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On NBUL class at specific age

M. A. W. Mahmoud and M. E. Moshref
Department of Mathematics, Faculty of Science, Al-Azhar University, Cairo, Egypt

A. M. Gadallah*
Department of Basic Sciences, Thebes Higher Institute for Engineering, Thebes Academy, Cairo, Egypt

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Abstract. New classes of life distributions called new better (worse) than used at age t_0 in Laplacetransform order, $\mathrm{NBUL} - t_0(\mathrm{NWUL} - t_0)$ are introduced. For the classes $\mathrm{NBUL} - t_0($ NWUL$- t_0)$, preservation under convolution, mixture, mixing and the homogeneous Poissonshock model are studied. In the sequel, we obtain a test for H_0: F is exponential versus H_1 : F is NBUL$- t_0$ and not exponential. The critical values and the powers of this test arecalculated to assess the performance of the test. It is shown that the proposed test has highefficiencies for some commonly used distributions in reliability. Sets of real data are used asexamples to elucidate the use of the proposed test for practical problems.

Key Words: *Convolution, hypothesis test, mixture, Poisson shock model, U-statistic*

1. INTRODUCTION

Numerous orderings have been recently appeared regarding life distributions. Most of these orderingsare defined by specifying a certain mode of aging. Before we go into the details, let us review some common notions of stochastic orderings (see Shaked and Shanthikumar (1994) and Barlow and Proschan (1981)).
Formally, if X and Y are two non-negative random variables with distributions F and G(survival functions $\bar{F}$ and $\bar{G}$), respectively, then we say X is smaller than Y in the

(i) Stochastic order (denoted by $X \leq_{\mathrm{st}} Y$) if
$E[\emptyset\,(X)] \leq E[\emptyset\,(Y)]$ for all increasing functions □.
(ii) Increasing concave order (denoted by $X \leq_{\mathrm{icv}} Y$) if
$E[\emptyset\,(X)] \leq E[\emptyset\,(Y)]$ for all increasing concave functions □.

*Corresponding Author.
E-mail address: *alaadean_mag@yahoo.com*

(iii) Laplace transform order (denoted by $X \leq_{Lt} Y$) if

$$\int_0^\infty e^{-\lambda x}\bar{F}(x)\,dx \leq \int_0^\infty e^{-\lambda x}\bar{G}(x)\,dx \quad \forall\lambda \geq 0. \tag{1.1}$$

Applications, properties and interrelations of the Laplace transform order in the statistical theory can be founded in Alzaid et al. (1991), Denuit (2001), Muller and Stoyan (2002), Klefsjo(1983), Shaked and Shanthikumar (1994) and Ahmad and Kayid (2004). In the context of lifetime distributions, the above orderings have been used to characterize several aging classes. We say X is

(i) New better than used (denoted by $X \in NBU$) if $\bar{F}(x+t) \leq \bar{F}(t)\bar{F}(x), \forall x, t \geq 0$.
(ii) New better ∫ than used in the increasing concave order (denoted by $X \in NBU(2)$) if $\int_0^x \bar{F}(u+t)\,dx \leq \bar{F}(t)\int_0^x \bar{F}(u)\,du, \forall x, t \geq 0$.
(iii) New better than used in the Laplace transform order (denoted by $X \in NBUL$) if $\int_0^\infty e^{-\lambda x}\bar{F}(x+t)\,dx \leq \bar{F}(t)\int_0^\infty e^{-\lambda x}\bar{F}(x)\,dx, \forall\lambda, t \geq 0$.

For more details about the above aging notions, one may refer to Bryson and Siddiqui (1969), Barlow and Proschan (1981), Deshpande et. al. (1986) and Wang (1996). Some interpretations, properties and closure properties of the $NBUL$ class have been discussed by Yue and Cao (2001) and Gao et. al. (2002), while Belzunce et. al. (1999) showed that the $NBUL$ class is preserved under both the pure birth shock model and Poisson shock model, Al-Wasel et. al. (2007) studied the $NBUL$ class and investigated a test statistic for it based on U- statistic by using expected value method, Diab et. al. (2009) investigated a test statistic for it based on moment inequality. Finally Diab (2010) investigated a test statistic for it based on U- statistic by using goodness of fit approach.
Statisticians and reliability analysts studied some aging classes of life distributions at specific age from various points of view. For more details we refer to Hollander et. al. (1986), Ebrahimi and Habbibullah (1990), Ahmad (1998) and Pandit and Anuradha (2007) for NBU $-t_0$ and Mahmoudet. al. (2013) for $NBUE - t_0$ and H$NBUE - t_0$.
In this paper we introduce a new class of life distribution namely NBUL $- t_0$, and its dual class NWUL $- t_0$.

Definition 1.1. X is new better (worse) than used at age t_0 in Laplace transform order (denoted by $X \in$ NBUL $- t_0$) if

$$\int_0^\infty e^{-\lambda x}\,\bar{F}(x+t_0)\,dx \leq \bar{F}(t_0)\int_0^\infty e^{-\lambda x}\,\bar{F}(x)\,dx, \quad \forall\lambda, t_0 \geq 0. \tag{1.2}$$

One can note that

$$NBU \Rightarrow NBU - t_0 \Rightarrow NBUL - t_0 \Rightarrow NBUE - t_0 \Rightarrow \mathrm{H}NBUE - t_0.$$

In the current investigation, preservation under convolution, mixture, mixing and the homogeneous Poisson shock model of the $NBUL - t_0$ ($NWUL - t_0$) classes are discussed in Section 2. In Section 3, based on U statistic we present a procedure to test that X is exponential versus that it is $NBUL - t_0$ and not exponential.

2. SOME PROPERTIES OF THE NBUL − $\mathbf{t_0}$ CLASS

In this section we discuss preservation and nonpreservation properties of the $NBUL - t_0$ and $NWUL - t_0$ classes.

2.1 Convolution, mixture and mixing properties

Our aim in this subsection is to discuss preservation under convolution, mixture and mixing properties of $NBUL - t_0$ and $NWUL - t_0$ classes.

Theorem 2.1. The $NBUL - t_0$ class is preserved under convolution.

Proof. Suppose that F_1 and F_2 are two independent $NBUL - t_0$ lifetime distributions then their convolution is given by:

$$\bar{F}(z) = \int_0^\infty \bar{\mathrm{F}}_1\,(z-y)dF_2(y).$$

And therefore:

$$\int_0^\infty e^{-\lambda x}\,\bar{F}(x+t_0)\,dx = \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_1(x+t_0-u)dF_2(u)\,dx$$
$$= \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_1(x+t_0-u)dxdF_2(u),$$

since F_1 is $NBUL - t_0$ then

$$\int_0^\infty e^{-\lambda x}\,\bar{F}(x+t_0)\,dx \leq \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_1(t_0)\bar{F}_1(x-u)dxdF_2(u)$$
$$= \bar{F}_1(t_{0)}\int_0^\infty e^{-\lambda x}\,\bar{F}(x)dx.$$

By using $\bar{F_\iota}(z) \leq \bar{F}(z)\;for\;i = 1,2$

$$\int_0^\infty e^{-\lambda x}\,\bar{F}(x+t_0)\,dx \leq \bar{F}(t_{0)}\int_0^\infty e^{-\lambda x}\,\bar{F}(x)dx,$$

which complete the proof.

The following example is presented to show that $NWUL - t_0$ class is not preserved under convolution.

Example 2.1. The convolution of the exponential distribution $F(x) = 1 - e^{-x}$ with itself yields the gamma distribution of order 2: $G(x) = 1 - (1+x)e^{-x}$, with strictly increasing failure rate.Thus $G(x)$ is not $NWUL - t_0$.

The following example shows that the $NBUL - t_0$ class is not preserved under mixtures.

Example 2.2. Let $\bar{\mathrm{F}}_\alpha(x) = e^{-\alpha x}$ and $\bar{G}(x) = \int_0^\infty \bar{\mathrm{F}}_\alpha\,(x)e^{-\alpha}d\alpha = (x+1)^{-1}$. Then the failure ratefunction is $\mathrm{r_g(x) = (x+1)^{-1}}$, which is strictly decreasing thus $\bar{G}(x)$ is not $NBUL - t_0$.

The following theorem is stated and proved to show that the $NWUL - t_0$ class is preserved under mixture.

Theorem 2.2. The $NWUL - t_0$ class is preserved under mixture.

Proof. Suppose F(x) is the mixture of F_α, where each F_α is $NWUL - t_0$ then,

$$\int_0^\infty e^{-\lambda x}\,\bar{F}(x+t_0)\,dx = \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_\alpha(x+t_0)dG(\alpha)dx$$
$$= \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_\alpha(x+t_0)dxdG(\alpha). \qquad (2.1)$$

Since F_α is $NWUL - t_0$ then

$$\int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_\alpha(x+t_0)dxdG(\alpha) \geq \int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_\alpha(t_0)\bar{F}_\alpha(x)dxdG(\alpha). \qquad (2.2)$$

Upon using Chebyschev inequality for similarity ordered functions we get

$$\int_0^\infty \int_0^\infty e^{-\lambda x}\,\bar{F}_\alpha(t_0)\bar{F}_\alpha(x)dG(\alpha)dx \geq \int_0^\infty e^{-\lambda x}\left\{\int_0^\infty \bar{F}_\alpha(t_0)dG(\alpha).\int_0^\infty \bar{F}_\alpha(x)dG(\alpha)\right\}dx. \qquad (2.3)$$

Upon using (2.1), (2.2) and (2.3) the proof is completed.

The following example illustrates that the $NBUL - t_0$ class is not preserved under mixing.

Example 2.3. Let $\bar{F}_1 = e^{-\delta x}$ and $\bar{F}_2 = e^{-\gamma x}$. Let $\bar{F} = \frac{1}{2}\bar{F}_1 + \frac{1}{2}\bar{F}_2$. It follows that both $\bar{F}_1$ and $\bar{F}_2$ are $NBUL - t_0$ but $\bar{F}$ is not $NBUL - t_0$.

2.2 Homogeneous Poisson shock model

The homogeneous Poisson shock model for the $NBUL$ class was proved by Belzunce et. al. (1999). Here our purpose is to check closure of the homogeneous Poisson shock model for $NBUL - t_0$.Suppose that a device is subjected to sequence shocks occurring randomly in the time accordingto a Poisson process with constant intensity s. Suppose further that the device has probability $\bar{p}_k$ of surviving the first k shocks, where $1 = \bar{p}_0 \geq \bar{p}_1 \geq \cdots$. Then the survival function of the deviceis given by

$$\bar{H}(t) = \sum_{k=0}^{\infty} \bar{p}_k \frac{(st)^k}{k!} e^{-st} \qquad (2.4)$$

This shock model has been studied by Esary et al. (1969) for IFR, $IFRA$, $DMRL$, NBU and $NBUE$ classes. Klefsjo (1981) for $HNBUE$ and Mahmoud et. al. (2009) for $NBURFR - t_0$.

Definition 2.1. A discrete distribution $p_k, k = 0,1,\cdots,\infty$ or its survival probabilities $\bar{p}_k, k = 0,1,\cdots,\infty$ is said to have discrete new better (worse) than used of age t_0 in Laplace transformorder ($NBUL - t_0$)($NWUL - t_0$) if

$$\sum_{r=0}^{\infty} \overline{p}_r + jz^r \leq (\geq) \overline{p}_j \sum_{r=0}^{\infty} \overline{p}_r z^r, 0 \leq z \leq 1, j = 0, 1, \cdots. \tag{2.5}$$

Now, let us introduce the following theorem.

Theorem 2.3. If P_k is discrete $NBUL - t_0$, then $\overline{H}(t)$ given by (2.4) is $NBUL - t_0$.

Proof. It must be shown that

$$\int_0^{\infty} e^{-\lambda x}\, \overline{H}(x+t_0)\, dx \leq \overline{H}(t_0) \int_0^{\infty} e^{-\lambda x}\, \overline{H}(x)\, dx.$$

Upon using (2.4), we get

$$\int_0^{\infty} e^{-\lambda x}\, \overline{H}(x+t_0)\, dx = \int_0^{\infty} e^{-\lambda x} \sum_{k=0}^{\infty} \overline{p}_k \frac{[s(t_0+x)]^k}{k!} e^{-s(x+t_{0)}}\, dx.$$

$$= e^{-st_0} \sum_{k=0}^{\infty} \overline{p}_k \sum_{r=0}^{k} \binom{k}{r} \frac{(st_0)^{k-r}}{k!} \int_0^{\infty} (sx)^r\, e^{-x(s+\lambda)} dx.$$

Integrating by parts yields

$$\int_0^{\infty} e^{-\lambda x}\, \overline{H}(x+t_0)\, dx = \frac{e^{-st_0}}{(s+\lambda)} \sum_{r=0}^{\infty} \sum_{k=r}^{\infty} \overline{p}_k \frac{(st_0)^{(k-r)}}{(k-r)!} \left(\frac{s}{s+\lambda}\right)^r$$

$$= \frac{e^{-st_0}}{(s+\lambda)} \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \overline{p}_{j+r} \frac{(st_0)^j}{j!} \left(\frac{s}{s+\lambda}\right)^r$$

$$\leq \frac{e^{-st_0}}{(s+\lambda)} \sum_{j=0}^{\infty} \overline{p}_j \sum_{r=0}^{\infty} \overline{p}_r \frac{(st_0)^j}{j!} \left(\frac{s}{s+\lambda}\right)^r$$

$$= \frac{1}{(s+\lambda)} \sum_{j=0}^{\infty} \frac{(st_0)^j}{j!} \overline{p}_j e^{-st_0} \sum_{r=0}^{\infty} \overline{p}_r \left(\frac{s}{s+\lambda}\right)^r$$

$$= \sum_{j=0}^{\infty} \frac{(st_0)^j}{j!} \overline{p}_j e^{-st_0} \sum_{r=0}^{\infty} \overline{p}_r \frac{s^r}{r!} \int_0^{\infty} x^r e^{-x(s+\lambda)}\, dx$$

$$= \overline{H}(t_0) \int_0^{\infty} e^{-\lambda x} \sum_{r=0}^{\infty} \frac{(sx)^r}{r!} \overline{p}_r\, e^{-sx}\, dx$$

$$= \overline{H}(t_0) \int_0^{\infty} e^{-\lambda x}\, \overline{H}(x)\, dx$$

The proof for the $NWUL - t_0$ class is obtained by reversing the inequality.

3. TESTING IN THE NBUL − t_0

Our goal in this section is to propose a test statistic for testing H_0 : F is exponential versus H_1 : F belongs to NBUL − t_0 class of life distributions and not exponential. Let

$$\Delta = \int_0^\infty e^{-\lambda x}\,\overline{F}(t_0)\overline{F}(x)dx - \int_0^\infty e^{-\lambda x}\,\overline{F}(x+t_0)dx, \tag{3.1}$$

it is clear that

$$\int_0^\infty e^{-\lambda x}\,\overline{F}(x+t_0)dx = \frac{1}{\lambda}E\left[1-e^{-\lambda(X-t_0)}\right], \tag{3.2}$$

and

$$\int_0^\infty e^{-\lambda x}\,\overline{F}(x)dx = \frac{1}{\lambda}E\left[1-e^{-\lambda X}\right]. \tag{3.3}$$

Upon using (3.2) and (3.3) in (3.1) we get

$$\Delta = \frac{1}{\lambda}E\left[\overline{F}(t_0) - 1 - e^{-\lambda X}\overline{F}(t_0) + e^{-\lambda(X-t_0)}\right]. \tag{3.4}$$

One can notice that the value of $\triangle$ under H_0 equals ζ, where

$$\zeta = \overline{F}(t_0) + \frac{e^{\lambda t_0} - \overline{F}(t_0)}{(1+\lambda)} - 1.$$

Setting the measure of departure from H_0 is $\delta = \triangle - \zeta$, then

$$\delta = \frac{1}{\lambda}\left[e^{\lambda t_0} - \overline{F}(t_0)\right]E\left[e^{-\lambda X} - \frac{1}{1+\lambda}\right]. \tag{3.5}$$

Note that under $H_0 : \delta = 0$, while under $H_1 : \delta > 0$.
To estimate δ, let $X_1, X_2, \cdots, X_n$ be a random sample from F, so the empirical form of δ in (3.5) is

$$\hat{\delta}_n = \frac{[e^{\lambda t_0} - \overline{F}(t_{(0)}]}{n\lambda}\sum_{i=1}^{n}\left[e^{-\lambda X_i} - \frac{1}{1+\lambda}\right]. \tag{3.6}$$

To find the limiting distribution of $\hat{\delta}_n$ we resort to the U-statistic theory. Let

$$\emptyset(X) = e^{-\lambda X} - \frac{1}{1+\lambda},$$

and define the symmetric kernel

$$\psi(X) = \sum_R \emptyset(X_i),$$

where the sum is over all arrangements of X_i, this leads that $\hat{\delta}_n$ in (3.6) is equivalent to U-statistic given by

$$U_n = \frac{1}{n}\sum_R \psi(X_i).$$

The next results summarizes the asymptotic normality of $\hat{\delta}_n$.

Theorem 3.1. (i) As $n \to \infty$, $\sqrt{n}(\hat{\delta}_n - \delta_1)$ is asymptotically normal with mean 0 and variance is

$$\sigma^2 = \mathrm{Var}\left\{\frac{\left(e^{\lambda t_0} - \overline{F}(t_0)\right)}{\lambda}\left[e^{-\lambda X} - \frac{1}{1+\lambda}\right]\right\} \tag{3.7}$$

(ii) Under H_0, the variance is reduced to

$$\sigma_0^2 = \frac{(e^{\lambda t_0} - \overline{F}(t_0))^2}{(1+2\lambda)(1+\lambda)^2} \tag{3.8}$$

Hence we reject H_0 if $\sqrt{n}\hat{\delta}_n / \sigma_0 > z_\alpha$, where z_α the standard normal variate.

3.1 The Pitman asymptotic efficiency (PAE) of δ_1

To asses how good this procedure is relative to others in the literature we employ the concept of Pittman asymptotic efficiency (PAE) for two alternatives these are:

1. Linear failure rate family (LFR): $\overline{F}_\theta(x) = \exp(-x - \frac{\theta}{2}x^2)$, $x > 0$, $\theta \geq 0$.
2. Makeham family: $\overline{F}_\theta = \exp(-x + \theta(x + e^{-x} - 1))$, $x > 0$, $\theta \geq 0$.

The PAE of δ is defined by

$$\mathrm{PAE}(\delta) = \frac{1}{\sigma_0}\left|\frac{d\delta}{d\theta}\right|_{\theta\to\theta_0}.$$

The following is PAE of δ for any alternative F,

$$\mathrm{PAE}(\hat{\delta}, F) = \frac{1}{\sigma_0}\left|e^{-t_0}\int_0^\infty e^{-\lambda x}\,\overline{F'}_\theta(x)dx + \frac{\overline{F}'_\theta(t_0)}{1+\lambda} - e^{-\lambda t_0}\int_{t_0}^\infty e^{-\lambda x}\overline{F}'_\theta(x)dx\right|.$$

So

$$\mathrm{PAE}(\hat{\delta}, \mathrm{LFR}) = \frac{1}{\sigma_0}\left|\frac{t_0 e^{-t_0}}{(1+\lambda)^2}\right|,$$

and

$$\mathrm{PAE}(\hat{\delta}, \mathrm{Makeham}) = \frac{1}{\sigma_0}\left|\frac{(\lambda+3)e^{-t_0}}{(1+\lambda)^2(2+\lambda)} + \frac{e^{-2t_0}}{(1+\lambda)(2+\lambda)}\right|.$$

After Searching, we find $\lambda = 0.001$ that maximizes the PAE($\hat{\delta}$,LFR) and the PAE($\hat{\delta}$,Makeham). Figure 3.1 shows the relation between efficiencies and $t_0 < 5$.

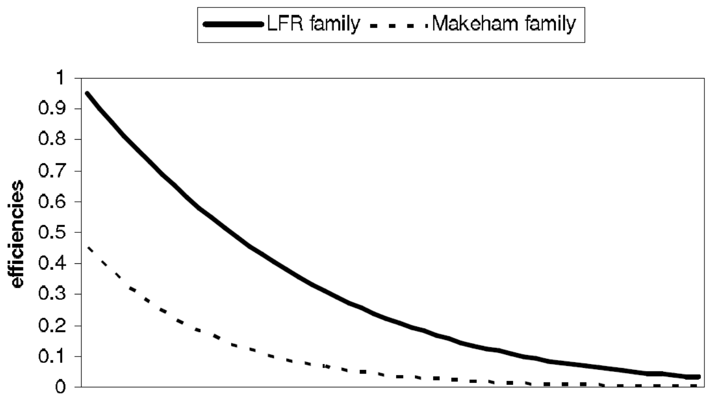

Figure 3.1. The relation between efficiencies and $t_0 < 5$ at $\lambda = 0.001$

In view of Figure 3.1 it is noticed that the PAE's of δ are decreasing as t_0 increasing and the PAE's for the LFR alternative is greater than the PAE's for Makeham alternative.
We compare the above PAE's at $t_0 = 0.1$ and $\lambda = 0.001$ with that of Hollander and Proschan (1972) and the results are shown in Table 3.1.

Table 3.1. The PAE's for LFR and Makeham families

Test LFR Makeham	LFR	Makeham
Hollander-Proschan	0.8660	0.2886
δ	0.9498	0.4521

Table 3.1 shows that our test here outperforms the other even though our class is much larger than that of the NBU.

3.2 Monte Carlo null distribution critical points

Many practitioners, such as applied statisticians, and reliability analysts are interested in simulated percentiles. Table 3.2 gives these percentile points of the statistic $\hat{\delta}_n$ given in (3.6) at t_0 = 0.1 and the calculations are based on 1000 simulated samples of sizes n = 2(1)50.

Table 3.2. Critical values of statistic $\hat{\delta}_n$ at $t_0 = 0.1$&$\lambda = 0.001$

n	0.01	0.05	0.10	0.90	0.95	0.99
2	-0.213	-0.137	-0.088	0.071	0.080	0.090
3	-0.177	-0.103	-0.075	0.061	0.070	0.084
4	-0.144	-0.086	-0.065	0.053	0.062	0.076
5	-0.126	-0.079	-0.060	0.048	0.057	0.069
6	-0.115	-0.074	-0.051	0.045	0.054	0.068
7	-0.102	-0.064	-0.047	0.043	0.051	0.066
8	-0.089	-0.060	-0.046	0.041	0.048	0.060
9	-0.086	-0.053	-0.042	0.038	0.045	0.057
10	-0.080	-0.053	-0.039	0.037	0.044	0.054
11	-0.084	-0.053	-0.037	0.036	0.043	0.052
12	-0.079	-0.046	-0.035	0.036	0.041	0.054
13	-0.075	-0.044	-0.033	0.034	0.041	0.053
14	-0.072	-0.046	-0.033	0.032	0.039	0.049
15	-0.063	-0.044	-0.032	0.030	0.036	0.049
16	-0.065	-0.042	-0.033	0.031	0.037	0.048
17	-0.063	-0.041	-0.030	0.029	0.036	0.047
18	-0.060	-0.039	-0.028	0.030	0.036	0.047
19	-0.061	-0.038	-0.029	0.028	0.034	0.043
20	-0.056	-0.037	-0.026	0.029	0.035	0.045
21	-0.058	-0.036	-0.027	0.027	0.032	0.040
22	-0.054	-0.034	-0.025	0.027	0.034	0.043
23	-0.054	-0.034	-0.025	0.025	0.032	0.043
24	-0.053	-0.033	-0.025	0.026	0.032	0.040
25	-0.050	-0.036	-0.026	0.026	0.031	0.041

26	-0.050	-0.031	-0.024	0.024	0.029	0.038
27	-0.050	-0.032	-0.025	0.024	0.030	0.040
28	-0.049	-0.030	-0.024	0.023	0.028	0.038
29	-0.047	-0.031	-0.024	0.024	0.030	0.038
30	-0.044	-0.033	-0.024	0.022	0.028	0.037
31	-0.043	-0.031	-0.021	0.021	0.027	0.037
32	-0.042	-0.028	-0.022	0.022	0.027	0.037
33	-0.043	-0.029	-0.022	0.021	0.027	0.035
34	-0.042	-0.028	-0.022	0.021	0.027	0.036
35	-0.042	-0.029	-0.022	0.021	0.026	0.033
36	-0.041	-0.027	-0.020	0.021	0.026	0.033
37	-0.042	-0.026	-0.020	0.020	0.025	0.033
38	-0.040	-0.027	-0.020	0.020	0.026	0.035
39	-0.040	-0.026	-0.019	0.020	0.025	0.031
40	-0.041	-0.027	-0.020	0.019	0.024	0.033
41	-0.038	-0.025	-0.020	0.019	0.023	0.031
42	-0.037	-0.025	-0.019	0.018	0.023	0.033
43	-0.039	-0.025	-0.019	0.019	0.023	0.032
44	-0.041	-0.025	-0.018	0.018	0.023	0.032
45	-0.036	-0.026	-0.019	0.019	0.023	0.030
46	-0.037	-0.026	-0.017	0.019	0.023	0.030
47	-0.038	-0.026	-0.019	0.018	0.023	0.029
48	-0.039	-0.024	-0.019	0.018	0.022	0.029
49	-0.037	-0.024	-0.018	0.018	0.022	0.029
50	-0.035	-0.023	-0.018	0.017	0.022	0.028

3.3 The power estimates

Table 3.3 shows the estimated power of the test statistic $\hat{\delta}_n$ given in (3.8) at the significant level 0.05 using LFR and Makeham distributions. The estimate are based on 1000 simulated samples for sizes n = 10, 20 and 30.

Table 3.3. Power estimates using $\alpha = 0.05$ at $t_0 = 0.1$

	n	$\theta = 1$ Powers	$\theta = 2$ Powers	$\theta = 3$ Powers	$\theta = 4$ Powers
LFR	10	1.000	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000
	30	1.000	1.000	1.000	1.000
Makeham	10	1.000	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000
	30	1.000	1.000	1.000	1.000

From Table 3.3 we can show that our test has perfect power.

3.4 Applications

Example 3.1. The following data represent 39 liver cancers patients taken from Elminia cancer center Ministry of Health – Egypt, which entered in (1999). The ordered life times (in years) are:

0.027	0.038	0.038	0.038	0.038	0.038	0.041	0.047	0.049	0.055
0.055	0.055	0.055	0.055	0.063	0.063	0.066	0.071	0.082	0.082
0.085	0.110	0.314	0.140	0.143	0.164	0.167	0.184	0.195	0.203
0.206	0.238	0.263	0.288	0.293	0.293	0.293	0.318	0.411	

It was found that $\hat{\delta}_n$ = 0.509 which is greater than the tabulated critical value in Table 3.2. There is enough evidence to accept H_1 which states that the data set has NBUL− t_0 property.

Example 3.2. Consider the well-known Darwin data (Fisher, (1966)) that represent the differences in heights between cross- and self-fertilized plants of the same pair grown together in one pot

4.9	-6.7	0.8	1.6	2.3	2.8	4.1	1.4
2.9	0.6	5.6	2.4	7.5	6.0	-4.8	

It was found that $\hat{\delta}_n$ = 0.902 and this value exceeds the tabulated critical value in Table 3.2. Then we conclude that this data set has NBUL − t_0 property.

Example 3.3. The data set of 40 patients suffering from blood cancer (Leukemia) from one of ministry of health hospitals in Saudi Arabia sees Abouammoh et al. (1994). The ordered life times (in years) are

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025
2.036	2.162	2.211	2.370	2.532	2.693	2.805	2.910
2.912	3.192	3.263	3.348	3.348	3.427	3.499	3.534
3.767	3.751	3.858	3.986	4.049	4.244	4.323	4.381
4.392	4.397	4.647	4.753	4.929	4.973	5.074	4.381

It was found that the value of test statistic for the data set by formula (3.6) $\hat{\delta}_n$ = -0.000211, which is smaller than the critical value in Table 3.2. Then we accept the null hypothesis which states that the data set has exponential property.

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