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On Generalized Intuitionistic Soft Equality

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Abstract

Park et al. (2011) introduced the concept of generalized intuitionistic fuzzy soft sets, which can be seen as an effective mathematical tool to deal with uncertainties. In this paper, the concept of generalized intuitionistic fuzzy soft equality is introduced and some related properties are derived. It is proved that generalized intuitionistic fuzzy soft equality is congruence relation with respect to some operations and the generalized intuitionistic fuzzy soft quotient algebra is established.

Key Words: Generalized intuitionistic fuzzy soft sets, Union, Intersection, Restricted union, Restricted intersection, Generalized intuitionistic soft equality.

1. Introduction

In 1999, Molodtsov [1] introduced the concept of soft set theory, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting the existing methods. The works on soft set theory are progressing rapidly. Soft set theory has a rich potential for applications to problems in real life situation. Lots of studies on this topic have been extended to fuzzy environment and its extended environments [1–8]. Park et al. [7] defined the concept of generalized intuitionistic fuzzy soft sets by combining the generalized intuitionistic fuzzy sets [9] and soft set models. Park et al. [10] described the application of generalized intuitionistic fuzzy soft sets to decision making problems. In [11], Park tried to find an answer to the question how the logic operations and their interrelations between each other correspond to generalized intuitionistic fuzzy set operations. He, in [12], also discussed the algebraic structure of generalized intuitionistic fuzzy soft sets. In this paper, the concept of generalized intuitionistic fuzzy soft equality is introduced and some related properties are derived. Some equivalent conditions for generalized intuitionistic fuzzy soft sets being generalized intuitionistic fuzzy soft equality are given. It is proved that generalized intuitionistic fuzzy soft equality is congruence relation with respect to some operations and the generalized intuitionistic fuzzy soft quotient algebra is established.

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2. Preliminaries

This section presents a review of some fundamental notions of generalized intuitionistic fuzzy soft sets. See [7-10] for further details and background.

Definition 2.1. [7] Let U be an initial universe and E be a set of parameters. $\mathcal{GIF}(U)$ denotes the set of all generalized intuitionistic fuzzy sets of U. Let $A \subseteq E$. A pair $\langle F, A\rangle$ is a generalized intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to \mathcal{GIF}(U)$.

In other words, a generalized intuitionistic fuzzy soft set is a parameterized family of generalized intuitionistic fuzzy subsets of U and thus its universe is the set of all generalized intuitionistic fuzzy sets of U, i.e., $\mathcal{GIF}(U)$.

Definition 2.2. [7] Let $E = \{e_1, e_2, \dots, e_n\}$ be a set of parameters. The *not* of E, denoted by $\rceil E$, is defined by $\rceil E = \{\neg e_1, \neg e_2, \dots, \neg e_n\}$, where $\neg e_i = not\ e_i$.

Definition 2.3. [7] The complement of a generalized intuitionistic fuzzy soft set $\langle F, A\rangle$, denoted by $\langle F, A\rangle^c$, is defined by $\langle F, A\rangle^c = \langle F^c, \rceil A\rangle$, where $F^c : \rceil A \to \mathcal{GIF}(U)$ is a mapping given by $F^c(e) = \langle x, \nu_{F(\neg e)}(x), \mu_{F(\neg e)}(x)\rangle$ for all $x \in U$ and $e \in \rceil A$.

Clearly, $(\langle F, A\rangle^c)^c = \langle F, A\rangle$ holds.

Definition 2.4. [11] The union of two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over a universe U, denoted by $\langle F, A\rangle \sqcup \langle G, B\rangle$, is a generalized intuitionistic fuzzy soft set $\langle H, C\rangle$, where $C = A \cup B$, and for all $\varepsilon \in C$ and $x \in U$,

$$\mu_{H(e)}(x) = \begin{cases} \mu_{F(e)}(x), & \text{if } e \in A - B, \\ \mu_{G(e)}(x), & \text{if } e \in B - A, \\ \max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}, & \text{if } e \in A \cap B, \end{cases}$$

$$\nu_{H(e)}(x) = \begin{cases} \nu_{F(e)}(x), & \text{if } e \in A - B, \\ \nu_{G(e)}(x), & \text{if } e \in B - A, \\ \min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}, & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.5. [11] The intersection of two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over a universe U, denoted by $\langle F, A\rangle \sqcap \langle G, B\rangle$, is a generalized intuitionistic fuzzy soft set $\langle H, C\rangle$, where $C = A \cup B$, and for all $\varepsilon \in C$ and $x \in U$,

$$\mu_{H(e)}(x) = \begin{cases} \mu_{F(e)}(x), & \text{if } e \in A - B, \\ \mu_{G(e)}(x), & \text{if } e \in B - A, \\ \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}, & \text{if } e \in A \cap B, \end{cases}$$

$$\nu_{H(e)}(x) = \begin{cases} \nu_{F(e)}(x), & \text{if } e \in A - B, \\ \nu_{G(e)}(x), & \text{if } e \in B - A, \\ \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}, & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.6. [12] The restricted union of two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over a universe U, denoted by $\langle F, A\rangle \tilde{\cup} \langle G, B\rangle$, is a generalized intuitionistic fuzzy soft set $\langle H, C\rangle$, where $C = A \cap B$, and for all $\varepsilon \in C$ and $x \in U$,

$$\mu_{H(e)}(x) = \max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\},$$
$$\nu_{H(e)}(x) = \min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}.$$

Definition 2.7. [12] The restricted intersection of two generalized intuitionistic fuzzy soft sets $\langle F, A\rangle$ and $\langle G, B\rangle$ over a universe U, denoted by $\langle F, A\rangle \tilde{\cap} \langle G, B\rangle$, is a generalized intuitionistic fuzzy soft set $\langle H, C\rangle$, where $C = A \cap B$, and for all $e \in C$ and $x \in U$,

$$\mu_{H(e)}(x) = \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\},$$
$$\nu_{H(e)}(x) = \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}.$$

Theorem 2.8. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then

(1) $(\langle F, A\rangle \sqcup \langle G, B\rangle)^c = \langle F, A\rangle^c \sqcap \langle G, B\rangle^c$.

(2) $(\langle F, A\rangle \sqcap \langle G, B\rangle)^c = \langle F, A\rangle^c \sqcup \langle G, B\rangle^c$.

(3) $(\langle F, A\rangle \tilde{\cup} \langle G, B\rangle)^c = \langle F, A\rangle^c \tilde{\cap} \langle G, B\rangle^c$.

(4) $(\langle F, A\rangle \tilde{\cap} \langle G, B\rangle)^c = \langle F, A\rangle^c \tilde{\cup} \langle G, B\rangle^c$.

3. The generalized intuitionistic fuzzy soft equality relation $\approx_{\mathrm{GIFS}}$

In this section, we introduce the concept of generalized intuitionistic fuzzy soft equality and study its related properties.

Let $\langle F, A\rangle$ be a generalized intuitionistic fuzzy soft set over the universe U. We consider the generalized intuitionistic fuzzy soft set $\langle G, E\rangle$, where $\mu_{G(e)}(x) = \mu_{F(e)}(x)$ and $\nu_{G(e)}(x) = \nu_{F(e)}(x)$ for all $e \in A$ and $x \in U$, and $\mu_{G(e)}(x) = 0$ and $\nu_{G(e)}(x) = 1$ for all $e \in E - A$ and $x \in U$. The parameter sets of $\langle F, A\rangle$ and $\langle G, E\rangle$ are different. It follows that F and G are different set-valued mappings and hence $\langle F, A\rangle$ and $\langle G, E\rangle$ are different generalized intuitionistic fuzzy soft sets. However, for any common parameter $e \in A$, the generalized intuitionistic fuzzy value sets of parameter e with respect to $\langle F, A\rangle$ and $\langle G, E\rangle$ are equal, and for all $e \in E - A$, $\mu_{G(e)}(x) = 0$ and $\nu_{G(e)}(x) = 1$ for any $x \in U$. Thus $\langle F, A\rangle$ and $\langle G, E\rangle$ may be considered to represent almost same generalized intuitionistic fuzzy soft sets of approximations and to some extent they are equal.

As illustration, we consider the following example.

Example 3.1. Suppose that there are six houses in the universe U given by $U = \{h_i | i = 1, 2, \ldots, 6\}$ and $E = \{e_j | j = 1, 2, \ldots, 8\}$ is the set of parameters, where e_1, e_2, e_3, e_4, e_5, e_6, e_7 and e_8 stand for the parameters 'expensive', 'beautiful', 'wooden', 'cheap', 'in the green surroundings', 'modern', 'in good repair', and 'in bad repair', respectively. The generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ describe the "attractiveness of the houses" to the decision maker. Suppose that $A = \{e_1, e_2, e_3, e_4\}$ and

$$\begin{aligned}
F(e_1) = \{&\langle h_1, 0.9, 0.2\rangle, \langle h_2, 0.7, 0.3\rangle, \langle h_3, 0.5, 0.4\rangle,\\
&\langle h_4, 0.6, 0.5\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.7, 0.4\rangle\};\\
F(e_2) = \{&\langle h_1, 0.8, 0.2\rangle, \langle h_2, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.4\rangle,\\
&\langle h_4, 0.8, 0.3\rangle, \langle h_5, 0.5, 0.4\rangle, \langle h_6, 0.5, 0.3\rangle\};\\
F(e_3) = \{&\langle h_1, 0.6, 0.2\rangle, \langle h_2, 0.5, 0.3\rangle, \langle h_3, 0.4, 0.5\rangle,\\
&\langle h_4, 0.7, 0.5\rangle, \langle h_5, 0.6, 0.4\rangle, \langle h_6, 0.8, 0.3\rangle\};\\
F(e_4) = \{&\langle h_1, 0, 1\rangle, \langle h_2, 0, 1\rangle, \langle h_3, 0, 1\rangle,\\
&\langle h_4, 0, 1\rangle, \langle h_5, 0, 1\rangle, \langle h_6, 0, 1\rangle\}.
\end{aligned}$$

It is worth noting that A is a proper subset of E. For a parameter $e \in E - A$, we may think that decision maker does not care about the attribute of e, or he does not take e into account. In this case, we may suppose that the decision maker thinks that $\mu_{F(e)}(h_i) = 0$ an $\nu_{F(e)}(h_i) = 1$ for all h_i.

Based on these observations, we introduce the concept of 'generalized intuitionistic fuzzy soft equality' as follows.

Definition 3.2. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then $\langle F, A\rangle$ is called generalized intuitionistic fuzzy soft equal to $\langle G, B\rangle$, denoted by $\langle F, A\rangle \approx_{\mathrm{GIFS}} \langle G, B\rangle$, if for all $e \in A \cup B$ and $x \in U$, (a) $e \in A \cap B$ implies $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, (b) $e \in A - B$ implies $\mu_{F(e)}(x) = 0$ and $\nu_{F(e)}(x) = 1$, and (c) $e \in B - A$ implies $\mu_{G(e)}(x) = 0$ and $\nu_{G(e)}(x) = 1$.

Example 3.3. We consider the generalized intuitionistic fuzzy soft set $\langle F, A\rangle$ given in Example 3.1. Let $\langle G, B\rangle$ be a generalized intuitionistic fuzzy soft set, where $B = \{e_1, e_2, e_3, e_5\}$, and $G(e_1) = F(e_1)$, $G(e_2) = F(e_2)$, $G(e_3) = F(e_3)$, and

$$\begin{aligned}
F(e_5) = \{&\langle h_1, 0, 1\rangle, \langle h_2, 0, 1\rangle, \langle h_3, 0, 1\rangle, \langle h_4, 0, 1\rangle,\\
&\langle h_5, 0, 1\rangle, \langle h_6, 0, 1\rangle\}.
\end{aligned}$$

Then $\langle F, A\rangle \approx_{\mathrm{GIFS}} \langle G, B\rangle$.

Theorem 3.4. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then $\langle F, A\rangle \approx_{\mathrm{GIFS}} \langle G, B\rangle$ if and only if $\langle F, A\rangle \sqcup \langle G, B\rangle \approx_{\mathrm{GIFS}} \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

Proof. Let $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle H, A \cup B\rangle$ and $\langle F, A\rangle \tilde{\cap} \langle G, B\rangle = \langle T, A \cap B\rangle$. Suppose that $\langle F, A\rangle \approx_{\mathrm{GIFS}} \langle G, B\rangle$. For all $e \in A \cap B$ and $x \in U$, by definition, we have $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, and hence

$$\begin{aligned}
\mu_{H(e)}(x) &= \max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}\\
&= \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}\\
&= \mu_{T(e)}(x),\\
\nu_{H(e)}(x) &= \min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}\\
&= \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}\\
&= \nu_{T(e)}(x).
\end{aligned}$$

For all $e \in A \cup B - A \cap B$ and $x \in U$, (a) if $e \in A - B$, then $\mu_{F(e)}(x) = 0$ and $\nu_{F(e)}(x) = 1$ and hence $\mu_{H(e)}(x) = \mu_{F(e)}(x) = 0$ and $\nu_{H(e)}(x) = \nu_{F(e)}(x) = 1$; (b) if $e \in B - A$, then $\mu_{G(e)}(x) = 0$ and $\nu_{G(e)}(x) = 1$ and hence $\mu_{H(e)}(x) = \mu_{G(e)}(x) = 0$ and $\nu_{H(e)}(x) = \nu_{G(e)}(x) = 1$. Thus, $\langle F, A\rangle \sqcup \langle G, B\rangle \approx_{\mathrm{GIFS}} \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

Conversely, suppose that $\langle F, A\rangle \sqcup \langle G, B\rangle \approx_{\text{GIFS}} \langle F, A\rangle \tilde{\cap}\langle G, B\rangle$. For all $e \in A \cap B$ and $x \in U$, $\max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\} = \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}$ and $\min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\} = \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}$ and hence $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$. For all $e \in A - B$ and $x \in U$, since $e \in A \cup B$ and $e \notin A \cap B$, $\mu_{F(e)}(x) = \mu_{H(e)}(x) = 0$ and $\nu_{F(e)}(x) = \nu_{H(e)}(x) = 1$. For all $e \in B - A$ and $x \in U$, since $e \in A \cup B$ and $e \notin A \cap B$, $\mu_{G(e)}(x) = \mu_{H(e)}(x) = 0$ and $\nu_{G(e)}(x) = \nu_{H(e)}(x) = 1$. Thus, $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$. □

Theorem 3.5. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U and $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$. Then

(1) $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle F, A\rangle \sqcap \langle G, B\rangle$.

(2) $\langle F, A\rangle \tilde{\cup} \langle G, B\rangle = \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

Proof. (1) Suppose that $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle H, A \cup B\rangle$ and $\langle F, A\rangle \sqcap \langle G, B\rangle = \langle T, A \cup B\rangle$. Let $e \in A \cup B$ and $x \in U$. If $e \in A \cap B$, then $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, and hence

$$\begin{aligned}\mu_{H(e)}(x) &= \max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\} \\ &= \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\} \\ &= \mu_{T(e)}(x), \\ \nu_{H(e)}(x) &= \min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\} \\ &= \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\} \\ &= \nu_{T(e)}(x).\end{aligned}$$

If $e \in A - B$ or $e \in B - A$, then, by definition, $\mu_{H(e)}(x) = \mu_{T(e)}(x)$ and $\nu_{H(e)}(x) = \nu_{T(e)}(x)$. Thus, $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle F, A\rangle \sqcap \langle G, B\rangle$.

(2) can be proved similarly. □

Theorem 3.6. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. If $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$, then $\langle F, A\rangle \tilde{\cup} \langle G, B\rangle \approx_{\text{GIFS}} \langle F, A\rangle \sqcap \langle G, B\rangle$.

Proof. It follows from Theorems 3.4 and 3.5. □

Now, we denote by $\mathcal{GIFS}(U, E)$ the set of all generalized intuitionistic fuzzy soft sets over the universe U and the parameter set E, that is

$$\mathcal{GIFS}(U, E) = \{\langle F, A\rangle | A \subseteq E, F : A \to \mathcal{GIF}(U)\}.$$

Theorem 3.7. $\approx_{\text{GIFS}}$ is an equivalence relation on $\mathcal{GIFS}(U, E)$.

Proof. It is trivial to verify that $\approx_{\text{GIFS}}$ is reflexive and symmetric. We prove that $\approx_{\text{GIFS}}$ is transitive.

Suppose that $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$ and $\langle G, B\rangle \approx_{\text{GIFS}} \langle H, C\rangle$. For all $e \in A \cap C$ and $x \in U$, if $e \in B$, then $e \in A \cap B$ and $e \in B \cap C$, it follows that $\mu_{F(e)}(x) = \mu_{G(e)}(x) = \mu_{H(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x) = \nu_{H(e)}(x)$; if $e \notin B$, then $e \in A - B$ and $e \in C - B$, it follows $\mu_{F(e)}(x) = 0 = \mu_{H(e)}(x)$ and $\nu_{F(e)}(x) = 1 = \nu_{H(e)}(x)$. For all $e \in A - C$ and $x \in U$, if $e \in B$, then $e \in A \cap B$ and $e \in B - C$, it follows that $\mu_{F(e)}(x) = \mu_{G(e)}(x) = 0$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x) = 1$; if $e \notin B$, then $e \in A - B$, it follows that $\mu_{F(e)}(x) = 0$ and $\nu_{F(e)}(x) = 1$. For all $e \in C - A$ and $x \in U$, if $e \in B$, then $e \in B \cap C$ and $e \in B - A$, it follows that $\mu_{H(e)}(x) = \mu_{G(e)}(x) = 0$ and $\nu_{H(e)}(x) = \nu_{G(e)}(x) = 1$; if $e \notin B$, then $e \in C - B$, it follows that $\mu_{H(e)}(x) = 0$ and $\nu_{H(e)}(x) = 1$. Hence $\langle F, A\rangle \approx_{\text{GIFS}} \langle H, C\rangle$ and thus $\approx_{\text{GIFS}}$ is transitive. □

Theorem 3.8. $\approx_{\text{GIFS}}$ is a congruence relation with respect to $\tilde{\cap}$ and $\sqcup$. That is to say, if $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$ and $\langle H, C\rangle \approx_{\text{GIFS}} \langle L, D\rangle$, then

(1) $\langle F, A\rangle \tilde{\cap} \langle H, C\rangle \approx_{\text{GIFS}} \langle G, B\rangle \tilde{\cap} \langle L, D\rangle$.

(2) $\langle F, A\rangle \sqcup \langle H, C\rangle \approx_{\text{GIFS}} \langle G, B\rangle \sqcup \langle L, D\rangle$.

Proof. (1) Suppose that $\langle F, A\rangle \tilde{\cap} \langle H, C\rangle = \langle M, A \cap C\rangle$ and $\langle G, B\rangle \tilde{\cap} \langle L, D\rangle = \langle N, B \cap D\rangle$. Let $e \in (A \cap C) \cup (B \cap D)$ and $x \in U$.

(a) If $e \in (A \cap C) \cap (B \cap D)$, then $e \in A \cap C$ and $e \in B \cap D$. It follows that $\mu_{F(e)}(x) = \mu_{G(e)}(x)$, $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, $\mu_{H(e)}(x) = \mu_{L(e)}(x)$ and $\nu_{H(e)}(x) = \nu_{L(e)}(x)$. Hence

$$\mu_{M(e)}(x) = \min\{\mu_{F(e)}(x), \mu_{H(e)}(x)\}$$

$$
\begin{aligned}
&= \min\{\mu_{G(e)}(x), \mu_{L(e)}(x)\}\\
&= \mu_{N(e)}(x),\\
\nu_{M(e)}(x) &= \max\{\nu_{F(e)}(x), \nu_{H(e)}(x)\}\\
&= \max\{\nu_{G(e)}(x), \nu_{L(e)}(x)\}\\
&= \nu_{N(e)}(x).
\end{aligned}
$$

(b) If $e \in (A \cap C) - (B \cap D)$, then $e \in A \cap C$ and $e \notin B \cap D$. If $e \notin B$, then $e \in A - B$ and so $\mu_{F(e)}(x) = 0$ and $\nu_{F(e)}(x) = 1$. Hence

$$
\begin{aligned}
\mu_{M(e)}(x) &= \min\{\mu_{F(e)}(x), \mu_{H(e)}(x)\} = 0,\\
\nu_{M(e)}(x) &= \max\{\nu_{F(e)}(x), \nu_{H(e)}(x)\} = 1.
\end{aligned}
$$

If $e \notin D$, then $e \in C - D$ and so $\mu_{H(e)}(x) = 0$ and $\nu_{H(e)}(x) = 1$. Hence

$$
\begin{aligned}
\mu_{M(e)}(x) &= \min\{\mu_{F(e)}(x), \mu_{H(e)}(x)\} = 0,\\
\nu_{M(e)}(x) &= \max\{\nu_{F(e)}(x), \nu_{H(e)}(x)\} = 1.
\end{aligned}
$$

(c) If $e \in (B \cap D) - (A \cap C)$, then $e \in B \cap D$ and $e \notin A \cap C$. If $e \notin A$, then $e \in B - A$ and so $\mu_{G(e)}(x) = 0$ and $\nu_{G(e)}(x) = 1$. Hence

$$
\begin{aligned}
\mu_{N(e)}(x) &= \min\{\mu_{G(e)}(x), \mu_{L(e)}(x)\} = 0,\\
\nu_{N(e)}(x) &= \max\{\nu_{G(e)}(x), \nu_{L(e)}(x)\} = 1.
\end{aligned}
$$

If $e \notin C$, then $e \in D - C$ and so $\mu_{L(e)}(x) = 0$ and $\nu_{L(e)}(x) = 1$. Hence

$$
\begin{aligned}
\mu_{N(e)}(x) &= \min\{\mu_{G(e)}(x), \mu_{L(e)}(x)\} = 0,\\
\nu_{N(e)}(x) &= \max\{\nu_{G(e)}(x), \nu_{L(e)}(x)\} = 1.
\end{aligned}
$$

Thus, $\langle F, A\rangle \tilde{\sqcap} \langle H, C\rangle \approx_{\text{GIFS}} \langle G, B\rangle \tilde{\sqcap} \langle L, D\rangle$.

(2) Suppose that $\langle F, A\rangle \sqcup \langle H, C\rangle = \langle S, A \cup C\rangle$ and $\langle G, B\rangle \sqcup \langle L, D\rangle = \langle T, B \cup D\rangle$. Let $e \in (A \cup C) \cup (B \cup D)$ and $x \in U$.

(a) If $e \in (A \cup C) \cap (B \cup D)$, then $e \in A \cup C$ and $e \in B \cup D$. Without loss of generality, we suppose that $e \in A$ and $e \in D$.

(i) If $e \in B$ and $e \in C$, then $e \in A \cap B$ and $e \in C \cap D$. It follows that $\mu_{F(e)}(x) = \mu_{G(e)}(x)$, $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, $\mu_{H(e)}(x) = \mu_{L(e)}(x)$ and $\nu_{H(e)}(x) = \nu_{L(e)}(x)$ and hence

$$
\begin{aligned}
\mu_{S(e)}(x) &= \max\{\mu_{F(e)}(x), \mu_{H(e)}(x)\}\\
&= \max\{\mu_{G(e)}(x), \mu_{L(e)}(x)\}\\
&= \mu_{T(e)}(x),\\
\nu_{S(e)}(x) &= \min\{\nu_{F(e)}(x), \nu_{H(e)}(x)\}\\
&= \min\{\nu_{G(e)}(x), \nu_{L(e)}(x)\}\\
&= \nu_{T(e)}(x).
\end{aligned}
$$

(ii) If $e \notin B$ and $e \in C$, then $e \in A - B$ and $e \in C \cap D$. It follows that $\mu_{F(e)}(x) = 0$, $\nu_{F(e)}(x) = 1$, $\mu_{H(e)}(x) = \mu_{L(e)}(x)$ and $\nu_{H(e)}(x) = \nu_{L(e)}(x)$ and hence

$$
\begin{aligned}
\mu_{S(e)}(x) &= \max\{\mu_{F(e)}(x), \mu_{H(e)}(x)\}\\
&= \mu_{H(e)}(x) = \mu_{L(e)}(x)\\
&= \mu_{T(e)}(x),\\
\nu_{S(e)}(x) &= \min\{\nu_{F(e)}(x), \nu_{H(e)}(x)\}\\
&= \nu_{H(e)}(x) = \nu_{L(e)}(x)\}\\
&= \nu_{T(e)}(x).
\end{aligned}
$$

(iii) If $e \in B$ and $e \notin C$, then $e \in A \cap B$ and $e \in D - C$. It follows that $\mu_{F(e)}(x) = \mu_{G(e)}(x)$, $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, $\mu_{L(e)}(x) = 0$ and $\nu_{L(e)}(x) = 1$ and hence

$$
\begin{aligned}
\mu_{S(e)}(x) &= \mu_{F(e)}(x) = \mu_{G(e)}(x)\\
&= \max\{\mu_{G(e)}(x), \mu_{L(e)}(x)\}\\
&= \mu_{T(e)}(x),\\
\nu_{S(e)}(x) &= \nu_{F(e)}(x) = \nu_{G(e)}(x)\}\\
&= \min\{\nu_{G(e)}(x), \nu_{L(e)}(x)\}\\
&= \nu_{T(e)}(x).
\end{aligned}
$$

(iv) If $e \notin B$ and $e \notin C$, then $e \in A - B$ and $e \in D - C$. It follows that $\mu_{F(e)}(x) = 0$, $\nu_{F(e)}(x) = 1$, $\mu_{L(e)}(x) = 0$ and $\nu_{L(e)}(x) = 1$ and hence

$$
\begin{aligned}
\mu_{S(e)}(x) &= \mu_{F(e)}(x) = 0 = \mu_{L(e)}(x) = \mu_{T(e)}(x),\\
\nu_{S(e)}(x) &= \nu_{F(e)}(x) = 1 = \nu_{L(e)}(x) = \nu_{T(e)}(x).
\end{aligned}
$$

(b) If $e \in (A \cup C) - (B \cup D)$, then $e \in A \cup C$, $e \notin B$ and $e \notin D$.

(i) If $e \in A$ and $e \in C$, then $e \in A - B$ and $e \in C - D$. It follows that $\mu_{F(e)}(x) = \mu_{H(e)}(x) = 0$ and $\nu_{F(e)}(x) = \nu_{H(e)}(x) = 1$, and hence

$$\mu_{S(e)}(x) = \max\{\mu_{F(e)}(x), \mu_{H(e)}(x)\} = 0,$$
$$\nu_{S(e)}(x) = \min\{\nu_{F(e)}(x), \nu_{H(e)}(x)\} = 1.$$

(ii) If $e \in A$ and $e \notin C$, then $e \in A - B$. It follows that $\mu_{F(e)}(x) = 0$ and $\nu_{F(e)}(x) = 1$, and hence

$$\mu_{S(e)}(x) = \mu_{F(e)}(x) = 0,$$
$$\nu_{S(e)}(x) = \nu_{F(e)}(x) = 1.$$

(iii) If $e \notin A$ and $e \in C$, then $e \in C - D$. It follows that $\mu_{H(e)}(x) = 0$ and $\nu_{H(e)}(x) = 1$, and hence

$$\mu_{S(e)}(x) = \mu_{H(e)}(x) = 0,$$
$$\nu_{S(e)}(x) = \nu_{H(e)}(x) = 1.$$

(c) If $e \in (B \cup D) - (A \cup C)$, $\mu_{T(e)}(x) = 0$ and $\nu_{T(e)}(x) = 1$ can be proved similarly.

Thus, $\langle F, A\rangle \sqcup \langle H, C\rangle \approx_{\text{GIFS}} \langle G, B\rangle \sqcup \langle L, D\rangle$. □

Let $\langle F, A\rangle_{\approx_{\text{GIFS}}} = \{\langle G, B\rangle | \langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle\}$ be the congruence class including $\langle F, A\rangle$ and

$$\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}} = \{\langle F, A\rangle_{\approx_{\text{GIFS}}} | \langle F, A\rangle \in \mathcal{GIF}(U)\}.$$

Now, we define operations $\sqcup_{\text{GIFS}}$ and $\tilde{\sqcap}_{\text{GIFS}}$ on $\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}}$ as follows:

$$\langle F, A\rangle_{\approx_{\text{GIFS}}} \sqcup_{\text{GIFS}} \langle G, B\rangle_{\approx_{\text{GIFS}}} = (\langle F, A\rangle \sqcup \langle F, A\rangle)_{\approx_{\text{GIFS}}},$$
$$\langle F, A\rangle_{\approx_{\text{GIFS}}} \tilde{\sqcap}_{\text{GIFS}} \langle G, B\rangle_{\approx_{\text{GIFS}}} = (\langle F, A\rangle \tilde{\sqcap} \langle F, A\rangle)_{\approx_{\text{GIFS}}}.$$

Then these two operations are well defined by Theorem 3.8 and thus we call $(\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}}, \sqcup_{\text{GIFS}}, \tilde{\sqcap}_{\text{GIFS}})$ the generalized intuitionistic fuzzy soft quotient algebra (with respect to $\approx_{\text{GIFS}}$) over the unverse U and the parameter set E.

Theorem 3.9. $(\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}}, \sqcup_{\text{GIFS}}, \tilde{\sqcap}_{\text{GIFS}})$ is distributive lattice.

Proof. We demonstrate that the distributive law holds in $\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}}$. In fact, for all $\langle F, A\rangle, \langle G, B\rangle, \langle H, C\rangle \in \mathcal{GIFS}(U, E)$,

$$\begin{aligned}
&\langle F, A\rangle_{\approx_{\text{GIFS}}} \sqcup_{\text{GIFS}} (\langle G, B\rangle_{\approx_{\text{GIFS}}} \tilde{\sqcap}_{\text{GIFS}} \langle H, C\rangle_{\approx_{\text{GIFS}}})\\
&= \langle F, A\rangle_{\approx_{\text{GIFS}}} \sqcup_{\text{GIFS}} (\langle G, B\rangle \tilde{\sqcap} \langle H, C\rangle)_{\approx_{\text{GIFS}}}\\
&= (\langle F, A\rangle \sqcup (\langle G, B\rangle \tilde{\sqcap} \langle H, C\rangle))_{\approx_{\text{GIFS}}}\\
&= ((\langle F, A\rangle \sqcup \langle G, B\rangle) \tilde{\sqcap} (\langle F, A\rangle \sqcup \langle H, C\rangle))_{\approx_{\text{GIFS}}}\\
&= (\langle F, A\rangle_{\approx_{\text{GIFS}}} \sqcup_{\text{GIFS}} \langle G, B\rangle_{\approx_{\text{GIFS}}})\\
&\qquad \tilde{\sqcap}_{\text{GIFS}} (\langle F, A\rangle_{\approx_{\text{GIFS}}} \sqcup_{\text{GIFS}} \langle H, C\rangle_{\approx_{\text{GIFS}}}).
\end{aligned}$$

By using similar techniques, we can prove that $\sqcup_{\text{GIFS}}$ and $\tilde{\sqcap}_{\text{GIFS}}$ are idempotent, associative and commutative. Furthermore, the absorption law with respect to $\sqcup_{\text{GIFS}}$ and $\tilde{\sqcap}_{\text{GIFS}}$ holds. Hence $(\mathcal{GIFS}(U, E)/\approx_{\text{GIFS}}, \sqcup_{\text{GIFS}}, \tilde{\sqcap}_{\text{GIFS}})$ is distributive lattice. □

4. The generalized intuitionistic fuzzy soft equality relation $\approx^{\text{GIFS}}$

In this section, we discuss the generalized intuitionistic fuzzy soft equality $\approx^{\text{GIFS}}$ and establish the generalized intuitionistic fuzzy soft quotient algebra with respect to $\tilde{\cup}^{\text{GIFS}}$ and $\sqcap^{\text{GIFS}}$.

Definition 4.1. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$, if for all $e \in A \cup B$ and $x \in U$, (a) $e \in A \cap B$ implies $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, (b) $e \in A - B$ implies $\mu_{F(e)}(x) = 1$ and $\nu_{F(e)}(x) = 0$, and (c) $e \in B - A$ implies $\mu_{G(e)}(x) = 1$ and $\nu_{G(e)}(x) = 0$.

Theorem 4.2. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$ if and only if $\langle F, A\rangle^c \approx_{\text{GIFS}} \langle G, B\rangle^c$.

Proof. Suppose that $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$. For all $\neg e \in \rceil A \cap \rceil B$ and $x \in U$, since $e \in A \cap B$, we have $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, and hence

$$\mu_{F^c(\neg e)}(x) = \nu_{F(e)}(x) = \nu_{G(e)}(x) = \mu_{G^c(\neg e)}(x),$$
$$\nu_{F^c(\neg e)}(x) = \mu_{F(e)}(x) = \mu_{G(e)}(x) = \nu_{G^c(\neg e)}(x).$$

For all $\neg e \in \rceil A - \rceil B$ and $x \in U$, since $e \in A - B$, we have $\mu_{F(e)}(x) = 1$ and $\nu_{F(e)}(x) = 0$, and hence $\mu_{F^c(\neg e)}(x) = \nu_{F(e)}(x) = 0$ and $\nu_{F^c(\neg e)}(x) = \mu_{F(e)}(x) = 1$. For all $\neg e \in \rceil B - \rceil A$ and $x \in U$, we can also prove that $\mu_{G^c(\neg e)}(x) = 0$ and $\nu_{G^c(\neg e)}(x) = 1$. Thus, $\langle F, A\rangle^c \approx_{\text{GIFS}} \langle G, B\rangle^c$.

Conversely, suppose that $\langle F, A\rangle^c \approx_{\text{GIFS}} \langle G, B\rangle^c$. For all $e \in A \cap B$ and $x \in U$, since $\neg e \in \rceil A \cap \rceil B$, we have $\mu_{F^c(\neg e)}(x) = \mu_{G^c(\neg e)}(x)$ and $\nu_{F^c(\neg e)}(x) = \nu_{G^c(\neg e)}(x)$, and hence

$$\mu_{F(e)}(x) = \nu_{F^c(\neg e)}(x) = \nu_{G^c(\neg e)}(x) = \mu_{G(e)}(x),$$
$$\nu_{F(e)}(x) = \mu_{F^c(\neg e)}(x) = \mu_{G^c(\neg e)}(x) = \nu_{G(e)}(x).$$

For all $e \in A - B$ and $x \in U$, since $\neg e \in \rceil A - \rceil B$, we have $\mu_{F^c(\neg e)}(x) = 0$ and $\nu_{F^c(\neg e)}(x) = 1$, and hence $\mu_{F(e)}(x) = \nu_{F^c(\neg e)}(x) = 1$ and $\nu_{F(e)}(x) = \mu_{F^c(\neg e)}(x) = 0$. For all $e \in B - A$ and $x \in U$, we can also prove that $\mu_{G(e)}(x) = 1$ and $\nu_{G(e)}(x) = 0$. Thus, $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$. □

Theorem 4.3. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U. Then

(1) $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$ if and only if $\langle F, A\rangle \sqcup \langle G, B\rangle \approx^{\text{GIFS}} \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

(2) $\langle F, A\rangle \approx_{\text{GIFS}} \langle G, B\rangle$ if and only if $\langle F, A\rangle \tilde{\cup} \langle G, B\rangle \approx_{\text{GIFS}} \langle F, A\rangle \sqcap \langle G, B\rangle$.

Proof. (1) Let $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle H, A \cup B\rangle$ and $\langle F, A\rangle \tilde{\cap} \langle G, B\rangle = \langle T, A \cap B\rangle$. Suppose that $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$. For all $e \in A \cap B$ and $x \in U$, we have $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$, and hence

$$\begin{aligned}\mu_{H(e)}(x) &= \max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}\\ &= \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}\\ &= \mu_{T(e)}(x),\\ \nu_{H(e)}(x) &= \min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}\\ &= \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}\\ &= \nu_{T(e)}(x).\end{aligned}$$

For all $e \in A \cup B - A \cap B$ and $x \in U$, (a) if $e \in A - B$, then $\mu_{F(e)}(x) = 1$ and $\nu_{F(e)}(x) = 0$ and hence $\mu_{H(e)}(x) = \mu_{F(e)}(x) = 1$ and $\nu_{H(e)}(x) = \nu_{F(e)}(x) = 0$; (b) if $e \in B - A$, then $\mu_{G(e)}(x) = 1$ and $\nu_{G(e)}(x) = 0$ and hence $\mu_{H(e)}(x) = \mu_{G(e)}(x) = 1$ and $\nu_{H(e)}(x) = \nu_{G(e)}(x) = 0$. Thus, $\langle F, A\rangle \sqcup \langle G, B\rangle \approx^{\text{GIFS}} \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

Conversely, suppose that $\langle F, A\rangle \sqcup \langle G, B\rangle \approx^{\text{GIFS}} \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$. For all $e \in A \cap B$ and $x \in U$, $\max\{\mu_{F(e)}(x), \mu_{G(e)}(x)\} = \min\{\mu_{F(e)}(x), \mu_{G(e)}(x)\}$ and $\min\{\nu_{F(e)}(x), \nu_{G(e)}(x)\} = \max\{\nu_{F(e)}(x), \nu_{G(e)}(x)\}$ and hence $\mu_{F(e)}(x) = \mu_{G(e)}(x)$ and $\nu_{F(e)}(x) = \nu_{G(e)}(x)$. For all $e \in A - B$ and $x \in U$, since $e \in A \cup B$ and $e \notin A \cap B$, $\mu_{F(e)}(x) = \mu_{H(e)}(x) = 1$ and $\nu_{F(e)}(x) = \nu_{H(e)}(x) = 0$. For all $e \in B - A$ and $x \in U$, since $e \in A \cup B$ and $e \notin A \cap B$, $\mu_{G(e)}(x) = \mu_{H(e)}(x) = 1$ and $\nu_{G(e)}(x) = \nu_{H(e)}(x) = 0$. Thus, $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$.

(2) can be proved similarly. □

Theorem 4.4. Let $\langle F, A\rangle$ and $\langle G, B\rangle$ be two generalized intuitionistic fuzzy soft sets over the universe U and $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$. Then

(1) $\langle F, A\rangle \sqcup \langle G, B\rangle = \langle F, A\rangle \sqcap \langle G, B\rangle$.

(2) $\langle F, A\rangle \tilde{\cup} \langle G, B\rangle = \langle F, A\rangle \tilde{\cap} \langle G, B\rangle$.

Proof. The proof is similar to that of Theorem 3.5. □

Theorem 4.5. $\approx^{\text{GIFS}}$ is an equivalence relation on $\mathcal{GIFS}(U, E)$.

Proof. It is trivial to verify that $\approx^{\text{GIFS}}$ is reflexive and symmetric. We only prove that $\approx^{\text{GIFS}}$ is transitive.

Suppose that $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$ and $\langle G, B\rangle \approx^{\text{GIFS}} \langle H, C\rangle$. By Theorem 4.2, $\langle F, A\rangle^c \approx_{\text{GIFS}} \langle G, B\rangle^c$ and $\langle G, B\rangle^c \approx_{\text{GIFS}} \langle H, C\rangle^c$ and hence by Theorem 3.7, $\langle F, A\rangle^c \approx_{\text{GIFS}} \langle H, C\rangle^c$. Consequently, $\langle F, A\rangle = (\langle F, A\rangle^c)^c \approx^{\text{GIFS}} (\langle H, C\rangle^c)^c = \langle H, C\rangle$. Thus, $\approx^{\text{GIFS}}$ is an equivalence relation on $\mathcal{GIFS}(U, E)$. □

Theorem 4.6. $\approx^{\text{GIFS}}$ is a congruence relation with respect to $\tilde{\cup}$ and $\sqcap$. That is to say, if $\langle F, A\rangle \approx^{\text{GIFS}} \langle G, B\rangle$ and $\langle H, C\rangle \approx^{\text{GIFS}} \langle L, D\rangle$, then

(1) $\langle F, A\rangle \tilde{\cup} \langle H, C\rangle \approx^{\text{GIFS}} \langle G, B\rangle \tilde{\cup} \langle L, D\rangle$.

(2) $\langle F, A\rangle \sqcap \langle H, C\rangle \approx^{\text{GIFS}} \langle G, B\rangle \sqcap \langle L, D\rangle$.

Proof. (1) Let $\langle F,A\rangle \approx^{\rm GIFS} \langle G,B\rangle$ and $\langle H,C\rangle \approx^{\rm GIFS} \langle L,D\rangle$, by Theorem 4.2, we have $\langle F,A\rangle^c \approx_{\rm GIFS} \langle G,B\rangle^c$ and $\langle H,C\rangle^c \approx_{\rm GIFS} \langle L,D\rangle^c$. By Theorem 3.8, we have $\langle F,A\rangle^c \tilde{\cap} \langle H,C\rangle^c \approx_{\rm GIFS} \langle G,B\rangle^c \tilde{\cap} \langle L,D\rangle^c$, and hence, by Theorem 2.8,

$$\begin{aligned}\langle F,A\rangle \tilde{\cup} \langle H,C\rangle &= (\langle F,A\rangle^c \tilde{\cap} \langle H,C\rangle^c)^c \\ &\approx^{\rm GIFS} (\langle G,B\rangle^c \tilde{\cap} \langle L,D\rangle^c)^c = \langle G,B\rangle \tilde{\cup} \langle L,D\rangle.\end{aligned}$$

(2) can be proved similarly. □

Let $\langle F,A\rangle_{\approx^{\rm GIFS}} = \{\langle G,B\rangle | \langle F,A\rangle \approx^{\rm GIFS} \langle G,B\rangle\}$ be the congruence class including $\langle F,A\rangle$ and

$$\mathcal{GIFS}(U,E)/\approx^{\rm GIFS} = \{\langle F,A\rangle_{\approx^{\rm GIFS}} | \langle F,A\rangle \in \mathcal{GIF}(U)\}.$$

Then, we define operations $\tilde{\cup}^{\rm GIFS}$ and $\sqcap^{\rm GIFS}$ on $\mathcal{GIFS}(U,E)/\approx^{\rm GIFS}$ as follows:

$$\langle F,A\rangle_{\approx^{\rm GIFS}} \tilde{\cup}^{\rm GIFS} \langle G,B\rangle_{\approx^{\rm GIFS}} = (\langle F,A\rangle \tilde{\cup} \langle F,A\rangle)_{\approx^{\rm GIFS}},$$

$$\langle F,A\rangle_{\approx^{\rm GIFS}} \sqcap^{\rm GIFS} \langle G,B\rangle_{\approx^{\rm GIFS}} = (\langle F,A\rangle \sqcap \langle F,A\rangle)_{\approx^{\rm GIFS}}.$$

These two operations are well defined by Theorem 4.6 and thus we call $(\mathcal{GIFS}(U,E)/\approx^{\rm GIFS}, \tilde{\cup}^{\rm GIFS}, \sqcap^{\rm GIFS})$ the generalized intuitionistic fuzzy soft quotient algebra (with respect to $\approx^{\rm GIFS}$) over the unverse U and the parameter set E.

Theorem 4.7. $(\mathcal{GIFS}(U,E)/\approx^{\rm GIFS}, \tilde{\cup}^{\rm GIFS}, \sqcap^{\rm GIFS})$ is distributive lattice.

Proof. The proof is similar to that of Theorem 3.9. □

5. Conclusions

Generalized intuitionistic fuzzy soft set theory has been regarded as an effective mathematical tool to deal with uncertainties. In this paper, we introduce the concept of generalized intuitionistic fuzzy soft equality and derive some related properties. It is proved that generalized intuitionistic fuzzy soft equality is congruence relation with respect to some operations and the generalized intuitionistic fuzzy soft quotient algebra is established.

References

[1] D. Molodtsov, "Soft set theory - first results," *Computers and Mathematics with Applications*, vol. 37, no. 4-5, pp. 19-31, 1999.

[2] P.K. Maji, R. Biswas and A.R. Roy, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589-602, 2001.

[3] P.K. Maji, R. Biswas and A.R. Roy, "Intuitionistic fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 677-692, 2001.

[4] P.K. Maji, A.R. Roy and R. Biswas, "On intuitionistic fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 12, no. 3, pp. 669-683, 2004.

[5] P.K. Maji, More on intuitionistic fuzzy soft sets. In: H. Sakai, M.K. Chakraborty, A.E. Hassanien, D. Slezak and W. Zhu, Editors, Proceedings of the 12th International Conference on Rough Sets, Fuzzy Sets, Data Mining and Granular Computing RSFDGrC 2009, *Lecture Notes in Computer Science*, vol. 5908, pp. 231-240, 2009.

[6] P. Majumdar and S.K. Samanta, "Generalised fuzzy soft sets," *Computers and Mathematics with Applications*, vol. 59, no. 4, pp. 1425-1432, 2010.

[7] J.H. Park, Y.C. Kwun and J.S. Hwang, "Generalized intuitionistic fuzzy soft sets," *Journal of Korean Institute of Intelligent Systems*, vol. 21, no. 3, pp. 389-394, 2011.

[8] W. Xu, J. Ma, S. Wang and G. Hao, "Vague soft sets and their properties," *Computers and Mathematics with Applications*, vol. 59, no. 2, pp. 787-794, 2010.

[9] T.K. Mondal and S.K. Samanta, "Generalized intuitionistic fuzzy sets," *Journal of Fuzzy Mathematics*, vol. 10, pp. 839-861, 2002.

[10] J.H. Park, Y.C. Kwun and M.J. Son, "A generalized intuitionistic fuzzy soft set theoretic approach to decision making problems," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 11, no. 2, pp. 71-76, 2011.

[11] J.H. Park, "On operations on generalized intuitionistic fuzzy soft sets," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 11, no. 3, pp. 184-189, 2011.

[12] J.H. Park, "Lattice structure of generalized intuitionistic fuzzy soft sets," *Journl of Korean Institute of Intelligent Systems*, vol. 24, no. 2, pp. 201-208.

[13] J.H. Park, O.H. Kim and Y.C. Kwun, "Some properties of equivalence soft set relations," *Computers and Mathematics with Applications*, vol. 63, no. 6, pp.1079-1088, 2012.

[14] A.R. Roy and P.K. Maji, "A fuzzy soft set theoretic approach to decision making problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412-418, 2007.

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