Synchronization of Dynamical Happiness Model

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Abstract

Chaotic dynamics is an active research area in fields such as biology, physics, sociology, psychology, physiology, and engineering. Interest in chaos is also expanding to the social sciences, such as politics, economics, and societal events prediction. Most people pursue happiness, both spiritual and physical in many cases. However, happiness is not easy to define, because people differ in how they perceive it. Happiness can exist in mind and body. Therefore, we need to be happy in both simultaneously to achieve optimal happiness. To do this, we need to synchronize mind and body. In this paper, we propose a chaotic synchronization method in a mathematical model of happiness organized by a second-order ordinary differential equation with external force. This proposed mathematical happiness equation is similar to Duffing's equation, because it is derived from that equation. We introduce synchronization method from our mathematical happiness model by using the derived Duffing equation. To achieve chaotic synchronization between the human mind and body, we apply an idea of mind/body unity originating in Oriental philosophy. Of many chaotic synchronization methods, we use only coupled synchronization, because this method is closest to representing mind/body unity. Typically, coupled synchronization can be applied only to non-autonomous systems, such as a modified Duffing system. We represent the result of synchronization using a differential time series mind/body model.

Keywords: Chaotic dynamics, Nonlinear dynamics, Happiness model, Duffing equation, Chaotic synchronization

1. Introduction

In the last two decades, chaotic dynamics has been used widely in real-world applications, such as biological systems [1], brain modeling [2], weather modeling [3], vibration modeling [4], mechanical and electrical engineering [5, 6], control and synchronization [7-11], robotics [12, 13], and others [14-18]. Currently, it is also being studied by many researchers in biology, physics, sociology, psychology, physiology, and engineering. However, this research is not limited to the natural sciences, but is spreading to the social sciences, such as politics, economics, and the prediction of societal events such as happiness and addiction [14, 19].

Most people pursue happiness. However, happiness is not easy to define, because people differ in how they perceive it. Research in mathematical models of happiness has been conducted by many researchers including physicists, psychologists, engineers, physiologists,
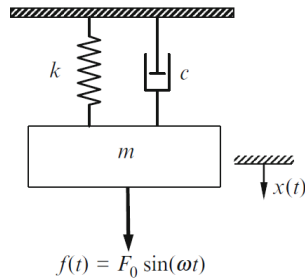
Figure 1. Duffing system.

and mathematicians. Many mathematical models of happiness have been introduced [15-17]. Sprott [15] proposed dynamical models of happiness in 2004 and in 2005. He described a simple linear model, a lottery-winning model, a drug or other addiction model, and an anticipation model for arousing happiness. Sinha et al.[14] proposed a dynamics of love and happiness and discussed the stability of their proposed model.

In this paper, we propose a chaotic synchronization method in a mathematical model of happiness organized by a second-order ordinary differential equation with external force. This proposed mathematical happiness equation is similar to Duffing's equation, as shown in Figure 1, because it is derived from that equation. We introduce our mathematical happiness model by using the derived Duffing equation, and then we demonstrate chaotic phenomena from the happiness model using a time series and phase portrait. We assume that the happiness model is divided into two parts, spiritual and physical.

To achieve chaotic synchronization between the human mind and body, we apply an idea of mind/body unity originating in Oriental philosophy. Of many chaotic synchronization methods [5-8], such as coupled, driven, coupled driven, phase, completed, and generalized synchronization, we use only coupled synchronization, because this method is closest to representing mind/body unity. Typically, coupled synchronization can be applied only to non-autonomous systems, such as a modified Duffing system. We represent the result of synchronization using a differential time series mind/body model.

2. Dynamical Happiness Model

The mathematical happiness models proposed by Sprott [15] in 2005 include simple linear, lottery-winning, drug and other addiction, real-life, nonlinear effect, and anticipation models.

The formulation of happiness is given by Eq. (1).

$$\frac{dx^2(t)}{dt^2} + \beta\frac{dx(t)}{dt} + \omega^2 x(t) = F(t) \tag{1}$$

where β is the damping (rate of decay), ω is the natural resonant frequency of oscillation in radians per unit time, and $F(t)$ is an external power. Without loss of generality, we can assume $\omega = 1$. Because $\omega = 2\pi f$, we can measure time in units of $\frac{1}{\omega}$.

However, Eq. (1) does not provide an exact definition of a dynamical happiness model for each parameter. To describe exact variables in such a model, we must define the parameter that relates happiness and perception. This is similar to current $\left(i = \frac{dq}{dt}\right)$ in an electrical RLC series circuit and velocity $\left(v = \frac{dx}{dt}\right)$ in a mechanical spring-damper-mass system. Hence, we can apply those ideas in our dynamical happiness model to derive a new formula.

In this paper, we define happiness (H) as the amount of perception variation (dp) per amount of time variation (dt), as in Eq. (2).

$$H = \frac{dp}{dt}[n/s] \tag{2}$$

where n is the amount of perception. Eq. (2) is equivalent to velocity in a spring-damper-mass model of a mechanical system. Perception in Eq. (2) includes human behavior, habituation, acclimation, and recognition adaption.

We define happiness (H) based on Eq. (2) to describe Eq. (3).

$$\begin{aligned} F_c(t) &= cp(t) \\ F_d(t) &= d\frac{dp(t)}{dt} \\ F_s(t) &= s\int p(t)dt \end{aligned} \tag{3}$$

where c, d, and s represent control, desire and sense coefficients, respectively.

The control coefficient (c) represents the fact that when humans feel happy, the sense coefficient naturally reduces with passing time. If this value is zero, there is no other response, because the simple harmonic motion continues forever with the number of vibrations, $\frac{1}{2\pi}\sqrt{\frac{s}{d}}$. This situation is identical to the phenomenon of a frictionless mechanical system. Of course, if the value of the control coefficient is sufficiently large, there is no variation in feeling. If the control coefficient has a reasonable or small value, a human being can have feeling for a long time.

The desire coefficient represents a criterion of human desire. It is related to will and gratitude, which are person-dependent.

If this value is sufficiently high, people tend to feel happy about minor things. As a result, their happiness will continue for a long time. The sense coefficients that represent the feeling of happiness differ greatly from person to person owing to differences in perceptivity.

In this paper, as vibrations result from the exchange of energy between the motion energy of a mass and the position energy of a spring in a spring-damper-mass system, we handle the occurrence of physical and mental fluctuation in terms of an exchange of energy or consciousness between the desire and sense coefficients in our dynamical happiness model.

With that theoretical background, we can finally derive a second-order equation for happiness with three independent parameters, the control, desire, and sense coefficients. This equation is given by (4).

$$d\frac{dp^2(t)}{dt^2} + c\frac{dp(t)}{dt} + sp(t) + \beta p(t)^3 = F(t) \qquad (4)$$

where, $F(t)$ is the power being applied externally. This external power is the cause of a happiness event. Examples may include periodic administration of medication or drugs, winning the lottery, or periodic gaming. In real life, it may be provided in the form of a sine or Gaussian wave. Periodic external power can be written as in Eq. (5).

$$F(t) = A\sin\omega t \qquad (5)$$

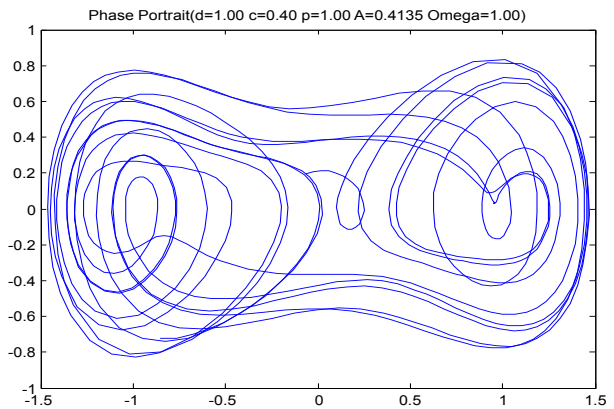From Eq. (4), we obtain a time series and phase portrait, as shown in Figures 2(a) and (b), respectively.

3. Chaotic synchronization between mind and body using linear couplings

In this section, we establish a chaotic synchronization technique between mind and body in human beings using an Oriental philosophical idea of mind/body unity. To achieve mind/body synchronization, we assume that mind and body each has a chaotic model, such as a Duffing equation similar to Eq. (4). Figure 3 shows a human mind/body synchronization device.

From Figure 3, we can distinguish three synchronization methods, linearly coupled, driven, and coupled-driven. However, in this paper, because we consider only non-autonomous systems, the modified Duffing type, we need apply only linearly coupled synchronization. Because driven and coupled-driven synchronization can be used only with autonomous systems, we do not consider applying them to a modified Duffing type.
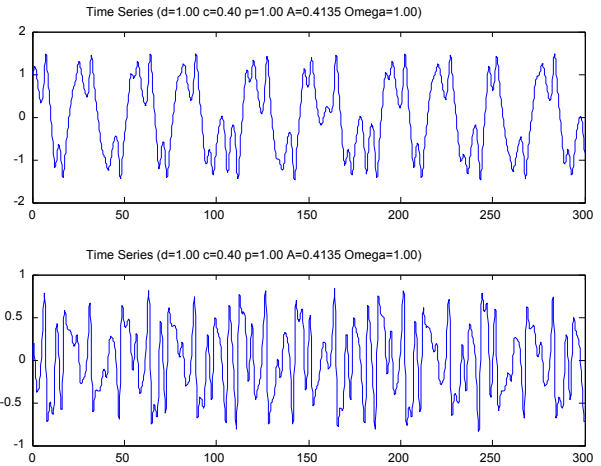
(a) Time series of happiness model from (4), when $d = 2$, $c = 0.40$, $s = 1.0$, $A = 0.4135$, $\omega = 1$.

(b) Phase portrait of happiness model from (4), when $d = 2$, $c = 0.40$, $s = 1.0$, $A = 0.4135$, $\omega = 1$.

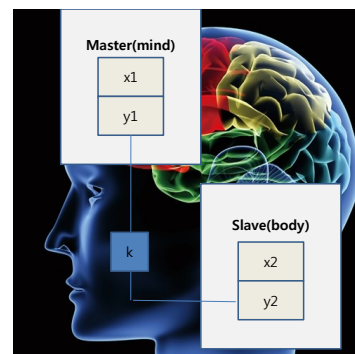Figure 2. Time series and phase portrait of happiness model form Eq. (4).

Figure 3. Synchronization device.

(a) Time series of mind as master (x_1), when $k = 0.10$.

(b) Time series of mind as master (y_1), when $k = 0.10$.

(c) Phase plane of mind as master (x_1, y_1), when $k = 0.10$.

(d) Time series of body as slave (x_2), when $k = 0.10$.

(e) Time series of body as slave (y_2), when $k = 0.10$.

(f) Phase plane of body as slave (x_2, y_2), when $k = 0.10$.

(g) Difference of time series ($x_1 - x_2$), when $k = 0.10$.

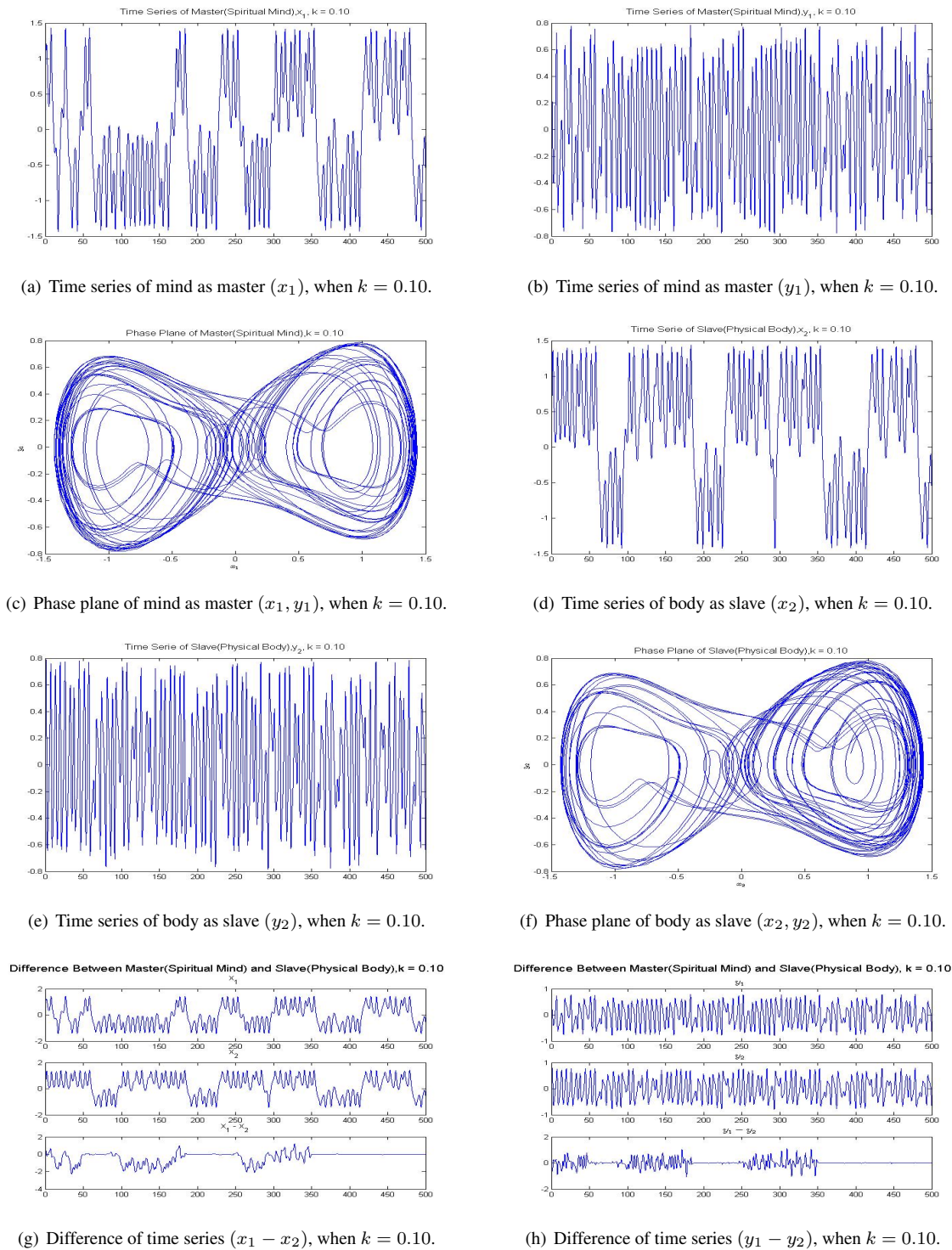(h) Difference of time series ($y_1 - y_2$), when $k = 0.10$.

Figure 4. Result of synchronization between the mind and body using linear coupling when $k = 0.10$.

Linearly coupled synchronization in two identical systems has been studied in several papers [20, 21]. It can be described in general form by Eq. (6).

$$
\begin{aligned}
\dot{x} &= f_1(x) \\
\dot{y} &= f_2(y) + K(x - y)
\end{aligned}
\tag{6}
$$

where $x, y \in \mathbb{R}^n$, and K is a real nonnegative parameter.

The synchronization procedure is formulated to find K such that $y(t) \to x(t)$ for $t \to \infty$. This means that the solution $y(t)$ will be synchronized to the signal $x(t)$.

To apply linear coupling to the human mind and body using the Duffing equation, we must define identical master and slave models as follows.

Master model

We assume that the master model is the mind model and that it can be expressed using the Duffing Eq. (7).

$$
\begin{aligned}
\dot{x} &= y \\
\dot{y} &= \frac{(-cy - sx - \beta x^3 + A\sin\omega t)}{d} + k(y - y')
\end{aligned}
\tag{7}
$$

Slave model

We also assume that the slave model is the physical model and that it can be expressed using the Duffing Eq. (8).

$$
\begin{aligned}
\dot{x'} &= y \\
\dot{y'} &= \frac{(-cy - sx - \beta x^3 + A\sin\omega t)}{d} + k(y' - y)
\end{aligned}
\tag{8}
$$

From Eqs. (7) and (8), we must find a stable solution for k. To obtain a stable solution for k, we use the characteristic Eqs. (7) and (8) and find a stable area using the Routh-Hurwitz condition. We were able to get $k > 0.10$. In a human being, we can think of k as representing breathing or meditation.

Because we assume that the master is the mind and the slave is the body, the parts of this system can communicate with each other by recognizing a human being. Finally, the mind and the body can be synchronized through action recognition. For these situations, we will call these the oneness of mind and body.

Figure 4 shows the results of mind/body synchronization, when $k = 0.10$. Figures 4(a) and (b) display (x_1, y_1) of a time series of the mind, respectively. Figure 4(c) represents (x_1, y_1) of the phase portrait of the mind. Figures 4(d) and (e) show x_2, y_2 of a time series of the body, respectively. Figure 4(e) represents (x_1, y_2) of the phase portrait of the body. Figures 4(f) and (g) show the results of synchronization as $(x_1 - x_2)$ and $(y_1 - y_2)$ of the differences of the time series, respectively.

From Figure 4 we recognize that synchronization between mind and body is not achieved completely. This means that mind and body have not become one.
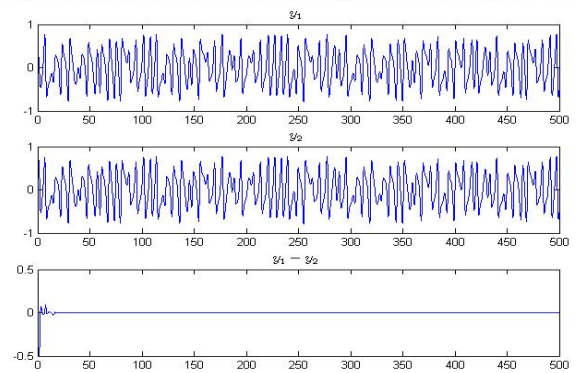
With the same condition, when we change to $k = 0.40$ and $k = 1.0$, we get the synchronization results shown in Figures 5 and 6, respectively.
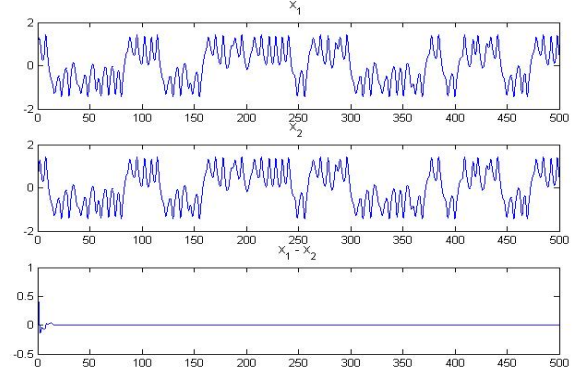
(a) Difference of time series $(x_1 - x_2)$, when $k = 0.40$.

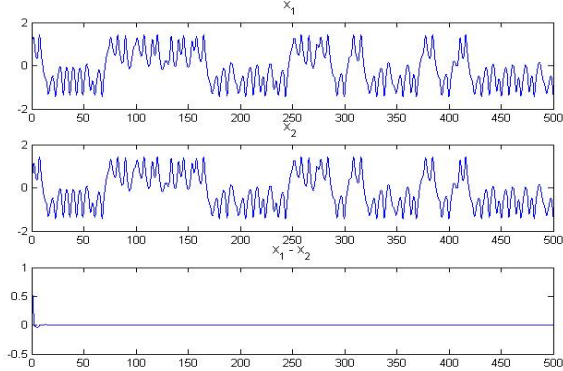(b) Difference of time series $(y_1 - y_2)$, when $k = 0.40$.

Figure 5. Result of synchronization between mind and body using linear coupling when $k = 0.40$.

From Figures 5 and 6, we see that synchronization between mind and body is achieved completely and is achieved more rapidly and fully, when the value of k is greater than 0.10. This means that mind and body have become one.

4. Conclusion

We proposed a chaotic synchronization method in a mathematical happiness model organized by a second-order ordinary differential equation with external force. This proposed mathematical happiness equation is similar to a modified Duffing's equation and is derived directly from that equation. We applied linear coupling synchronization to the modified Duffing equation to demonstrate the result of synchronization by determining the feedback gain, k. Finally, we showed the result of synchronization through a time series and phase portrait of

(a) Difference of time series $(x_1 - x_2)$, when $k = 1.0$.
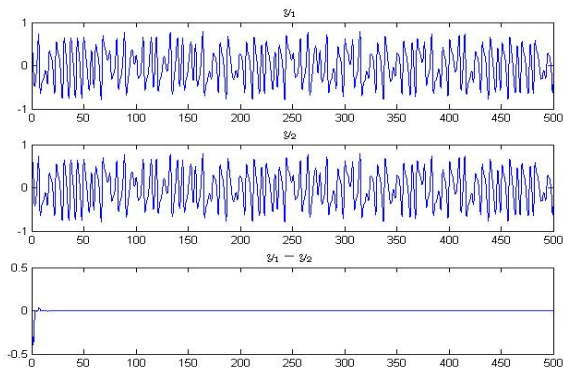
(b) Difference of time series $(y_1 - y_2)$, when $k = 1.0$.

Figure 6. Result of synchronization between mind and body using linear coupling when $k = 1.0$.

master and the slave models, as well as the difference of the two time series.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

References

[1] A. Garfinkel, M. Spano, W. Ditto, and J. Weiss, "Controlling cardiac chaos," *Science*, vol. 257, no. 5074, pp. 1230-1235, Aug. 1992. http://dx.doi.org/10.1126/science.1519060

[2] P. Faure and H. Korn, "Is there chaos in the brain? I. Concepts of nonlinear dynamics and methods of investigation,"

Comptes Rendus de l'Acadmie des Sciences - Series III - Sciences de la Vie, vol. 324, no. 9, pp. 773-793, Sep. 2001. http://dx.doi.org/10.1016/S0764-4469(01)01377-4

[3] R. M. May, "Simple mathematical models with very complicated dynamics," *Nature*, vol. 261, no. 5560, pp. 459-467, Jun. 1976. http://dx.doi.org/10.1038/261459a0

[4] Y. C. Bae, "A study on chaotic phenomenton in rolling mill bearing," *Journal of Korean Institute of Intelligent Systems*, vol. 11, no. 4, pp. 315-319, Apr. 2001.

[5] Y. C. Bae, J. Kim, Y. Kim, and Y. W. Shon, "Secure communication using embedding drive synchronization," *Journal of Korean Institute of Intelligent Systems*, vol. 13, no. 3, pp. 310-315, Jun. 2003.

[6] Y. C. Bae, Y. G. Kim, and M. Tinduka, "A study on generalized synchronization in the state-controlled cellular neural network (SC-CNN)," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 5, no. 4, pp. 291-296, Dec. 2005. http://dx.doi.org/10.5391/IJFIS.2005.5.4.291

[7] S. H. Yu, C. H. Hyun, and M. Park, "Backstepping control and synchronization for 4-D Lorenz-Stenflo chaotic system with single input," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 11, no. 3, pp. 143-148, Sep. 2011. http://dx.doi.org/10.5391/IJFIS.2011.11.3.143

[8] S. H. Yu, C. H. Hyun, and M. Park, "Control and synchronization of new hyperchaotic system using active backstepping design," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 11, no. 2, pp. 77-83, Jun. 2011. http://dx.doi.org/10.5391/IJFIS.2011.11.2.077

[9] J. H. Kim, C. W. Park, E. Kim, and M. Park, "T-S fuzzy model-based adaptive synchronization of chaotic system with unknown parameters," *Journal of Korean Institute of Intelligent Systems*, vol. 15, no. 2, pp. 270-275, Apr. 2005.

[10] E. S. Kim, "Dynamics of multi-delayed feedback neural nets for the nonlinear capability and noise characteristics," *Journal of Korean Institute of Intelligent Systems*, vol. 9, no. 2, pp. 144-156, Apr. 1999.

[11] Y. H. Choi and S. M. Kim, "Radial basis function network based predictive control of chaotic nonlinear systems," *Journal of Korean Institute of Intelligent Systems*, vol. 13, no. 5, pp. 606-613, Oct. 2003. http://dx.doi.org/10.5391/JKIIS.2003.13.5.606

[12] Y. C. Bae, Y. Kim, and Y. Koo, "The target searching method in the chaotic mobile robot embedding BVP model," *Journal of Korean Institute of Intelligent Systems*, vol. 17, no. 2, pp. 259-264, Apr. 2007.

[13] Y. C. Bae, Y. Kim, and C. Kim, "Obstacle avoidance method in the chaotic unmanned aerial vehicle," *Journal of Korean Institute of Intelligent Systems*, vol. 14, no. 7, pp. 883-888, Dec. 2004.

[14] Y. C. Bae, "Chaotic phenomena in addiction model for digital leisure," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 13, no. 4, pp. 291-297, Dec. 2013. http://dx.doi.org/10.5391/IJFIS.2013.13.4.291

[15] J. C. Sprott, "Dynamical models of happiness," *Nonlinear Dynamics, Psychology, and Life Sciences*, vol. 9, no. 1, pp. 23-36, Jan. 2005.

[16] D. Satsangi and A. K. Sinha, "Dynamics of love and happiness: a mathematical analysis," *International Journal of Modern Education and Computer Science*, vol. 4, no. 5, pp. 31-37, Jun. 2012. http://dx.doi.org/10.5815/ijmecs.2012.05.05

[17] D. Lykken and A. Tellegen, "Happiness is a stochastic phenomenon," *Psychological Science*, vol. 7, no. 3, pp. 186-189, May 1996. http://dx.doi.org/10.1111/j.1467-9280.1996.tb00355.x

[18] Y. C. Bae, "Chaotic phenomena in MEMS with duffing equation," *Journal of the Korea Institute of Electronic Communication Sciences*, vol. 6, no. 5, pp. 709-716, 2011.

[19] S. Metin and N. S. Sengor, "Dynamical system approach in modeling addiction," in *Proceedings of the Brain Inspired Cognitive Systems*, Madrid, Spain, July 14-16, 2010.

[20] L. O. Chua, L. Kocarev, K. Eckert, and M. Itoh, "Experimental chaos synchronization in Chua's circuit," *International Journal of Bifurcation and Chaos*, vol. 2, no. 3, pp. 705-708, Sep. 1992. http://dx.doi.org/10.1142/S0218127492000811

[21] L. Kocarev, K. S. Halle, K. Eckert, L. O. Chua, and U. Parlitz, "Experimental demonstration of secure communications via chaotic synchronization," *International Journal of Bifurcation and Chaos*, vol. 2, no. 3, pp. 709-713, Sep. 1992. http://dx.doi.org/10.1142/S0218127492000823

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