East Asian Math. J.
Vol. 30 (2014), No. 5, pp. 599–607
http://dx.doi.org/10.7858/eamj.2014.039

LIGHTLIKE HYPERSURFACES OF INDEFINITE KAEHLER MANIFOLDS OF QUASI-CONSTANT CURVATURES

Dae Ho Jin

Abstract. We study lightlike hypersurfaces M of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature subject to the condition that the curvature vector field of $\bar{M}$ belongs to the screen distribution $S(TM)$. We provide several new results on such lightlike hypersurfaces M.

1. Introduction

B.Y. Chen and K. Yano [1] introduced the notion of a *semi-Riemannian manifold of quasi-constant curvature* as a semi-Riemannian manifold $(\bar{M}, \bar{g})$ endowed with the curvature tensor $\bar{R}$ satisfying the following equation:

$$\begin{aligned} \bar{R}(X,Y)Z = \ell\{\bar{g}(Y,Z)X - \bar{g}(X,Z)Y\} \\ + \hbar\{\bar{g}(Y,Z)\theta(X)\zeta - \bar{g}(X,Z)\theta(Y)\zeta \\ + \theta(Y)\theta(Z)X - \theta(X)\theta(Z)Y\}, \end{aligned} \tag{1.1}$$

for any vector fields X, Y and Z, where ℓ and $\hbar$ are smooth functions, ζ is a non-vanishing smooth unit spacelike vector field, which is called the *curvature vector field* of $\bar{M}$, and θ is a 1-form associated with ζ by

$$\theta(X) = \bar{g}(X, \zeta).$$

It is well known that if the curvature tensor $\bar{R}$ is of the form (1.1), then $\bar{M}$ is conformally flat. If $\hbar = 0$, then $\bar{M}$ is a space of constant curvature ℓ.

The theory of lightlike hypersurfaces is an important topic of research in differential geometry due to its application in mathematical physics, especially in the general relativity. The study of such notion was initiated by Duggal and Bejancu [2] and later studied by many authors (see two books [3, 4]).

The purpose of this paper is to study lightlike hypersurfaces M of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature such that the curvature vector field of $\bar{M}$ belongs to the screen distribution $S(TM)$. We provide several new results on such lightlike hypersurfaces M subject such that the structure

Received June 13, 2014; Accepted July 29, 2014.
2010 *Mathematics Subject Classification.* Primary 53C25, 53C40, 53C50.
Key words and phrases. quasi-constant curvature, parallel vector and tensor fields, lightlike hypersurface, indefinite Kaehler manifold.

(pISSN 1226-6973, eISSN 2287-2833)

fields $\{F, U, V\}$ on M, induced from the indefinite Kaehler structure J of $\bar{M}$, are parallel with respect to the induced connection ∇ on $\bar{M}$.

2. Lightlike hypersurfaces

Let (M, g) be a lightlike hypersurface of a semi-Riemannian manifold $(\bar{M}, \bar{g})$. Then the normal bundle $TM^{\perp}$ of M is a subbundle of the tangent bundle TM, of rank 1, and coincides with the radical distribution $Rad(TM) = TM \cap TM^{\perp}$. A complementary vector bundle $S(TM)$ of $TM^{\perp}$ in TM is non-degenerate distribution on M, which is called a *screen distribution* on M, such that

$$TM = TM^{\perp} \oplus_{orth} S(TM),$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M, by $\Gamma(E)$ the $F(M)$ module of smooth sections of any vector bundle E over M and by $(-.-)_i$ the i-th equation of $(-.-)$. We use same notations for any others. It is known [2] that, for any null section ξ of $TM^{\perp}$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique lightlike vector bundle $tr(TM)$ in $S(TM)^{\perp}$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \quad \forall\, X \in \Gamma(S(TM)).$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen distribution $S(TM)$, respectively. Then the tangent bundle $T\bar{M}$ of $\bar{M}$ is decomposed as follow:

$$T\bar{M} = TM \oplus tr(TM) = \{TM^{\perp} \oplus tr(TM)\} \oplus_{orth} S(TM).$$

From now and in the sequel, let X, Y, Z and W be the vector fields on M, unless otherwise specified. Let $\bar{\nabla}$ be the Levi-Civita connection of $\bar{M}$ and P the projection morphism of TM on $S(TM)$. Then the local Gauss and Weingartan formulas of M and $S(TM)$ are given respectively by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N, \tag{2.1}$$
$$\bar{\nabla}_X N = -A_N X + \tau(X)N; \tag{2.2}$$
$$\nabla_X PY = \nabla^*_X PY + C(X, PY)\xi, \tag{2.3}$$
$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \tag{2.4}$$

where ∇ and ∇^* are the liner connections on TM and $S(TM)$ respectively, B and C are the local second fundamental forms on TM and $S(TM)$ respectively, which are called the *lightlike* and *screen* second fundamental forms of M, A_N and A^*_ξ are the shape operators on TM and $S(TM)$ respectively and τ is a 1-form on TM. Since the connection $\bar{\nabla}$ of $\bar{M}$ is torsion-free, the induced connection ∇ of M is also torsion-free and B is symmetric on TM.

The induced connection ∇ of M is not metric and satisfies

$$(\nabla_X g)(Y, Z) = B(X, Y)\,\eta(Z) + B(X, Z)\,\eta(Y), \tag{2.5}$$

where η is a 1-form on M such that

$$\eta(X) = \bar{g}(X, N).$$

From the fact that $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we show that B is independent of the choice of a screen distribution $S(TM)$ and satisfies

$$B(X, \xi) = 0. \tag{2.6}$$

The above second fundamental forms are related to their shape operators by

$$B(X,Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{2.7}$$

$$C(X, PY) = g(A_N X, PY), \quad \bar{g}(A_N X, N) = 0. \tag{2.8}$$

From (2.7), A^*_ξ is $\Gamma(S(TM))$-valued self-adjoint on $\Gamma(TM)$ with respect to the induced metric g on M such that

$$A^*_\xi \xi = 0. \tag{2.9}$$

Denote by $\bar{R}$, R and R^* the curvature tensors of the Levi-Civita connection $\bar{\nabla}$ of $\bar{M}$, the induced connection ∇ on M and the induced connection ∇^* on $S(TM)$ respectively. Using the Gauss-Weingarten formulas for M and $S(TM)$, we obtain the Gauss-Codazzi equations for M and $S(TM)$ such that

$$\begin{aligned} \bar{R}(X,Y)Z = {} & R(X,Y)Z + B(X,Z)A_N Y - B(Y,Z)A_N X \\ & + \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ & + \tau(X)B(Y,Z) - \tau(Y)B(X,Z)\}N, \end{aligned} \tag{2.10}$$

$$\begin{aligned} \bar{R}(X,Y)N = {} & -\nabla_X(A_N Y) + \nabla_Y(A_N X) + A_N[X,Y] \\ & + \tau(X)A_N Y - \tau(Y)A_N X \\ & + \{B(Y, A_N X) - B(X, A_N Y) + 2d\tau(X,Y)\}N, \end{aligned} \tag{2.11}$$

$$\begin{aligned} R(X,Y)PZ = {} & R^*(X,Y)PZ + C(X,PZ)A^*_\xi Y - C(Y,PZ)A^*_\xi X \\ & + \{(\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) \\ & - \tau(X)C(Y,PZ) + \tau(Y)C(X,PZ)\tau(Y)\}\xi, \end{aligned} \tag{2.12}$$

$$\begin{aligned} R(X,Y)\xi = {} & -\nabla^*_X(A^*_\xi Y) + \nabla^*_Y(A^*_\xi X) + A^*_\xi[X,Y] \\ & - \tau(X)A^*_\xi Y + \tau(Y)A^*_\xi X \\ & + \{C(Y, A^*_\xi X) - C(X, A^*_\xi Y) - 2d\tau(X,Y)\}\xi. \end{aligned} \tag{2.13}$$

In case $R = 0$, we say that M is a *flat* manifold.

3. Indefinite Kaehler manifolds of quasi-constant curvatures

Let $\bar{M} = (\bar{M}, \bar{g}, J)$ be an indedinite Kaehler manifold, where $\bar{g}$ is a semi-Riemannian metric and J is an indefinite almost complex structure satisfying

$$J^2 = -I, \quad \bar{g}(JX, JY) = \bar{g}(X,Y), \quad (\bar{\nabla}_X J)Y = 0, \tag{3.1}$$

for any vector fields X and Y of $\bar{M}$ [2, 4, 5].

Suppose that M is a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$. Then the screen distribution $S(TM)$ splits as follows [2, 5, 6, 7]:

If ξ and N are local sections of $TM^{\perp}$ and $tr(TM)$ respectively, then

$$\bar{g}(J\xi,\xi)=\bar{g}(J\xi,N)=\bar{g}(JN,\xi)=\bar{g}(JN,N)=0, \quad \bar{g}(J\xi,JN)=1.$$

It follow that the vector fields $J\xi$ and JN belong to $S(TM)$. Thus $J(TM^{\perp})$ and $J(tr(TM))$ are distributions on M of rank 1 such that $TM^{\perp}\cap J(TM^{\perp})=\{0\}$ and $TM^{\perp}\cap J(tr(TM))=\{0\}$. Hence $J(TM^{\perp})\oplus J(tr(TM))$ is a subbundle of $S(TM)$, of rank 2. Therefore, there exists a non-degenerate almost complex distribution D_o on M with respect to J, that is, $J(D_o)=D_o$, such that

$$TM=TM^{\perp}\oplus_{orth}\{J(TM^{\perp})\oplus J(tr(TM))\oplus_{orth}D_o\}.$$

Consider the 2-lightlike almost complex distribution D such that

$$D=\{TM^{\perp}\oplus_{orth}J(TM^{\perp})\}\oplus_{orth}D_o, \qquad TM=D\oplus J(tr(TM)).$$

Consider two local lightlike vector fields U and V on $S(TM)$, and two 1-forms u and v defined by

$$U=-JN, \quad V=-J\xi, \quad u(X)=g(X,V), \quad v(X)=g(X,U). \tag{3.2}$$

Denote by S the projection morphism of TM on D. Let F be a tensor field of type $(1,1)$ globally defined on M by $F=J\circ S$. Then JX is expressed as

$$JX=FX+u(X)N, \quad \forall\, X\in\Gamma(TM). \tag{3.3}$$

Applying J to (3.3) and using (3.1) and (3.2), we have

$$F^2X=-X+u(X)U, \qquad FU=0, \qquad u(U)=1. \tag{3.4}$$

Therefore, (F, u, U) defines an indefinite almost contact structure on M. The vector field U is called the *structure vector field* of M. Applying $\bar{\nabla}_X$ to (3.2) and (3.3) by turns and using (2.1)∼(2.8) and (3.1)∼(3.3), we have

$$B(X,U)=C(X,V)\equiv\sigma(X), \tag{3.5}$$

$$\nabla_XU=F(A_{N}X)+\tau(X)U, \tag{3.6}$$

$$\nabla_XV=F(A^*_{\xi}X)-\tau(X)V, \tag{3.7}$$

$$(\nabla_XF)(Y)=u(Y)A_{N}X-B(X,Y)U. \tag{3.8}$$

From now and in the sequel, let $\bar{M}$ be an indefinite Kaehler manifold of quasi-constant curvature. In the entire discussion of this article, we shall assume that ζ belongs to $S(TM)$. In this case, comparing the tangential and transversal components of two equations (1.1) and (2.10), we obtain

$$\begin{aligned} R(X,Y)Z \;=\; &\ell\{\bar{g}(Y,Z)X-\bar{g}(X,Z)Y\} \\ &+\hbar\{\bar{g}(Y,Z)\theta(X)\zeta-\bar{g}(X,Z)\theta(Y)\zeta \\ &\qquad +\theta(Y)\theta(Z)X-\theta(X)\theta(Z)Y\} \\ &+B(Y,Z)A_{N}X-B(X,Z)A_{N}Y, \end{aligned} \tag{3.9}$$

$$(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + \tau(X)B(Y,Z) - \tau(Y)B(X,Z) = 0, \tag{3.10}$$

respectively. Taking the scalar product with N to (2.12), we have

$$\bar{g}(R(X,Y)PZ,\, N) = (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) - \tau(X)C(Y,PZ) + \tau(Y)C(X,PZ).$$

Substituting (3.9) into the last equation, we see that

$$\begin{aligned}(\nabla_X C)(Y,PZ) &- (\nabla_Y C)(X,PZ) \\ &- \tau(X)C(Y,PZ) + \tau(Y)C(X,PZ) \\ = \ell\{g(Y,PZ)\eta(X) &- g(X,PZ)\eta(Y)\} \\ &+ \hbar\{\theta(Y)\eta(X) - \theta(X)\eta(Y)\}\theta(PZ).\end{aligned} \tag{3.11}$$

Replacing Z by N to (1.1) and then, comparing the tangential and transversal components of this resulting equation and (2.11), we obtain

$$\begin{aligned}&\ell\{\eta(Y)X - \eta(X)Y\} + \hbar\{\eta(Y)\theta(X) - \eta(X)\theta(Y)\}\zeta \\ &= - \nabla_X(A_N Y) + \nabla_Y(A_N X) + A_N[X,Y] \\ &\qquad + \tau(X)A_N Y - \tau(Y)A_N X,\end{aligned} \tag{3.12}$$

$$B(Y,A_N X) - B(X,A_N Y) + 2d\tau(X,Y) = 0. \tag{3.13}$$

Theorem 3.1. *Let M be a lightlike hypersurface of an indefinite Kaehler manifold of quasi-constant curvature. If ζ belongs to $S(TM)$, then*

$$\ell = 0, \qquad \hbar\theta(V) = 0. \tag{3.14}$$

Proof. Applying ∇_X to (3.5): $B(Y,U) = C(Y,V)$, we have

$$(\nabla_X B)(Y,U) = (\nabla_X C)(Y,V) + g(A_N Y, \nabla_X V) - g(A^*_\xi Y, \nabla_X U).$$

Using (2.8), (3.1), (3.3) and (3.5) $\sim$ (3.7), the last equation is reduced to

$$\begin{aligned}(\nabla_X B)(Y,U) \;=\; &(\nabla_X C)(Y,V) - 2\tau(X)C(Y,V) \\ &- g(A^*_\xi X, F(A_N Y)) - g(A^*_\xi Y, F(A_N X)).\end{aligned}$$

Substituting this equation and (3.5) into (3.10) with $Z = U$, we get

$$(\nabla_X C)(Y,V) - (\nabla_Y C)(X,V) - \tau(X)C(Y,V) + \tau(Y)C(X,V) = 0.$$

Comparing this equation with (3.11) with $PZ = V$, we obtain

$$\ell\{u(Y)\eta(X) - u(X)\eta(Y)\} + \hbar\{\theta(Y)\eta(X) - \theta(X)\eta(Y)\}\theta(V) = 0.$$

Replacing Y by ξ to this equation, we get

$$\ell u(X) + \hbar\theta(X)\theta(V) = 0.$$

Taking $X = V$ to this, we get $\hbar\theta(V) = 0$. Therefore, we have $\ell u(X) = 0$. Taking $X = U$ to this result, we obtain $\ell = 0$. □

Definition 1. Let $\nabla^{\perp}_X N = \pi(\bar{\nabla}_X N)$ for any $X \in \Gamma(TM)$, where π is the projection morphism of $T\bar{M}$ on $tr(TM)$. Then $\nabla^{\perp}$ is a linear connection on the transversal vector bundle $tr(TM)$ of M. We say that $\nabla^{\perp}$ is the *transversal connection* of M. We define the curvature tensor $R^{\perp}$ of $tr(TM)$ by

$$R^{\perp}(X,Y)N = \nabla^{\perp}_X \nabla^{\perp}_Y N - \nabla^{\perp}_Y \nabla^{\perp}_X N - \nabla^{\perp}_{[X,Y]} N.$$

The transversal connection $\nabla^{\perp}$ is called *flat* if $R^{\perp}$ vanishes identically [5].

As $\nabla^{\perp}_X N = \tau(X)N$, we show [5] that the transversal connection of M is flat if and only if the 1-form τ is closed, i.e., $d\tau = 0$, on any $\mathcal{U} \subset M$.

Theorem 3.2. *Let M be a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature such that ζ belongs to $S(TM)$. If one of the following three conditions*

(1) *F is parallel with respect to the connection ∇,*
(2) *U is parallel with respect to the connection ∇, and*
(3) *V is parallel with respect to the connection ∇*

is satisfied, then $\bar{M}$ is flat and the transversal connection of M is flat. Moreover, in case (1)*, M is also flat.*

Proof. (1) If F is parallel with respect to ∇, that is, $\nabla_X F = 0$ for any vector field X of M, then, replacing Y by U to (3.8), we get

$$A_N X = \sigma(X)U. \tag{3.15}$$

Taking the scalar product with V to (3.8), we have $B(X,Y) = u(Y)\sigma(X)$, that is, $g(A^*_\xi X, Y) = g(\sigma(X)V, Y)$. As $S(TM)$ is non-degenerate, we obtain

$$A^*_\xi X = \sigma(X)V. \tag{3.16}$$

From (3.6), (3.15) and the fact that $FU = 0$, we get

$$\nabla_X U = \tau(X)U. \tag{3.17}$$

Substituting (3.15) into (3.12) and using (3.17) and the fact that $\ell = 0$, we get

$$\hbar\{\eta(X)\theta(Y) - \eta(Y)\theta(X)\}\zeta = 2d\sigma(X,Y)U,$$

due to $FU = 0$. Taking the scalar product to the last equation with V and using $(3.14)_2$, we see that $d\sigma = 0$. therefore, we have

$$\hbar\{\eta(Y)\theta(X) - \eta(X)\theta(Y)\} = 0.$$

Taking $X = \zeta$ and $Y = \xi$ to this, we obtain $\hbar = 0$. Thus $\bar{M}$ is flat.

Substituting (3.15) and (3.16) into (3.13) and using (2.7), we obtain $d\tau = 0$. Therefore, the transversal connection of M is flat.

As $\ell = \hbar = 0$, substituting (3.15) and (3.16) into (3.9), we get

$$R(X,Y)Z = \{\sigma(Y)\sigma(X) - \sigma(X)\sigma(Y)\}u(Z)U = 0,$$

for all X, Y, $Z \in \Gamma(TM)$. Therefore $R = 0$ and M is also a flat manifold.

(2) If U is parallel with respect to ∇, then, from (3.3) and (3.6), we have

$$J(A_N X) - u(A_N X)N + \tau(X)U = 0.$$

Taking the scalar product with V to this equation and using (3.1) and (3.2), we get $\tau = 0$. As $\tau = 0$, we obtain $d\tau = 0$. Thus the transversal connection of M is flat. Applying J to the last equation and using (3.1), we get

$$A_N X = \sigma(X)U. \tag{3.18}$$

Substituting (3.18) into (3.12) and using the fact that U is parallel, we get

$$\hbar\{\eta(Y)\theta(X) - \eta(X)\theta(Y)\}\zeta = 2d\sigma(X,Y)U.$$

Taking the scalar product to the last equation with V and using $(3.14)_2$, we see that $d\sigma = 0$. therefore, we have

$$\hbar\{\eta(Y)\theta(X) - \eta(X)\theta(Y)\} = 0.$$

Taking $X = \zeta$ and $Y = \xi$ to this, we obtain $\hbar = 0$. Thus $\bar{M}$ is flat.

(3) If V is parallel with respect to ∇, then, from (3.3) and (3.7), we have

$$J(A^*_\xi X) - u(A^*_\xi X)N - \tau(X)V = 0. \tag{3.19}$$

Taking the scalar product with U to this, we have $\tau = 0$. Thus $d\tau = 0$ and the transversal connection is flat. As V is parallel with respect to ∇, we see that

$$R(X,Y)V = 0. \tag{3.20}$$

Replacing Z by V to (3.9) and using $(3.14)_{1,2}$ and (3.20), we have

$$\hbar\{u(Y)\theta(X) - u(X)\theta(Y)\}\zeta = \rho(X)A_N Y - \rho(Y)A_N X, \tag{3.21}$$

where $\rho(X) = B(X,V)$. Taking $Y = V$ to this and using $(3.14)_2$, we get

$$\rho(V)A_N X = \rho(X)A_N V.$$

In case that $\rho(V) \neq 0$: From the last equation, we obtain

$$A_N X = \lambda\rho(X)A_N V,$$

where we set $\lambda = \rho(V)^{-1}$. Substituting this result into (3.21), we have

$$\hbar\{u(Y)\theta(X) - u(X)\theta(Y)\} = 0.$$

Taking $X = \zeta$ and $Y = U$, we obtain $\hbar = 0$.

In case that $\rho(V) = 0$: Applying J to (3.19) and using (3.11), we have

$$A^*_\xi X = \rho(X)U. \tag{3.22}$$

Replacing X by V to (3.22) and using the fact that $\rho(V) = 0$, we have $A^*_\xi V = 0$. Taking the scalar product with ζ to this result, we obtain

$$\rho(\zeta) = 0. \tag{3.23}$$

Taking the scalar product with U to (3.22), we have $B(X,U) = 0$. Replacing X by V to this result, we obtain

$$\rho(U) = B(V,U) = 0. \tag{3.24}$$

Taking $X = \zeta$ and $Y = U$ to (3.21) and using $(3.14)_2$, (3.23), (3.24) and the fact that $u(\zeta) = \theta(V)$, we obtain $\hbar = 0$. Therefore, $\bar{M}$ is flat. □

Definition 2. The structure tensor field F on M is said to be *recurrent* [10] if there exists a 1-form ω on M such that

$$(\nabla_X F)Y = \omega(X)FY, \quad \forall\, X,\, Y \in \Gamma(TM).$$

Theorem 3.3. *Let M be a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature such that ζ belongs to $S(TM)$. If F is recurrent, then it is parallel with respect to the induced connection ∇ on M, $\bar{M}$ and M are flat manifolds, and the transversal connection of M is flat.*

Proof. Assume that F is recurrent. From (3.8), we get

$$\omega(X)FY = u(Y)A_N X - B(X,Y)U.$$

Replacing Y by ξ to this and using (2.6), (3.2) and the fact that $F\xi = -V$, we get $\omega(X)V = 0$ for all $X \in \Gamma(TM)$. Taking the scalar product with U to this result, we obtain $\omega = 0$. It follows that $\nabla_X F = 0$ for all $X \in \Gamma(TM)$. Therefore, F is parallel with respect to the induced linear connection ∇. By (1) of Theorem 3.2, we obtain our assertions. □

Definition 3. The structure vector field U on a lightlike hypersurface M of an indefinite almost complex manifold $\bar{M}$ is called *principal* [10], with respect to the shape operator A^*_ξ, if there exists a smooth function α such that

$$A^*_\xi U = \alpha U. \tag{3.25}$$

Taking the scalar product with X to (3.25) and using (2.7) and (3.5), we get

$$\sigma(X) = B(X,U) = C(X,V) = \alpha v(X). \tag{3.26}$$

Theorem 3.4. *Let M be a lightlike hypersurfaces of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature. If F is parallel with respect to ∇ and U is principal, then $\alpha = 0$ and M is totally geodesic and screen totally geodesic.*

Proof. From (3.15), (3.16) and (3.26), we get

$$A_N X = \alpha v(X)U, \qquad A^*_\xi X = \alpha v(X)V. \tag{3.27}$$

Replacing X by U to $(3.27)_1$, we have $A_N U = 0$. As $\nabla_X F = 0$, taking the scalar product with V to (3.8), we have

$$B(X,Y) = u(Y)u(A_N X).$$

Replacing X by U to this and using the fact that $u(A_N U) = 0$, we obtain

$$\sigma(X) = B(X,U) = 0.$$

From this equation and (3.26), we get $\alpha = 0$. It follows from (3.27) that $A_N = 0$ and $A^*_\xi = 0$. Therefore, M is totally geodesic and screen totally geodesic. □

Theorem 3.5. *Let M be a lightlike hypersurface of an indefinite Kaehler manifold $\bar{M}$ of quasi-constant curvature. If U is parallel with respect to ∇ on M and U is principal, then $S(TM)$ is an integrable distribution.*

Proof. As U is parallel with respect to ∇, from (3.18) and (3.26), we obtain

$$A_{N}X = \alpha v(X)U.$$

Taking the scalar product with Y to the last equation, we see that

$$g(A_{N}X, Y) = \alpha v(X)v(Y).$$

It follow that A_{N} is self-adjoint operator with respect to g. Consequently, C is symmetric on $S(TM)$ due to (2.8). By using (2.3) we obtain

$$\eta([X,Y]) = C(X,Y) - C(Y,X) = 0,$$

which implies that $[X,Y] \in \Gamma(S(TM))$ for any $X, Y \in \Gamma(S(TM))$. Therefore, $S(TM)$ is an integrable distribution. □

References

[1] B.Y. Chen and K.Yano, *Hypersurfaces of a conformally flat space*, Tensor (N. S.) 26, 1972, 318-322.

[2] K.L. Duggal and A. Bejancu, *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Acad. Publishers, Dordrecht, 1996.

[3] K.L. Duggal and D.H. Jin, *Null curves and Hypersurfaces of Semi-Riemannian Manifolds*, World Scientific, 2007.

[4] K.L. Duggal and B. Sahin, *Differential geometry of lightlike submanifolds*, Frontiers in Mathematics, Birkhäuser, 2010.

[5] D.H. Jin, *Screen conformal lightlike real hypersurfaces of an indefinite complex space form*, Bull. Korean Math. Soc. 47(2), 2010, 341-353.

[6] D.H. Jin, *Lightlike real hypersurfaces with totally umbilical screen distributions*, Commun. Korean Math. Soc. 25(3), 2010, 443-450.

[7] D.H. Jin, *Lightlike hypersurfaces of an indefinite Kaehler manifold*, Commun. Korean Math. Soc. 27(2), 2012, 307-315.

[8] D.H. Jin, *Lightlike hypersurfaces of a semi-Riemannian manifold of quasi-constant curvature*, Commun. Korean Math. Soc. 27(4), 2012, 763-770.

[9] D.H. Jin and J.W. Lee, *Lightlike submanifolds of a semi-Riemannian manifold of quasi-constant curvature*, Journal of Applied Mathematics, 2012, Art ID 636782, 1-18.

[10] G. Kaimakamis and K. Panagiotidou, *Real hypersurfaces in a non-flat complex space form with Lie recurrent structure Jacobi operator*, Bull. Korean Math. Soc. 50(6), 2013, 2089-2101.

Department of Mathematics
Dongguk University
Gyeongju 780-714, Republic of Korea
E-mail address: jindh@dongguk.ac.kr