http://dx.doi.org/10.7468/jksmed.2014.18.3.187
J. Korean Soc. Math. Educ., Ser. D, Res. Math. Educ.
Vol. 18, No. 3, September 2014, 187–208

"Heart beating" of the classroom-Interaction in mathematics lessons as reflected in classroom discourse

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(Received July 27, 2013; Revised August 28, 2014; Accepted September 25, 2014)

This study engages in the features of interaction in elementary school mathematics lessons as reflected in the class discourse. 28 pre-service teachers documented the discourse during observation of their tutor-teachers' lessons. Mapping the interaction patterns was performed by a unique graphic model developed for that purpose and enabled providing a spatial picture of the discourse conducted in the lesson. The research findings present the known discourse pattern "initiation–response–evaluation / feedback" (IRE/F) which is recurrent in all the lessons and the teacher's exclusive control over the class discourse patterns. Hence, the remaining time of the lesson for the pupils' discourse is short and meaningless.

Keywords: classroom interaction, mathematics discourse, discourse pattern
MESC Classification: B52, B59, C60, C62, C70, C72
MSC2010 Classification: 97B50, 97C40, 97C60

0. INTRODUCTION

This study aims to characterise interaction patterns in elementary school mathematics lessons. It explores and presents the discourse types, using unique graphic model representation which facilitates the identification of these types and the mapping thereof. The assumption is that, except for the overt mathematical contents, the way by which the discourse is conducted can shed light on the method of teaching these contents. The term 'discourse' encompasses the issue of teachers' communication with their class. It comprises monologues and dialogues as well as communication in mathematical and daily language. In fact, the term relates to the wide variety of communicational acts which transpire in the course of the lesson.

The various approaches to mathematics teaching at the end of the 20th century and consequently the changes teachers have been required to introduce, forced teachers to

cope also with the need to generate changes in the class discourse. This concerns a change whereby teachers have to shift from conventional teaching patterns such as monologues and dialogues in a pattern of initiation–response–evaluation / feedback (IRE/F) to discourse patterns which consist of real discussions involving all the partners. This is part of adopting the constructivist approach which advocates that teachers' role is to navigate the class discourse, listen to the learners and help building the mathematical knowledge.

Study of the communication in class

Ever since classrooms have existed[1], one can distinguish three types of discourse which take place in them[2]:

1. Teacher's monologue: the teacher is speaking and the pupils only listen but are not allowed to speak.
2. Teacher-pupil dialogue: a dialogue between teachers and pupils which takes place only when pupils are given the permission to speak (until today in some classes pupils even must stand up when answering the teacher).
3. Discourse (class discussion): a discussion which involves several interlocutors. The teacher manages the discussion and determines who will be given the right to speak.

1. THEORETICAL BACKGROUND

Starting from the 1970s, researchers in the field of education extensively explored teachers' behaviour and to what extent it affected the pupils' attainments (Kounin, 1970; Good & Grouws, 1979). The overarching goal of most studies was mapping all the features of good teachers (based on their pupils' attainments) in order to propose their teaching patterns to others. The most common research method was observations.

At the end of the 1980s and beginning of the 1990s, the need arose to examine the class discourse through other approaches (Cazden, 1988; Cobb, Wood, Yackel & McNeal, 1992). The change in the approach stemmed from the introduction of new teaching and learning methods. In these years the motto was shifting from frontal teaching to teaching in groups and adopting the constructivist approach. Another approach on which educators based themselves in this period was the socio-linguistic approach which advocated that 'a linguistic discourse weaves the fabric of a class culture… language functions as a mediator in understanding' (Alvermann & Hayes, 1989, p. 307). This approach placed the patterns of the class discourse and the said contents at the centre of the empirical interest.

[1] This refers to a traditional class structure whereby one teacher stands vis-à-vis the pupils.
[2] The first two types were the most common whereas the third was rarer.

The socio-cultural approach to the study of cognition also allocates special place to the discourse (Halliday & Hasan, 1989). Speech in its wider sense is considered as some sort of over-activity which accompanies all other human activities, including thinking and attributes to them their special nature. Consequently, a new meaning is assigned to various terms such as 'teaching', 'learning' or 'knowledge acquisition'.

Implications of the investigated constructivist principles were manifested in the new mathematics curricula designed around the world. This refers to the standards issued by the National Council of Teachers of Mathematics in the United States (NCTM, 1989) and to its later updates and extensions (NCTM, 2000). The standards encompass almost every area associated with the teaching and learning of mathematics and they constitute for the mathematics teacher community a 'guide' in their practice. Below are several of the principles relations to the class discourse:

1. Learning transpires when pupils **build their knowledge in an active manner.**
2. **Learning must be done through discourse** in order to be effective.
3. Teachers **should avoid lecturing** and instead they should encourage learners to formulate problems and try solving them by themselves.
4. Teachers should conduct many **mathematical discussions** in class.
5. Mathematics teachers' role is not to deliver knowledge but rather to **guide and navigate learning processes**.

Hence, if the goal is to view teachers as 'change agents' whose role is to introduce new patterns of discourse, then they should be aware of the importance of the class interactions and the existence of socio-cultural and socio-mathematical norms which they inculcate by their words. These norms have a strong impact on the pupils' level of performance and attainments.

The constructivist approach as an epistemological orientation in mathematical education presented to the researchers another important issues, namely the way by which learners build their knowledge (Cobb, Wood & Yackel, 1993). This issue evoked greater interest in what is going on in the class, mainly from the point of view of the discourse. The discourse provides information not only about the way pupils think but also about the way teachers think of teaching in general and mathematics teaching in particular. Documenting the teacher-pupil interactions offers extensive information about the teachers. Holzman (1986) reinforces this approach, saying that the class discourse affects the insight of teachers themselves about the teaching methods and even widens it. In fact, teachers usually navigate and guide the class discourse but being partners to the occurrences they become part of the learning class community.

Patterns of class communication

Communication in a mathematics class is more difficult than in other lessons. Mathematics teachers teach not only a new language (this is also done by language teachers) but also terms, symbols, definitions and mainly thinking ways which are unique to this subject. Class communication is essential for understanding the learnt material. As indicated by Sierpinska (1998), 'We can consolidate the meaning of mathematical expressions — words, equations, diagrams — only when they become part of a discourse in which the pupil participates together with others' (Sierpinska, 1998, p. 55).

In their study, Brendefur & Frykhol (2000) investigated the standards' recommendations regarding mathematical communication in class. Below are three main patterns of this communication:

1. **Unidirectional communication** – this is the most common communication pattern. The teachers are 'masters' of the discourse and they mostly lecture. Sometimes they ask open-ended questions and enable a small number of pupils to respond in order to express their ideas. According to Thompson (1992), teachers adhere to this communication pattern because of the way they perceive the subject which they teach. They view mathematics as a 'static body of knowledge' and believe that the pupils' role is to passively absorb this knowledge.
2. **Contributive communication** – this communication pattern focuses on the interactions between teachers or pupils (dialogue) or between the pupils themselves. The discourse has a superficial nature and is mainly used for the purpose of clarifications or comments for correction. Cobb, Boufi, McClain & Whitenack (1997) discuss this type of discourse which is conducted between pupils during group work. This primarily concerns an informal discourse between the group members and the talks typically focus on corrections and comments.
3. **Instructive communication**– this is a pattern which relates to the highest level of teacher-pupil communication. This is an in-depth conversation which allows teachers to comprehend the pupils' way of thinking and to relate to it.
   The third pattern was not found in any of the classes and therefore the first two patterns will be further elaborated.

Unidirectional communication was for many years the prevalent pattern in class. Teaching in this way was considered 'effective' and the teacher-lecturer was considered the effective teacher (Smith, 1996). It was rather difficult to make teachers abandon this

communication pattern. Adhering to a conventional structure of a mathematics lesson[3] has contributed to the fact that teachers are the sole speakers. In the frontal teaching pattern teachers control the class and use monologues quite often. They were accustomed to inculcate knowledge in this way since they could maintain their status and exclusivity as the source of authority and knowledge. According to this approach there is hardly any room for other people's discourse. The demand (dictated to teachers) to generate a change in the lesson structure (which will entail a change in the class discourse) on the one hand and teachers' inability to abandon the conventional pattern of lessons on the other, form unclear discourse patterns. For example, moving from long monologues of the teacher to very rapid teacher-pupil dialogues which miss the point.

Contributive communication mainly relates to teacher-pupil discourse, known in the field of research as IRE pattern. This communication pattern is conducted in the following way: the teacher is the initiator who usually raises a question, one of the pupils responds, getsa feedback or immediate evaluation from the teacher. This cycle of question-response and then evaluation or feedback is repeated. Due to the fact that it is very rapid, this pattern is also metaphorically called 'Ping-Pong'. This is also the most common pattern in teaching and as such it aas been widely researched (Lemke, 1990; Shiniak, 1990; Cazden, 2001).

The IRE pattern is also referred to as the **triad dialogue**in the literature. It consists of three elements: **Initiation, Response** and **Evaluation.** This pattern was defined first by the researchers Sinclair & Coulthard (1975) after observing classrooms. This is an interaction pattern which repeats itself over and over during the lesson. Later the definition was expanded so that the teachers' response to the pupil's words would include not only an explicit (positive or negative) evaluation but also a feedback. Hence, this pattern is sometimes presented as IRE/F.

Various researchers (Wood, 1998; Peled & Blum-Kulka, 1997; Vardi-Rat, 2002; Nissan, 2009) corroborate the existence of the IRE 'cyclic pattern' and define it as a key element of the class discourse. This sequence is meant to help teachers to maintain the power relations in those classes where there are many learners and only one teacher. Moreover, it enables teachers' complete control over the class discourse since this discourse must pass through them. The main argument in favour of the wide use of this discourse pattern is that in regular learning teachers should keep and control over 'just' power relations among all the partners (Buzzelli & Johnston, 2001).

The element of initiation which is the first in the triad sequence serves as an essential

[3] Checking the homework given in the previous lesson, checking representative examples from this work on the board, presenting a new topic, illustration of the new assignments pupils are required to submit and, towards the end of the lesson, giving pupils individual work designed to practice the new topic.

and principal element of the class discourse. Teachers' initiation is usually manifested by a question or statement designed to lead and promote the topic of the lesson.

The second stage of the pattern relates to pupils' response which follows the teacher's initiation. Apparently pupils' responses are generally short and are sometimes even formulated (and sound) like a question or an incomplete assertion. The pupils' responses can be classified into those which meet the teacher's expectations, those which meet them only partially or those which do not meet the teacher's expectations at all (Lemke, 1990; Cazden, 2001).

The third stage of the pattern is the evaluation. At this stage of the triad discourse, similarly to the previous stages, different types of teachers' responses can be distinguished. These responses can range between providing a short and clear evaluation which constitutes a positive or negative reinforcement of the speaker and up to a total lack of response. The intermediate stages between these two options can be expressed by various ways, such as revoicing pupils' words, interpreting them, expanding and adding to them, different body movements and so on.

Lemke (1990) related extensively to the issue of interactions between partners to the class discourse in the contributive communication pattern. He underscored the fact that in the contributive communication teachers have a long series of exclusive activities which demonstrate their control in this type of communication. For example, controlling the lesson structure, starting and ending a topic, controlling the discourse pattern (monologue, dialogue), controlling the time allocated to pupils' speech, stimulating or stopping an activity, controlling behaviour and controlling the content area of the lesson.

Determination of pupils' turn to speak

One of the salient aspects of class interactions in which teachers have the right to consolidate their control, is determining the pupils' turn to speak. Keeping the order in which people will speak in class and supervising the time allocated to each speaker are exclusively decided by the teacher. There are several patterns by means of which teachers determine the pupils' turn to speak. For example:

Personal appointment: Granting the pupil the turn to speak is done by addressing the pupil personally, by name or implicitly by making a sign and establishing eye contact with the speaker.

Appointment after declaring an initiation: The teacher presents a question (initiation), inviting or encouraging pupils' participation. When pupils wish to take part they raise their hand and then the teacher chooses one of them.

Group appointment: This appointment does not always stem from the teacher's

wish. It concerns a joint response by several pupils who respond simultaneously to the teacher's initiation. In fact this is a case of taking the turn to speak without an appointment.

A learning framework with a large number of partners obliges the teachers to function as keepers of the power relations. They usually do it by applying the triad discourse. Thus, all the assertions pass through them and pupils get every turn to speak only by the teachers and under their supervision. They are the only ones who grant and control the turn to speak (Vardi-Rath & Blum-Kulka, 2005). This control stems from the teachers' authority and their specialisation in the studied subject.

Investigating the issue of determining the pupils' turn to speak enhances the problematic embodied in the fact that the teacher is alone opposite a large group of pupils. Cazden (2001) calls it the 'discourse traffic'. Teachers should be responsible for navigating the discourse traffic. By the very fact of being the teachers' responsibility, the role of determining the pupils' turn to speak becomes both a means of control and a means of defence of an individual versus a group.

Only a few means are available to the 'other partners', namely the pupils, in order to win control over the class discourse. These means are neither customary nor expected. Needless to say they are not prevalent in the lower grades of elementary school. When pupils use those means which are available to them, their action surprises the teacher. The asymmetry in the teacher-pupils interaction during the class discourse is perceived as normative and acceptable. Hence, every deviation from the norm not only surprises the teachers but also forces them to cope with unexpected situations.

Class discourse

The various reforms and the constructivist approach adopted in teaching emphasise the need to shift from frontal teaching to the mathematical-class discussion with all its components. A class discussion which is appropriately conducted should lead pupils to discuss the learnt subject in a rich language which includes the terms associated with the subject, in this case mathematical terms (Sfard, 2008; Howe, 2009).

Levenberg (1998) indicated that teachers do not know what is the exact meaning of the term 'class discussion' and in what way it should be implemented in class. The term 'mathematical discussion' is perceived as close to the term IRE. Similarly, Larson (2000), who investigated the perception of the term 'class discussion', founda vast differentiation in the way the term is perceived. Among the perceptions one can mention for example: pupils' recitation of the teacher's words, teacher-guided conversation, collection of responses to challenge questions, guided delivery of knowledge and others.

Cazden (1988) describes discussion as a sort of class discourse whereby pupils are not

captive in the familiar IRE discourse pattern; rather, they freely present ideas and converse with each other. This is a change in the role played by teachers, turning them from the source of authority and knowledge into partners to the discourse. In a subject like mathematics which is considered unique and difficult, the knowledgeable teachers naturally become the exclusive source of authority. Therefore changing the teachers' role in this lesson is not easy.

Lampert (1998), who explored in-depth this issue of mathematics teachers' role in class discourse, based herself on the assumption that teachers should not adhere to the traditional IRE pattern. In order to conduct a proper class discussion about mathematics, teachers must plan well the questions suitable to the discussion and choose the way of managing the discussion. For example, they should decide whether to grant the turn to speak to a specific and elected pupil or to conduct an open discussion with anybody who wants to participate in it. Teachers must identify in real time when to be involved and when to step aside and allow the transpiring process to flow with no intervention. They have to see to it that the mathematical contents are correct and the mathematical language is accurate (Nahlieli & Regev, 2009).

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2. THE RESEARCH METHODOLOGY

During the mathematics teaching education programme, the pre-service teachers [hereunder – students] observe for many hours their tutors. In one of the teaching workshops they were asked to document (record or write down) all the occurrences of the lessons which they were observing, in order to learn the interaction patterns.

The research question presented to the students was: 'What are the interaction patterns in elementary school mathematics lessons?'

2.1. Research population and research tools

28 pre service teachers (students) fully documented, each one in their class, everything that went on during one mathematics lesson. Overall, a population of 28 mathematics teachers in the 1st – 6th grades in five different schools was investigated.

The research tool was observations. Everything said in the lessons was recorded or written during the observation. Moreover, a full documentation of the events and occurrences during the lesson was performed, including what was written on the board. Twelve students recorded the lesson with a sound system and 16 of them wrote down every word said both by the teacher and by the pupils. All the lessons were entirely transcribed.

The observations and the lesson transcriptions made by the students transpired in the following grades:

Five 1st grade classes (three classes learnt arithmetic and two others learnt geometry).
Four 2nd grade classes (all the classes learnt arithmetic).
Four 3rd grade classes (three classes learnt arithmetic and one class learnt geometry).
Nine 4th grade classes (six classes learnt arithmetic and three classes learnt geometry).
Four 5th grade classes (all the classes learnt arithmetic).
Two 6th grade classes (all the classes learnt arithmetic).

2.2. Data processing method

2.2.1. Transcription

The transcription included all the events which transpired during the lesson, paying full and accurate attention to everything that was said, written and done in the lesson. All the transcriptions are illustrated in Table 1.

Table 1. Example of the method of documenting the observations

	Speaker	**Statement**	**Events, action, board and comments**
1	Teacher	-	Writing the following exercise on the board $3 + 4 \times 2$
2	Teacher	'What did I write on the board?'	-
3	Rami	'3 plus 4 multiplied by 2'	-
4	Teacher	'What is this exercise?'	-
5	Adam	'Expansion'	-

The column 'statement' documents everything said aloud. The other occurrences in

the class, including comments, were written in the column of 'Events, action, board and comments'. This column reflects in fact all the non-verbal events in the lesson. The information included in these tables serves as the main data of this study (see Appendix A).

The speakers' number in the first column is used for the representations of the various graphs. In each graphic pattern or citation one can identify what is being said by the number indicated next to it, this number matching the numbering in the lesson transcription (see Appendix A).

2.2.2. Choosing a transcription excerpt

After performing a full transcription of the lessons and inserting them in the table, each of the students chose an excerpt of the transcription of a discourse which was conducted in the class. The chosen excerpt lasted about seven minutes of the lesson duration. Some of the students meticulously indicated the duration of the discourse. Those who recorded the lesson, measured the duration of the discourse by means of a stopwatch.

The considerations for choosing the excerpts were the following:

1. The chosen excerpt included a continuous class discourse, without prolonged breaks for individual work or disruption of the lesson by external elements.
2. The chosen excerpt lasted about seven minutes for each participant.
3. In the chosen excerpt both the teacher and the pupils took part and it contained exchanges of words.
4. According to the student, the chosen excerpt reflects most faithfully the way mathematics is learnt in this class.

In light of these considerations, the chosen excerpts are not entirely identical from the point of view of the teaching activity duration. However, the duration gaps do not exceed 1–3 minutes.

2.3. The graphic tool

The graphic tool was built for the purpose of making a visual mapping of the discourse and it presents in a concise and holistic way what transpires during the class discourse. Using the tool one can notice the dynamics of the class discourse and the discourse patterns. After explaining the signs appearing in the graphic description this tool will be used for presenting the discourse of three teachers. Moreover, the tool sensitivity and uniqueness will be explained.

2.3.1. Explanation of the graphic representation

Below is the explanation of the way by which the graphic description was built:

1. The teacher is located at the centre of the page and is marked with a square. The teacher's name is indicated at the centre (all the names are fictitious).
2. The class pupils were marked around the teacher. Each of the pupils is indicated by an ellipse. The number of elliptic signs is identical to the number of the class pupils.
3. If the name of the speaking pupil is known, he or she is indicated within the ellipse (the name appears also in the lesson transcription).
4. If the pupil's name is unknown, the words 'male-pupil/female-pupil' are written in the elliptic sign.
5. An empty elliptic sign symbolises a pupil who was present in class but did not speak throughout the lesson.
6. The teacher's monologue is marked by a **purple arc** which goes out of the teacher's square and goes back to it.
7. A teacher addressing a pupil is marked by a **red arrow** which goes out of the teacher's square to the ellipse of the pupil addressed by the teacher.
8. A teacher addressing a pupil is marked by a **blue arrow** when the teacher's words are a feedback to the pupil's words.
9. A pupil addressing a teacher is marked by a **green arrow** which goes out of the pupil's ellipse in the direction of the teacher.
10. A dotted **orange** circle marked around the teacher indicates a teachers' question addressed to the entire class.
11. A full **black** circle around the teacher indicates a joint statement of the class pupils.
12. A numerical marking is presented above all the graphic signs, relating to the number of the line in the lesson transcription.
13. Every graphic sign represents a **whole turn to speak*** regardless of the **amount of utterances**** which is includes.
    ***Whole turn to speak:** from the moment of getting (or taking) the permission to speak until the end of the spoken words. In the transcription every turn to speak is marked by a different number.
    ****Utterance**: is the smallest unit of discourse which has a meaning and is followed by a break. This break closes the unit of meaning.

This pattern which presents 'as a picture' all the speaking events in class by a time unit, enables fast 'reading' of the discourse events during the lesson. Presentation of these graphic patterns facilitates not only quick acquaintance with all the discourse events but

also comparison of lessons delivered by different teachers.

After receiving an explanation about the graphic tool[4], each of the students drew the graphic model which represents the class discourse according to the transcription.

2.3.2. Classification of the pupils' turn to speak

According to the transcription and the graphic representation, the number of the turns to speak of all the speaker in the class was also investigated. There was a separate counting of the turns to speak granted to the entire class and of the turns to speak granted to a single pupil, namely the monologues and dialogues were counted (see Table 2).

Table 2. Classification of the pupils' turn to speak during the lessons

	Teachers			**Pupils**				
	No of times a teacher spoke in a **monologue**	No of times a teacher spoke in a **dialogue**	Total no. of turns to speak **teacher**	No of times a pupil spoke in a **dialogue**	No of times the class addressed the teacher ('choir')	No of times a **pupil addressed another pupil**	Total no of turns to speak **pupils**	No of speaking pupils out of the **entire class (%)**
Rachel	**10**	**4**	**20**	**5**	**5**	**-**	**10**	**17%**
Limor	**12**	**9**	**22**	**15**	**4**	**-**	**19**	**38%**
Savrin	**8**	**7**	**21**	**11**	**2**	**---**	**13**	**28%**

Table 2 illustrates the data collected about the number of the turns to speak and the addressee to whom the speaker speaks. These data are based on the chosen part of the lessons of each of the three teachers.

2.3.3. Findings of the graphic tool

Below are three graphs chosen as an example out of the total 28 graphs which were drawn. Figure 1 represents the discourse in a *geometry* lesson in the 1st grade of the teacher Rachel.

Figure 2 and Figure 3 (resp.) represent the discourse in arithmetic lesson in the 4th grade of the teachers Limor and Savrin (resp.).

[4] The graphic tool was developed within the framework of a research work designed to investigate discourse in secondary education (Levenberg, 2010)

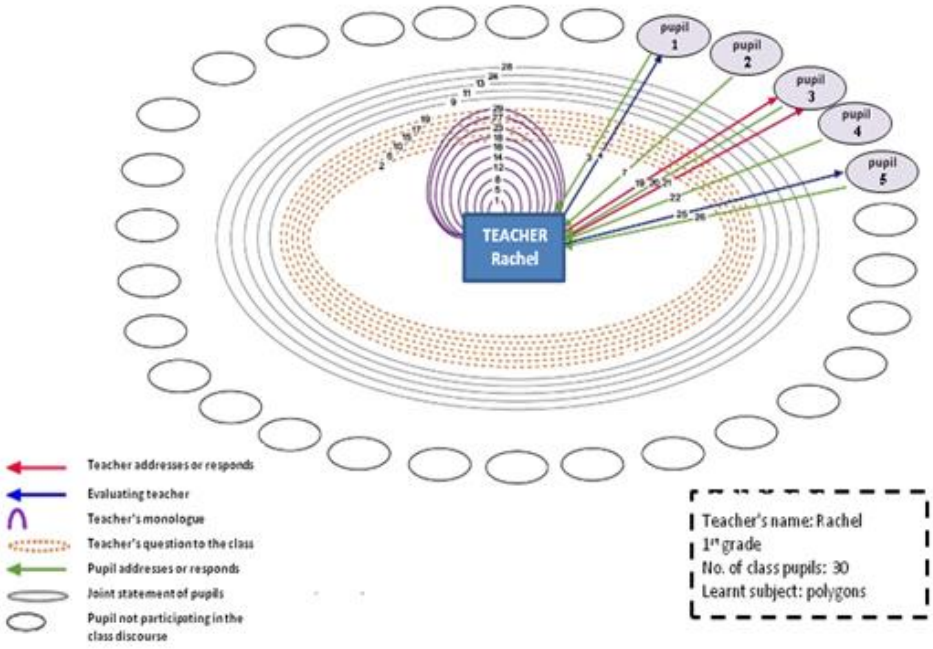

Figure 1. Mapping the speaking events in Rachel's lesson – 1st grade

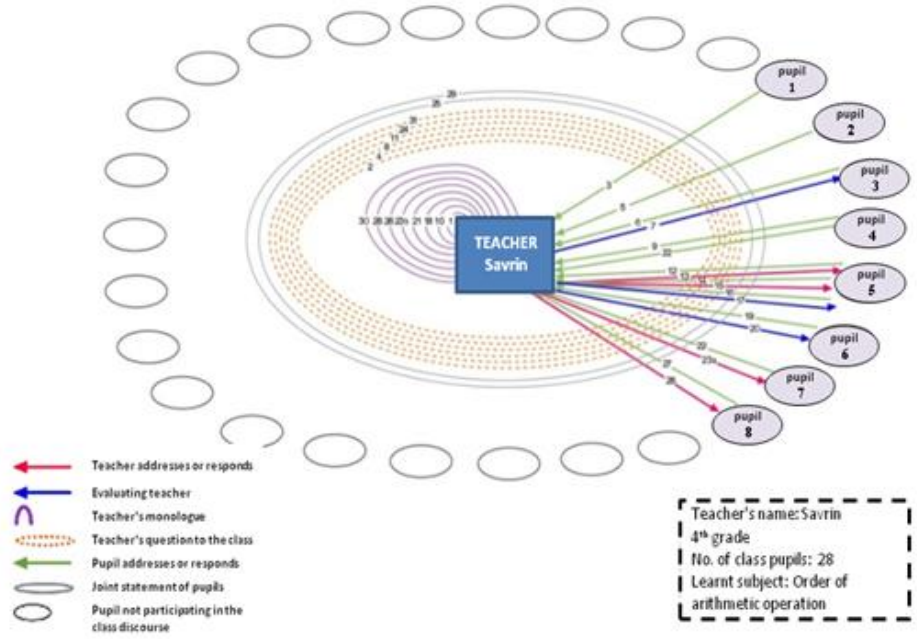

Figure 2. Mapping the speaking events in Savrin's lesson – 4th grade

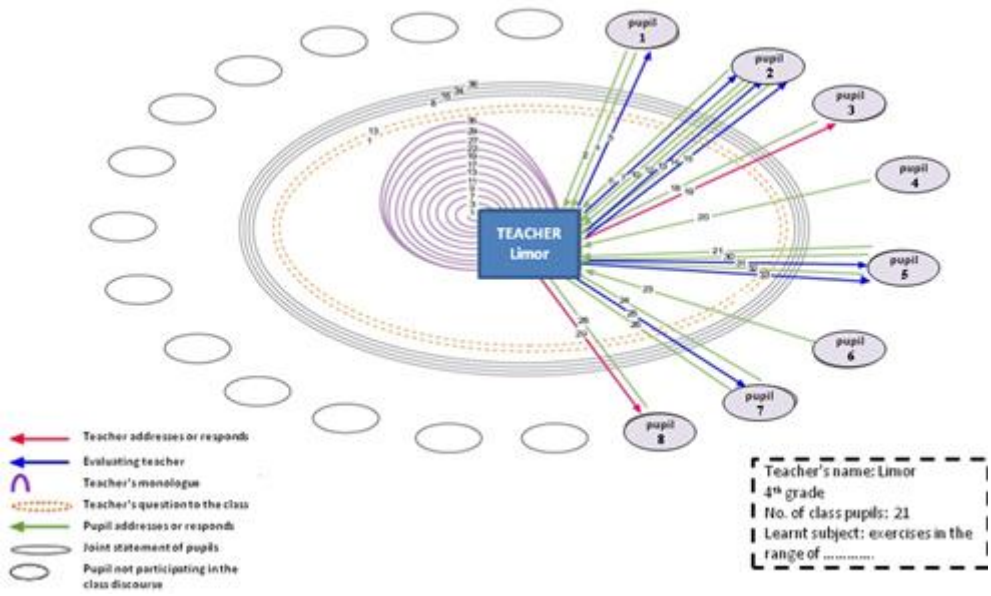

Figure 3. Mapping the speaking events in Limor's lesson – 4th grade

3. FINDINGS

Based on the entirety of collected data and graphic representations built for all the transcriptions, three teachers were chosen as a representative sample. The following findings were illustrated regarding the interactions between the partners to the class discourse.

a. The number of the teachers' turns to speak was double that of the pupils, mainly in the 1st and 2nd grades.
b. In the high grades (4th to 6th), the number of the turns to speak of all the class pupils combined together was on average about 55% of the total number of the turns to speak in the class.
c. The number of speaking pupils did not exceed 38% in any of the classes (the highest percentage was in Limor's class). In most of the classes the number of speakers amounted to about 30% of the total number of the class pupils. This fact is mainly salient when we examine the graphic representations. Each of them comprises a large number of 'empty ellipses' which symbolise the pupils who usually did not take part in the discourse during the lesson. This finding is clearly manifested in the graphic representation and therein resides the uniqueness of the tool.

d. There were no pupils who talked with each other during the lesson. All the class discourse was conducted through the teacher. None of the pupils directly addressed another pupil in any of the classes.
e. When checking the number of times the entire class addressed the teacher (pupils who speak in a 'choir') the finding show that in the lower grades (1st or 2nd) speaking in a choir is more prevalent and the teachers even encourage that. In the higher grades the frequency of speaking in a choir decreases and some teachers even remarked that responses in a choir are prohibited.
f. The number of times when the teacher talked to the entire class (monologues – long or short) was not uniform. Some teachers address the class a lot whereas others do it only seldom. This difference constitutes one of the indicators which demonstrate the teachers' personality and the professional conduct which they have adopted.
g. After counting the turns to speak, a gap (albeit not very big) was found between the pupils who were granted the right to speak. However, the findings illustrate that the number of turns cannot express the true ratio between the teacher's part and that of the pupils in the class discourse. The pupils' turn to speak usually comprises a very small number of words (sometimes even one word only). Conversely, the teachers' turn lasts much longer (starting from several sentences said consecutively and up to monologues which are several minutes long).
h. The graphic representation enables us to realise immediately that in every class there are 'active pupils'. Those are the pupils with whom the teacher spoke more than once in a lesson. They are prominent in the number of dialogues they conducted with the teacher as well as in the number of feedbacks they received from him or her.

4. CONCLUDING REMARKS

This study aimed to explore the interaction patterns in elementary school mathematics lessons. Two main patterns were found: the teacher's monologue and the teacher-pupil dialogue. The class discourse passed always through the teachers and no discourse among the pupils was shown. In order to map the class discourse a graphic model was built and it illustrates the discourse patterns and the turns to speak.

The graphic tool developed for investigating the interactions in class clearly indicates the exchange of the turns to speak between the teacher and the pupils. Moreover, it allows identification of the groups of pupils who were not involved in the class discourse as well as the 'active' pupils with whom the teacher spoke more than once. When examining the

discourse in the classes which participated in the study, the graphic representation serves as a 'mirror' of all the interaction patterns transpiring in the class. More than everything it presents the teacher's full control over the discourse.

By means of the graphic tool we identified the IRE discourse pattern which was recurrent in all the classes as well as the extent to which each of the teachers applied it. This discourse pattern is marked by red, green and blue arrows which are repeated in a cyclic way, frequently several times with the same pupil as can be seen in the graph of Limor. A sequence of dialogue as an IRE pattern illustrates not only the features of the interaction in class and the teacher's control over the discourse. It mainly shows the way mathematics is studied and the way the mathematical class discourse is conducted.

When conducting an interaction in the IRE discourse pattern we clearly see in fact what Peled & Blum-Kulka (1997) refer to as 'apparent dialogism'. The IRE pattern is built of a question of knowledge initiated by the teacher, a short answer by the pupil and the teachers' feedback. A dialogue of this type cannot be considered as a real dialogue between interlocutors. According to Lemke (1990), cycles of these patterns constitute in fact a controlled continuation of the teachers' monologue. Nevertheless, in an educational framework with a large number of participants (as was the case in most of the investigated classes), learning can transpire when the power relations in the class are maintained. The use of the IRE/F pattern enables teachers to maintain their power and every discourse begins and ends with them and occurs by their initiative. As already mentioned, this finding is in line with findings of other studies which examined interactions in the class.

Consequently, teachers control the class discourse with regard to the number of times they grant themselves the right to speak and of the duration of time they act as speakers. Hence, the remaining time of the lesson which is dedicated to the pupils' discourse is short. This situation does not allow a real discussion as advocated by the constructivists approach. This occurrence was found in all the classes, regardless of the pupils' age and the learnt subject.

One of the interesting finding was the way the IRE discourse pattern is inculcated in the pupils already at the 1st grade. In this grade teachers tend to allow pupils to speak together in a choir more than in higher grades. However, the number of teacher-pupil dialogues in these classes is lower than in the higher grades. The discourse patterns and the teacher's full control over this discourse are taught to the pupils in the elementary school grades. Hence, it is likely to assume it would be hard to change them later on in mathematics studies. Findings of this study support and even explain the findings of an extensive study of interaction patterns in secondary education (Levenberg, 2010).

This study found that the flow of the teacher's questions and the pupils' short answers in such a short time unit constitute some kind of a race to keep up the pace and avoid losing control. Cazden cites the study conducted by Rowe (1986), emphasising that the fast

pace stemming from the need to control prevents a true interaction between the partners to the discourse and the pupils are not given sufficient time for thinking in-depth. The question raised is: 'Why is this race happening already in elementary school grades?' Indeed, elementary school teachers have to cope with a loaded syllabus. However, it cannot be compared to the pressure to cover the learning material with which high school teachers deal when preparing students for their matriculation examinations.

The interactions analysis gave rise to another finding, namely the number of pupils who did not take part at all in the discourse. Only about 35% of the class pupils were 'partners for a moment' to the class discourse. In crowded classes (like some of the investigated classes), it was not always possible to involve everyone in the discourse. However, the small number of active pupils presents a class in which most of the 'audience' came to watch a 'performance' of the teacher alone. The limited and passive participation of most of the class pupils in the discourse raises doubts as to the effectiveness of learning mathematics in lessons with this type of interactional structure. Most probably those pupils complement the learning in another framework but they still lack the insights and competences which only an active participation in a mathematical class discourse can provide them.

In the investigated classes a prominent asymmetry was demonstrated between the partners to the discourse: the teacher spoke a lot while the pupils had no room for being active partners and expressing their thoughts. This picture is not different from what was customary and acceptable in the traditional teaching in the past. It seems that the waves of changes in the spirit of the reforms and the constructivist approach are rather far away from the classes investigated in this study.

To sum up, beyond what the students experienced while making the transcriptions and drawing the graphic representations, it is important to mention that they too were surprised by the research findings. The visual picture obtained following the mapping of the class discourse patterns was different than their estimation of the interaction patterns in the lesson which they documented.

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APPENDIX A:
EXAMPLE OF A CLASS DISCOURSE DOCUMENTATION

4th grade - Teacher's name: Savrin**. 28 pupils**

Lesson subject: Order of the four rules of mathematics

	Speaker	Statement	Events, action, board and comments
1	Teacher	-	Writing the following exercise on the board $3 + 4 \times 2$
2	Teacher	'What did I write on the board?'	-
3	Rami	'3 plus 4 multiplied by 2'	-
4	Teacher	'What is this exercise?'	-
5	Adam	'Expansion'	-
6	Amir	'Two rules'	-
7	Teacher	'Nice, we are using the terms, right?'	-
8	Teacher	'Can we solve it? And how can we solve it?'	-
9	Simi	'We solve according to the order'	(Starts solving) '3 plus 4 are 7, and 7 multiplied by 2 are 14, so the answer is 14'
10	Teacher	-	Writes the answer on the board
11	Teacher	'Is there another way?'	Silence in the classroom and the teacher tries reading Simi's solution
12	Yarin	'You multiply 7 by 2 and the result is 14'	The teacher writes on the board: 2*7=14
13	Teacher	'So what, Yarin, what do we do now?'	-
14	Yarin	'We can switch!!'	Without permission
15	Teacher	'Switch what?'	-
16	Yarin	'Between the rules?'	-
17	Teacher	'You cannot change the rules, that is how the exercise is written'	Shouts in the classroom

	Speaker	Statement	Events, action, board and comments
18	Teacher	'Give me another way which does not consist of switching the rules'	-
19	Daniel	'You multiply 2 by 4 and add 3'	The teacher writes on the board
20	Teacher	'Nice. Why do you add 3?'	Comments on the solution and writes on the board the solution method
21	Teacher	'You are very close to the solution'	Whispers in the classroom
22	Sandi and Simi	2 + 4 × 3	The teacher says it in words
23a	Teacher (addresses Sandi)	'I don't switch rules!'	And emphasises again the first solution
23b	Teacher	'Let's try'	Writes the solution on the board
24	Teacher	'Why do we start with the multiplication?'	Addresses the class
25	All the class	Speak together, unclearly	Shouts in the classroom and wrong answers. The teacher makes no comment.
26	Teacher	'Right, the multiplication answer is bigger than the addition'	Comments, writes on the board 4*2=8, 2+4=6
27	Meir	'The multiplication comes before the addition'	-
28	Teacher	'Right. What about the division and subtraction?'	-
29	Class	The class shouts something unintelligible	Answers emerge in the classroom
30	Teacher	'The addition and subtraction are "weaker" than the multiplication and the division'	Start solving with the class. Writes the answer on the board

	Speaker	Statement	Events, action, board and comments
31	Teacher	"And if we don't have an–mark how do we solve? Or [that] the exercise is only division and multiplication? You start according to the order now. Open your notebooks, write down the exercises and solve them by yourselves	Quiet in the classroom and the teacher writes: solve the following exercises: Example: $7 \times 3 - 1 + 10$ $8 + 4 \times 9$ $5 + 6 \times 6$ $3 + 3 + (4 \times 5)$