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On Regular Generalized b-Continuous Map in Topological Space

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Abstract. In this paper, we introduce a new class of regular generalized b-continuous map and study some of their properties as well as inter relationship with other continuous maps.

1. Introduction

Continuous map was studied for different types of closed sets by various researchers for past many years. In 1996, Andrijevic introduced new type called b-open sets. A.A.Omari and M.S.M. Noorani were introduced and studied b-continuous map and b-closed map.

The aim of this paper is to continue the study of regular generalized b-continuous map, regular generalized b-closed map have been introduced and studied their relations with various generalized closed maps. Throughout this paper (X, τ) and (Y, τ) represents the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned.

2. Preliminaries

Definition 2.1. Let a subset A of a topological space (X, τ) is called

(1) a *pre-open set* [13] if $A \subseteq int(cl(A))$.

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(2) a *semi-open set*[8] if $A \subseteq cl(int(A))$.

(3) a *α-open set* [14] if $A \subseteq int(cl(int(A)))$.

(4) a *b-open set* [3] if $A \subseteq cl(int(A)) \cup int(cl(A))$.

(5) a *generalized closed set* (briefly g-closed) [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(6) a *generalized α closed set* (briefly $g\alpha$-closed) [10] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α open in X.

(7) a *generalized b-closed set* (briefly gb-closed) [1] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(8) a *generalized semi-closed set* (briefly gs-closed) [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(9) a *semi generalized closed set* (briefly sg-closed) [5] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

(10) a *generalized αb-closed set* (briefly $g\alpha b$-closed) [15] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in X.

(11) a *generalized pre regular closed set* (briefly gpr-closed) [6] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

(12) a *semi generalized b- closed set* (briefly sgb- closed) [7] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in X.

(13) a *regular generalized b-closed set* (briefly rgb-closed set)[12] if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

3. Regular Generalized b-Continuous Maps

In this section we introduce regular generalized b-continuous map and investigate some of their properties.

Definition 3.1. Let X and Y be topological spaces. A map $f : (X, \tau) \to (Y, \tau)$ is said to be regular generalized b- continuous map if the inverse image of every open set in Y is rgb-open in X.

Theorem 3.2. *If a map $f : (X, \tau) \to (Y, \sigma)$ from a topological space X into a topological space Y is continuous, then it is rgb-continuous but not conversely.*

Proof. Let V be an open set in Y. Since f is continuous, then $f^{-1}(V)$ is open in X. As every open set is rgb-open, $f^{-1}(V)$ is rgb-open in X. Therefore f is rgb-continuous.

Example 3.3. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$,

$f(c) = c$, then f is rgb-continuous but not continuously as the inverse image of an open set $\{a, c\}$ in Y is $\{b, c\}$ which is not open set in X.

Theorem 3.4. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is b-continuous, then it is rgb-continous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is b-continuous. Let V be an open set in Y, Since f is b-continuous then $f^{-1}(V)$ is b-open. Hence every b-open is rgb-open in X. Therefore f is rgb-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$, $f(c) = a$, then f is rgb-continuous but not b-continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{a, b\}$ which is not b-open set in X.

Theorem 3.6. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is α-continuous then it is rgb-continous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is α-continuous. Let V be an open set in Y, Since f is α-continuous then $f^{-1}(V)$ is α-open. Hence every α-open is rgb-open in X. Therefore f is rgb-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.7. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \varphi, \{c\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$, then f is rgb-continuous but not α-continuously as the inverse image of an open set $\{b, c\}$ in Y is $\{a, b\}$ which is not α-open set in X.

Theorem 3.8. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is semi continuous, then it is rgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is semi continuous. Let V be an open set in Y, Since f is semi continuous then $f^{-1}(V)$ is semi open. Hence every semi open is rgb-open in X. Therefore f is rgb-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.9. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = c$, $f(c) = b$, then f is rgb-continuous but not semi-continuously as the inverse image of an open set $\{c\}$ in Y is $\{b\}$ which is not semi-open set in X.

Theorem 3.10. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is pre continuous, then it is rgb-continuous but not conversely.*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is pre continuous. Let V be an open set in Y, Since f is pre continuous, then $f^{-1}(V)$ is pre open. Hence every pre open is rgb-open in X. Therefore f is rgb-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.11. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$, then f is rgb-continuous but not pre-continuously as the inverse image of an open set $\{a, c\}$ in Y is $\{a, b\}$ which is not semi-open set in X.

Theorem 3.12. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha$ continuous, then it is rgb-continuous but not conversely*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is $g\alpha$ continuous. Let V be an open set in Y, Since f is $g\alpha$ continuous then $f^{-1}(V)$ is $g\alpha$ -open. Hence every $g\alpha$ -open is rgb-open in X. Therefore f is rgb-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.13. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = c$, then f is $g\alpha$-continuous but not rgb-continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{b, c\}$ which is not $g\alpha$-open set in X.

Theorem 3.14. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is gpr continuous then it is rgb-continuous but not conversely*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is *gpr* continuous. Let V be an open set in Y, Since f is *gpr* continuous then $f^{-1}(V)$ is *gpr* open. Hence every *gpr* open is rgb-open in X. Therefore f is rgb-continuous
The converse of above theorem need not be true as seen from the following example.

Example 3.15. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$, then f is rgb-continuous but not *gpr*-continuously as the inverse image of an open set $\{a, c\}$ in Y is $\{a, c\}$ which is not *gpr*-open set in X.

Theorem 3.16. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous, then it is gb-continuous but not conversely .*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous. Let V be an open set in Y, since f is rgb continuous then $f^{-1}(V)$ is rgb open. Hence every rgb open is *gb*-open in X. Therefore f is *gb*-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.17. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, then f is *gb*-continuous but not rgb -continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{a, c\}$ is not rgb-open set in X.

Theorem 3.18. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous, then it is gsp-continuous but not conversely .*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous. Let V be an open set in Y, Since f is rgb continuous then $f^{-1}(V)$ is rgb open. Hence every rgb open is *gsp*-open in X. Therefore f is *gsp*-continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.19. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$, then f is rgb-continuous but not gsp -continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{b, c\}$ which is not rgb-open set in X.

Theorem 3.20. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous then it is $g\alpha b$ -continuous but not conversely*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is rgb continuous. Let V be an open set in Y, Since f is rgb continuous then $f^{-1}(V)$ is rgb open. Hence every rgb open is $g\alpha b$ open in X. Therefore f is $g\alpha b$ -continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.21. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$, then f is rgb-continuous but not $g\alpha b$ -continuously as the inverse image of an open set $\{a, b\}$ in Y is $\{b, c\}$ which is not rgb-open set in X.

Theorem 3.22. *Let X and Y be topological spaces. If a map $f : (X, \tau) \to (Y, \sigma)$ is sgb continuous then it is rgb -continuous but not conversely*

Proof. Let us assume that $f : (X, \tau) \to (Y, \sigma)$ is sgb continuous. Let V be an open set in Y, Since f is sgb continuous then $f^{-1}(V)$ is sgb open. Hence every sgb open is rgb open in X. Therefore f is rgb -continuous.
The converse of above theorem need not be true as seen from the following example.

Example 3.23. Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = a$, $f(c) = b$, then f is sgb-continuous but not rgb -continuous as the inverse image of an open set $\{a, c\}$ in Y is $\{a, b\}$ is not sgb-open set in X.

Remark 3.24. The following examples show that rgb continuous and rg continuous maps are independent.

Example 3.24. **(a)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, then f is rg-continuous but not rgb-continuous as the inverse image of and $\{a, c\}$ in Y is $\{b, c\}$ is not rgb-continuous.

Example 3.24. **(b)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, then f is rgb-continuous but not rg -continuous as the inverse image of and $\{b, c\}$ in Y is $\{a, b\}$ is not rg-continuous.

Remark 3.25. The following examples show that rgb continuous and sg continuous maps are independent.

Example 3.25. **(a)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = a$,

$f(c) = b$, then f is sg-continuous but not rgb -continuous as the inverse image of and $\{a, c\}$ in Y is $\{a, c\}$ is not rgb-continuous.

Example 3.25. **(b)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c\}\}$ and $\sigma = \{Y, \phi, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$, then f is rgb-continuous but not sg -continuous as the inverse image of and $\{a, c\}$ in Y is $\{b, c\}$ is not sg-continuous.

Remark 3.26. The following examples show that rgb continuous and gs continuous maps are independent.

Example 3.26. **(a)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = c$, $f(b) = b$, $f(c) = a$, then f is gs-continuous but not rgb -continuous as the inverse image of and $\{a, c\}$ in Y is $\{a, c\}$ is not rgb-continuous.

Example 3.26. **(b)** Let $X = Y = \{a, b, c\}$ with $\tau = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = c$, $f(c) = c$, then f is rgb-continuous but not gs -continuous as the inverse image of and $\{b, c\}$ in Y is $\{a, b\}$ is not gs-continuous.

4. Applications

Theorem 4.1. *If a map $f : (X, \tau) \to (Y, \sigma)$ then (i) the following are equivalent (a) f is rgb-continous, (b) The inverse image of open set in Y is rgb-open in X. (ii) If $f : (X, \tau) \to (Y, \sigma)$ is rgb-continous, then $f(b^*(A)) \subset cl(f(A))$ for every subset A of X*

Proof. (i) Let us assume that $f : X \to Y$ be rgb - continuous. Let F be open in Y. Then F^c is closed in Y. Since f is rgb-continous, $f^{-1}(F^c)$ is rgb-closed in X. But $f^{-1}(F^c) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is rgb-closed in X. So $f^{-1}(F)$ is rgb-open in X. Hence $(a) \Longrightarrow (b)$. Conversely, Let us assume that the inverse image of each open set in Y is rgb -open in X. Let G be closed in Y. Then G^c is open in Y. By assumption $X - f^{-1}(G)$ is open in X. So $f^{-1}(G)$ is rgb-closed in X. There fore f is rgb-continous. Hence $(b) \Longrightarrow (a)$. We have (a) and (b) are equivalent.

(ii) Let us assume that f is rgb-continous. Let A be any subset of X. Then $cl(f(A))$ is closed in Y. Since f is rgb-continous, $f^{-1}(cl(f(A)))$ is rgb- closed in X and it contains A. But $b^*(A)$ is the intersection of all b^* closed sets containing. There fore $b^*(A) \subset f^{-1}(cl(f(A)))$. So that $f(b^*(A)) \subset cl(f(A))$.

Theorem 4.2. Pasting Lemma for rgb-continous maps
Let $X = A \cup B$ be a topological space with topology τ and Y be a topological space with σ. Let $f : (A, \tau/A) \to (Y, \sigma)$ and $g : (B, \tau/B) \to (Y, \sigma)$ be rgb-continous map such that $f(x) = g(x)$ for every $x \in A \cup B$. Suppose that A and B are rgb-closed sets in X, Then $\alpha : (X, \tau) \to (Y, \sigma)$ is rgb-continous.

Proof. Let F be any closed set in Y. Clearly $\alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D$, where $C = f^{-1}(F)$ and $D = g^{-1}(F)$. But C is rgb closed in A and A is rgb-closed in X. So C is rgb-closed in X. Since we have prove the result, if $B \subseteq A \subseteq X$, B is rgb-closed in A and A is rgb-closed in X, then B is rgb-closed in X. Also $C \cup D$ is rgb-closed in X. There fore $\alpha^{-1}(F)$ is rgb-closed in X. Hence α is rgb-continous.

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