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A new model based on Lomax distribution

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Abstract. In this article, a new model based on Lomax distribution is introduced. This new model is both useful and practical in areas such as economic, reliability and life testing. Some statistical properties of this model are presented including moments, hazard rate, reversed hazard rate, mean residual life and mean inactivity time functions, among others. It is also shown that the distributions of the new model are ordered with respect to the strongest likelihood ratio ordering. The method of moment and maximum likelihood estimation are used to estimates the unknown parameters. Simulation is utilized to calculate the unknown shape parameter and to study its properties. Finally, to illustrate the concepts, the appropriateness of the new model for real data sets are included.

Key Words: *Hazard rate, maximum likelihood estimates, mean inactivity time, mean residual life, moment estimation, reversed hazard rate, stochastic order*

1. INTRODUCTION AND MOTIVATION

Numerous classical distributions have been used extensively over the past decades for modeling data in areas such as engineering, actuarial, environmental and medical sciences, biological studies, demography, economics, finance and insurance. Of these, the Lomax distribution (LD) is a widely used model event that occurs in fields such as reliability, actuarial science, queuing problems and biological sciences. A random variable X is said to have Lomax distribution if its probability distribution function (pdf) is

$$g(x, \alpha, \lambda) = \frac{\lambda\alpha}{(1+\lambda x)^{\alpha+1}}, \qquad x \geq 0, \lambda, \alpha > 0$$

where λ is the scale parameter and α is the shape parameter. The LD has been used in the literature in a number of ways. For example, it has been extensively used for reliability modeling and life testing (Balkema and de Haan (1974)). It also has been used as an alternative to the exponential distribution when data are heavy tailed (Bryson (1974)).

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Ahsanullah (1991) studied the record values of LD, and Balakrishnan and Ahsanullah (1994) introduced some recurrent relationships between the moments of record values from LD. The order statistics from non-identical right-truncated Lomax
random variables were studied by Childs et al. (2001). Many authors (Arnold et al. (1998), El-Din et al. (2013)) have studied LD from a Bayesian perspective. Howlader and Hossain (2002) presented Bayesian estimation of the survival function of the LD. Ghitany et al. (2007) extended LD via the Marshall−Olkin approach. Cramer and Schmiedt (2011) considered data on progressively type-II censored competing risks from Lomax distribution. The LD also has several applications in economics, actuarial modeling, queuing problems and biological sciences (Johnson et al. (1994)).
In many applied areas such as lifetime analysis, finance and insurance, there is a clear need for extended form of LD distribution. One and well-known method to extend the distributions is the length-biased (LB) distributions. Formally, if Y is a lifetime random variable (r.v) with probability distribution function (pdf) $f_Y(y)$, then the weighted version of Y with weight function $w(y)$, which we denote by the r.v Y_w and whose distribution is called the weighted distribution, has pdf given by

$$f_Y(y) = \frac{w(y)f_Y(y)}{E[w(y)]},\ y > 0 \tag{1.1}$$

by assuming the first moment of $w(Y)$ exists. A particular case of the weighted distributions is obtained when we substitute $w(Y) = Y$ in Eq. (1.1). In this case, is called the size-biased or length-biased (LB) version of Y, denoted by the r.v. T, which has pdf expressed as

$$f_T(t) = \frac{tf_Y(t)}{\mu},\ t > 0$$

where $0 < \mu = E[Y] < \infty$.
LB distributions have been applied in various fields such as biometry, ecology, environmental sciences, reliability, and survival analysis. A review of these distributions and their applications is included in Gupta and Kirmani (1995). LB distribution occurs naturally in many situations, because sometimes it is not possible to work with a truly random sample from the population of interest. In particular, in the environmental field, Patil (2002) mentioned that observations might fall in non-experimental, non-replicated, and non-random categories, thereby making random selection from the target population impossible. Thus, in this case, model specification and data interpretation problems acquire great importance. One way of confronting this problem is by considering observations selected with probability proportional to their length. The resulting distribution is called an LB distribution, which adjusts the probabilities of the actual occurrence of events to arrive at a specification of the probabilities of those events as observed and recorded. Failure to make such adjustments can lead to invalid conclusions.
In this article, we propose a new model called length-biased Lomax (LBL) distribution. In Section 2, we introduce the new model and present some basic properties and characterizations. In Section 3, we carry out a survival analysis based on some reliability functions such as: hazard rate, reversed hazard, mean residual life, and mean inactivity time. In that section, we show that the new model is ordered with respect to the strongest likelihood ratio ordering. In Section 4, methods of moment estimates and maximum likelihood methods are used for estimating the shape parameter of the proposed model. In Section 5, we provide some applications with real data. Finally, in Section 6, we give a

brief conclusion and some remarks on current and future research.

2. THE NEW MODEL

Suppose that the lifetimes of a given sample of items follow a LD, and that the items do not have the same chance of being selected but that each one is selected according to its lifespan. Then, the resulting distribution does not follow the LD but follows the length-biased Lomax (LBL) distribution. LB versions for several distributions, such as Weibull, inverse Gaussian (IG), Sinhnormal (SN), and Birnbaum–Saunders(BS) distributions, have been developed in the literature (cf. Sansgiry and Akman (2001), Boudrissa and Shaban (2007), Leiva et al. (2009)). Following the same method, we propose a new model as follows.

Definition 2.1. A non-negative random variable T is said to have LBL distribution with scale parameter λ and shape parameter α if its probability density function is given by

$$f(t,\lambda,\alpha) = \frac{\lambda^2\alpha(\alpha-1)t}{(1+\lambda t)^{\alpha+1}},\ t \geq 0,\ \alpha > 1, \lambda > 0 \tag{2.1}$$

For this new model the notation $f(t,\lambda,\alpha)$ will be used.
Clearly, $f(t,\lambda,\alpha)$ vanishes as $t \to 0$ and $t \to \infty$. Taking the derivative of this function with respect to t yields

$$\frac{d}{dt}f(t,\lambda,\alpha) = \lambda^2\alpha(\alpha-1)t\left(\frac{1-\lambda\alpha t}{(1+\lambda t)^{\alpha+2}}\right)$$

$$\frac{d^2}{dt^2}f(t,\lambda,\alpha) = \lambda^3\alpha(\alpha-1)(\alpha+1)\left(\frac{\lambda\alpha t-2}{(1+\lambda t)^{\alpha+3}}\right)$$

which has a maximum at $t_0 = \frac{1}{\lambda\alpha}$. The shapes of the pdf for special values of the shape parameter α and λ = 1 are illustrated in Figure 2.1.

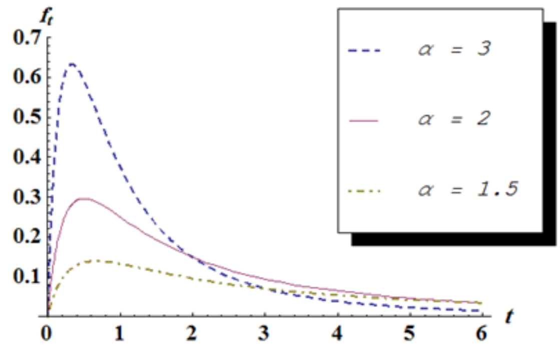

Figure 2.1. The pdf of LBL for different α and λ=1

The positive integer moments are useful for inference and model fitting (cf., Johnson et al. (1994) p. 23, Sanhueza et al. (2001)). The next result allows us to compute the moments of the LBL distribution.

Lemma 2.1. If $T \sim LBL(t;\lambda,\alpha)$, then the γ^{th} moment is given by

$$E(T^r) = \frac{\alpha(\alpha-1)}{\lambda^{\gamma}} \sum_{i=1}^{\gamma+1} \binom{r+1}{i} (-1)^{r+1-i} \left[\frac{1}{\alpha-i}\right], \quad \alpha > r+1$$

From the above result, the mean and variance are given respectively by

$$E(T) = \frac{2}{\lambda(\alpha-2)}, \quad \alpha > 2$$

and

$$Var(T) = \frac{2\alpha}{\lambda^2(\alpha-2)^2(\alpha-3)}, \quad \alpha > 3$$

The coefficient of skewness γ_1 of the LBL distribution is given by

$$\gamma_1 = \frac{12(\alpha-2)^3(\alpha-3)^{\frac{1}{2}}}{\sqrt{2}a^{\frac{3}{p}}(\alpha-4)}, \qquad \alpha > 4$$

which is independent of λ. Since λ_1 is positive, the distribution is skewed to the right, which is also clear from the plot of the pdf in Figure 1. The coefficient of kurtosis λ_2 of the LBL distribution isgiven by

$$\gamma_2 = \frac{27.5(\alpha-2)^3(\alpha-3)}{\alpha^2(\alpha-4)(\alpha-5)} - 3, \quad \alpha > 5$$

which is positive. Figure 2.2 shows graphical representations of λ_1 and λ_2. From Figure 2, it is observed that λ_1 and λ_2 decrease as α increases.

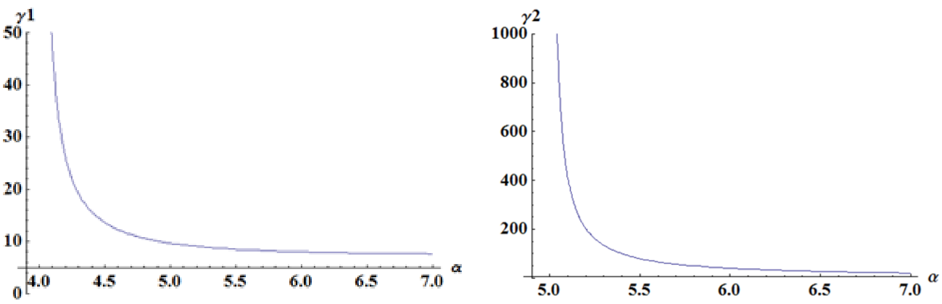

Figure 2.2. The skewness and the kurtosis measures

3. SURVIVAL ANALYSIS

Survival analysis is commonly used in the reliability literature to analyze the determinants of firm failure. In addition, it can be used for socio-economic research to investigate complex phenomena such as employment, supply and demand for bank loans, life expectancy of products, the producer and consumer, etc. In reliability theory, there are some additional functions of interest that are based on the distributions and densities. Intuitively, the survival rate at time x gives the probability that the value of a random variable will exceed x, while the hazard rate (HR) is the probability of observing an outcome within a neighborhood of x, conditional on the outcome being no less than x.

Finally, the reverse hazard (RH) rate is the probability of observing an outcome in a neighborhood of x, conditional on the outcome being no more than x. Several studies have applied these functions to reliability models (cf. Barlow and Proschan (1975)). The survival, HR, and RHR functions of the LBL distribution are given by

$$S(t,\lambda,\alpha) = \frac{\alpha\lambda t+1}{(1+\lambda t)^{\alpha}} \tag{3.1}$$

$$h(t,\lambda,\alpha) = \frac{\lambda^2\alpha(\alpha-1)t}{(1+\lambda t)(\alpha\lambda t+1)} \tag{3.2}$$

$$r(t,\lambda,\alpha) = \frac{\lambda^2\alpha(\alpha-1)t}{(1+\lambda t)((1+\lambda t)^{\alpha}-\lambda\alpha t-1)}$$

respectively.

In the next theorem, we discuss the behavior of the HR of the LBL distribution.

Theorem 3.1. Let T be a non negative random variable following the LBL distribution, then

(i) if $t < \frac{1}{\sqrt{\alpha}\lambda}$, then T is increasing hazard rate (IHR)

(ii) if $t > \frac{1}{\sqrt{\alpha}\lambda}$, then $T$is decreasing hazard rate (DHR).

Proof.

The first and second derivatives of (3.2) are given by

$$\frac{d}{dt}[h(t,\lambda,\alpha)] = \lambda^2\alpha(\alpha-1)\left[\frac{1-\alpha\lambda^2t^2}{(1+\lambda t)^2(1+\lambda\alpha t)^2}\right],$$

and

$$\frac{d^2}{dt^2}[h(t,\lambda,\alpha)] = 2\lambda^3\alpha(\alpha-1)\left[\frac{\alpha^2\lambda^3t^3-1-\alpha-3\alpha\lambda t}{(1+\lambda t)^3(1+\alpha\lambda t)^3}\right],$$

which has a maximum at $t_0 = \frac{1}{\lambda\sqrt{\alpha}}$

The shapes of the HR and RHR of the LBL distribution for different values of α and at λ=1 are illustrated in Figure 3.

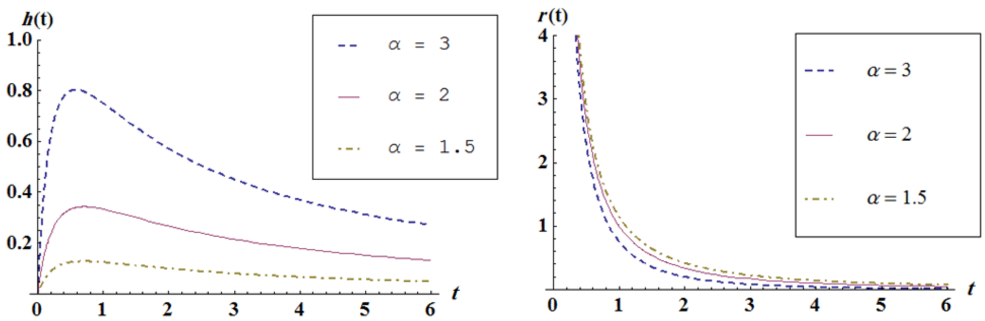

Figure 3.1. The HR and RHR of LBL distribution for different α and λ= 1

The mean residual life (MRL) function is very important in survival analysis. It is well known that MRL uniquely determines the distribution function, i.e. it contains all the information about the model (cf. Barlow and Proschan (1975), Navarro et al. (2002)). Let X be a lifetime random variable with survival function S(t). If E(X) is finite, the MRL of X is defined by

$$m(t) = \frac{\int_t^{\infty} S(x)\,\mathrm{dx}}{S(t)}$$

The MRL of the new model is given by

$$m(t,\lambda,\alpha) = \frac{(\alpha\lambda t+2)(\lambda t+1)}{\lambda(\alpha-2)(\lambda\alpha t+1)}, \qquad \alpha > 2 \tag{3.3}$$

It is well known that decreasing (increasing) HR implies increasing (decreasing) MRL (see Barlow and Proschan (1975)). This reciprocal property may not hold for non-monotone hazard rates. The shapes of the MRL of the LBL distribution for di¤erent values of α and λ= 1 are illustrated in Figure 3.2.

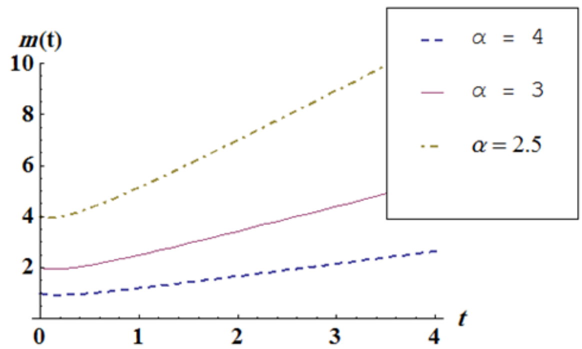

Figure 3.2. The MRL of LBL distribution of different α and λ= 1

In addition to the above reliability functions, the inactivity time, also known as reversed residual life or waiting time, has been a topic of increasing interest in the literature. Although inactivity time has mainly been used to study reliability, it has also been useful to describe the behavior of lifetime random variables in survival retrospective studies, and some applications have been derived in risk theory and econometrics (cf. Kayid and Ahmad (2004)). The mean inactivity time (MIT) of non-negative continuous random variable T is defined by

$$\mu(t) = \frac{\int_0^t F(x)dx}{F(t)}, \; F(t) > 0$$

The MIT of LBL distribution is given by

$$\mu(t,\lambda,\alpha) = \frac{\lambda t(\alpha-2)(1+\lambda t)^{\alpha}-2(1+\lambda t)^{\alpha}+(2+\alpha\lambda t)(1+\lambda t)}{\lambda(\alpha-2)[(1+\lambda t)^{\alpha}-\alpha\lambda t-1]}, \; \alpha > 2$$

On the other hand, there are many situations in reliability and economics where it is useful to make comparisons between two distributions. Consider two random variables, X and Y; assumethat they both represent the same mapping from the same sample space into the real line, but are governed by differing probability laws. More specifically, suppose that the realizations of X are typically higher than those of Y. One way to rigorously define this property is in terms of stochastic dominance. For the following definitions, let X and Y be two non-negative random variables having distribution functions F_X and F_Y, respectively; assume $\overline{F}_X = 1 - F_X$ and $\overline{F}_Y = 1 - F_Y$ as their respective survival functions, and f_X and f_Y as corresponding density functions.

Definition 3.2. A non-negative random variable X is said to be smaller than a random variable in the:

(i) First-order stochastic dominance (denoted as $X \leq_{fosd} Y$) if

$$\overline{F}_X(x) \leq \overline{F}_Y(x) \text{for all } x \in \mathbb{R}$$

(ii) Hazard rate order (denoted as $X \leq_{hr} Y$) if,

$$\overline{G}(x)/\overline{F}(x) \geq g(x)/f(x), \text{ for all } x \in \mathbb{R}$$

(iii) Reversed hazard rate order (denoted as $\leq_{rhr} Y$) if,

$$\mathrm{G}(x)/\mathrm{F}(x) \leq \mathrm{g}(x)/f(x), \text{ for all } x \in \mathbb{R}$$

(iv) Likelihood ratio order (denoted as $X \leq_{tr} Y$), if

$$f(x)/\, \mathrm{g}(x) \text{ decreasing in x} \in \mathbb{R}$$

The four orders of stochastic dominance defined above are related to one another, as the following implications:

$$X \leq_{rhr} Y \Longleftarrow X \leq_{lr} Y \Longrightarrow X \leq_{hr} Y \Longrightarrow X \leq_{fosd} Y. \tag{3.2}$$

Several applications of stochastic dominance in reliability, game theory, and industrial organization are found in the literature (cf. Shaked and Shanthikumar (2007)). The next theorem shows that the LBL distributions are ordered with respect to the strongest likelihood ratio ordering.

Theorem 3.2. Let $X \sim \mathrm{LBL}(x; \lambda, \alpha_1)$ and $\mathrm{Y} \sim \mathrm{LBL}\ (x; \lambda, \alpha_2)$ .If $\alpha_2 < \alpha_1$ then $X \leq_{lr} Y (X \leq_{hr} Y, X \leq_{rhr} Y, X \leq_{fosd} Y)$.

Proof.

First let $g(x) = f_x(x)/\, f_y(x)$ then

$$g(x) = \left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{\alpha_1 - 1}{\alpha_2 - 1}\right)(1 + \lambda \mathrm{t})^{\alpha_1 - \alpha_2}$$

Since

$$\frac{d}{dx}[\mathrm{g}(x)] = \lambda(\alpha_2 - \alpha_1)\left(\frac{\alpha_1}{\alpha_2}\right)\left(\frac{\alpha_1 - 1}{\alpha_2 - 1}\right)(1 + \lambda \mathrm{t})^{\alpha_2 - \alpha_1 - 1}$$

then $g(x)$ is decreasing in x for $\alpha_2 < \alpha_1$, that is $X \leq_{lr} Y$. The remaining statements follows from the implications (3.2).

4. PARAMETERS ESTIMATION

The problem of estimation is of more central importance. In this section, we consider the estimation of the unknown parameters λ and α of the LBL distribution. Two methods of estimation are used, namely the method of moments and the maximum likelihood. The first method yields an explicit solution, in contrast to the second method.

4.1 Moment estimates

In the method of moments, we have to solve the equation

$$m'_r = \mu'_r \,, r = 1,2,..., \tag{4.1}$$

Where $m_r = \frac{1}{n}\sum_{i=1}^{n} t_i^r$ is the sample moment and $\mu_r = E(T^r)$ is the population moment. From (4.1), we have

$$\frac{2}{\lambda(\alpha-2)} = \bar{T} \tag{4.2}$$

and

$$\frac{6}{\lambda^2(\alpha-2)(\alpha-3)} = \frac{1}{n}\sum_{i=1}^{n} t_i^2 \,.$$

From (4.1), we obtain:

$$\lambda = \frac{2}{\bar{T}(\alpha-2)}. \tag{4.3}$$

Using Eqs. (4.3) and (4.2), we have

$$\alpha = \frac{6((\bar{T})^2 - \frac{1}{n}\sum_{i=1}^{n} T^2)}{(3(\bar{T})^2 - \frac{2}{n}\sum_{i=1}^{n} T^2)} \tag{4.4}$$

and

$$\lambda(\alpha) = \frac{2}{\bar{T}(\alpha-2)}. \tag{4.5}$$

4.2 Maximum likelihood estimates

Let $T_1, \dots, T_n$ be a random sample from LBL distribution. The likelihood function is given by

$$\begin{aligned} L(t_1, \dots, t_n|\lambda, \alpha) &= \prod_{i=1}^{n}(t_1, \dots, t_n|\lambda, \alpha) \\ &= \lambda^{2n}\alpha^n(\alpha-1)^n\left(\prod_{i=1}^{n} t_i\right)\left(\prod_{i=1}^{n}\frac{1}{(1+\lambda t_i)^{\alpha+1}}\right) \end{aligned} \tag{4.6}$$

The logarithm of the likelihood function is then given by

$$\ell(t_1, \dots, t_n|\lambda, \alpha) = 2n\ln\lambda + n\ln\alpha + n\ln(\alpha-1) + \sum_{i=1}^{n}\ln t_i - (\alpha+1)\sum_{i=1}^{n}\ln(1+\lambda t_i) \tag{4.7}$$

The maximum likelihood estimators (MLEs) of λ and α can be obtained by solving the next two nonlinear equations

$$\frac{\partial l}{\partial \lambda} = \frac{2n}{\lambda} - \lambda(\alpha+1)\sum_{i=1}^{n}\frac{1}{(1+\lambda t_i)} = 0 \tag{4.8}$$

and

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \frac{n}{\alpha-1} - \sum_{i=1}^{n} ln(1+\lambda t_i) = 0. \tag{4.9}$$

Clearly there are no explicit solutions for (4.8) and (4.9), and they must therefore be solved numerically.

4.3 Asymptotic confidence intervals

From Eqs. (4.8) and (4.9) we have

$$\frac{\partial^2 l}{\partial \lambda^2} = -\frac{2n}{\lambda^2} - (\alpha+1)\sum_{i=1}^{n}\frac{1}{(1+\lambda t_i)} + \lambda^2(\alpha+1)\sum_{i=1}^{n}\frac{1}{(1+\lambda t_i)^2},$$

$$\frac{\partial^2 l}{\partial\lambda\partial\alpha} = \frac{\partial^2 l}{\partial\alpha\partial\lambda} = -\lambda\sum_{i=1}^{n}\frac{1}{(1+\lambda t_i)}$$

Denote the MLE of (λ, α) by (λ, α), which are approximately bivariate normal with mean $(\lambda, \alpha)$and covariance matrix I_0^{-1}, whereI^{-1} is the inverse of the observed information matrix (Lawless (2003)).

$$I_0^{-1}(\theta) = \begin{pmatrix} -\frac{\partial^2 l}{\partial\lambda^2} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\alpha^2} \end{pmatrix} = \begin{pmatrix} Var(\lambda) & Cov(\lambda,\ \alpha) \\ Cov(\lambda,\ \alpha) & Var(\alpha) \end{pmatrix}$$

The approximate $(1-\delta)$ 100% confidence intervals (CIs) for parameters α and λ, respectivelyare

$$\alpha \pm Z_{\delta/2}V(\alpha)\ and\ \lambda \pm Z_{\delta/2}V(\lambda)$$

Where $V(\alpha)$ and $V(\lambda)$ are the variances of α and λ, which are given by the first and the second diagonal elements of $I_0^{-1}(\theta)$, and $Z_{\delta/2}$ is the upper $(\delta/2)$ percentile of the standard normal distribution.

5. SOME APPLICATIONS

5.1 Numerical example

In this subsection, for a given known scale parameter ($\lambda = 1$), 1000 different samples are simulated from LBL with different sizes. The behaviors of the moment estimate (ME) and the maximum likelihood estimates (MLE) are studied from unknown shape parameter α. Tables 5.1 and 5.2 present the ME and MLE of parameter α, respectively.

Table 5.1. Moment estimate of the parameter

α	n	Estimate	Bias	MSE
3	30	3.51782	0.517823	0.925202
	50	3.37115	0.371148	0.380695
	70	3.35259	0.352589	0.309089
	100	3.28857	0.288571	0.204088
3.7	30	4.08862	0.388616	0.556141
	50	3.99281	0.292814	0.305385
	70	3.99446	0.29446	0.254348
	100	3.93735	0.23735	0.16902
4	30	4.40196	0.401956	0.614413
	50	4.32295	0.1322948	0.367236
	70	4.3142	0.314201	0.287914
	100	4.26025	0.260248	0.197522

From Table 5.1, it is observed that the mean square errors of the moment estimates of parameter decrease with increasing sample size (n). There is an overestimate in all of the chosen values of the parameter here.

Table 5.2. Maximum likelihood estimate of the parameter

α	n	Estimate	Bias	MSE
3	30	3.04062	0.040616	0.103192
	50	3.01327	0.013271	0.0574656
	70	3.02257	0.022571	0.0445872
	100	3.00174	0.001744	0.0304342
3.7	30	3.75213	0.052129	0.1768380
	50	3.71672	0.016724	0.0985985
	70	3.72910	0.029099	0.0765368
	100	3.70197	0.001970	0.0522825
4	30	4.06203	0.062027	0.2174300
	50	4.02266	0.022664	0.1205650
	70	4.03635	0.0363492	0.0938624
	100	4.00656	0.006564	0.0640049

From Table 5.2, the mean square errors of the maximum likelihood estimate of parameter decrease with increasing sample size (n). In addition, the estimation of α is better for small values of α in terms of the mean square error. Comparing Table 5.1 with Table 5.2, it is clear that the maximum likelihood estimation provides much better results than the method of moment estimation in terms of mean square error and bias.

5.2 Real data

In this subsection, we analyze a real data set to show how the new model works in practice. The data set obtained from Hubble (1929) represents the distance between extra-galactic nebulae and Earth, measured in mega parsecs

Data Set

0.032	0.034	0.214	0.263	0.275	0.275	0.450	0.500
0.500	0.630	0.800	0.900	0.900	0.900	0.900	1.000
1.100	1.100	1.400	1.700	2.000	2.000	2.000	2.000

The data were fitted to LD and LBL distributions. We use the Kolmogorov−Smirnov (K−S) distances between the empirical distribution function and the fitted distribution function to determine the appropriateness of the model. The K−S values are presented in Table 5.3.

Table 5.3. The K-S value of distributions

distribution	K-S
LD	0.196049
LBL	0.172812

The K−S for LBL is smaller than that for LD, which indicates that these data ⋯t the LBL better than the LD.

6. CONCLUSION

The proposed length-biased Lomax distribution has several desirable properties and useful physical interpretations. This model is useful and practical in areas such as physics, reliability, and life testing. The model has a unimodal pdf and an eventually decreasing hazard rate function. Such characteristics are useful for modeling continuous data from life testing experiments. Analysis of real data sets demonstrates that the proposed distribution can provide a better fit than other well-known distributions. Interesting future applications include studying length-biased versions of log normal and inverse Gaussian distributions, among others.

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