Generalized Likelihood Ratio Test For Cyclostationary Multi-Antenna Spectrum Sensing

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Abstract

In this paper, a generalized likelihood ratio test (GLRT) is proposed for cyclostationary multi-antenna spectrum sensing in cognitive radio systems, which takes into account the cyclic autocorrelations obtained from all the receiver antennas and the cyclic cross-correlations obtained from all pairs of receiver antennas. The proposed GLRT employs a different hypotheses problem formulation and a different asymptotic covariance estimation method, which are proved to be more suitable for multi-antenna systems than those employed by the Dandawaté-Giannakis algorithm. Moreover, we derive the asymptotic distributions of the proposed test statistics, and prove the constant false alarm rate property of the proposed algorithm. Extensive simulations are conducted, showing that the proposed GLRT can achieve better detection performance than the Dandawaté-Giannakis algorithm and its extension for multi-antenna cases.

Keywords: Cyclostationarity, generalized likelihood ratio test, cognitive radio, multi-antenna spectrum sensing

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1. Introduction

Recently, cognitive radio (CR) has drawn significant attention from academic and industrial communities to meet the ever-growing needs for spectrum resources [1]- [4]. In CR systems, cognitive unlicensed users are allowed to identify and exploit the local and instantaneous spectrum white space where no licensed user is present. A CR user is required to perform spectrum sensing [5]- [8] periodically to avoid significant interference to the licensed systems. If an idle channel is detected, the CR user can transmit or receive data on the channel; whereas, if a licensed user is detected, the CR user avoids data transmission on the channel and tunes to another idle channel. Cyclostationarity-based signal detection is one of the widely considered spectrum sensing techniques for CR systems, due to its capability of differentiating noise from licensed user signals. The cyclostationary spectrum sensing methods exploit the cyclic statistics such as cyclic auto-correlation for the detection of licensed user, which are nonzero at particular cyclic frequencies for the licensed user signal [9], [10].

Many existing cyclostationarity-based spectrum sensing methods [11]- [21] have employed the Dandawaté-Giannakis algorithm [22] due to its constant false alarm rate (CFAR) property and robustness in the low signal-to-noise ratio (SNR) regime. The Dandawaté-Giannakis algorithm can be viewed as the generalized likelihood ratio test (GLRT) for the presence of cyclostationarity. It has assumed that the distributions of the cyclic auto-correlation estimations under the null hypothesis and the alternative hypothesis have the same asymptotic covariance and differ only in mean. Based on this, the asymptotic covariance estimation can be generalized regardless of the hypothesis, which leads to a final test statistic having a squared Mahalanobis distance form.

The performance of spectrum sensing can be seriously degraded in Rayleigh fading situations. To remedy this problem, multi-antenna spectrum sensing that exploits spatial diversity could be employed. Recently, how to improve the performance of cyclostationary spectrum sensing by using multiple antennas has also been studied by researchers. In [20], we have extended the Dandawaté-Giannakis algorithm to allow multi-antenna spectrum sensing, which takes into account the cyclic auto-correlations obtained from all receiver antennas and the cyclic cross-correlations obtained from all pairs of receiver antennas. Maximum radio combining was proposed in [21], which requires the channel state information as *a priori* information. In [23], the spectral correlation function (SCF) is used to estimate the phase difference between the channel responses of different antennas, and the test statistic is formed by combining the SCFs of the received signals from all the antennas. Collaborative spectrum sensing [11] can be viewed as a special case of multi-antenna spectrum sensing, which drops the cyclic cross-correlations in detection.

In this paper, we propose a novel GLRT for cyclostationary multi-antenna spectrum sensing. The framework of the proposed algorithm is similar to that in [20], i.e., taking into account all the achievable cyclic auto-correlations and cyclic cross-correlations. However, it is different with [20] in hypotheses problem formulation and asymptotic covariance estimation. More specifically, the the asymptotic covariances among the cyclic cross-correlations and auto-correlations under the null hypothesis are assumed to different from those under the alternative hypothesis, while they are assumed to be the same in the Dandawaté-Giannakis algorithm and its extension [20]. We use an example, where the additive noise is low-pass zero-mean Gaussian noise, to illustrate the validity and necessity of this difference and conduct the proposed GLRT. The distribution and the computational complexity of the proposed test statistic are also derived. The proposed algorithm maintains the CFAR property and has better detection performance and much lower computational complexity than those of [20]. For example, a performance gain of 1.7dB is achieved for a four-antenna system. The necessity of taking into account the cyclic cross-correlations is also demonstrated via simulation experiments.

The rest of this paper is organized as follows: In Section 2, the multi-antenna spectrum sensing model is presented. In Section 3, we propose the GLRT for cyclostationary multi-antenna spectrum sensing, and then demonstrate how to construct the proposed GLRT under the assumption that the additive noise is low-pass zero-mean Gaussian noise with uncertain power. An example is presented in Section 4 to further illustrate the proposed test. In section 5, asymptotic distribution and computational complexity of the proposed test statistic are given. Simulation results and discussions are presented in Section 6. Finally, conclusions are drawn in Section 7.

2. System Model of Multi-Antenna Spectrum Sensing

As shown in **Fig. 1**, we consider a CR receiver with $N\,(N \geq 2)$ antennas. We assume an independent flat Rayleigh fading channel [24] for each pair of antennas between the transmitter and receiver in the following derivations.

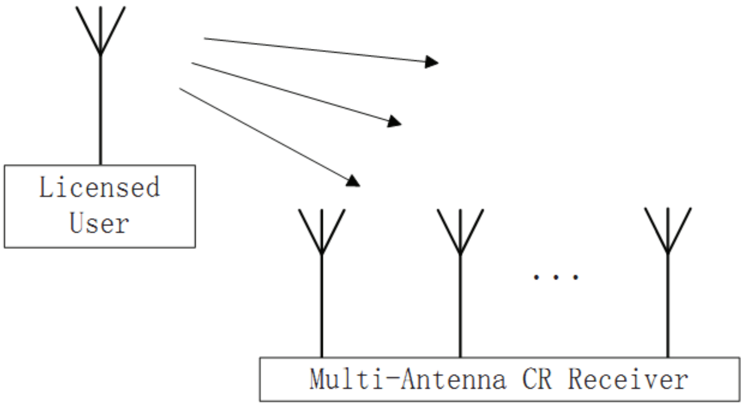

Fig. 1. Multi-antenna spectrum sensing model.

At each sensing period, the CR tries to distinguish between the following two hypotheses:

$$\begin{aligned} H_0 &: \quad x_i(t) = w_i(t), \quad i = 1,2,\ldots,N, \\ H_1 &: \quad x_i(t) = h_i s(t) + w_i(t), \quad i = 1,2,\ldots,N, \end{aligned} \tag{1}$$

where $s(t)$ is the signal transmitted by the licensed user; *i* denotes the index of antenna; $x_i(t)$ and $w_i(t)$ denote the received signal and the low-pass zero-mean complex Gaussian noise at the *i*-th antenna, respectively; h_i denotes the channel response for the *i*-th antenna, which can be further expressed as

$$h_i = \gamma_i \exp\{-j2\pi\theta_i\}, \tag{2}$$

where γ_i is Rayleigh distributed with the probability density function (PDF) as

$$f(\gamma_i)=\frac{\gamma_i}{\sigma^2}\exp\left\{-\frac{\gamma_i^2}{2\sigma^2}\right\},\gamma_i>0, \tag{3}$$

and θ_i denotes the phase offset of the channel response, which is uniformly distributed with the PDF as

$$f(\theta_i)=\begin{cases}\frac{1}{2\pi}, & 0\le\theta_i<2\pi,\\ 0\ , & \text{otherwise}.\end{cases} \tag{4}$$

Throughout this paper, we assume that h_i is invariant over the sensing period, and both $w_i(t)$ and h_i are independent for different antennas.

3. Cyclostationarity-Based Multi-Antenna Spectrum Sensing

3.1 Cyclostationarity

A continuous-time random process $x(t)$ is a wide sense second-order cyclostationary process, if its mean and auto-correlation are periodical with some periods [10]. Define the time varying auto-correlation of $x(t)$ as $R_{xx}(t,\tau)\overset{\Delta}{=}E\{x(t)x^*(t+\tau)\}$, where τ denotes the lag. Then, due to its periodicity, $R_{xx}(t,\tau)$ can be represented as a Fourier series as

$$R_{xx}(t,\tau)=\sum_{\alpha}R_{xx}^{\alpha}(\tau)e^{j2\pi\alpha t} \tag{5}$$

where τ is called the cyclic frequency and the Fourier coefficients are called the cyclic auto-correlation functions, which are given by

$$R_{xx}^{\alpha}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}R_{xx}(t,\tau)e^{-j2\pi\alpha t}dt. \tag{6}$$

Another useful function that can characterize the second-order cyclostationary process $x(t)$ is called the conjugated cyclic auto-correlation (CCA) function, which is given by

$$R_{xx^*}^{\alpha}(\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}R_{xx^*}(t,\tau)e^{-j2\pi\alpha t}dt\,, \tag{7}$$

where $R_{xx^*}(t,\tau)\overset{\Delta}{=}E\{x(t)x(t+\tau)\}$. In the rest of this paper, we focus on the case using the CCA functions, while the one concerning cyclic auto-correlation functions can be derived similarly.

The discrete time version of the CCA estimation, $\hat{R}_{xx^*}^{\alpha}(\nu)$, can be calculated as

$$\hat{R}_{xx^*}^{\alpha}(\nu)=\frac{1}{M}\sum_{m=0}^{M-1}x[m]x[m+\nu]e^{-j2\pi\alpha m}, \tag{8}$$

where M is the number of available signal samples, ν is the discrete version of the lag parameter, and m is the discrete time index, i.e., $x[m]=x(mT_s)$, where T_s denotes the sampling period.

As for the multi-antenna spectrum sensing problem where there are N antennas at the CR receiver, N CCA functions and $N(N-1)/2$ conjugated cyclic cross-correlation (CCC) functions can be obtained, which can be defined together as follows:

$$R^{\alpha}_{x_i x_j^*}(\tau) \overset{\Delta}{=} \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} R_{x_i x_j^*}(t,\tau)e^{-j2\pi\alpha t}dt, i \le j \le N, \tag{9}$$

where $R_{x_i x_j^*}(t,\tau) \overset{\Delta}{=} E\left[x_i(t)x_j(t+\tau)\right]$. Note that when $i=j$, $R^{\alpha}_{x_i x_j^*}(\tau)$ denotes the CCA functions, and when $i<j$, $R^{\alpha}_{x_i x_j^*}(\tau)$ denotes the CCC functions. Also note that we only take into account $\hat{R}_{x_i x_j^*}$ and neglect $\hat{R}_{x_j x_i^*}$ ($i<j$), since it is easy to verify that

$$\lim_{M\to\infty}\hat{R}^{\alpha}_{x_i x_j^*}(\tau) = \lim_{M\to\infty}\hat{R}^{\alpha}_{x_j x_i^*}(\tau). \tag{10}$$

Similar to (8), a discrete time version estimation of $R^{\alpha}_{x_i x_j^*}(\tau)$ with M samples is given by

$$\hat{R}^{\alpha}_{x_i x_j^*}(\nu) = \frac{1}{M}\sum_{m=0}^{M-1} x_i[m]x_j[m+\nu]e^{-j2\pi\alpha m}. \tag{11}$$

3.2 GLRT for the Multi-Antenna Cyclostationary Spectrum Sensing

Taking into account all CCA and CCC estimations, we can define a vector $\hat{\mathbf{r}}_{\text{mul}}$ as

$$\begin{aligned}\hat{\mathbf{r}}_{\text{mul}} \overset{\Delta}{=} \Big[&\hat{\mathbf{r}}_{x_1 x_1^*}, \hat{\mathbf{r}}_{x_1 x_2^*}, \hat{\mathbf{r}}_{x_1 x_3^*}, \ldots, \hat{\mathbf{r}}_{x_1 x_N^*},\\ &\hat{\mathbf{r}}_{x_2 x_2^*}, \hat{\mathbf{r}}_{x_2 x_3^*}, \ldots, \hat{\mathbf{r}}_{x_2 x_N^*},\\ &\ldots\\ &\hat{\mathbf{r}}_{x_N x_N^*}\Big],\end{aligned} \tag{12}$$

where

$$\begin{aligned}\hat{\mathbf{r}}_{x_i x_j^*} \overset{\Delta}{=} \Big[&\text{Re}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_1)\right\}, \text{Re}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_2)\right\}\ldots, \text{Re}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_P)\right\},\\ &\text{Im}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_1)\right\}, \text{Im}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_2)\right\}\ldots, \text{Im}\left\{\hat{R}^{\alpha}_{x_i x_j^*}(\nu_P)\right\}\Big].\end{aligned} \tag{13}$$

By using $\hat{\mathbf{r}}_{\text{mul}}$, the hypotheses problem for multi-antenna spectrum sensing is now formulated as follows:

$$\begin{aligned}H_0 &: \hat{\mathbf{r}}_{\text{mul}} = \boldsymbol{\epsilon}_{\text{mul},0},\\ H_1 &: \hat{\mathbf{r}}_{\text{mul}} = \mathbf{r}_{\text{mul}} + \boldsymbol{\epsilon}_{\text{mul},1},\end{aligned} \tag{14}$$

where $\mathbf{r}_{\text{mul}}$ is the asymptotic value of $\hat{\mathbf{r}}_{\text{mul}}$ under H_1, $\boldsymbol{\epsilon}_{\text{mul},0}$ is the estimation error vector resulted from $w_i(t)$ and $\boldsymbol{\epsilon}_{\text{mul},1}$ is the estimation error vector resulted from both $w_i(t)$ and $s(t)$, which are asymptotically distributed as $\lim_{M\to\infty}\sqrt{M}\boldsymbol{\epsilon}_{\text{mul},0} \overset{D}{=} \mathcal{N}(\mathbf{0},\boldsymbol{\Sigma}_{\text{mul},0})$ and $\lim_{M\to\infty}\sqrt{M}\boldsymbol{\epsilon}_{\text{mul},1} \overset{D}{=} \mathcal{N}(\mathbf{0},\boldsymbol{\Sigma}_{\text{mul},1})$, respectively. Different from the Dandawaté-Giannakis algorithm and its extension proposed in [20], we do not assume that the asymptotic covariance matrix under H_0 and H_1 is the same, i.e., we assume $\boldsymbol{\Sigma}_{\text{mul},0} \neq \boldsymbol{\Sigma}_{\text{mul},1}$ in our hypotheses problem. Our hypotheses problem is more realistic, due to the fact that both $s(t)$ and $w_i(t)$ contributes to the asymptotic covariance matrix under H_1, whereas only $w_i(t)$ contributes to H_0. We will use an example, where the additive noise is the low-pass zero-mean complex Gaussian noise, to illustrate its validity and necessity in the next subsection.

According to the binary hypotheses given in (14), the generalized likelihood ratio (GLR) for the test problem is given by

$$\tilde{\Lambda}_{\text{mul}} = \frac{\sup_{\boldsymbol{\theta}_1} f(\sqrt{M}\hat{\mathbf{r}}_{\text{mul}} \mid \boldsymbol{\theta}_1, \boldsymbol{\Sigma}_{\text{mul},1}, H_1)}{\sup_{\boldsymbol{\theta}_0} f(\sqrt{M}\hat{\mathbf{r}}_{\text{mul}} \mid \boldsymbol{\theta}_0, \boldsymbol{\Sigma}_{\text{mul},0}, H_0)}.$$

By substituting $\hat{\mathbf{r}}_{\text{mul}}$ for $\boldsymbol{\theta}_1$ and $\mathbf{0}$ for $\boldsymbol{\theta}_0$, we can obtain that

$$\begin{aligned}\tilde{\Lambda}_{\text{mul}} &= \frac{\dfrac{1}{(2\pi)^P \det(\boldsymbol{\Sigma}_{\text{mul},1})^{1/2}}}{\dfrac{1}{(2\pi)^P \det(\boldsymbol{\Sigma}_{\text{mul},0})^{1/2}}} \\ &\times \frac{\exp\left\{-\dfrac{1}{2}M(\hat{\mathbf{r}}_{\text{mul}}-\hat{\mathbf{r}}_{\text{mul}})\boldsymbol{\Sigma}_{\text{mul},1}^{-1}(\hat{\mathbf{r}}_{\text{mul}}-\hat{\mathbf{r}}_{\text{mul}})^{\mathrm{T}}\right\}}{\exp\left\{-\dfrac{1}{2}M(\hat{\mathbf{r}}_{\text{mul}}-\mathbf{0})\boldsymbol{\Sigma}_{\text{mul},0}^{-1}(\hat{\mathbf{r}}_{\text{mul}}-\mathbf{0})^{\mathrm{T}}\right\}} \\ &= \frac{\det(\boldsymbol{\Sigma}_{\text{mul},0})^{1/2}}{\det(\boldsymbol{\Sigma}_{\text{mul},1})^{1/2}} \exp\left\{\frac{1}{2}M\hat{\mathbf{r}}_{\text{mul}}\boldsymbol{\Sigma}_{\text{mul},0}^{-1}\hat{\mathbf{r}}_{\text{mul}}^{\mathrm{T}}\right\}. \end{aligned} \tag{15}$$

Taking the logarithm transformation of the GLR gives the test statistic as follows:

$$\mathcal{T}_{\text{mul}} = M\hat{\mathbf{r}}_{\text{mul}}\boldsymbol{\Sigma}_{\text{mul},0}^{-1}\hat{\mathbf{r}}_{\text{mul}}^{\mathrm{T}} + 2\ln\frac{\det(\boldsymbol{\Sigma}_{\text{mul},0})^{1/2}}{\det(\boldsymbol{\Sigma}_{\text{mul},1})^{1/2}}. \tag{16}$$

Since the second term of this test statistic is of the same value under H_1 and H_0, it has no contribution to the detection and can be eliminated, which yields the final test statistic as

$$\mathcal{T}_{\text{mul}} = M\hat{\mathbf{r}}_{\text{mul}}\boldsymbol{\Sigma}_{\text{mul},0}^{-1}\hat{\mathbf{r}}_{\text{mul}}^{\mathrm{T}}. \tag{17}$$

Remark 1: The GLR test statistic only uses the asymptotic covariance matrix under H_0, $\boldsymbol{\Sigma}_{\text{mul},0}$, and it is unrelated to the asymptotic covariance matrix under H_1, $\boldsymbol{\Sigma}_{\text{mul},1}$.

Remark 2: The GLR test statistic suggests that knowledge or partial knowledge of the statistics and distributions of the additive noise can be exploited to improve the detection performance. In the next section, it is shown in an example that if the additive noise is low-pass zero-mean complex Gaussian, and its bandwidth and power spectral density is known, $\mathbf{\Sigma}_{\text{mul},0}$ can be calculated.

Remark 3: If no assumption about the additive noise can be made in the detection, the estimated $\hat{\mathbf{\Sigma}}_{\text{mul}}$ calculated from the most recent detection that was decided to be under H_0 can be employed as a good approximation of $\mathbf{\Sigma}_{\text{mul},0}$ of the current detection. Since the noise statistic does not change rapidly in practical situations, $\hat{\mathbf{\Sigma}}_{\text{mul}}$ calculated from the recent detection is a good approximation of current $\mathbf{\Sigma}_{\text{mul},0}$. The test statistic of Dandawaté-Giannakis algorithm employs current $\hat{\mathbf{\Sigma}}_{\text{mul}}$, whereas our proposed method suggests using the $\hat{\mathbf{\Sigma}}_{\text{mul}}$ calculated from the most recent detection that was decided to be under H_0.

4. An Example of $\Sigma_{\text{mul},0}$ Estimation and the Corresponding Test

In this section, we demonstrate how to estimate $\mathbf{\Sigma}_{\text{mul},0}$ and construct the GLRT, with the partial knowledge that the noise is a low-pass zero-mean complex Gaussian noise with uncertain power, which is a typical and practical scenario for wireless communication (it models the equivalent baseband signal of bandpass additive white Gaussian noise). Moreover, we demonstrate the necessity of assuming $\mathbf{\Sigma}_{\text{mul},0} \neq \mathbf{\Sigma}_{\text{mul},1}$ during the derivation of the covariance matrix estimation.

With the assumption that $w(t)$ is a low-pass zero-mean complex Gaussian noise, the power spectral density (PSD) of $w(t)$ is given by

$$\Phi_{ww}(f) \stackrel{\Delta}{=} \int_{-\infty}^{\infty} E\{w(t)w^*(t+\tau)\}e^{-j2\pi f\tau}d\tau = \begin{cases} N_0, & \text{if } |f| \leq \frac{1}{2}B, \\ 0, & \text{else} \end{cases} \tag{18}$$

where B is the bandwidth of the equivalent band-pass signal of $w(t)$ and N_0 is assumed to be unknown due to noise power uncertainty. The auto-correlation function and the conjugated auto-correlation function of $w(t)$ are defined as

$$R_{ww}(\tau) \stackrel{\Delta}{=} E\{w(t)w^*(t+\tau)\} = 2N_0 \frac{\sin(\pi B\tau)}{\pi\tau}, \tag{19}$$

and

$$R_{ww^*}(\tau) \stackrel{\Delta}{=} E\{w(t)w(t+\tau)\} = 0, \tag{20}$$

respectively, whose discrete time versions are given by

$$R_{ww}(\nu) \stackrel{\Delta}{=} E\{w[m]w^*[m+\nu]\} = 2N_0 \frac{\sin(\pi B\nu T_s)}{\pi\nu T_s}, \tag{21}$$

and

$$R_{ww^*}(\nu) \stackrel{\Delta}{=} E\{w[m]w[m+\nu]\} = 0, \tag{22}$$

where T_s denotes the sampling period, ν is the discrete time version of lag, and $w[m]$ is the discrete time version of $w(t)$, i.e., $w[m]=w(mT_s)$.

Theorem 1: Suppose the PSD of $w_i(t)$ is

$$\Phi_{w_iw_i}(f)=\begin{cases} N_i, & (|f|\leq \frac{1}{2}B),\\ 0, & (|f|>\frac{1}{2}B).\end{cases}$$

Then, when $i<j$, the asymptotic covariance of $\hat{R}^{\alpha}_{w_iw_j^*}(\nu)$ is given by

$$\lim_{M\to\infty} M\mathrm{cum}\left\{\hat{R}^{\alpha}_{w_iw_j^*}(\nu),\hat{R}^{\alpha}_{w_iw_j^*}(\rho)\right\}=0, \tag{23}$$

$$\lim_{M\to\infty} M\mathrm{cum}\left\{\hat{R}^{\alpha}_{w_iw_j^*}(\nu),\left\{\hat{R}^{\alpha}_{w_iw_j^*}(\rho)\right\}^*\right\}=4N_iN_j\sum_{k=-\infty}^{\infty}\left\{\left(\frac{\sin\pi BkT_s}{\pi kT_s}\right)\right. \times\left.\left(\frac{\sin\pi B(k+\rho-\nu)T_s}{\pi(k+\rho-\nu)T_s}\right)\right\}e^{-j2\pi\alpha kT_s}. \tag{24}$$

Proof: Following the procedure presented in the proof of Theorem 1 of [22], and applying the PSD of $w_i(t)$, (23) and (24) are proved.

Theorem 2: When $i=j$, the asymptotic covariance of $\hat{R}^{\alpha}_{w_iw_j^*}(\nu)$ is given by

$$\lim_{M\to\infty} M\mathrm{cum}\left\{\hat{R}^{\alpha}_{w_iw_i^*}(\nu),\hat{R}^{\alpha}_{w_iw_i^*}(\rho)\right\}=0, \tag{25}$$

$$\lim_{M\to\infty} M\mathrm{cum}\left\{\hat{R}^{\alpha}_{w_iw_i^*}(\nu),\left\{\hat{R}^{\alpha}_{w_iw_i^*}(\rho)\right\}^*\right\}=4N_i^2\sum_{k=-\infty}^{\infty}\left\{\left(\frac{\sin\pi BkT_s}{\pi kT_s}\right)\left(\frac{\sin\pi B(k+\rho-\nu)T_s}{\pi(k+\rho-\nu)T_s}\right)\right. +\left.\left(\frac{\sin\pi B(k+\rho)T_s}{\pi(k+\rho)T_s}\right)\left(\frac{\sin\pi B(k-\nu)T_s}{\pi(k-\nu)T_s}\right)\right\} \times e^{-j2\pi\alpha kT_s}. \tag{26}$$

Proof: Similar to that of Theorem 1.

It can be seen from (23) to (26) that only N_i needs to be estimated to calculate the asymptotic covariance of $\hat{R}^{\alpha}_{w_iw_i^*}(\nu)$ and $\hat{R}^{\alpha}_{w_iw_j^*}(\nu)$, since the cyclic frequency of interest and the lag parameters are given in prior, and the bandwidth B and the sampling period T_s are available as for a specific CR receiver. Under H_0, when $x_i(t)=w_i(t)$, N_i can be consistently estimated by

$$\hat{N}_i=\frac{\frac{1}{M}\sum_{m=0}^{M-1}x_i[m]x_i^*[m]}{2B}. \tag{27}$$

Under H_1, using (27) to estimate N_i tends to yield a larger result. However, in the regime of low SNR, which is the critical situation in spectrum sensing, the error introduced is not significant. Since this approximation error only happens under H_1, it has no impact on the CFAR property of the proposed algorithm.

By substituting (27) into (24) and (26), the asymptotic covariance of $\hat{R}^{\alpha}_{w_i w_i^*}(\nu)$ and $\hat{R}^{\alpha}_{w_i w_j^*}(\nu)$ can be estimated. Actually, this covariance estimation method can be viewed as a special case of that in the Dandawaté-Giannakis algorithm [22]. No matter it is under H_0 or H_1, N_i is estimated as the energy of $x_i(t)$. This approximation is essentially equivalent to the assumption in the Dandawaté-Giannakis algorithm that the cyclic correlation estimation has the same asymptotic covariance under H_0 and H_1. It cannot be avoided, since under H_1 it is impossible to consistently estimate N_i.

Theorem 3: The asymptotic covariance between $\hat{R}^{\alpha}_{w_i w_j^*}(\nu)$ and $\hat{R}^{\alpha}_{w_a w_b^*}(\rho)$ ($(i,j)\neq(a,b)$) is given by

$$\lim_{M\to\infty} M\text{cum}\left\{\hat{R}^{\alpha}_{w_i w_j^*}(\nu), \hat{R}^{\alpha}_{w_a w_b^*}(\rho)\right\} = 0, \tag{28}$$

$$\lim_{M\to\infty} M\text{cum}\left\{\hat{R}^{\alpha}_{w_i w_j^*}(\nu), \left\{\hat{R}^{\alpha}_{w_a w_b^*}(\rho)\right\}^*\right\} = 0. \tag{29}$$

Proof: Similar to that of Theorem 1, omitted due to space limit.

Theorem 3 indicates that under H_0 the asymptotic covariance between $\hat{R}^{\alpha}_{w_i w_j^*}(\nu)$ and $\hat{R}^{\alpha}_{w_a w_b^*}(\nu)$ ($(i,j)\neq(a,b)$) can be simply and correctly estimated as zero under the low-pass zero-mean complex Gaussian noise. Apparently, the asymptotic covariance between $\hat{R}^{\alpha}_{x_i x_j^*}(\nu)$ and $\hat{R}^{\alpha}_{x_a x_b^*}(\nu)$ ($(i,j)\neq(a,b)$) can not be guaranteed to be equal to zero under H_1. Therefore, it is necessary to assume that the asymptotic covariance is different under H_0 and H_1.

As for the estimation of $\mathbf{\Sigma}_{\text{mul},0}$, which contains all the asymptotic covariance among the CCA and CCC estimations, the covariance matrix estimator [20] with the Dandawaté-Giannakis algorithm is not valid, since under H_1 it fails to give good approximations for the entries of $\mathbf{\Sigma}_{\text{mul},0}$ corresponding to the asymptotic covariance between $\hat{R}^{\alpha}_{w_i w_j^*}(\nu)$ and $\hat{R}^{\alpha}_{w_a w_b^*}(\nu)$ ($(i,j)\neq(a,b)$). This also demonstrate the necessity to formulate the hypotheses of cyclostationary multi-antenna spectrum sensing as in (14).

Denote $\hat{\mathbf{\Sigma}}_{\text{mul},0}$ as the estimation of the asymptotic covariance matrix $\mathbf{\Sigma}_{\text{mul},0}$, which is of size $((N^2+N)P)\times((N^2+N)P)$, and can be divided into $((N^2+N)/2)\times((N^2+N)/2)$ blocks as follows, one block for each pair of $\left((i,j),(a,b)\right)$, where $i\leq j\leq N$ and $a\leq b\leq N$:

$$\boldsymbol{\Sigma}_{\mathrm{mul},0}=\begin{bmatrix} \Sigma_{x_1x_1^*,x_1x_1^*,0} & \cdots & \Sigma_{x_1x_1^*,x_1x_N^*,0} & \Sigma_{x_1x_1^*,x_2x_2^*,0} & \cdots & \Sigma_{x_1x_1^*,x_2x_N^*,0} & \cdots & \Sigma_{x_1x_1^*,x_Nx_N^*,0} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Sigma_{x_1x_N^*,x_1x_1^*,0} & \cdots & \Sigma_{x_1x_N^*,x_1x_N^*,0} & \Sigma_{x_1x_N^*,x_2x_2^*,0} & \cdots & \Sigma_{x_1x_N^*,x_2x_N^*,0} & \cdots & \Sigma_{x_1x_N^*,x_Nx_N^*,0} \\ \Sigma_{x_2x_2^*,x_1x_1^*,0} & \cdots & \Sigma_{x_2x_2^*,x_1x_N^*,0} & \Sigma_{x_2x_2^*,x_2x_2^*,0} & \cdots & \Sigma_{x_2x_2^*,x_2x_N^*,0} & \cdots & \Sigma_{x_2x_2^*,x_Nx_N^*,0} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Sigma_{x_2x_N^*,x_1x_1^*,0} & \cdots & \Sigma_{x_2x_N^*,x_1x_N^*,0} & \Sigma_{x_2x_N^*,x_2x_2^*,0} & \cdots & \Sigma_{x_2x_N^*,x_2x_N^*,0} & \cdots & \Sigma_{x_2x_N^*,x_Nx_N^*,0} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Sigma_{x_Nx_N^*,x_1x_1^*,0} & \cdots & \Sigma_{x_Nx_N^*,x_1x_N^*,0} & \Sigma_{x_Nx_N^*,x_2x_2^*,0} & \cdots & \Sigma_{x_Nx_N^*,x_2x_N^*,0} & \cdots & \Sigma_{x_Nx_N^*,x_Nx_N^*,0} \end{bmatrix}, \tag{30}$$

where $\boldsymbol{\Sigma}_{x_ix_j^*,x_ax_b^*,0}$ is the asymptotic covariance matrix between $\hat{\mathbf{r}}_{x_ix_j^*}$ and $\hat{\mathbf{r}}_{x_ax_b^*}$ under H_0, which can be calculated as

$$\boldsymbol{\Sigma}_{x_ix_j^*,x_ax_b^*,0}=\begin{bmatrix} \mathrm{Re}\left\{\dfrac{\mathbf{Q}_{ij,ab}}{2}\right\} & \mathrm{Im}\left\{-\dfrac{\mathbf{Q}_{ij,ab}}{2}\right\} \\ \mathrm{Im}\left\{\dfrac{\mathbf{Q}_{ij,ab}}{2}\right\} & \mathrm{Re}\left\{\dfrac{\mathbf{Q}_{ij,ab}}{2}\right\} \end{bmatrix}, \tag{31}$$

where $\mathbf{Q}_{ij,ab}$ is a $P \times P$ matrix with the (p,q)-th entry defined as follows:

$$\mathbf{Q}_{ij,ab}(p,q)\overset{\Delta}{=}\lim_{M\to\infty} M\mathrm{cum}\left\{\hat{R}^{\alpha}_{w_iw_j^*}(\nu_p),\left\{\hat{R}^{\alpha}_{w_aw_b^*}(\nu_q)\right\}^*\right\}. \tag{32}$$

According to Theorem 3, $\boldsymbol{\Sigma}_{x_ix_j^*,x_ax_b^*,0}$ with $(i,j)\neq(a,b)$ is a $2P\times 2P$ zero matrix, thus, we can rewrite $\boldsymbol{\Sigma}_{\mathrm{mul},0}$ as

$$\begin{aligned}\boldsymbol{\Sigma}_{\mathrm{mul},0}=\mathrm{diag}\Big\{&\boldsymbol{\Sigma}_{x_1x_1^*,x_1x_1^*,0},\boldsymbol{\Sigma}_{x_1x_2^*,x_1x_2^*,0},\ldots,\boldsymbol{\Sigma}_{x_1x_N^*,x_1x_N^*,0},\\ &\boldsymbol{\Sigma}_{x_2x_2^*,x_2x_2^*,0},\boldsymbol{\Sigma}_{x_2x_3^*,x_2x_3^*,0},\ldots,\boldsymbol{\Sigma}_{x_2x_N^*,x_2x_N^*,0},\\ &\cdots\cdots\\ &\boldsymbol{\Sigma}_{x_Nx_N^*,x_Nx_N^*,0}\Big\}.\end{aligned} \tag{33}$$

Combining (24), (26) and the equations from (31) to (33), the estimator of $\boldsymbol{\Sigma}_{\mathrm{mul},0}$, $\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}$, can be calculated.

In summary, the proposed algorithm for cyclostationary multi-antenna spectrum sensing, under low-pass zero-mean complex Gaussian noise with uncertain power, can be implemented using the following steps:

Step 1 Declare a cyclic frequency α and a set of lags.

Step 2 Compute the CCA and CCC estimations as in (11) and construct $\hat{\mathbf{r}}_{\mathrm{mul}}$ as in (12).

Step 3 Calculate $\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}$, by using (24), (26), and (31) to (33).

Step 4 Substitute $\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}$, for $\boldsymbol{\Sigma}_{\mathrm{mul},0}$ in (17) and calculate the test statistic as

$$\mathrm{T}_{\mathrm{mul,Prop}} = M\hat{\mathbf{r}}_{\mathrm{mul}}\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}\hat{\mathbf{r}}_{\mathrm{mul}}^{\mathrm{T}}. \tag{34}$$

Step 5 Let Γ denote the threshold that satisfies the required detection performance. If $\mathrm{T}_{\mathrm{mul,Prop}} > \Gamma$, accept H_1; if $\mathrm{T}_{\mathrm{mul,Prop}} \le \Gamma$, accept H_0.

5. Asymptotic Distribution and Computational Complexity of the Proposed Test Statistic

5.1 Asymptotic Distribution of $\mathrm{T}_{\mathrm{mul,Prop}}$

To derive the asymptotic distribution of $\mathrm{T}_{\mathrm{mul,Prop}}$, we follow [11] and borrow the following theorem from [25]:

Theorem 4: Let $\mathbf{x} \sim \mathrm{N}(\boldsymbol{\mu},\mathbf{V})$, where $\mathbf{V}$ is $L \times L$ nonsingular, suppose that the real $L \times L$ matrix $\mathbf{A}$ is symmetric, and let $r(\mathbf{A})$ denote its rank. Then the quadratic form $\mathbf{xAx}^{\mathrm{T}}$ follows a chi-square distribution if and only if $\mathbf{AV}$ is idempotent, in which case $\mathbf{xAx}^{\mathrm{T}}$ has $r(\mathbf{A})$ degrees of freedom and noncentrality parameter $\boldsymbol{\mu}\mathbf{A}\boldsymbol{\mu}^{\mathrm{T}}$.

As for the proposed test statistic under H_0, let $\mathbf{x} = \sqrt{M}\hat{\mathbf{r}}_{\mathrm{mul}}$, $\boldsymbol{\mu} = \mathbf{0}$, $\mathbf{V} = \boldsymbol{\Sigma}_{\mathrm{mul},0}$ and $\mathbf{A} = \hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}$. Since $\hat{N}_i$ is a mean-square sense consistent estimation of N_i under H_0 and $\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}$ is only determined by $\hat{N}_i$, $\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}$ is also mean-square sense consistent under H_0. Thus, $\lim_{M\to\infty}\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}\boldsymbol{\Sigma}_{mul,0} \overset{P}{=} \hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}\boldsymbol{\Sigma}_{mul,0} = \mathbf{I}$, i.e., the matrix product, $\mathbf{AV}$, is asymptotically idempotent. The convergence in probability ($\overset{P}{=}$ above) follows from application of a Cramer-Wold device [26] and from the fact that convergence in the mean-square implies convergence in probability. Hence, from Theorem 4, it follows that under H_0

$$\lim_{M\to\infty} \mathrm{T}_{\mathrm{mul,Prop}} \overset{D}{=} \chi^2_{2PN_{\mathrm{mul}}}, \tag{35}$$

where $N_{mul} = (N^2 + N)/2$. Clearly, this distribution is not related to the noise power, so uncertainty of the noise power has no impact on the CFAR property of the proposed algorithm. Deriving the distribution of the proposed test statistic under H_1 needs to introduce the following definition (See p. 67-88 of [27]):

Definition 1: If $\mathbf{x}$ has a multivariate normal distribution, $\mathrm{N}(\boldsymbol{\mu},\mathbf{V})$, then the value of the following form $(\mathbf{x}+\mathbf{a})\mathbf{A}(\mathbf{x}+\mathbf{a})^{\mathrm{T}} + \mathbf{b}^{\mathrm{T}}\mathbf{x}$, where $\mathbf{A}$ is a square matrix, has a generalized chi-square distribution.

As for the proposed test statistic under H_1, let $\mathbf{a} = \mathbf{b} = \mathbf{0}$, $\mathbf{x} = \sqrt{M}\hat{\mathbf{r}}_{\mathrm{mul}}$ and $\mathbf{A} = \hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}$. Under H_1, $\lim_{M\to\infty}\sqrt{M}\hat{\mathbf{r}}_{\mathrm{mul}} \overset{D}{=} \mathrm{N}(\boldsymbol{\theta}_1,\boldsymbol{\Sigma}_{\mathrm{mul},1})$, thus, the test statistic $\mathrm{T}_{\mathrm{mul,Prop}} = M\hat{\mathbf{r}}_{\mathrm{mul}}\hat{\boldsymbol{\Sigma}}_{\mathrm{mul},0}^{-1}\hat{\mathbf{r}}_{\mathrm{mul}}^{\mathrm{T}}$ is asymptotically generalized chi-square distributed.

5.2 Computational Complexity

Denote the test static of the extended Dandawaté-Giannakis algorithm proposed in [20] as $\mathcal{T}_{\text{mul,Dan}}$. The computational complexity of $\mathcal{T}_{\text{mul,Dan}}$ focuses on the calculation of the estimated asymptotic covariance $\hat{\boldsymbol{\Sigma}}_{\text{mul}}$, which is given as follows when only multiplication is considered:

$$\Omega_{\text{mul,Dan}} = \mathcal{O}\left(N_{\text{mul}} P M_{\text{FFT}} \log_2 \frac{M_{\text{FFT}}}{2}\right) = \mathcal{O}\left(\frac{N^2}{2} P M_{\text{FFT}} \log_2 \frac{M_{\text{FFT}}}{2}\right). \tag{36}$$

where M_{FFT} is the length of the Fast-Fourier transform (FFT), which is employed for the calculation of $\hat{\boldsymbol{\Sigma}}_{\text{mul}}$ (Refer to [20] and [22] for details). Note that M_{FFT} must not be less than the sample size, i.e., $M_{\text{FFT}} \geq M$.

As for the proposed test statistic given in (34), the computational complexity is

$$\Omega_{\text{mul}} = \mathcal{O}\left(N_{\text{mul}} P M\right) = \mathcal{O}\left(\frac{N^2}{2} P M\right), \tag{37}$$

Apparently, the calculation of $\mathcal{T}_{\text{mul,Prop}}$ has a lower complexity compared with $\mathcal{T}_{\text{mul,Dan}}$.

6. Simulation Results

The licensed communication system considered in the following simulations is a simplified GSM system with GMSK modulated signal of symbol rate $f_{\text{GSM}} = 1/T_{\text{GSM}} = 270.833\text{kbit/s}$. The baseband GMSK modulated signal is given by

$$s(t) = \exp\left\{ j2\pi h_m \sum_{k=-\infty}^{\infty} I_k \int_{-\infty}^{t} g(\tau - kT_{\text{GSM}}) d\tau \right\}, \tag{38}$$

where I_k is the k th data symbol, and $I_k \in \{-1,1\}$; $h_m = 0.5$ is the modulation index; $g(t)$ is the impulse function given by

$$g(t) = \frac{1}{T_{\text{GSM}}} \text{rect}\left(\frac{t}{T_{\text{GSM}}}\right) * p(t), \tag{39}$$

where $\text{rect}(t)$ is the rectangular pulse function of unit length, and $p(t)$ is a Gaussian impulse function with the time bandwidth product $B_{\text{Gaussion}} T_{\text{GSM}} = 0.3$. We assume that all the time slots of the simplified GSM system are occupied. It has been derived in [13] that a GMSK signal exhibits cyclostationarity at the cyclic frequency $\alpha = f_{\text{GSM}}/2$ and the lag $\tau = 0$, which are employed in the following simulations as the cyclic frequency and lag of interest, respectively.

We assume flat Rayleigh fading channels in the simulations of **Fig. 3** to **Fig. 8**, and the average SNR in dB of the received signal is defined by

$$\text{SNR(dB)} = 10\log_{10} \frac{E[|hs(t)|^2]}{P_N}, \tag{40}$$

where P_N is the average power of the low-pass zero-mean complex Gaussian noise. The bandwidth of the equivalent band-pass Gaussian noise is $B = 223.437\text{kHz}$. The sampling rate of the CR receiver is set to be 10 times the symbol rate of the GSM system, i.e., $f_s = 10 f_{\mathrm{GSM}}$ and $T_s = T_{\mathrm{GSM}}/10$.

For the estimation of the asymptotic covariance matrix with the Dandawaté-Giannakis algorithm, a length-2049 Kaiser window with β parameter of 10 is used [11].

We mainly use the false alarm rate, P_f, and the detection probability, P_d, to measure the detector performance, which are defined respectively as

$$P_f \overset{\Delta}{=} \mathrm{Prob}(\mathrm{T} > \Gamma \mid H_0), \tag{41}$$

and

$$P_d \overset{\Delta}{=} \mathrm{Prob}(\mathrm{T} > \Gamma \mid H_1), \tag{42}$$

where T is the test statistic.

6.1 Distributions of the Proposed Test Statistics under H_0

To verify the distributions of the proposed test statistics given in (35), we plot in **Fig. 2** the simulated cumulative distribution functions (CDFs) of $\mathrm{T}_{mul,\mathrm{Prop}}$ for different numbers of antennas under H_0. The theoretical CDFs of χ^2 distribution with corresponding degrees of freedom are also presented for comparison. The sample size is $M = 4000$.

It is observed from **Fig. 2** that the curves of the simulated CDFs and the theoretical CDFs nearly coincide, which confirms that the proposed test statistics given in (34) are χ^2 distributed as (35) under H_0. Moreover, since these distributions are not related to the noise power, the CFAR property can be guaranteed for spectrum sensing based on the proposed test statistics.

6.2 Multi-Antenna Cyclostationary Spectrum Sensing

We investigate the detection performance of the proposed multi-antenna cyclostationary spectrum sensing with flat Rayleigh fading channel in this subsection. We first compare the proposed algorithm using the test statistic $\mathrm{T}_{mul,\mathrm{Prop}}$ with the Dandawaté-Giannakis algorithm for single-antenna sensing, whose test statistic is denoted as $\mathrm{T}_{\mathrm{Dan}}$, and the one simply extending the Dandawaté-Giannakis algorithm for multi-antenna sensing, which was given in [20] and whose test statistic is denoted as $\mathrm{T}_{mul,\mathrm{Dan}}$. Next, we show the necessity of taking into account the CCCs by comparing the proposed algorithm with the one excluding all the CCCs.

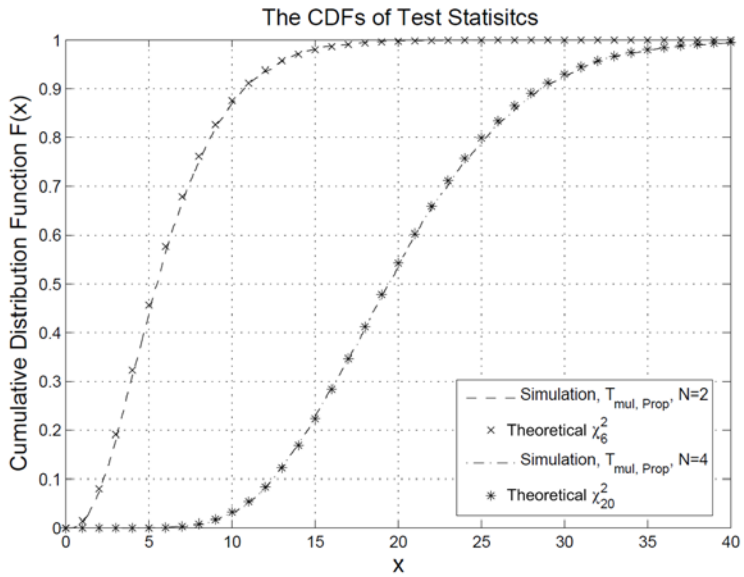

Fig. 2. Distributions of the proposed test statistics under H_0.

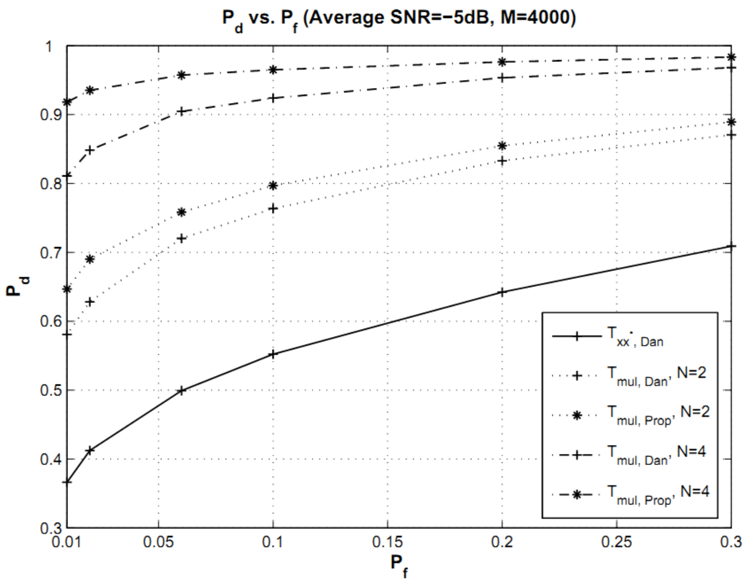

Fig. 3. P_d vs. P_f with an average SNR of -5dB and a sample size of 4000, under Rayleigh fading channel.

Fig. 3 plots the curves of detection probability vs. false alarm rate for an average SNR of -5dB and different numbers of antennas, and **Fig. 4** plots the curves of detection probability vs. average SNR with a false alarm rate of 0.01 and different numbers of antennas. The sample size of both figures is 4000. From **Fig. 3** and **Fig. 4**, it observed that as the number of antennas increases, the detection performance increases significantly. These two figures also demonstrate that our proposed algorithm has better detection performance than the extended Dandawaté-Giannakis algorithm, i.e., the detection probability using $T_{mul,\mathrm{Prop}}$ is larger than the one using $\mathrm{T}_{mul,\mathrm{Dan}}$ under the same number of antennas. Most importantly, the performance gap becomes more significant as the number of antennas increases. For a four-antenna system, the performance gain is about 1.7dB, which is much more significant than that for a single-antenna system. The reason is that the proposed algorithm gives better estimation of the asymptotic covariance matrices of $\hat{\mathbf{r}}_{\mathrm{mul}}$ under H_0, i.e., $\boldsymbol{\Sigma}_{\mathrm{mul},0}$, than the Dandawaté-Giannakis algorithm does, and the improvement increases as the number of antennas and the dimension of the matrices increase, which are verified in **Fig. 5**.

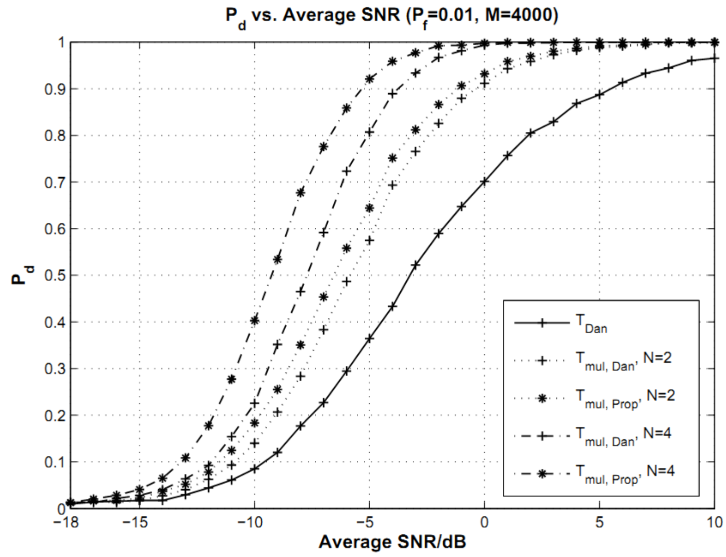

Fig. 4. P_d vs. average SNR with a false alarm rate $P_f = 0.01$ and a sample size of 4000, under Rayleigh fading channel.

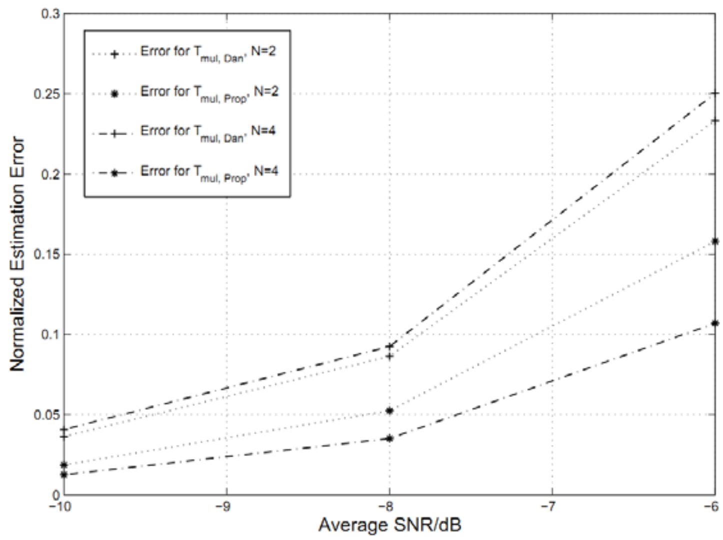

Fig. 5. Normalized Estimation Error (normalized with antenna number *N*) vs. average SNR for the estimation of $\boldsymbol{\Sigma}_{\mathrm{mul},0}$, with a sample size of 4000, under Rayleigh fading channel.

To demonstrate the contribution of CCCs, we simulate the performance of the multi-antenna cyclostationary spectrum sensing algorithms excluding all CCCs, which uses $\hat{\mathbf{r}}_{\text{mul,CCA}}$ as following instead of $\hat{\mathbf{r}}_{\text{mul}}$ in the proposed algorithm:

$$\hat{\mathbf{r}}_{\text{mul,CCA}} \stackrel{\Delta}{=} \left[\hat{\mathbf{r}}_{x_1 x_1^*}, \hat{\mathbf{r}}_{x_2 x_2^*}, \ldots, \hat{\mathbf{r}}_{x_N x_N^*}\right].$$

Fig. 6 plots the curves of detection probability vs. false alarm rate for the proposed algorithm without the CCCs with an average SNR of -5dB and different numbers of antennas ($N = 2$ and $N = 4$). It shows that the multi-antenna spectrum sensing algorithms that take into account all CCAs and CCCs outperform the ones excluding the CCCs, which reveals the contribution of CCCs. **Fig. 7** plots the curves of detection probability vs. average SNR for the proposed algorithm without CCCs with a false alarm rate of 0.01 and different numbers of antennas ($N = 2$ and $N = 4$), and **Fig. 8** shows a zoom of the important area illustrating the differences in performance more clearly. From **Fig. 8**, it is observed that the proposed algorithm obtains a performance gain of nearly 2.0dB over the one excluding the CCCs for four antennas. Moreover, it is observed that the performance gain provided by the CCCs is more significant for the proposed algorithm than that for the extended Dandawaté-Giannakis algorithm. As the number of antennas increases, the performance gain provided by CCCs also increases for the proposed algorithm, but stays unchanged for the extended Dandawaté-Giannakis algorithm. This demonstrates that the extended Dandawaté-Giannakis algorithm fails to take full advantages of the CCCs due to the false assumption that the asymptotic covariance matrix of $\hat{\mathbf{r}}_{\text{mul}}$ is the same under H_0 and H_1 for spectrum sensing.

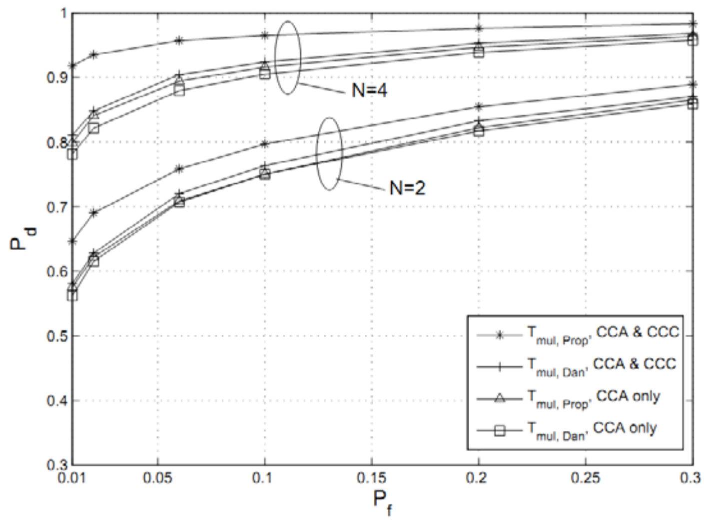

Fig. 6. P_d vs. P_f for the proposed multi-antenna cyclostationary spectrum sensing algorithms, with an average SNR of -5dB and a sample size of 4000, under Rayleigh fading channel.

7. Conclusion

In this paper, we proposed a novel GLRT algorithm for cyclostationary multi-antenna spectrum sensing. Using an example where the additive noise is low-pass zero-mean Gaussian noise, we verified that the proposed GLRT has a more reasonable hypotheses problem formulation and employs a more suitable asymptotic covariance estimator than those of the Dandawaté-Giannakis algorithm and its extension. We have also derived the asymptotic distributions of the proposed test statistics, and proved the CFAR property of the proposed algorithm. Theoretical analysis and simulation results showed that the proposed algorithm has better performance of detection probability and lower computational complexity compared with the Dandawaté-Giannakis algorithm and its extension. Moreover, simulation results also verified the necessity of taking into account the CCCs in multi-antenna cyclostationary spectrum sensing.

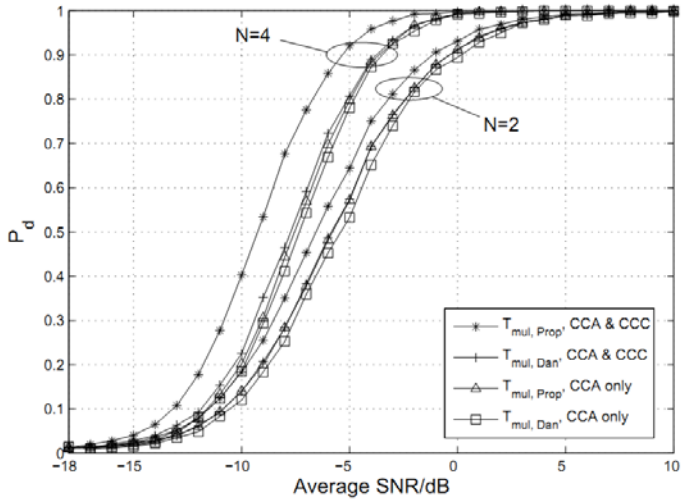

Fig. 7. P_d vs. average SNR for the proposed multi-antenna cyclostationary spectrum sensing algorithms, with a false alarm rate $P_f = 0.01$ and a sample size of 4000, under Rayleigh fading channel.

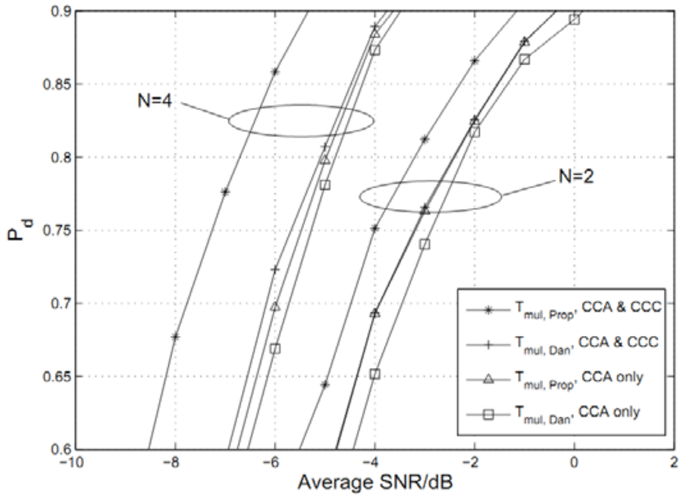

Fig. 8. Zoom of the important region of Fig. 7. P_d vs. average SNR for the proposed multi-antenna cyclostationary spectrum sensing algorithms, with a false alarm rate $P_f = 0.01$ and a sample size of 4000, under Rayleigh fading channel.

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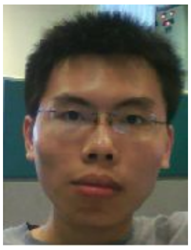

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