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Experimental Studies of Swing Up and Balancing Control of an Inverted Pendulum System Using Intelligent Algorithms Aimed at Advanced Control Education

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Abstract

This paper presents the control of an inverted pendulum system using intelligent algorithms, such as fuzzy logic and neural networks, for advanced control education. The swing up balancing control of the inverted pendulum system was performed using fuzzy logic. Because the switching time from swing to standing motion is important for successful balancing, the fuzzy control method was employed to regulate the energy associated with the angular velocity required for the pendulum to be in an upright position. When the inverted pendulum arrived within a range of angles found experimentally, the control was switched from fuzzy to proportional-integral-derivative control to balance the inverted pendulum. When the pendulum was balancing, a joystick was used to command the desired position for the pendulum to follow. Experimental results demonstrated the performance of the two intelligent control methods.

Keywords: Swing up control, Inverted pendulum, Fuzzy control, Neural network control, Joystick control

1. Introduction

The inverted pendulum system is a prototype of dynamical systems for control education. The system is nonlinear and has a single input multiple output structure, and control of this system is quite challenging. Both the pendulum angle and cart position should be regulated to satisfy desired specifications.

Various types of inverted pendulums have been presented in the literature for either educational or research purposes [1-13]. Pendulums with multiple degrees of freedom have been presented and controlled to verify control algorithms. Experimental results have been presented for a pendulum that moves not only on a single track, but also in the x-y plane while balancing [3]. A rotating structure of the pendulum system, known as the Furuta pendulum, has been presented as well [4]. Remote swing up control of an inverted pendulum system has also been presented [5].

Inverted pendulum systems with multiple serial linkage structures have been presented to solve challenging control performance by mimicking an acrobat system with two links [7-13]. Simulation studies of the swing up motion of an acrobat have been performed using the partial feedback linearization method [11] and the energy-based control method [12]. Swing up con-

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trol of the Acrobot mimicking gymnasts is experimentally demonstrated by considering compliance [13]. Modifications and variations of the inverted pendulum system demand more complicated and sophisticated control algorithms other than the conventional proportional-integral-derivative (PID) control methods.

The PID controller works well for balancing the inverted pendulum and is sufficient for teaching undergraduate students about PID control algorithms. However, advanced control tasks of controlling both the angle and position as desired, such as swing up control or cart position tracking control while balancing, are quite difficult and challenging. In addition, the system is required to be robust enough under external disturbance.

Thus, advanced control methods are required for advanced control education. Among many advanced control methods, an intelligent control method is one of the promising tools for controlling complicated systems, including robots and mechatronics systems. Many successful control performances by intelligent control algorithms, including fuzzy logic and neural network control, have been reported in the literature [1, 3, 9, 10, 14-16].

We have learned the pending necessity of educating control engineers from the aforementioned pioneering research results. One of the merits of the inverted pendulum system is its simplicity to be used as a test bed for control algorithms or control education.

In the control education framework, the inverted pendulum system as a test bed should be simple and portable so that it can be easily utilized. Therefore, an inverted pendulum system was built as an experimental kit consisting of three parts: an inverted pendulum, control hardware, and a case. Because the pendulum kit is designed to be light and portable, it can be used anywhere with ease. However, because of its small size, the range of the moving distance of the pendulum system is quite limited.

Two experimental studies were performed with the experimental kit: one on swing up balancing control of the inverted pendulum system using fuzzy control and another on remote position control using a joystick. Swing up control is required to increase the energy to enter the angle range where it is possible to have a stand up position. Swing motions of the inverted pendulum were regulated by fuzzy rules to control the energy of the pendulum and to control the switching time from the swing to the balancing motion. When the pendulum successfully maintained an upright position, desired position tracking control was done remotely using a joystick, as in [5]. Here, the neural network was used as a control tool to give robustness against disturbance to the system.

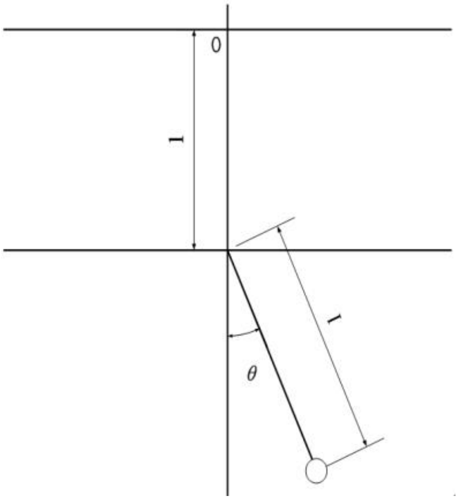


Figure 1. Swing motion.

Experimental studies validated the theoretical energy required to have an upright position and proved the robustness of the intelligent controllers for the cart to follow the desired trajectory while balancing.

2. Fuzzy Control for Swing Up Motion

2.1 Fuzzy Control

Successful swing up control of the inverted pendulum system requires accurate control of the switching time from the swing to the balancing motion. It is important to determine the appropriate condition for switching control and it is necessary to make the pendulum arrive at the condition for balancing.

The energy-based control method is a popular approach for swing up control. Successful swing up control starts from zero energy at the down position and reaches maximum energy at the upright position. The energy of the pendulum in the swing up motion is described as the sum of the kinetic energy and the potential energy:

$$\mathrm{E} = \frac{1}{2}J\dot{\theta}^2 + mgL(1-\cos\theta) \tag{1}$$

where J is the angular moment, m is the mass, L is the length, and θ is the angle of the pendulum.

The maximum energy of the pendulum can be calculated as $E = 2mgL$ when the pendulum is at the upright position at rest, as shown in Figure 1. However, relying on angle control

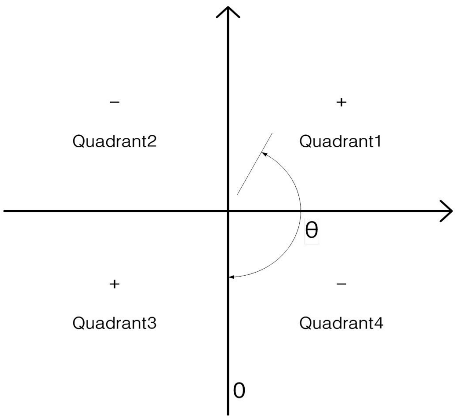
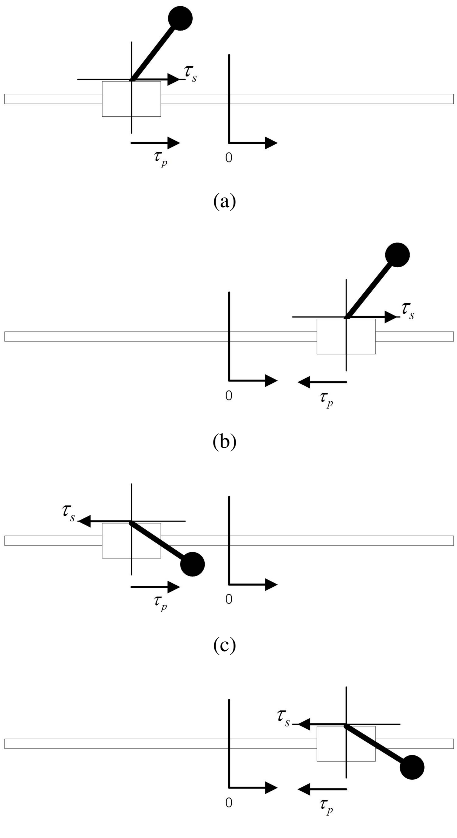


Figure 2. Signs of the torque relative to the pendulum angle θ.



(a)

(b)

(c)

(d)

Figure 3. Direction of torques. (a) 1st quadrant and left position, (b) 1st quadrant and right position, (c) 4th quadrant and left position, (d) 4th quadrant and right position.

alone may result in failure of standing up. Eq. (1) shows that the energy depends on the angular velocity as well as the angle. If the angular velocity $\dot{\theta}$ is too large around the upright position to make $E > 2mgL$, the pendulum passes over the upright position and rotates instead of staying in the upright position, and then swing up control fails.

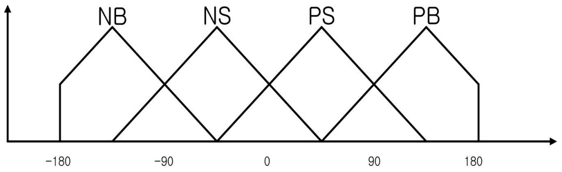
Because the control input torque to the system is the sum of two torques, the angle and the position control torques, the signs of the two torques play an important role to balance the pendulum. The angle control torque τ_s controls not only the angle but also the angular velocity of the inverted pendulum. It is therefore important for successful swing up control. Because the angular velocity plays an important role, the angular velocity of the pendulum should be controlled such that the angular velocity is reduced gradually around the upright region in order not to pass through.

Another consideration from a control point of view is the positional control torque τ_p that regulates the moving distance in the reference to a center point. The cart should not deviate too much from the center position, because the distance is limited. Figure 2 explains the sign of the torques with respect to the pendulum angle. The sign of $\tau_s$is positive in both the first and the third quadrants, and negative in both the second and fourth quadrants (Figure 2).

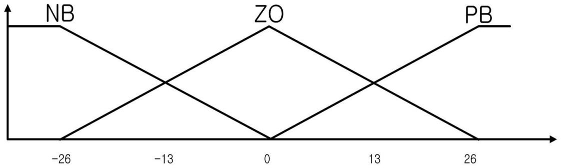
Figure 3 shows torque configurations in the first and fourth quadrants. When the pendulum is in the first quadrant and the cart position is located on the left side (Figure 3a), the cart should move to the right with a large torque because the signs of $\tau_s$and $\tau_p$are the same. When the pendulum is in the first quadrant and the cart position is located on the right side (Figure 3b), the cart should move to the left with less torque compared to the case of Figure 3a because the signs are opposite. Figures 3c and d can be explained in a similar manner.

Therefore, an important control issue is the sign of the torques for both the angle and the angular velocity control in the four areas shown in Figure 3. In each quadrant, a different configuration of torque control should be applied. For example, the pendulum angle θ is positive in the first quadrant and the sign of the torque is positive to generate torques to the right. If the pendulum is located in the second quadrant, the cart should move to the left because the sign of the torque is negative. The fuzzy control method is suitable for considering all the conditions effectively as a gain scheduling control method.

To generate appropriate control rules, four fuzzy membership functions for the angle and three membership functions for the angular velocity were designed, as shown in Figure 4. The membership function of the control input u is a singleton indicating only direction. Therefore, fuzzy logic determines the sign of the torque applied with respect to pendulum location



(a) Angle



(b) Angular velocity

Figure 4. Fuzzy membership functions. NB, negative big; NS, negative small; PS, positive small; PB, positive big; ZO, zero.

Table 1. Fuzzy rules

u		$\dot{\theta}$		
		NB	ZO	PB
θ	NB	1	-1	1
	NS	-1	1	-1
	PS	1	-1	1
	PB	-1	1	-1

NB, negative big; NS, negative small; PS, positive small; PB, positive big; ZO, zero.

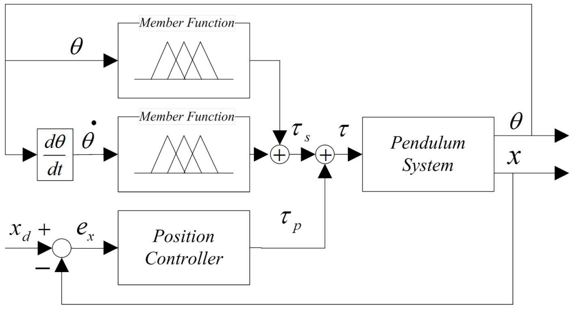


Figure 5. Fuzzy control block diagram for swing up.

and angular velocity. The corresponding fuzzy rules are listed in Table 1.

A control block diagram is shown in Figure 5. The control input for swing up control is given by

$$\tau = \tau_s + \tau_p \tag{2}$$

where τ_s is the torque of the fuzzy controller for angle control and τ_p the torque for position control. The angle control torque is a constant with a different sign:

$$\tau_s = (-1)^s K \tag{3}$$

where K is a torque constant, which was 350 in the experiment. The position control torque used the proportional-derivative (PD) control method:

$$\tau_p = k_p e_p + k_d \dot{e}_p \tag{4}$$

where $e_p = x_d - x$.

2.2 Determination of the Region of Upright Position

As the switching time is a key issue, determining the region of upright position of the pendulum is critical for successful swing up control. Here we found the critical angle by trial and error through experimental studies. The region of the pendulum angle was about 20° (0.349 rad). As soon as the pendulum reached that region, switching control from swing up control to balancing control was activated.

3. Neural Network Control

After successful upright swing up control, the inverted pendulum should maintain balance. Linear controllers may work for balancing, but are not robust enough to follow the desired trajectory of the cart given remotely by a joystick. To obtain robust balancing performance while following desired trajectories, neural network control was applied.

A neural network is a nonlinear controller that learns, adapts, and generalizes for complicated systems. Here, neural network control was added to the PID controlled system, as shown in Figure 6, to give robustness to the system. The scheme known as the reference compensation technique is shown in Figure 6 [14, 17]. Neural network control modifies the desired trajectories to minimize the output errors. Compensated signals pass through PID controllers to generate the control input. The control input signal is modified by the neural network to minimize the output errors. Compensated errors pass through the PID controller to generate a control input signal for the system.

$$e = z_d - z + \phi_N \tag{5}$$

where z_d is the desired input and ϕ_N is the neural network output. The error passes through the PID controller.

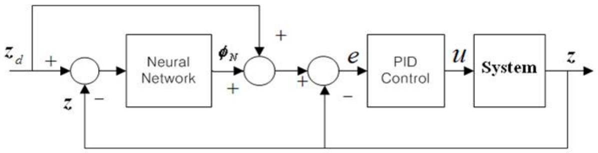


Figure 6. Neural network control block diagram.

The detailed PID controller output becomes

$$u_\theta = k_{p\theta}(e_\theta + \phi_1) + k_{d\theta}(\underset{\theta}{\dot{e}} + \phi_2) + k_{i\theta}(\int e_\theta dt + \phi_3) \quad (6)$$

$$u_x = k_{px}(e_x + \phi_4) + k_{dx}(\underset{x}{\dot{e}} + \phi_5) + k_{ix}(\int e_x dt + \phi_6) \quad (7)$$

The total control input is

$$\begin{aligned} u &= u_\theta + u_x \\ &= u_{c\theta} + u_{cx} + \phi_\theta + \phi_x \end{aligned} \quad (8)$$

where $u_{c\theta} = k_{p\theta}e_\theta + k_{d\theta}\dot{e}_\theta + k_{i\theta}\int e_\theta dt$, $u_{cx} = k_{px}e_x + k_{dx}\dot{e}_x + k_{ix}\int e_x dt$, $\phi_\theta = k_{p\theta}\phi_1 + k_{d\theta}\phi_2 + k_{i\theta}\phi_3$, $\phi_x = k_{px}\phi_4 + k_{dx}\phi_5 + k_{ix}\phi_6$.

From Eq. (8), the training signal is defined as

$$v = u - (\phi_\theta + \phi_x) \quad (9)$$

where $v = u_{c\theta} + u_{cx}$.

The objective function is defined as

$$E = \frac{1}{2}v^2 \quad (10)$$

The back-propagation algorithm requires differentiating (10) with respect to the weight vector *w* as

$$\frac{\partial E}{\partial w} = \frac{\partial E}{\partial v}\frac{\partial v}{\partial w} = v\frac{\partial v}{\partial w} = -v(\frac{\partial \phi_\theta}{\partial w} + \frac{\partial \phi_x}{\partial w}) \quad (11)$$

Then, weights are updated as

$$w(t+1) = w(t) - \eta\frac{\partial E}{\partial w} \quad (12)$$

where η is the learning rate.

First, simulation studies of neural network control for the inverted pendulum system were performed. Figure 7 shows the simulation block diagram. Figure 8 shows the simulation results for balancing the pendulum. The cart is required to follow the sinusoidal trajectory while balancing.

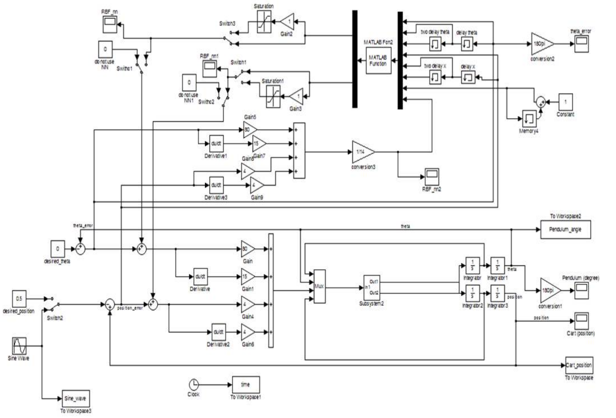


Figure 7. Simulink control block diagram.

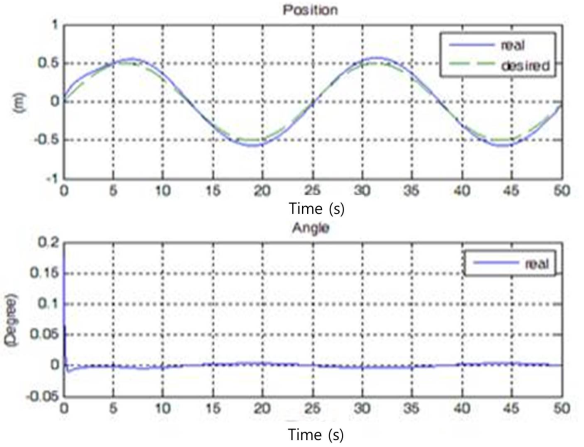


Figure 8. Simulation result of neural network control.

Table 2. Parameters of the inverted pendulum system

$J = 0.008kgm^2$	Moment of the pendulum
M = 0.2 kg	Mass of the pendulum
M = 0.5 kg	Mass of the cart
L = 0.2 m	Length of the pendulum
x	Movement of the cart
θ	Angle of the pendulum

4. Experimental Setup

The experimental setup is shown in Figure 9. The system consists of the pendulum, a guide rail, a DC motor, control hardware, and a joystick. The inverted pendulum system is driven by a DC motor through a timing belt. The pendulum angle is measured by the encoder attached to the pendulum. Table 2 lists the parameters of the pendulum system.

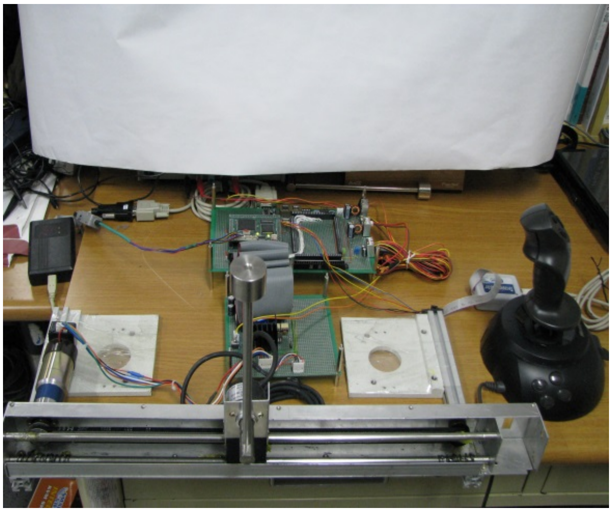


Figure 9. Experimental setup. FPGA, field programmable gate array; DSP, digital signal processing.

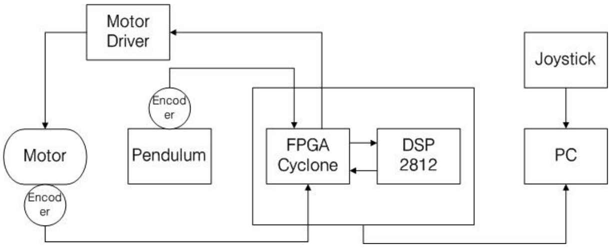


Figure 10. Overall system configuration.

The overall system structure is shown in Figure 10. The control algorithm is embedded into a DSP 2812 board. Encoder counters are embedded into an FPGA chip. The sampling time is 100 Hz.

Figure 11 shows the process of swing up control. The switching angle here is 20°, which was determined via experiments.

5. Experiment of Swing Up Motion Control by Fuzzy Logic

Experimental results of the swing up control task are shown in Figure 12. Initially the pendulum was downward and at rest. After shaking six times by itself to generate energy, the pendulum successfully maintained balance. Figure 13 shows the angle of the pendulum, and Figure 14 shows the cart position. Because our system has movement limitations, the pendulum failed several times by going beyond the limit.

The torque constant was K = 350 in Eq. (3) and $k_p = 81, k_d =$

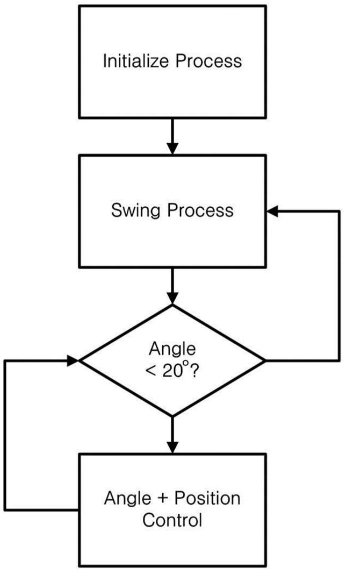


Figure 11. Flow chart of swing up control.

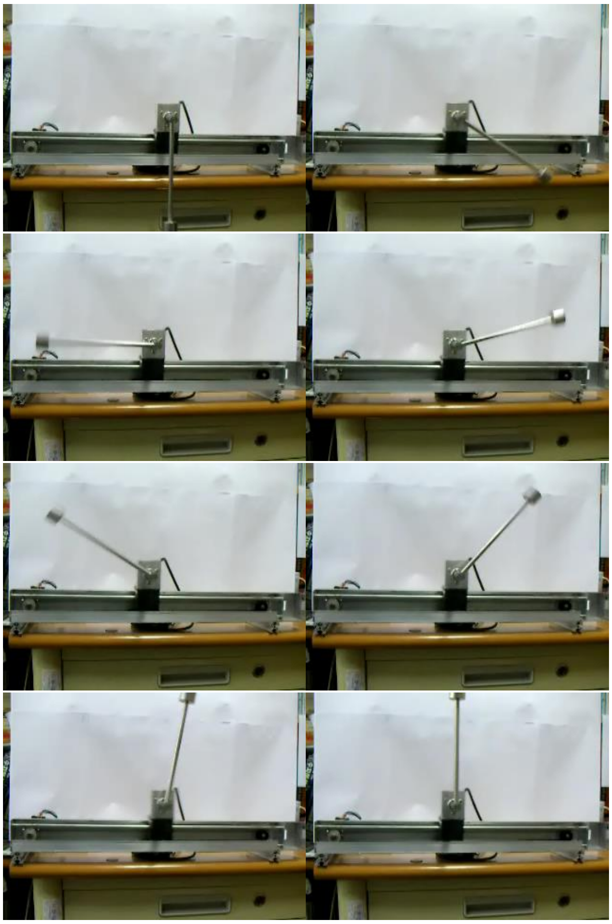


Figure 12. Demonstration of swing up control.

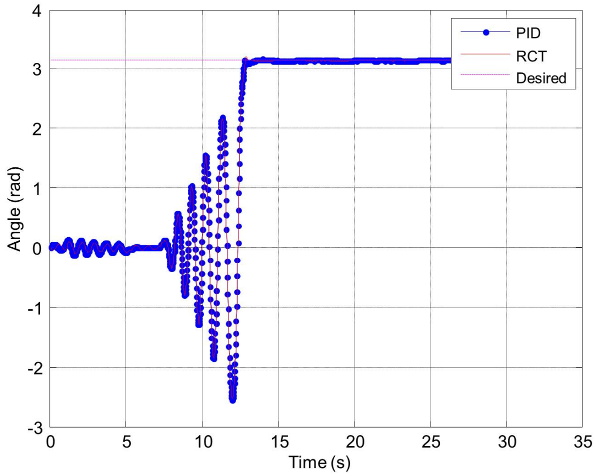


Figure 13. Angle of the pendulum. PID, proportional-integral-derivative; RCT, radial basis function.

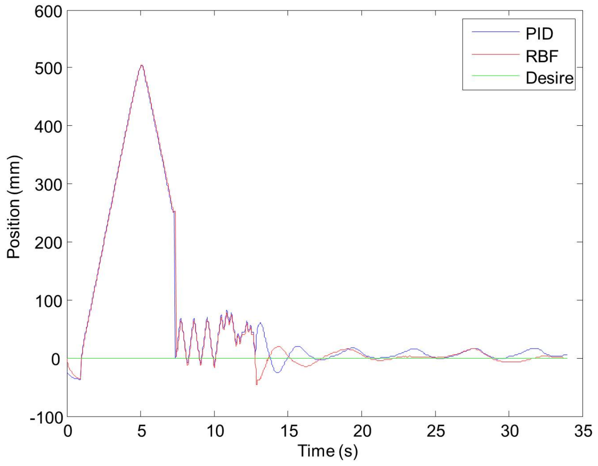


Figure 14. Position of the cart. PID, proportional-integral-derivative; RCT, radial basis function.

10 were used in Eq. (4). After successful swing up control by fuzzy logic, two control methods were tested: PID control and neural network control. The results of both PID control and neural network control are plotted together for comparison (Figures 13 and 14). No clear distinction was observed in this experiment, because the requirement was balancing without moving the cart, and there was no disturbance. If there was a disturbance or a requirement of following the desired trajectory, PID control may have shown inferior performance. Therefore, the neural network was employed for joystick control, in the next section.

Table 3. Controller gains

Angle		Position	
$k_{p\theta}$	20.0	k_{px}	4.5
$k_{d\theta}$	1.1	k_{dx}	1.5
$k_{i\theta}$	2.0	k_{ix}	2.0

Table 4. Learning rates

Learning rates	Value
η_ω	0.0001
η_θ	0.0001
η_μ	0.0001
η_σ	0.0001

6. Experiment of Joystick Position Control by Neural Network Control

The next experiment was a position control test of following desired trajectories commanded by a joystick. Command by the joystick may add disturbance to the system such that PID control shows poor performance. The pendulum is required to follow the desired position while balancing. Table 3 lists the PID controller gains for angle and position. Gains were selected by trial and error procedures through experimental studies. Table 4 lists the values of the learning rate for the radial basis function neural network.

Figure 15 shows pictures of an experimental demonstration. As the joystick moved, the pendulum also moved in the same direction. However, if the joystick moved too fast, the pendulum could not follow the position. Figure 16 shows the corresponding tracking plot. It can be clearly observed that the pendulum followed the commanded trajectory.

7. Conclusions

Intelligent control methods were tested with an inverted pendulum system embedded in an experimental kit. Fuzzy logic was used for swing up control and neural network control was used for desired position tracking control by a joystick. Experimental results demonstrated the performance of the intelligent control methods. For swing up control, it was found that the switching angle was important, and controlling angular velocity played an important role. Remote tracking control by the joystick was successfully performed by the neural network control method. Therefore, it can be expected that a combination of two intelligent control methods will provide the most effective

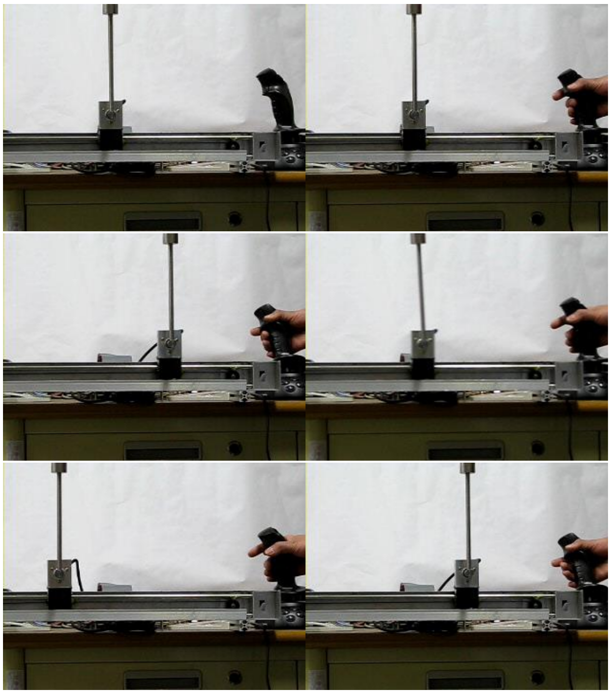


Figure 15. Demonstration of joystick control.

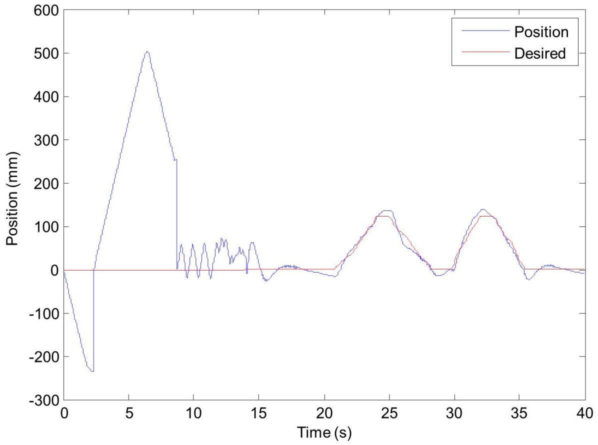


Figure 16. Position tracking by joystick control.

results.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

Acknowledgements

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References

[1] F. L. Lewis, S. Jagannathan, and A. Yesildirek, *Neural network control of robot manipulators and nonlinear systems*,London: Taylor & Francis, 1999

[2] I. Fantoni and R. Lozano, “Global stabilization of the cart-pendulum system using saturation functions,” in *Proceedings of the 42nd IEEE Conference on Decision and Control*, Maui, HI, December 9-12, 2003, pp. 4393-4398. http://dx.doi.org/10.1109/CDC.2003.1272193

[3] S. Jung, H. T. Cho, and T. C. Hsia, “Neural network control for position tracking of a two-axis inverted pendulum system: experimental studies,” *IEEE Transactions on Neural Networks*, vol. 18, no. 4, pp. 1042-1048, Jul. 2007. http://dx.doi.org/10.1109/TNN.2007.899128

[4] M. Yamakita, K. Nonaka, and K. Furuta, “Swing up control of a double pendulum,” in *Proceedings of the American Control Conference*, San Francisco, CA, June 2-4, 1993, pp. 2229-2233.

[5] M. Koga and S. Kawakami, “Real-time remote control system in Java and its application to swing up control of inverted pendulum,” in *Proceedings of the SICE Annual Conference*, Fukui, Japan, August 4-6, 2003, pp. 358-361.

[6] J. Ahn and S. Jung, “Swing-up fuzzy control of an inverted pendulum system for control education with an experimental kit,” in *Proceedings of the International Conference on Control*, Automation and Systems, Seoul, Korea, October 14-17, 2008, pp. 1709-1714. http://dx.doi.org/10.1109/ICCAS.2008.4694503

[7] R. Yang, Y. Y. Kuen, and Z. Li, “Stabilization of a 2-DOF spherical pendulum on X-Y table,” in *Proceedings of the IEEE International Conference on Control Applications*, Anchorage, AK, September 25-27, 2000, pp. 724-729. http://dx.doi.org/10.1109/CCA.2000.897522

[8] R. J. Wai, J. D. Lee, and L. J. Chang, “Development of adaptive sliding-mode control for nonlinear dual-axis inverted-pendulum system,” in *Proceedings of the*

IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Kobe, Japan, July 20-24, 2003, pp. 815-820. http://dx.doi.org/10.1109/AIM.2003.1225447

[9] M. E. Magana and F. Holzapfel, "Fuzzy-logic control of an inverted pendulum with vision feedback," *IEEE Transactions on Education*, vol. 41, no. 2, pp. 165-170, May 1998. http://dx.doi.org/10.1109/13.669727

[10] S. Omatu, T. Fujinaka, and M. Yoshioka, "Neuro-PID control for inverted single and double pendulums," in *Proceedings of the IEEE International Conference on Systems*, Man, and Cybernetics, Nashville, TN, October 8-11, 2000, pp. 2685-2690. http://dx.doi.org/10.1109/ICSMC.2000.884401

[11] M. W. Spong, "The swing up control problem for the acrobat," *IEEE Control Systems Magazine*, vol. 15, pp. 72-79, Feb. 1995.

[12] X. Xin and M. Kaneda, "New analytical results of the energy based swinging up control of the Acrobot," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, Nassau, Bahamas, December 17, 2004, pp. 704-709. http://dx.doi.org/10.1109/CDC.2004.1428728

[13] M. Kanazawa, R. Anami, S. Nakaura, and M. Sampei, "Swing up control experiment aimed at energy interaction between the acrobot and compliance," in *Proceedings of the SICE Annual Conference*, Takamatsu, Japan, September 17-20, 2007, pp. 1236-1241. http://dx.doi.org/10.1109/SICE.2007.4421173

[14] D. H. Song, G. H. Lee, and S. Jung, "Neural network compensation technique for standard PD-like fuzzy controlled nonlinear systems," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 8, no. 1, pp. 68-74, Mar. 2008

[15] H. Lee and S. Jung, "Neuro-fuzzy control of inverted pendulum system for intelligent control education," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 9, no. 4, pp. 309-314, Dec. 2009.

[16] W. Kim and S. Jung, "Fuzzy logic application to a two-wheel mobile robot for balancing control performance," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 12, no. 2, pp. 154-161, Jun. 2012. http://dx.doi.org/10.5391/IJFIS.2012.12.2.154

[17] S. Jung and S. S. Kim, "Hardware implementation of a real-time neural network controller with a DSP and an FPGA for nonlinear systems," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 1, pp. 265-271, Feb. 2007. http://dx.doi.org/10.1109/TIE.2006.888791

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