블라인드 등화를 위한 상수 모듈러스 오차의 영-확률 추정 방법

Estimation of Zero-Error Probability of Constant Modulus Errors for Blind Equalization

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요 약

상수 모듈러스 오차가 영이 될 확률을 최대화하도록 설계된 블라인드 등화 알고리듬은 한 반복시간에서 합산과정을 수행하여 큰복잡성을 유발한다. 합산과정에서 생기는 이러한 계산상의 부담을 줄여보고자 상수 모듈러스 오차 (CME)의 영확률(ZEP)과 그것의 기울기를 추정하는 새로운 접근 방법을 이 논문에서 제안하였다. 다음 반복시간에서 CME의 ZEP는 현행 CME의 ZEP를 기반으로 하여반복적으로 계산될 수 있음을 보였다. 알고리듬의 가중치 계산을 위한 기울기도 반복적 추정 방법에 의해 구해진 CME의 ZEP를 미분하여 구해질 수 있음을 제시하였다. 시뮬레이션에서 기존의 블록 처리에 의해 구하던 방법과 비교하였을 때, 제안한 방법에 의해 구해진 CME의 ZEP와 기울기가 상당히 줄인 계산량에도 불구하고 완전히 동일한 추정 결과를 보였다.

☞ 주제어 : 상수 모듈러스 오차; 계산복잡도; 기울기; 충격성 잡음; 영확률.

ABSTRACT

Blind algorithms designed to maximize the probability that constant modulus errors become zero carry out some summation operations for a set of constant modulus errors at an iteration time inducing heavy complexity. For the purpose of reducing this computational burden induced from the summation, a new approach to the estimation of the zero-error probability (ZEP) of constant modulus errors (CME) and its gradient is proposed in this paper. The ZEP of CME at the next iteration time is shown to be calculated recursively based on the currently calculated ZEP of CME. It also is shown that the gradient for the weight update of the algorithm can be obtained by differentiating the ZEP of CME estimated recursively. From the simulation results that the proposed estimation method of ZEP-CME and its gradient produces exactly the same estimation results with a significantly reduced computational complexity as the block-processing method does.

🖙 keyword : Constant modulus error; Computational Complexity; Gradient; Impulsive noise; Zero-error probability.

1. INTRODUCTION

In the blind signal processing applications to broadcast and the wireless/mobile networks, a training data set is not available [1][2][3]. For blind equalization the constant modulus error (CME) is widely utilized that is defined the difference between the instant output power and a constant modulus predefined according to the modulation schemes [4]. The most commonly used constant modulus algorithm (CMA) for blind equalization is based on mean squared

error (MSE) criterion and minimize the average of the constant modulus error. The averaging operation of the MSE criterion can mitigate the influence of Gaussian noise on the algorithm.

However, many communication systems are interfered with not only Gaussian noise but also impulsive noise from a variety of impulse noise sources [5][6]. Impulsive noise induces large instantaneous system output and error which often makes the system fail.

In impulsive noise environment the CMA based on MSE criterion is revealed to fail. Instead of the MSE criterion that utilizes error energy, the correntropy concept has been proposed as one of the information-theoretic learning (ITL) method to cope with impulsive noise problems [7][8]. The correntropy blind method known to be effective in

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correlative signaling systems has shown dissatisfying performance in the communication systems employing the identical, independently distributed symbol points.

In our previous work [9], as an alternative to the correntropy algorithm that is well known for robustness to impulsive noise, a new blind algorithm designed to maximize the zero-error probability for CME (ZEP-CME) has been proposed to cope with impulsive noise problem as well as channel distortions. The algorithm based on ZEP-CME has shown superior performance in ISI and impulsive noise environments, but it has one drawback of requiring heavy computations due to some summation operations at each iteration time, and that problem hinders the efficient implementation of the algorithm.

In this paper, for the purpose of reducing the computational complexity, a recursive approach to the ZEP-CME estimation and its gradient calculation for the weight update of the blind algorithm is proposed by investigating how the computation of ZEP-CME is done and whether any simpler methods can be possible based on the analysis. And then it is experimented whether the proposed method of recursive estimation of the ZEP-CME and its gradient produces the same equalization performance having no summation operations, compared to the original method introduced in [9].

This paper is organized as follows. Section 2 presents the definition of MSE and ZEP-CME. The ZEP-CME is shown to be able to be estimated recursively in Section 3, and the recursive estimation of the gradient of the ZEP-CME for the weight update is introduced in Section 4. Section 5 presents simulation results and discussions. Finally, concluding remarks are given in Section 6.

2. MSE AND ZERO-ERROR PROBABILITY OF CONSTANT MODULUS ERRORS

The MSE criterion P_{CME} for constant modulus error $e_{CME,k}$ at time k has been employed in many blind equalization algorithms as

$$e_{CME,k} = \left| y_k \right|^2 - R_2 \tag{1}$$

$$P_{CME} = E[e_{CME,k}^2] \tag{2}$$

$$P_{CME}|_{timeAVE} = \frac{1}{N} \sum_{i=k-N+1}^{k} e_{CME,i}^2$$
 (3)

where $R_2 = E[|d_i|^4]/E[|d_i|^2]$ and $\{d_i\}$ is a set of symbol points predefined for modulation. By taking the instant power of CME $P_{CME} = e_{CME,k}^2$ and minimizing it with respect to weight, the well-known CMA algorithm is derived [4]. While the ensemble averaging operation in (2) or time averaging (3) can mitigate the influence of the Gaussian noise, even a single large impulse in impulsive noise environments can defeat the averaging causing equalizers employing the MSE criterion to be unstable.

On the other hand, the ZEP-CME criterion for blind algorithms have been shown to be well adequate in impulsive noise channels since the Gaussian kernel embedded in the criterion has the effect of reducing the contribution of outlying samples that are located far away from zero [9]. This characteristic makes algorithms based on the ZEP-CME criterion insensitive to large errors induced mostly from impulsive noise.

The probability density function for CME $f_E(e_{CME})$ can be constructed by the kernel density estimation method where outliers can be cut out by the Gaussian kernel [10].

$$f_{E}(e_{CME}) = \frac{1}{N} \sum_{i=k-N+1}^{k} \frac{1}{\sigma \sqrt{2\pi}} \exp\left[\frac{-(e_{CME} - [|y_{i}|^{2} - R_{2}])^{2}}{2\sigma^{2}}\right]$$

$$= \frac{1}{N} \sum_{i=k-N+1}^{k} G_{\sigma}(e_{CME} - e_{CME,i})$$
(4)

The zero-error probability of CME is obtained by letting $e_{\it CME}$ be zero as

$$f_E(e_{CME} = 0) = \frac{1}{N} \sum_{i=k-N+1}^{k} G_{\sigma}(e_{CME,i}^2)$$
 (5)

The criterion of ZEP-CME (5) retains the Gaussian function of $e_{CME,i} = \left|y_i\right|^2 - R_2$ so that output samples where the instant output power $\left|y_i\right|^2$ is much greater than R_2 are cut out. Usually the large instant output power comes from impulsive noise. This implies that blind algorithms designed

based on maximizing the criterion ZEP-CME can be stable contrary to the MSE-based cost function (2) or (3) [9].

One of the basic ideas of the ITL method is that data samples can be considered as physical particles in physics. That is, the CME sample $e_{CME,i}$ in (5) is considered as information particles placed at the locations of $e_{CME,i}$. The Gaussian kernel $G_{\sigma}(e_{CME,i}^2)$ has an exponential decay with the distance between the particle $e_{CME,i}^2$ and the particle located at zero. This leads us to consider the Gaussian kernel as a potential field inducing interaction among the information particles. The right term $\frac{1}{N}\sum_{i=k-N+1}^k G_{\sigma}\left(e_{\mathit{CME}_{-i}i}^2\right)$ in (5) is corresponding to the sum of interactions on the $\text{particle}\, e_{CME,i}^2 \,. \,\, \text{The inequality} \,\, \frac{1}{N} \sum_{i=k-N+1}^k G_\sigma(e_{CME,i}^2) < \frac{1}{N} \sum_{i=k-N+1}^k G_\sigma(0)$ indicates that maximization of ZEP-CME (MZEP-CME) leads to the result that all $G_{\sigma}(e_{CME,i}^2)$ become $G_{\sigma}(0)$, that is, all CME samples concentrate on zero. Therefore, blind algorithms designed on the MZEP-CME can have the capability of cancelling ISI as well as the immunity to impulsive noise.

One drawback of the criterion ZEP-CME can be the computational burden from the summation operations in (5) at each iteration time. This burden hinders the efficient implementation of blind equalizers based on the criterion ZEP-CME. In the following section, a method of reducing the computational complexity significantly by investigating how the computation of ZEP-CME, that is, the sum of interactions $\frac{1}{N} \sum_{i=k-N+1}^{k} G_{\sigma}\left(e^{i}_{CME-i,i}\right)$ is done and whether any simpler methods can be possible based on the analysis.

3. PROPOSED ESTIMATION METHOD OF ZEP-CME

According to the concept of information particle, the ZEP-CME (5) can be regarded as the sum of interactions among a block data of N squared CME samples $\left\{e_{CME,k}^2,e_{CME,k-1}^2,...,e_{CME,k-N+1}^2\right\}$. When a new squared CME sample $e_{CME,k+1}^2$ comes into the data block at time k+1, the data block becomes $\left\{e_{CME,k+1}^2,e_{CME,k+2}^2,...,e_{CME,k-N+2}^2\right\}$, and the

sum of interactions is renewed by computing and summing all interactions among the squared CME samples. It is observed that the old squared CME sample $e^2_{CME,k-N+1}$ leaves the data block as the new squared CME sample $e^2_{CME,k+1}$ comes into it. This indicates that the interactions between $e^2_{CME,k-N+1}$ and the other squared CME samples are discarded while the new interactions between $e^2_{CME,k-N+1}$ and the others are added to the sum. It is noticeable that each new ZEP-CME at time k+1 might be calculated just by adding new interactions related with $e^2_{CME,k+1}$ to the current sum, and subtracting old interactions related with $e^2_{CME,k-N+1}$ from the current sum. This points out that a simpler method for ZEP-CME calculation can be possible than the block processing method in (5).

Since the data block for the summation is not filled full in the initial state $1 \le k \le N$, two cases will be discussed separately as the $\int_E^I (e_{CME} = 0)|_k$ is for the initial state and $\int_E^S (e_{CME} = 0)|_k$ is for the steady state k > N.

$$f_E^I(e_{CME} = 0)\Big|_k = \frac{1}{k} \sum_{i=1}^k G_\sigma(e_{CME,i}^2)$$
 (6)

$$f_E^S(e_{CME} = 0)\Big|_k = \frac{1}{N} \sum_{i=k-N+1}^k G_\sigma(e_{CME,i}^2)$$
 (7)

Firstly, the initial state ZEP-CME at time k+1 $f_E^I(e_{\rm CME}=0)\big|_{k+1}$ will be divided into some terms related with $e_{\rm CME,k+1}^2$ and the remaining.

$$\begin{split} f_{E}^{I}(e_{CME} = 0)\Big|_{k+1} &= \frac{1}{k+1} \sum_{i=1}^{k+1} G_{\sigma}(e_{CME,i}^{2}) \\ &= \frac{k}{k+1} \frac{1}{k} \sum_{i=1}^{k+1} G_{\sigma}(e_{CME,i}^{2}) \\ &= \frac{k}{k+1} \left[\frac{1}{k} \sum_{i=1}^{k} G_{\sigma}(e_{CME,i}^{2}) + \frac{1}{k} G_{\sigma}(e_{CME,k+1}^{2}) \right] \\ &= \frac{k}{k+1} \left[f_{E}^{I}(e_{CME} = 0) \Big|_{k} + \frac{1}{k} G_{\sigma}(e_{CME,k+1}^{2}) \right] \\ &= \frac{k}{k+1} \cdot f_{E}^{I}(e_{CME} = 0) \Big|_{k} + \frac{1}{k+1} \cdot G_{\sigma}(e_{CME,k+1}^{2}) \end{split}$$

$$(8)$$

In the steady state,

$$\begin{split} f_E^S(e_{CME} = 0) \Big|_{k+1} &= \frac{1}{N} \sum_{i=k-N+2}^{k+1} G_{\sigma}(e_{CME,i}^2) \\ &= \frac{1}{N} \Big[\sum_{i=k-N+2}^{k} G_{\sigma}(e_{CME,i}^2) + G_{\sigma}(e_{CME,k+1}^2) \Big] \\ &= \frac{1}{N} \Big[\sum_{i=k-N+1}^{k} G_{\sigma}(e_{CME,i}^2) + G_{\sigma}(e_{CME,k+1}^2) - G_{\sigma}(e_{CME,k-N+1}^2) \Big] \end{split}$$

That is,

$$\begin{aligned} f_E^S(e_{CME} = 0) \Big|_{k+1} &= f_E^S(e_{CME} = 0) \Big|_k \\ &+ \frac{1}{N} [G_{\sigma}(e_{CME,k+1}^2) - G_{\sigma}(e_{CME,k-N+1}^2)] \end{aligned}$$
(9)

The equations (8) for the initial state shows that the next ZEP-CME $f_E^I(e_{CME}=0)\big|_{k+1}$ can be calculated with the squared CME $e_{CME,k+1}^I$ and the current ZEP-CME $f_E^I(e_{CME}=0)\big|_k$. The equation (9) indicates that in the steady state, ZEP-CME at the next iteration time k+1 can be estimated recursively with the current ZEP-CME and a combination of interactions related with the newly coming squared CME and the discarded one. Furthermore, this recursive estimation of ZEP-CME contains no summation operations at all, contrary to the block-processing method of ZEP-CME (5).

4. GRADIENT OF ZEP-CME FOR BLIND EQUALIZATION

Assuming the structure of tapped delay line (TDL) with L weights is employed, the output y_k at time k with the input vector $\mathbf{x}_k = [x_k, x_{k-1}, ..., x_{k-L+1}]^T$ and equalizer weight vector $\mathbf{w}_k = [w_{0,k}, w_{1,k}, ..., w_{L-1,k}]^T$ can be $y_k = \mathbf{w}_k^T \mathbf{x}_k$. Blind algorithms designed based on the performance criterion ZEP-CME can be derived from the MZEP-CME with respect to the equalizer weights, in which the gradient of the criterion and the steepest ascent method are used.

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu \cdot \frac{\partial f_E(e_{CME} = 0)|_k}{\partial \mathbf{W}}$$
(10)

where μ is the step-size for convergence control.

When we utilize the block processing method (5), the following MZEP-CME algorithm can be obtained.

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu_{ZEPCME} \cdot \frac{2}{\sigma^2 N} \sum_{i=k-N+1}^k G_{\sigma}(e_{CME,i}^2)$$
$$\cdot (R_2 - |y_i|^2) \cdot y_i \cdot \mathbf{X}_i^*$$
(11)

On the other hand, it can be inferred that the gradient at time k+1 $\frac{\partial f_E(e_{CME}=0)|_{k+1}}{\partial \mathbf{W}}$ might also be calculated by differentiating the ZEP-CME in (8) and (9) instead of (5). By differentiating (8), the gradient in the initial state $\frac{\partial f_E'(e_{CME}=0)|_{k+1}}{\partial \mathbf{W}}$ becomes

$$\frac{\partial f_{E}^{I}(e_{CME} = 0)|_{k+1}}{\partial \mathbf{W}}$$

$$= \frac{k}{k+1} \cdot \frac{\partial f_{E}^{I}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}} + \frac{1}{k+1} \cdot \frac{\partial}{\partial \mathbf{W}} G_{\sigma}(e_{CME,k+1}^{2})$$

$$= \frac{k}{k+1} \cdot \frac{\partial f_{E}^{I}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}}$$

$$+ \frac{1}{k+1} \cdot \frac{G_{\sigma}(e_{CME,k+1}^{2})}{-2\sigma^{2}} \frac{\partial e_{CME,k+1}^{2}}{\partial \mathbf{W}}$$

$$= \frac{k}{k+1} \cdot \frac{\partial f_{E}^{I}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}}$$

$$- \frac{2}{(k+1)\sigma^{2}} G_{\sigma}(e_{CME,k+1}^{2}) e_{CME,k+1} y_{k+1} \mathbf{X}_{k+1}$$
(12)

On the other hand, the gradient in the steady state $\frac{\partial f_E^S(e_{\text{CME}}=0)\big|_{k+1}}{\partial \mathbf{W}} \quad \text{can be obtained By differentiating (9) as}$

$$\frac{\partial f_{E}^{S}(e_{CME} = 0)|_{k+1}}{\partial \mathbf{W}} = \frac{\partial f_{E}^{S}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}} + \frac{1}{N} \left[\frac{\partial}{\partial \mathbf{W}} G_{\sigma}(e_{CME,k+1}^{2}) - \frac{\partial}{\partial \mathbf{W}} G_{\sigma}(e_{CME,k-N+1}^{2}) \right] \\
= \frac{\partial f_{E}^{S}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}} + \frac{1}{N} \left[\frac{G_{\sigma}(e_{CME,k+1}^{2})}{-2\sigma^{2}} \frac{\partial e_{CME,k+1}^{2}}{\partial \mathbf{W}} - \frac{G_{\sigma}(e_{CME,k-N+1}^{2})}{\partial \mathbf{W}} \right] \\
= \frac{\partial f_{E}^{S}(e_{CME} = 0)|_{k}}{\partial \mathbf{W}} - \frac{2}{\sigma^{2}N} \left[G_{\sigma}(e_{CME,k+1}^{2}) + e_{CME,k+1} \cdot y_{k+1} \mathbf{X}_{k+1} - G_{\sigma}(e_{CME,N+1}^{2}) \cdot e_{CME,k-N+1} \cdot y_{k-N+1} \mathbf{X}_{k-N+1} \right] \tag{13}$$

With (12) and (13), the weight update operation in (10) is now carried out recursively. As opposed to the block-processed gradient (11) having a summation operation, the recursive equations (12) and (13) show that they have no summations. That is, the recursive gradient estimation can significantly reduce its computational complexity.

The computational complexity analysis could be carried out in the aspects of time complexity and number of operations (usually multiplications), but in this paper the number of operations is analyzed according to conventional complexity analysis in the research field of equalization [11][12].

For the sake of convenience and comparison, we consider the common term $\frac{2}{N\sigma^2}G_{\sigma}(e_{CMEJ}^2)$ in (11) as a constant. Then the number of multiplications for the conventional gradient estimation of (11) is 4N. Similarly, defining $\frac{2}{N\sigma^2}G_{\sigma}(e_{CME,k-1}^2)$ and $\frac{2}{N\sigma^2}G_{\sigma}(e_{CME,k-N+1}^2)$ in (13) as a constant C_2 and C_3 , respectively, the number of multiplications for the proposed estimation of (13) becomes 6. The comparison of multiplication operations with respect to the data block size N is described in Fig. 1. It is noticeable that the number of multiplications of the conventional method increases linearly proportional to the data block size N while that of the proposed estimation remains as a constant 6 being independent of the data block size N. When N=30 which will be used in the simulation, the complexity ratio of the conventional method to the proposed estimation is 20:1.

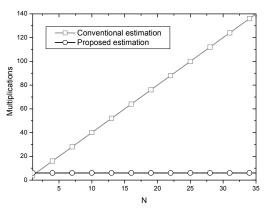


Fig. 1. Comparison of the number of multiplications for the data block size N.

5. SIMULATION RESULTS AND DISCUSSION

In this section, it will be investigated whether the proposed estimation of ZEP-CME yields the same estimation results as the block-processed ZEP-CME estimation method. Also the recursively estimated gradient of ZEP-CME for the weight update will be shown to have equal results to that of the block-processing method.

We use the same blind equalization environment as in [9]. The parameters for the simulation are the same too as the data-block size N = 30, the convergence parameter $\mu = 0.02$, and the kernel size is set to be $\sigma = 6$. In Fig. 2, the trace of ZEP-CME is described for the two methods; the block-processing method by (5), and the recursive estimation by (8) and (9). The probability ZEP-CME decreases rapidly with iterations as the algorithm (11) designed to maximize the ZEP-CME converges. The ZEP-CME converges at about iteration number 3000 complying exactly with the MSE convergence result appeared in [9]. However, in Fig. 3 focused in the initial part of iteration $(1 \le k < N = 30)$, a slight difference is observed that initially the two methods produce different ZEP-CME estimation results. But the two estimation traces get closer to each other and in the steady state $(k \ge N = 30)$ they are exactly the same. This difference is considered to be due to the condition of the data block, that is, the difference depends on whether the data block for the summation operation is filled full or not.

In Fig. 2 and 3, the block processing method and the recursive estimation method produce exactly the same learning curves right after N=30 regardless of how different the two curves are. Unless the time region before convergence is important for any other operations, the normal signal

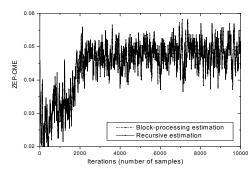


Fig. 2. The ZEP-CME estimation results of the blockprocessing method and the recursive one.

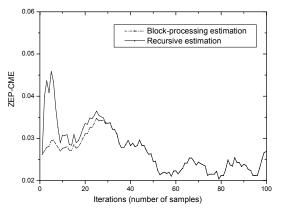


Fig. 3. The comparison of ZEP-CME results in the initial part of iteration.

processing operations are carried out after convergence. This indicates that the difference in the initial part $(1 \le k < N = 30)$ can be ignored because this time region is way before convergence.

The gradient is estimated separately by the block-processing method (11), and the recursive estimation (12) and (13). It is desirable to present gradient results for all (L=11) weights for fair comparison, but only the center weight gradient is shown in Fig. 4 and 5 just for the limited space of this paper. The Fig. 4 shows similar results that the traces of center weight gradient by the two methods go together except the initial part

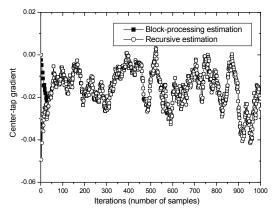


Fig. 4. Gradient for the center weight update for the two estimation methods of the block-processing method and the recursive one.

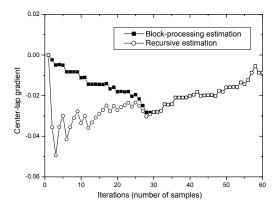


Fig. 5. Gradient comparison in the initial part of iteration. $(1 \le k < N = 30)$

In Fig. 5, it is observed apparently that the two methods yield exactly the same gradient estimation results in the steady state ($k \ge 30$) as observed in Fig. 3 for ZEP-CME estimation.

Recursive gradient estimation approaches to zero-error probability for CME have not been reported in the scientific literature as far as we know. So the convergence performance of the proposed method is compared with other recently introduced CMA algorithms based on recursive estimation of step size. In the recent works [13][14], modified CMA algorithms with variable step size (VSS) show improved convergence performance. The CMA with VSS (referred to as VSS-CMA in this simulation) can be viewed as utilizing recursively estimated autocorrelation of error signal \boldsymbol{e}_k as summarized in the followings.

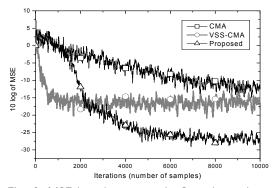


Fig. 6. MSE learning curves in Gaussian noise environment.

$$e_k = decision(y_k) - y_k \tag{14}$$

$$p_{k} = \alpha \cdot p_{k-1} + (1 - \alpha)e_{k}e_{k-1}$$
 (15)

$$\mu_{CMA,k} = \beta \cdot p_k^2 \tag{16}$$

$$\mathbf{W}_{k+1} = \mathbf{W}_k - 2\mu_{CMAk} \cdot y_k \cdot \mathbf{X}_k^* \cdot (|y_k|^2 - R_2)$$
(17)

Unlike the proposed method is for impulsive noise environments as in [9], the VSS-CMA algorithm has been designed only for Gaussian noise environment so that the MSE results are shown in two cases: the Gaussian noise case in Fig. 5 and the case of impulsive noise in Fig. 6. The parameters μ_{CMA} , for CMA, α , and β are set to be 0.000001, 0.999, and 0.000000001, respectively. The parameter values were chosen when they converged and yielded the lowest MSE curves.

In Fig. 6, VSS-CMA shows a very improved convergence performance speed compared to CMA thanks to the usage of the error correlation. In the impulsive noise case, however, the error correlation information causes the VSS to be highly unstable as depicted in Fig. 7. This implies that in (15) the multiplication of the two adjacent error samples e_k and e_{k-1} can yield an uncontrollably big value when they are afflicted by impulsive noise. This problem is considered to be part of the reason for the instability of VSS-CMA. On the other hand, the proposed algorithm converges well showing no sensitivity against impulsive noise.

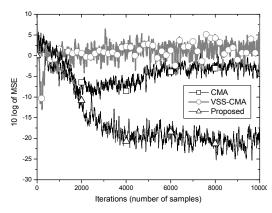


Fig. 7. MSE learning curves in impulsive noise environment.

6. CONCLUSION

The blind algorithms developed based on the criterion of ZEP-CME yield superior performance but one of its drawbacks is a heavy computational burden due to some summation operations at an iteration time. In this paper, a recursive estimation method of ZEP-CME is proposed and its gradient is derived by taking derivation operation to the recursive ZEP-CME. The recursive gradient is directly applied to the blind algorithm replacing the original block-processed gradient with the recursive one. This approach significantly reduces the computational complexity removing the summation operation. From the simulation results that the proposed estimation method of ZEP-CME and its gradient produces exactly the same estimation results in the steady state as the block-processing method, it can be concluded that the proposed method having a significantly reduced computational complexity can lead blind algorithms based on the ZEP-CME to their efficient implementation.

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