0$, $\left[-1+\exp\left(\frac{T}{L}\right)\right] > 0$, and $\exp\left(\frac{b-T}{L}-\frac{b}{F}\right) > 0$ for $\forall T > 0, K > 0$, $L > 0$. $F > 0$. Thus, equation (15) is positive in the cases of E and F while the equation is negative in the cases of G and H for all T, K, L, $F > 0$ as shown below :

- Case E : $\left(\frac{b^2}{F}-2b\right)$ is positive, i.e., $b > 2F$, and $L\cdot\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is positive, i.e., $\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right) > -\frac{T}{L}$

- Case F : $\left(\frac{b^2}{F}-2b\right)$ is negative, i.e., $b < 2F$, and $L\cdot\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is negative, i.e., $\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right) > -\frac{T}{L}$

- Case G : $\left(\frac{b^2}{F}-2b\right)$ is positive, i.e., $b > 2F$, and $L\cdot\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is negative, i.e., $\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right) < -\frac{T}{L}$

- Case H : $\left(\frac{b^2}{F}-2b\right)$ is negative, i.e., $b < 2F$, and $L\cdot\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right)+T$ is positive, i.e., $\exp\left(\frac{b}{L}\right)\left(\exp\left(-\frac{T}{L}\right)-1\right) > -\frac{T}{L}$

Therefore, if the second derivative of $PO(T)$ is positive, it is concave up (convex) with respect to F. By Jensen's inequality, it comes up with $\overline{PO(T|F)}_b \ge PO(T|\overline{F})_b$, which indicates that increasing the variance in the forgetting rate while the other parameters are being constant, is expected to have the tendency to increase overall production. However, if (15) is negative, $PO(T)$ becomes concave down (concave) with respect to F. Applying Jensen's inequality leads to $\overline{PO(T|F)}_b \le PO(T|\overline{F})_b$, which implies that increasing the variance in the forgetting rate while the other parameters are constant, is expected to have the tendency to decrease overall production. Therefore, more heterogeneous workforce in terms of forgetting would lower system productivity.

4. Conclusions and Future Studies

Worker productivity and experience levels have been recognized as important factors in a wide range of managerial topics including call center operations or assembly line layout decisions. Shafer et al. [19] and Buzacott [1] show that individual differences and workforce heterogeneity should be considered when measuring productivity. In the course of addressing this specific issue, several research questions are emerged such as what kinds of effects can be expected from a more diverse workforce versus a more homogeneous workforce in terms of performance and experience levels, and more specifically, what factors in a L/F model have more significant impacts on productivity or system outputs. In order to understand the performance implications of workforce, this paper examines analytically how heterogeneity in each of the four parameters of the exponential L/F model affects system performance in three cases: consecutive assignments with no break, n breaks of s-length each, and total b break-periods occurred over T periods. This paper aims to assist manager's decision-making in worker-task assignments by providing more knowledge on the impact of the heterogeneity of workforce on producivity. The study presents the direction of change in worker performance under different assignment schedules as the variance in initial expertise, steady-state productivity, learning or forgetting increases. Thus, it implies whether having more heterogenous workforce in terms of each of four parameters in the L/F model is desired or not in different schedules from the perspective of system productivity measurement.

This research can be extended in several ways. It would be needed to validate our analytical results through the simulation models informed by the empirical data. It would be worthwhile to investigate what circumstances it is reasonable to use a more homogeneous workforce versus a more heterogeneous workforce. For example, what kind of impact does the heterogeneity of workforce with respect to the productivity level, learning/forgetting behaviors, and skill sets have when demand or product types are uncertain. Another research is to further investigate the cases in which the number of breaks and their patterns follow a certain stochastic process such as a Poisson Process. In a sense, it could be the current research approach that allow us analytically to identify the most significant driver of productivity among worker parameters in terms of the level of its heterogeneity as such could lead to providing more accurate cost and productivity estimates and implementing process improvements.

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