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Teacher Change in Teaching Practices Towards Developing Students' Reasoning in Mathematics[1]

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Research shows that formative assessment has a more powerful effect on student learning than summative assessment. This case study of an 8th grade algebra classroom focuses on how the implementation of *Formative Assessment Lessons* (FALs) and the participation in teacher learning communities related to FALs changed in the teacher's instructional practices, over the course of a year, to promote students' mathematical reasoning and justification. Two classroom observations are analyzed to identify how the teacher elicited and built on students' mathematical reasoning, and how the teacher prompted students to respond to and develop one another's mathematical ideas. Findings show that the teacher solicited students' reasoning more often as the academic year progressed, and students also began developing mathematical reasoning in meaningful ways, such as articulating their mathematical thinking, responding to other students' reasoning, and building on those ideas leading by the teacher. However, findings also show that teacher change in teaching practices is complicated and intertwined with various dimensions of teacher development. This study contributes to the understanding of changes in teaching practices, which has significant implications for teacher professional development and frameworks for investigating teacher learning.

Keywords: mathematical reasoning, formative assessment, classroom practice, accountable discussions
MESC Classification: C70, E50
MSC2010 Classification: 97C70, 97D99

1. INTRODUCTION

Recent reform movements of teaching and learning mathematics have focused on stu-

[1] A draft version of the article was presented at the 2013 Joint International Conference on Mathematics Education held at Seoul National University, Seoul 151-742, Korea; November 1–2, 2013 (*cf.* Kim, 2013).

dents' mathematical reasoning and justification (CCSSI, 2011; Kilpatrick, Swafford & Findell, 2001; NCTM, 1991; NCTM, 2000). In order for students to develop mathematical reasoning and justification, students should have opportunities to develop their mathematical thinking, present their ideas, analyze others' mathematical arguments, and evaluate strategies of others (NCTM, 2000). These particular classroom practices require traditional teachers to shift their teaching practices toward bringing their students' mathematical thinking to the surface and promoting students' accountable discussions. However, teachers seem to have difficulties in changing their teaching practices to the direction intended by reformers (Cuban, 1984; Gross, Giaquinta & Bernstein, 1971). Formative Assessment Lessons (FALs)[2] are developed with an intention of supporting teachers in their ability to elicit, challenge, and refine student mathematical thinking with contextually rich problems.

Formative assessment, also known as a classroom assessment or diagnostic test, is a reformed view of assessment. Unlike educational tests used as achievement tests, which measure how much students have learned up to a particular point in time, formative assessment considers assessment as parts of the learning process (Fleming & Chamber, 1983; Shepard, 2000). Research shows that assessment should be an integral part of instruction and it has more powerful effect on students learning (Black & Wiliam, 2004; NCTM, 1995; Kilpatrick, Swafford & Findell, 2001; Donovan & Bransford, 2005). Formative assessment supports both the teaching and learning processes in classrooms. When instructional practice engages students to reason and to confront and resolve their own misconceptions through discussion, it leads to substantial and long-term learning as students' thinking takes on a more central role in classroom teaching and learning (de Lange, 1999; Shafer & Romberg,1999; Shepard, 2000; also see reviews by Black & Wiliam, 1998).

Formative Assessment Lessons (FALs) are developed aligned with this classroom assessment for students' learning, especially focused on eliciting students' mathematical thinking and building on the ideas presented by students. Teachers who have used them reported that FALs provide a significant and productive learning environment: students' understanding of the mathematics expands and deepens by talking, analyzing, justifying, and/or revising their mathematical thinking. All students of different abilities can participate in mathematical activities by discussing their reasoning; and teachers can hear and see their students in new ways, thereby illuminating what they know and can do mathematically (MAP, 2012).

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[2] The *Formative Assessment Lessons* are devised by the Mathematics Assessment Project (see, http://map.mathshell.org) which is a project of U.C. Berkeley and the University of Nottingham. These free lessons have become a national resource, with an average of 80,000 downloads each month.

This case study demonstrates how a traditional teacher, Ms. Lee[3], changed her teaching practices toward revealing and building on students' mathematical reasoning. It shows how Ms. Lee's classroom interactions changed as she implemented FALs and participated in teacher learning communities.

2. PRIOR RESEARCH AND THEORETICAL FRAMEWORK

2.1. Teacher Change

Traditionally, research has argued that it is hard for teachers to change their teaching practices to the direction intended by reformers (Cuban, 1984; Gross, Giaquinta, & Bernstein, 1971). "The case of Mrs. Oublier" (Cohen, 1990) shows that the issue of teacher change is complicated. While Mrs. Oublier believed that she taught in a reformed way, observation found that her teaching still remained traditional. Though she utilized innovative approaches such as a new curriculum and new classroom organization, her teaching ways were traditional, with a belief about mathematics as a fixed body of right answers and old ways of classroom discourse. Mrs. Oublier used drills and right-answer questions instead of accountable classroom discussions about mathematical ideas. Mrs. Oublier did not have opportunities to learn more mathematics, to get feedback on her teaching in light of the new mathematics framework, or any other resources for improving her instruction. Learning from "The case of Mrs. Oublier", we provided supports for the teacher, Ms. Lee, to study mathematics in FALs, new teaching ways, and possible new ways and ideas for students to think about mathematics using FALs. As did Mrs. Oublier's case, the case of Ms. Lee in this study adds to the growing body of research that explores the complicated issue of teacher change.

2.2. Mathematical Reasoning and Accountable Talks

Developing and presenting students' mathematical reasoning in classroom discussion can provide rich opportunities for them to engage in coherent mathematical understanding and proficiency (CCSSI, 2011; Kilpatrick, Swafford & Findell, 2001; NCTM, 1991, 2000). This requires teachers to focus on students' mathematical understanding, such as encouraging students to reason mathematically, justify and present their own ideas, listen to the thoughts of others, and analyze and build on the ideas of others. These particular classroom practices also encourage teachers to promote Accountable Talk (Institute for Learning, 2010) by students, in which they respond to and further develop what others have said. To promote accountable discussions in the classroom, teachers need to recog-

[3] The names of the teacher and students are all pseudonyms.

nize and reinforce the strengths and abilities of each student by finding ways to help individual students engage in mathematical learning in the classroom when they are reluctant to do so on their own (Boaler, 2008; Cohen & Lotan, 1997; Horn, 2007; Schoenfeld, 2013). Teachers accomplish this accountable discussion by eliciting students' mathematical reasoning and making their thinking public to other classroom members. Therefore, accountable discussion provides rich opportunities for students to engage and develop their mathematical reasoning and justification in classrooms.

3. METHODS

3.1. Contexts and Data Collection

This case study employs several data sources: classroom observations, qualitative teacher interviews, ethnographic teacher learning community observations, and artifacts[4]. The participant teacher, Ms. Lee, is an 8th grade mathematics teacher with ten years of teaching experience in urban school district located in northern California. Ms. Lee taught three FALs in their regular teaching across the school year and participated in the teacher learning community offered by the research team, in which community members study lessons and pedagogies together and focus on how to elicit students' mathematical thinking. In this learning community, teachers discussed their students' mathematical thinking, pedagogical strategies and knowledge for teaching, and mathematical knowledge relevant to the lesson. Specifically, teachers studied FALs, rehearsed their teaching of FALs, analyzed their own students' pre-thoughts about the lesson, and read research articles that support their teaching. These learning communities were ethnographically observed and field-note was taken. The author observed and video-recorded all FALs classes and four regular classes (two were at the beginning of the year and two were at the end of the year). Teacher interview were conducted before and after every classroom observation and at the beginning and end of the school year. Several artifacts, such as students' pre-assessments and post-assessments of FALs, teaching materials, students' worksheet, posters, and so on were gathered.

3.2. Analysis

To investigate classroom interactions and teaching practices, qualitative measurement

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[4] The data for this study are drawn from a larger study, the Mathematics Assessment Project (Bill and Melinda Gates Foundation Grant OPP53342 to Alan Schoenfeld, UC-Berkeley, and Hugh Burkhardt & Malcolm Swan, University of Nottingham). http://map.mathshell.org/materials/index.php

tools emerged (i.e., the coding scheme) by iterative passes through the classroom video data. The coding scheme has two different dimensions: five different kinds of teaching practices and four different kinds of student discourse (Table 1 and Table 2). The video data gathered in the classroom observations (two FALs classes and two regular lessons) are analyzed using this coding scheme and then quantified according to how often a particular classroom practice occurs in each lesson segment.

Table 1. Identifying classroom practices focusing on teaching practice

Teaching Practice	Description and Examples in the Data
A. Explaining	Teacher explains concepts/methods. e.g., "Let's do the multiplication on your paper: 6 times 5 is 30."
B. Eliciting Students' Reasoning	Teacher elicits students' reasoning and justification. e.g., "Can you explain why this is up to 50?"
C. Known Answer Questions	Teacher asks students known answer questions WITHOUT leaving room for students to explain their reasoning. e.g., "**What's the percent of 1/2?**" "50 percent." "Excellent."
D. Promoting Accountable Talk and Authority	Teacher asks students to reframe someone else's strategies, prompt students for further participation, or ascribe students' ideas. e.g., "Do you agree with him? Why?" or "Can you expand on what Anna said or why it doesn't make sense to you?"

Table 2. Identifying classroom practices focusing on student talk

Student Talk	Description and Examples in the Data
A. Explaining Own Reasoning to the Teacher	Students' answer or talk about their own strategies and reasoning. e.g., "How do we know that when we went from 100 to 150 there's an increase of 50?" "**Okay, because 50% of 100 is 50, so you add that other 50% to the next one. Then, 100 plus 50 is 150.**"
B. Answering 'known answer questions'	Students answer '*known answer questions*' WITHOUT providing their reasoning. e.g., "What's the percent of 1/2?" "**50 percent.**" "Excellent."
C. Accountable Talk and Authority	Students explain/reframe/revoice what other students say. e.g.,"What Aria said was, so 100% and when you have 50% of it, you have 150%."

Teaching practices and student discussions across two regular lessons—one at the beginning of the year (October) and another lesson at the end of the year (May)—were segmented by whole classroom discussion sessions and coded by A, B, C, or D, as shown in Table 1 and Table 2. If the discourse was not counted, it was coded as others in order to compute the ratio.

4. FINDINGS

Analyses showed some changes towards eliciting student reasoning and promoting accountable discussion in Ms. Lee's teaching practices, as revealed through teacher discourse, but the students' accountable talks did not change meaningfully. Table 3 and Table 4 show the frequencies of the different types of teaching practices and student discussion. The author computed the ratio of A, B, C, or D to all coded discussion in each lesson episode.

Table 3. Frequencies of matched teaching practices

Type of Teaching Practices	**Regular Lessons in October (at the beginning of the year)**		**Regular Lessons in May (at the end of the year)**	
	Frequency	Percent	Frequency	Percent
A. Explaining	13	16.25%	11	14.47%
B. Eliciting Students' Reasoning	14	17.50%	22	28.95%
C. Known Answer Questions	45	56.25%	29	38.15%
D. Promoting Accountable Talk and Authority	8	10.00%	14	18.42%
Others	11	13.75%	8	10.53%

Table 4. Frequencies for matched student talk

Type of Students' Talk	**Regular Lessons in October (at the beginning of the year)**		**Regular Lessons in May (at the end of the year)**	
	Frequency	Percent	Frequency	Percent
A. Explaining Own Reasoning to the Teacher	12	20.69%	15	60.00%
B. Answering 'Known Answer Questions'	44	75.86%	9	36.00%
C. Accountable Talk and Authority	2	3.45%	3	12.00%

Figure 1 graphically illustrates how the teacher's practice shifted between regular lessons throughout the year. As shown in Figure 1 and in Table 3, Type C, "Known answer questions," significantly decreased (–18.42%). In contrast, Type B, "Eliciting students reasoning" increased (11.45%). Furthermore, while Type A, "Explaining," decreased (–1.78%), Type D, "Prompting accountable talk and authority," increased (8.42%).

Student discourse in the same lessons was analyzed. As the teacher elicited students' reasoning more often, students began responding to other students' reasoning and building on it. Figure 3 graphically illustrates the shift in students' classroom practices between the two lessons in October and in May. As shown in Figure 2 and in Table 4, Type A, "Explaining own reasoning to teacher," occurred more frequently (39.31%), whereas

Type B, "Answering to the 'known answer questions," decreased (–38.86%). However, Type C, "Student accountable talk and authority," did not show meaningful differences.

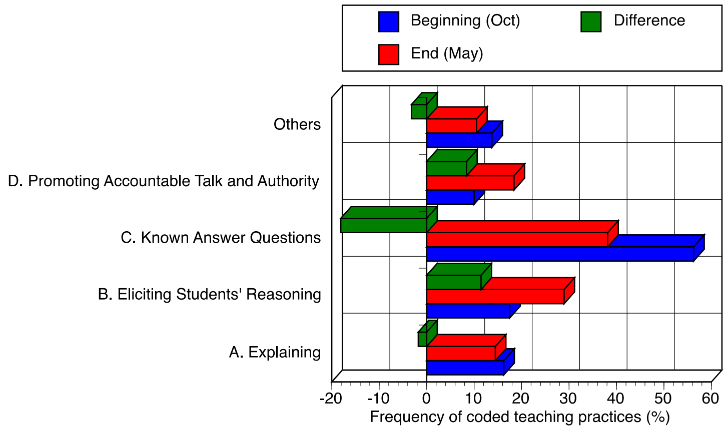

Figure 1. Differences in Ms. Lee's teaching practice in October and in May

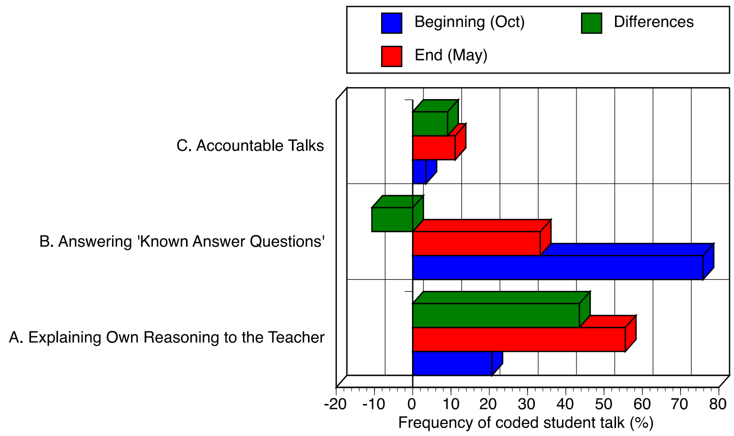

Figure 2. Differences in student discourse October and in May in Ms. Lee's classroom

As Table 3, Table 4, Figure 1, and Figure 2 illustrate, Ms. Lee's questioning style differed greatly between the two classes. In the October lesson, Ms. Lee asked mostly known-answer questions, which the students answered without reasoning, as in the following episode:

Ms. Lee: Why did you put in the middle?

Jacob (J): Uh…Because it's one half.

Ms. Lee: Okay, so where in the middle?

Here? [*Pointing one point in the middle of the number line*]

J: Uh… yeah.

Ms. Lee: Did you convert it or just put one half like that? [*Writes ½ in the middle of the number line*]

J: Just like that.

However, in the May lesson, the questions were very different and the students' answers were elaborate. Ms. Lee tried prompting students to have accountable discussion. For example,

Ms. Lee: That's one way. Anyone else show something else? [*Several students raise hands*] Go ahead, Eli.

Eli (E): I chose c.

Ms. Lee: Can you tell me how?

E: Well, 2 time 3 equals 6. So, if you already had that as 6, then you could just do 2*x* plus 6.

Ms. Lee: Philip, I just want you to explain what Eli was saying this, right here.
And another way to explain that is…Rosa, I want you to come back
[*Rosa is walking around the classroom*].
Remember the models we've worked on? The distributive property…

There were a few follow-up student discussions in Type C (Table 4), "Accountable talk." In the last episode of teacher discussion, Philip did not explain what Eli said because Ms. Lee's attention moved to Rosa's behavior. She then explained to the students instead of asking for Philip's reasoning. Ms. Lee's question functioned to promote students' accountable discussion, but the students' authority was not actually distributed. This example shows possible reasons for why the students' discourse of authority (Type C in Table 4) did not change meaningfully.

From the teacher interview data, Ms. Lee reported that her teaching has changed over the course of year. However, she faced a new challenge at the end of the year as reported as follows:

Ms. Lee: I feel like my regular teaching has changed a lot, because of the Common Core and because of the things that I'm doing in the group. I was like one of those, you know, the very traditional teachers. In the front give the examples, and "okay, here's the steps to follow, the steps, okay. This is the

> part you need to watch out, look at this." You know, I feel very comfortable with that part. The part that I would try to work with (as participating in this project) is where inside giving them information and asking them questions, in having students talk more. Try having them talk more and having them listen to each other. And then, that brought up another struggle… I feel like my management was so… more suffering. That's the part that I really kind of have to think about, you know, and I think about that with our current teachers too.

5. CONCLUSION AND DISCUSSION

Ms. Lee had been notably successful in changing in her teaching practices. The results indicate that some aspects of Ms. Lee's teaching practice shifted as a result of which she participated in the learning community and implemented FALs over the course of year. The collective classroom practice shifted toward soliciting students' reasoning, and the students also began to articulate their reasoning as FALs intended. However, with regard to students' accountable discussion and authority, although Ms. Lee often attempted to promote accountable discussion, the students did not build on each other's ideas or agree on shared ideas spontaneously. *Classroom interaction occurred between the teacher-student discourse rather than student-student interactions.* Furthermore, Ms. Lee had a new challenge in her teaching practice: loss of control of her class. Though she attempted to elicit students' mathematical reasoning, classroom management issues came up. At the beginning of the year, students engaged in mathematics activities well in classroom. However, at the end of the year, there were more students talking off-topic. It affected students' accountable discussion and created authority issues. As shown in the examples above, Ms. Lee tried to promote students' accountable talks when she asked Philip to explain what Eli said and tried to ask another student to explain other methods. However, Rosa was walking around the classroom while Ms. Lee indicated. Ms. Lee was distracted by Rosa's behavior and resorted to a traditional practice of explaining directly rather than allowing Philip or other students to share their reasoning.

There are at least two issues with characterizing the teacher change in practices from this case study. First, teacher change and development may have several dimensions and the change occurs not at one time, but interrelated with the various dimensions. Ms. Lee became a temporary novice teacher with the new curriculum and new pedagogies exploring the innovation through participating in learning communities and implementing FALs.

It is reminiscent of the three planes of teachers' professional activity described by Schoenfeld (2011): *classroom management, implementing engaging activities*, and *engaging in diagnostic teaching*. Teachers' instructional practices take place in these three planes. Relatively novice teachers allocate more of their teaching activities and time to the first plane, classroom management. More proficient teachers spend more time on the two other planes by engaging in students' understanding of the content. Good teachers can modify their curriculum for students to engage in content-related-activities, although they typically focus on activities themselves rather than on eliciting and using students' own ideas. Ms. Lee was neither a novice teacher nor a highly proficient teacher but could be considered a good teacher in terms of these three planes theory. Through implementation of FALs, she made changes in her teaching practices toward the third plane using students' mathematical thinking. Her teaching evolved between October (beginning of the school year) and May (end of the school year), but she also needed to allocate more time and efforts to classroom management. Ms. Lee's case suggests that she may have become a novice teacher in terms of the three planes paradigm, at least temporarily due to shifts in classroom circumstance. That is, the trajectory of teacher change in teaching practices may not occur sequentially or in a straightforward manner but intertwined. Her teaching seems to be still evolving in adapting the innovation toward students' mathematical reasoning.

Second, balance and flexibility between old habits and new practices are required for teachers to change their practices. The case study of Mrs. Oublier (Cohen, 1990) showed that the teacher's old classroom practices cannot easily shift to the new practices. As Cohen pointed out using personal conversation with Cuban (1984),

> "many teachers constructed hybrids of particular progressive practices grafted onto what they ordinarily did in classrooms" (recitation, Cohen, 1990, p. 323).

On the other hand, Ms. Lee's case indicated comparatively radical change in her teaching practices and it resulted in a new challenge of classroom management issue. The cases of both teachers seem to indicate that teachers may be required to have a flexibility to maintain some of their old routines while developing and enacting innovative teaching practices in order to sustain a balance between radical change with risks and too slow change (or not change)

These findings also have significant implications for the design of professional development that supports teachers to create mathematically productive environments. Unlike Mrs. Oublier, Ms. Lee had opportunities to study mathematics relevant to FALs, and to discuss her own students' mathematical thinking, her pedagogical strategies and knowledge for teaching in teacher learning communities. She rehearsed her teaching of FALs, analyzed her own students' pre-thoughts about the lesson, and read research articles that support their use in teaching. Furthermore, Ms. Lee had teaching experiences implementing FALs during her regular curriculum teaching. These professional development opportunities seemed to help Ms. Lee change in her teaching practices and it demonstrates how professional development can be designed for teachers to improve their classroom practices.

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