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A Study of Sales Increase and/or Decrease by Campaign Using a Differential Equation Model of the Growth Phenomenon

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ABSTRACT

With society becoming more advanced and complex, the required management engineering makes essential the development of human resources that can propose solutions for problems of new phenomena from a different perspective. As an example of such phenomena, we note a consumer electronics 'Eco-point' system campaign in this study. To mitigate global warming, revitalize the economy, and encourage the adoption of terrestrial digital compatible TVs, the consumer electronics Eco-point system campaign was implemented in May 2009 in Japan. In this study, we note a model which is constant term with exponential curve with notion of the growth phenomenon (Nakagiri and Kurita, Journal of the Operations Research Society of Japan, 2002). In our study, we call this model the 'differential equation model of the growth phenomenon.' This model represents a phenomenon with a hierarchical structure for capturing the properties of n species. In this study, we propose a new model which can represent not only the impact of large-scale campaigns but also seasonal factors. Accordingly, we understand the phenomenon of fluctuation of sales of some products caused by large-scale campaigns and predict the fluctuation of sales. The final goal of this study is to develop human resources that can propose provision and solution for pre-consumption and reactionary decline in demand by understanding the impact of large-scale campaigns. As the first step of this goal, our objective is to propose a new regression method with different conventional perspective that can describe the fluctuation of sales caused by large-scale campaigns and show the possibility of new management engineering education.

Keywords: Regression, Prediction, Differential Equation Model, Subsidiary Growth Phenomenon

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1. INTRODUCTION

With society becoming more advanced and complex, the required management engineering makes essential the development of human resources that can propose solutions for problems of new phenomena from a different perspective. As an example of the phenomenon, we note a consumer electronics Eco-point system campaign in this study. To mitigate global warming, revitalize the economy, and encourage the spread of terrestrial digital compatible televisions (TVs), the consumer electronics Eco-point system campaign was implemented in May 2009 in Japan. This consumer electronics Eco-point system granted points to customers who bought products with excellent energy savings (Aoshima, 2010). The target products in this campaign were digital TVs, air conditioners, and refrigerators. The consumer electronics Eco-point system campaign started in 2009 was one of the most effective campaigns in Japan (Suehiro, 2009). If the campaigns were effective, the growth rate will high. There were many growth model studies regarding rumors (Huo *et al*., 2012, Kawachi, 2006, 2008a, 2008b; Kawachi *et al*., 2008). The growth model study of rumors stemmed from a mathematical epidemic model (Kermack and McKendrick, 1991; Murray, 1989). The linear models were adaptable for epidemics (Hosono and Ilyas, 1995; Kallen 1984). The model can be applied to consumer goods amount growth (Nishiura and Aihara, 2009). The mathematical models for BSE (Nakagiri and Kurita, 2002b) and epidemic models (Inaba, 2008; Nishiura and Inaba, 2006). We consider that these models cannot apply the trends of before and after the effective campaigns.

There was a great influence on sales of not only the target products, but also of other products which were not involved with the campaign. However, one of the major problems that arose was the pre-consumption and reactionary decline in demand caused by such a large-scale campaign. Furthermore, the pre-consumption and reactionary decline in demand caused the problem of surplus stock in shops and a decrease of sales of the companies making the products. In addition to that, the appreciation of the yen and inexpensive products from overseas further spurred a decrease in sales, causing losses to companies making products.

In this study, we note a model based on the idea of introducing stepwise growth to exponential curve with a constant term (Kurita and Nakagiri, 2002a). We call this model the 'differential equation model of the growth phenomenon' in this study. This model represents the phenomenon with a hierarchical structure for capturing the properties of *n* species. This model defines the growth rate of the number of individuals that acquire the property of a certain stage as depending on the number of individuals who have obtained the property at the present and previous stages (Kurita and Nakagiri, 2002a).

The supposition we use in this model is that quantitative data related to growth phenomena which are the population size, the area of land or sales of products are at the certain stage. This means that the data were in the previous stages before and have obtained the property of the present stages now. That is, the data have a hierarchical structure. The models which follow this supposition fit to the real data more closely than the other data which do not make this supposition. For example, Koto (1990) made the model which can show the change of urbanization by defining the supposition that after residential land gets bigger, the area of urbanization get bigger. Also, Koto (1993) showed transition of the number of population using same model and supposition.

In this study, we propose new model which can represent not only the impact of large-scale campaigns but also seasonal factors. Accordingly, we understand the phenomenon of fluctuation of sales of some products caused by large-scale campaigns and predict the fluctuation of sales.

The final goal of this study is to develop human resources that can propose countermeasures and solutions for pre-consumption and reactionary decline in demand by understanding the impact of large-scale campaigns. As the first step of this goal, our objective is to propose a new regression method with different conventional perspective that can describe the fluctuation of sales caused by large-scale campaigns and show the possibility of new management engineering education.

2. GROWTH PHENOMENON AND DIFFERENTIAL EQUATION

In this section, we describe the differential equation model of the growth phenomenon on which the idea of our proposed model is based.

2.1 The Differential Equation Model of the Growth Phenomenon

We now introduce the differential equation model of the growth phenomenon on which this study is based. Figure 1 shows a series of flow of growth phenomenon affected by campaigns.

In this study, the equations are formulated under the assumption that growth occurs in a series of stages. That is, the premise is that each stage occurs after the growth of the previous stage has happened. Furthermore, the rate of the growth is defined in proportion to the state of the previous stage. $y_i(t)$ denotes the population in the state of stage *i* at time *t* (see Figure 1). Therefore, we formulate the following equation (Kurita and Nakagiri, 2002a).

$$\frac{dy_i(t)}{dt} = b_i(y_{i-1}(t) - y_i(t)) \quad (i = 1, 2, \cdots) \tag{1}$$

In addition, we define the equation below.

$$y_0 = S \tag{2}$$

And, in Eq. (1), we define the expression relating *S* and parameters of b_i are given below.

S : The number of individuals when $i = 0$
b_i : Sales growth rate of $y_i(t)$ from $y_{i-1}(t)$

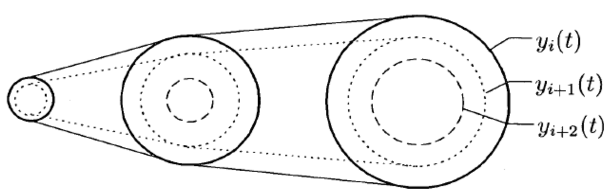

Figure 1. Flow of the model of the growth phenomenon.

3. SUBSIDIARY GROWTH PHENOMENON AND DIFFERENTIAL EQUATION

In this section, we describe the detail of our proposed model.

3.1 The Differential Equation Model of the Subsidiary Growth Phenomenon

We introduced the differential equation model of the growth phenomenon in Section 2. It shows the phenomenon with a hierarchical structure.

In this section, we propose a new differential equation model. This is the model which shows fluctuation of sales of product 1 and fluctuation of sales of product 2 during and after a large-scale campaign. Fluctuations of sales of product 1 are influenced by the campaign. Fluctuations of sales of product 2 are influenced by the fluctuation of sales of product 1. Also, this model adds seasonal factors to the previous model, which is the differential equation model of the growth phenomenon. We call this new model the 'differential equation model of the subsidiary growth phenomenon' in this study.

Figure 2 shows the flow of the model of the subsidiary growth phenomenon affected by campaigns. In Figure 2, the diagrams are uneven because we add the seasonal factors to the previous model. We describe this further in Section 3.3.

3.2 States of Sales

We define that state of sales is either of the following three states, these being, with the passage of time,

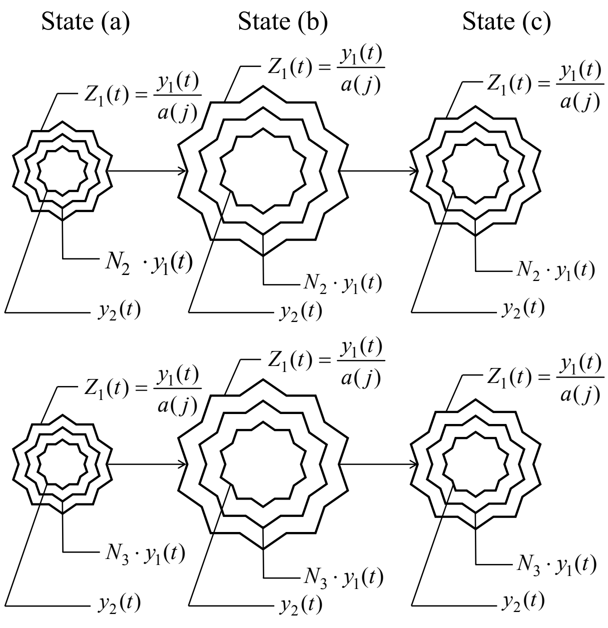

Figure 2. Flow of the model of the subsidiary growth phenomenon affected by the campaigns.

(a) before the campaign, (b) during the campaign and (c) after the campaign.

State (a): Before the campaign. The campaign has not started and the number of sales of products is stable.
State (b): During the campaign. The campaign starts and the number of sales of products is increasing with the passage of time. We could see that sales increased as the campaign approaches the end.
State (c): After the campaign. The campaign is finished and the number of sales of products is decreasing, returning to a steady state.

3.3 Notation

In this section, we define the notation of the differential equation model of the subsidiary growth phenolmenon. We define the time (t) as $t = 0$ when the Eco-point system campaign starts and as $t = T$ when it finishes.

In this section, we determine the ratio of the current sales of product 1 to sales thereof in the previous year, and we apply this to the differential equation model of the subsidiary growth phenomenon in order to describe the fluctuation of sales of product 1 caused by seasonal factors. $Z_1(t)$ in Figure 2 is the result that the number of sales of product 1 divided by number of sales of product 1 per month in previous year. Therefore, the diagrams of Figure 2 are not smooth curves, but are rather curves with convexities.

We define signs below in this study.

$Z_1(t)$: Ratio of the current sales of product 1 to their sales in previous year at time t ($t \le 0$, $0 \le t$)
$a(j)$: Sales of product 1 per month in previous year ($j = 0, 1, 2, \cdots, 11$)
$y_1(t)$: Sales of product 1 at time t ($0 \le t$)
$y_2(t)$: Sales of product 2 at time t ($0 \le t$)
N : Proportion of customers who want to buy product 2 to

$y_1(t)$ ($0 \le N \le 1$)

$N \cdot y_1(t)$: Number of customers who want to buy product 2

The expression relating $Z_1(t)$ and number of $a(j)$ are given below.

$$Z_1(t) = \frac{y_1(t)}{a(j)} \tag{3}$$

$$y_1(t) = Z_1(t)a(j) \tag{4}$$

3.4 Quantity of Sales of the Previous Year

In this section, we describe $a(j)$, which is the sales of product 1 per month in the previous year before the campaign. In the differential equation model of the subsidiary growth phenomenon, we first determine $a(j)$ in order to grasp the quantity of sales of the previous year. $a(j)$ is not influenced by the campaign. Now, we can determine the ratio of quantity of increment of sales of product 1 to quantity of sales of product 1 per month in the previous year after we divide $y_1(t)$ by $a(j)$ (formula 3). We define that these value are $Z_1(t)$, where $Z_1(t)$ is ratio of the current sales of product 1 to the sales in the previous year. Also, $a(j)$ is a constant because the sales of products before the campaign have been constant since the campaign started.

We define $a(j)$ in this study as follows.

$a(0)$: Sales of product 1 in April *Y*
$a(1)$: Sales of product 1 in May *Y*
$a(2)$: Sales of product 1 in June *Y*:
$\cdots$
$a(10)$: Sales of product 1 in February *Y*+1
$a(11)$: Sales of product 1 in March *Y*+1

We define *Y* in this study as follows.

Y: Previous fiscal year ($1 \le Y$)

We add $a(j)$ ($a(0)$ to $a(11)$) to the differential equation model of the subsidiary growth phenomenon. According to this, we can describe the fluctuation of sales of products in each season that we could not represent using the previous model.

3.5 Formulation of the State Transition

In this section, we describe the state transition of the large-scale campaign. The state of the campaign is either of following two states.

1) Before the campaign to during the campaign
   At first the number of sales of product 1 is stable. By the influence of the campaign, it starts to increase. Also, the numbers of sales of product 2 start to increase by the influence of fluctuation of sales of product 1.
2) During the campaign to after the campaign
   The number of sales of product 1 finishes increasing and then begins to decrease sharply. Also, the number of sales of product 2 starts to decrease by the influence of fluctuation of sales of product 1.

By considering the state transition along with the above assumptions, we arrive at the differential equation model below (Formula 6 to Formula 9).

We use

$$Z_1(0) : \text{Initial value of } Z_1(t) \ (t = 0) \tag{5}$$

as the initial condition.

[During the large-scale campaign $(0 \le t \le T)$]

$$\frac{dZ_1(t)}{dt} = b_2(Z_1(0) - Z_1(t)) \tag{6}$$

$$\frac{dy_2(t)}{dt} = B_2(N_2 y_1(t) - y_2(t)) \tag{7}$$

[After the campaign $(T \le t)$]

$$\frac{dZ_1(t-T)}{dt} = -b_3 \cdot Z_1(t-T) \tag{8}$$

$$\frac{dy_2(t-T)}{dt} = B_3\left(N_3 \cdot y_1(t-T) - y_2(t-T)\right) \tag{9}$$

3.6 Parameters

In this section, we define each of the parameters as follows.

b_2 : Sales growth rate of product 1; the ratio of the current sales to that in the previous year 'during the campaign' ($-1 \le b_2 \le 0$)
B_2 : Sales growth rate of product 2 'during the campaign' ($0 \le B_2 \le 1$)
N_2 : Proportion of customers who want to buy product 2 to $y_1(t)$ 'during the campaign' ($0 \le N_2 \le 1$)
b_3 : Sales growth rate product 1; the ratio of the current sales to that in the previous year 'after the campaign' ($0 \le b_3 \le 1$)
B_3 : Sales growth rate of product 2 'after the campaign' ($0 \le B_3 \le 1$)
N_3 : Proportion of customers who want to buy product 2 to $y_1(t)$ 'after the campaign' ($0 \le N_3 \le 1$)

3.7 Solutions of the Differential Equation

We obtain an analysis solution easily if we remove simultaneous Eqs. (6) to (9) by the cause of Eq. (3) and initial value 5.

[During the large-scale campaign $(0 \le t \le T)$]

As for Eq. (3) and $Z_1(t) = Z_1(0) - C_1 e^{-b_2 t}$,

$$y_1(t) = a(j)(Z_1(0) - C_1 e^{-b_2 t}) \tag{10}$$

$$y_2(t) = N_2 a(j) Z_1(0) - \frac{B_2}{B_2 - b_2} N_2 a(j) C_1 e^{-b_2 t} - C_2 e^{-B_2 t} \tag{11}$$

[After the campaign $(T \le t)$]

$$y_1(t) = a(j) C_3 e^{-b_3(t-T)} \tag{12}$$

$$y_2(t) = \frac{B_2}{B_2 - b_2} N_3 a(j) C_3 e^{-b_3(t-T)} - C_4 e^{-B_3(t-T)} \tag{13}$$

4. EVALUATION AND CONSIDERATION

In this section, in order to evaluate our proposed model, we show the results of the prediction by using actual data and we discuss the results.

4.1 Acquisition of Actual Data

In this study, using the Eco-point system campaign for Japanese households that operated from May 2009 to December 2010, we examined the change in the sales of two products during and after the campaign. We defined product 1 as digital TVs and product 2 as Blu-ray Disc recorders. We applied the new differential equation model of growth to the sales of digital TVs and Blu-ray Disc recorders during and after the campaign and verified the validity of the new model. We defined Y (the previous fiscal year) as 2008.

We acquired the data for changes in sales per month from Japan Electrics and Information Technology Industries Association (consumer electronic equipment 2008–2013, http://www.jeita.or.jp/english/stat). This source contains listings of sales per month during and after the campaign. In Figure 3 below, the vertical axis represents the quantity of product sales (quantity of sales/1000) and the horizontal axis represents time (bi-monthly basis).

4.2 Appearance of Fluctuation in the Sales of Digital TVs

In the case of digital TVs, it seems possible to approximate transitory fluctuation of sales caused by such a campaign and by seasons because the assumption refers to actual data.

The customer demand for digital TVs in June-July 2011 was high because of the change from terrestrial analog broadcasting to terrestrial digital broadcasting that happened on July 24, 2011. Therefore, we infer that this sudden increase in sales was not caused by the campaign in June-July 2011.

The values of each Parameters were $b_2 = -0.3073$ and $b_3 = 0.0640$.

The sales fell below sales before the campaign $a(j)$, and pre-consumption and reactionary decline in demand occurred in 2012 and 2013.

4.3 Appearance of Fluctuation in the Sales of Blu-ray Disc Recorders

In the case of Blu-ray Disc recorders, it appears possible to approximate the transitory fluctuation of sales caused by such a campaign and by seasons because the assumption refers to actual data. There was a high customer demand for the Blu-ray Disc recorders during June-July 2011 because of the change from terrestrial analog broadcasting to terrestrial digital broadcasting on July 24, 2011. Therefore, we infer that the sudden increase in sales was not due to the campaign in June-July 2011.

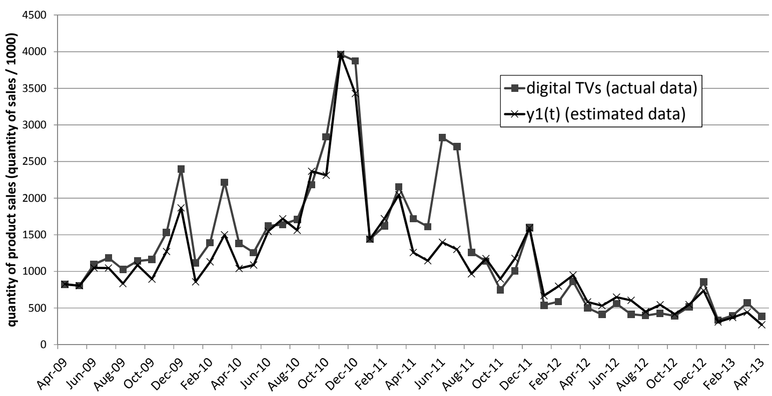

Figure 3. Actual data and estimated data of digital TVs.

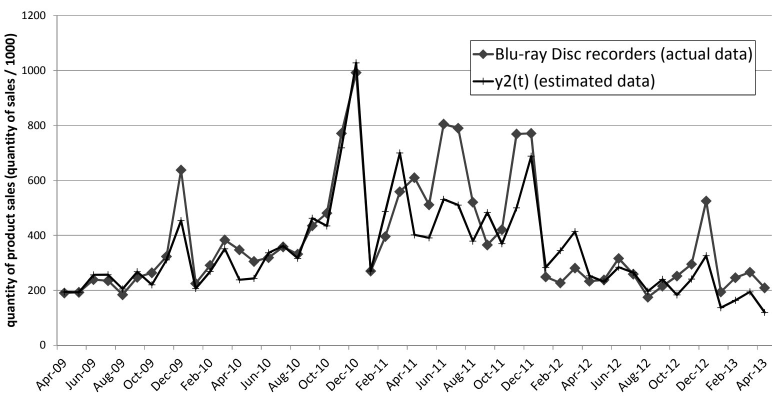

Figure 4. Actual data and estimated data of Blu-ray Disc recorders.

The values of each parameters were $B_2 = 0.4682$, $N_2 = 0.2503$, $B_3 = 0.2883$ and $N_3 = 0.3442$.

The sales of Blu-ray Disc recorders in 2012 and 2013 did not fall below the sales level before the campaign because there was little pre-consumption and the sales of Blu-ray Disc recorder were related to that of digital TVs.

In addition, the reactionary decline in demand did not occur, for the same reason given above. Moreover, the market for Blu-ray Disc recorders itself became much larger result from that sales of Blu-ray Disc overtake $a(j)$ every month after the campaign and the numerical value of N increased more than that during the campaign.

4.4 Comparison of Two Products

As seen in Figure 5, there is some correlation between sales of digital TV and sales of Blu-ray Disc recorders. We create a new curve that can reflect seasonality better than the previous model after we adopt 'seasonal factors' in the previous model. The curve is 'the differential equation model of the subsidiary growth phenomenon'. We determined that the high sales values in June-July 2011 were outliers, resulting not from the campaign but from the other causes mentioned earlier.

The numerical values of N were different during and after the campaign because the Blu-ray Disc recorder market itself became much bigger and newer models were more superior to the older ones.

Table 1. Comparisons of each parameter

Parameter	b	B	N
State (b)	$b_2 = -0.3073$	$B_2 = 0.4682$	$N_2 = 0.2503$
State (c)	$b_3 = 0.0640$	$B_3 = 0.2883$	$N_3 = 0.3442$

Table 2. Evaluation of the model using AIC

AIC	Previous model	New model
State (b)		
$y_1(t)$	305.14	284.11
$y_2(t)$	246.44	213.19
State (c)		
$y_1(t)$	447.41	435.44
$y_2(t)$	387.15	365.67

AIC: Akaike information criteria.

4.5 Evaluation of the Model

Table 2 shows a comparison of accuracy of previous model and the differential equation model of the subsidiary growth phenomenon using the Akaike information criteria (AIC).

We found that the new model is more accurate than previous model as seen as Table 2. Therefore, we could prove the superiority of new model.

4.6 Result of the Prediction

Figure 6 is the result of prediction of sales of digital TVs during the campaign. Figure 7 is the result of prediction of sales of Blu-ray Disc recorders during the campaign.

In Figure 6, the results of prediction of sales of digital TVs were few bigger than the actual data. But they were approximately near to the actual data. Therefore, we could predict the fluctuation of sales of digital TVs which is influenced by the campaign and the factor of seasonal changes. The cause of few differences between prediction and actual data is expected to not considering to the factor of "last-minute demand." In the case of large-scale campaigns, we could imagine "last-minute demand" that the customer begin to buy products

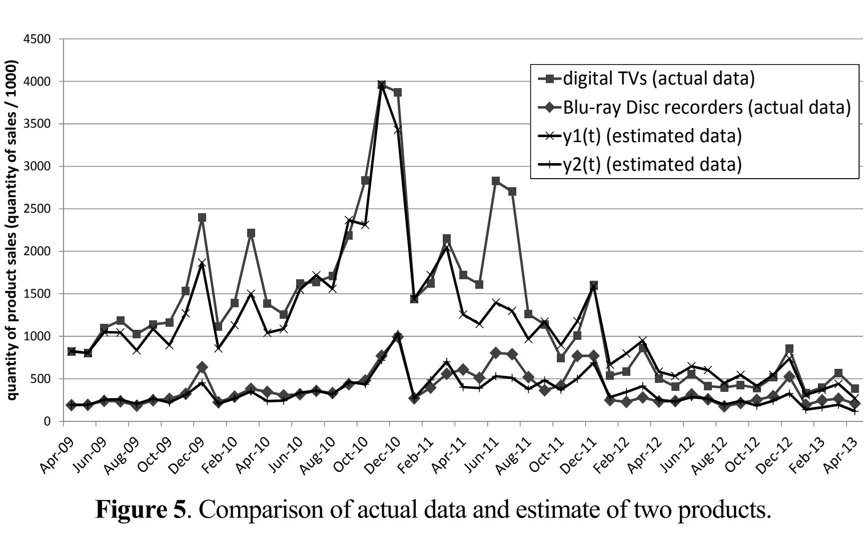

Figure 5. Comparison of actual data and estimate of two products.

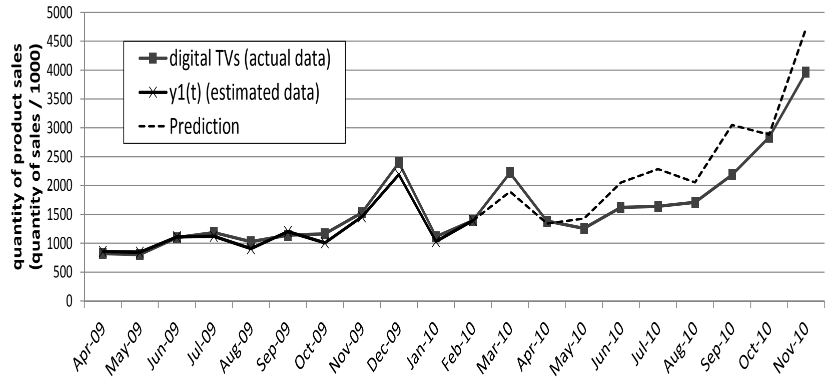

Figure 6. Prediction of sales of digital TVs 'during the campaign ($t = 10$).'

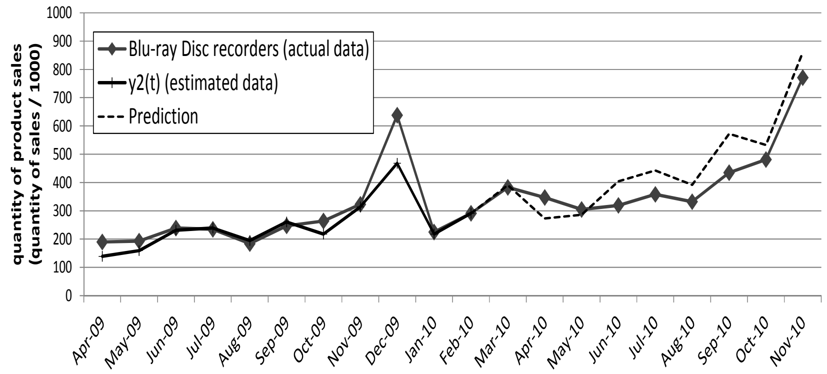

Figure 7. Prediction of sales of Blu-ray Disc recorders 'during the campaign ($t = 10$).'

just before a campaign ends. If we add this factor to the differential equation model of the subsidiary growth phenomenon, we could get better results than results of this study.

In Figure 7, the result of prediction of sales of Blu-ray Disc recorders were few bigger than the actual data. We think the prediction become smaller as a 'last-minute demand' makes the actual sales bigger just before the campaign ends. Therefore, we could predict the fluctuation of sales of Blu-ray Disc recorders which is influenced by the campaign and the factor of seasonal changes. The cause of difference between prediction and actual data is expected that it is influenced by the result of prediction of digital TVs. The result of prediction of Blu-ray Disc recorders was bigger than actual data because the result of prediction of digital TVs was few bigger than actual data. Above-mentioned, if we add the factor of 'last-minute demand' to the differential equation model of the subsidiary growth phenomenon, we could get better result than result of this study.

The values of each parameters were $b_2 = -0.2183$, $B_2 = 0.6221$ and $N_2 = 0.2261$.

5. SUMMARY

In this study, we considered the change in the sales of two products under the Eco-point system campaign. We discovered that it is possible to predict not only the quantity of sales due to the campaign but also the quantity of sales due to seasonality.

We found that the pre-consumption and reactionary decline in demand for sales of digital TVs was influenced by the campaign. However, there was no pre-consumption and reactionary decline in the demand for Blu-ray Disc recorders directly due to the campaign, even though the sales did increase.

In conclusion, we were able to create a new curve in section 3 that follows the fluctuation due to seasons better than the previous model by considering 'the change resulting from the season' in it. We referred to this curve as the 'differential equation model of the subsidiary growth phenomenon.' This also could predict the future sales of products.

We ignored external factors such as those related to psychology and society. In the future, these factors should be considered to better predict demand. This will also enable human resources to propose the development of better countermeasures and plans for out-of-stock and surplus products as well as pre-consumption and reactionary decline in demand.

This study has the possibility to analyze various phenomena and another field of studies. We hope that subsequent studies in this area will demonstrate the possibility of new management engineering education and contribute to various fields of society.

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