Optimal Production Planning for Remanufacturing with Quality Classification Errors under Uncertainty in Quality of Used Products

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ABSTRACT

This paper discusses a green supply chain with a manufacturer and a collection trader, and it proposes an optimal production planning for remanufacturing of parts in used products with quality classification errors made by the collection trader. When a manufacturer accepts an order for parts from a retailer and procures used products from a collection trader, the collection trader might have some quality classification errors due to the lack of equipment or expert knowledge regarding quality classification. After procurement of used products, the manufacturer inspects if there are any classification errors. If errors are detected, the manufacturer reclassifies the misclassified (overestimated) used products at a cost. Accordingly, the manufacturer decides to remanufacture from the higher-quality used products based on a remanufacturing ratio or produce parts from new materials. This paper develops a mathematical model to find how quality classification errors affect the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of the manufacturer. Numerical analysis investigates how quality of used products, the reclassification cost and the remanufacturing cost of used products affect the optimal products of a manufacturer.

Keywords: Green Supply Chain, Uncertainty in Quality, Quality Classification Errors

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1. INTRODUCTION

The amount of consumption of resources has been continuously increasing over the last 50 years. Consequently, serious environmental problems such as waste of used natural resources and depletion of natural resources have been occurring worldwide (Pochampally *et al.*, 2009). Under these situations, environmental protection and sustainable development have become increasingly important. Driven by legislation, the rise of social concern about the environment problem and social pressure to mitigate the environmental problems, more and more firms have been starting to engage in remanufacturing operations in supply chain management in addition to the traditional operations in supply chain management (Tang and Teunter, 2006; Savaskan and Van Wassenhove, 2006; Yan and Sun, 2012; Zhang *et al.*, 2014).

In recent years, the concept of a new supply chain management combining traditional operations and remanufacturing operations has been important in optimally controlling a supply chain including traditional forward chains/logistics and reverse chains/logistics.

The traditional operations are implemented in the traditional forward chains/logistics. The traditional forward chains/logistics consist of the flows from procurement of new materials through production of new products to selling them. The remanufacturing operations are implemented in the reverse chains/logistics. The reverse chains/logistics consist of the flows from collection of used products through recycling parts from used prod-

ucts to reuse the recycled parts (Fleischman *et al.*, 1997; Guide, 2000; Guide and Jayaraman, 2000; Guide and Van Wassenhove, 2001; Aras *et al.*, 2004; Inderfurth, 2005; Behret and Korugan, 2009; Ferguson *et al.*, 2009; Pochampally *et al.*, 2009; Mukhopadhyay and Ma, 2009; Konstantaras *et al.*, 2010; Nenes *et al.*, 2010; Pokharel and Liang, 2012; Teunter and Flapper, 2011; Wei *et al.*, 2011; Wu, 2012).

Also, a new framework of supply chain management which consists of the forward operation management and the reverse operation management has been called a closed-supply chain, reverse supply chain or a green supply chain (Bakal and Akcali, 2006; Fleischman et al., 1997; Guide and Van Wassenhove, 2001; Inderfurth, 2005; Kaya, 2010; Lee et al., 2011; Shi et al., 2010, 2011; Tagaras and Zikopoulos, 2008; Thierry et al., 1995; Van Wassenhove and Zikopoulos, 2010; Wei et al., 2012; Yan and Sun, 2012; Zikopoulos and Tagaras, 2008). In this paper, a supply chain including the forward chains and the reverse chains is referred to as a green supply chain (GSC). The manufacturing to reuse recycled parts is called the remanufacturing. The concept and management of GSC has been evolving to assist academic researchers and real-world policymakers regarding production planning in a GSC which try to take some measures and policies in order to promote 3R activities (Reuse-Recycle-Reduce) in the GSC. It is necessary for academic researchers and real-world policymakers to consider the optimal operations for GSC, and the uncertainty in remanufacturing. Several previous papers have dealt with the optimal operations for GSC, and the uncertainty in remanufacturing has been attracting more attention in recent papers.

Inderfurth (2005), Lee *et al.* (2011), Mukhopadhyay and Ma (2009), Shi *et al.* (2010, 2011), and Wei *et al.* (2011) discussed the incorporation of the uncertainty in demands of products/parts and collection quantity of used products into GSC. Also, Bakal and Akcali (2006), Pokharel and Liang (2012), Shi *et al.* (2010), Teunter and Flapper (2011), Wei *et al.* (2012), and Yan and Sun (2012) discussed the incorporation of the price-sensitivity in collection quantity of used products and demands of products/parts into the optimal tactical production planning GSC.

When the GSC is operated, the issue regarding the uncertainty in quality of used products which are collected from a market has been attracting more attention among both practitioners and academics. Some previous papers have discussed the optimal tactical production planning by incorporating uncertainty in quality of used products into the GSC.

Aras *et al.* (2004) investigated the effect of the stochastic nature of product returns on the optimal production planning and verified conditions under which quality-based categorization was most cost effective. Zikopoulos and Tagaras (2007) investigated how uncertainty regarding the quality of returned products in two collection sites affected the profitability of reuse activities and derived the unique optimal solution (procurement and production quantities). In Guide and Van Wassenhove, (2001) and Ferguson et al. (2009), returned products were assumed to have N quality categories, and the procurement prices and the remanufacturing costs were different based on the corresponding quality level. Behret and Korugan (2009) discussed a remanufacturing stage with uncertainties in the quality of remanufacturing products, return rates and return times of returned products. After returned products were classified by considering the uncertainty in quality of the returned product, remanufacturing processing times, material recovery rates, the remanufacturing costs and disposal costs were determined by using the ARENA simulation program. Mukhopadhyay and Ma (2009) discussed a GSC model consisting of a retailer who sold a single product and a manufacturer who collected used products from a market, remanufactured parts from used products and then produced products. Two situations were assumed regarding the remanufacturing ratio between reuse parts and used products: one was a constant situation and the other was an uncertain situation. Under each situation, the optimal production strategy was proposed regarding the procurement quantity of used products, the remanufacturing quantity of parts from used products and the production quantity of new parts from new materials. Nenes et al. (2010) found that both quality and quantity of returns (used products) were substantially high stochastic. Under the situations, the optimal policies were verified regarding both ordering quantities of new products and remanufacturing of products so as to enhance the firms' performances such as minimization of their expected cost or maximization of their expected profit. Teunter and Flapper (2011) investigated how a large change of quality of cores (i.e., products supplied for remanufacturing) affected not only the cost related to the remanufacturing, and derived the optimal policies regarding acquisition and remanufacturing for both deterministic and uncertain demand.

In general, it is necessary to consider a situation where either inspection errors of used products or classification errors in quality grading of used products occur after classifying them for recycling due to a lack of classification equipment or expert knowledge of a collection trader. The effects of inspection and sorting of used products on the optimal tactical production planning in GSC have been discussed by Aras *et al.* (2004), Behret and Korugan (2009), Ferguson et al. (2009), Guide and Van Wassenhove (2001), Konstantaras et al. (2010), Nenes et al. (2010), Tagaras and Zikopoulos (2008). Zikopoulos and Tagaras (2007, 2008) and Van Wassenhove and Zikopoulos (2010) incorporated some quality classification errors into a GSC. Here, quality classification errors were dealt as an uncertain value. Zikopoulos and Tagaras (2008) handled quality classification errors as Type I error and Type II error based on a concept of quality control. However, in their previous studies, possible errors in inspection and sorting of used products were considered as a given Type I error and a given Type II error, and those errors were incorporated into the GSC.

Besides, in the above previous studies, a concept of the optimal decision-ma-king for the lower limit of quality to recycle used products was not considered for a GSC.

This paper focuses on the optimal production planning for a GSC to operate remanufacturing of parts used in products such as consumer electronics (mobile phone, personal computer), semiconductor and electronic component (Guide, 2000; Guide and Jayaraman, 2000; Guide and Van Wassenhove, 2001; Guide *et al.*, 2003; Ferguson *et al.*, 2009; Van Wassenhove and Zikopoulos, 2010). Also, this paper is motivated from a real example, the ReCellular Inc., which is a firm that remanufacturers and sells used mobile telephones, discussed in previous studies by Guide (2000), Guide and Jayaraman (2000), Guide and Van Wassenhove (2001), Guide *et al.* (2003), Ferguson *et al.* (2009), and Van Wassenhove and Zikopoulos (2010).

When the GSC is operated to remanufacture parts extracted from used products, it is necessary for practitioners and academics to discuss the issue regarding the quality of used products collected from a market. Firms may need to determine the optimal production planning when used products are classified into a small number Nof quality categories according to a quality classification list provided by a firm which remanufacturers parts/ products (Guide and Van Wassenhove, 2001; Guide et al., 2003; Ferguson et al., 2009; Van Wassenhove and Zikopoulos, 2010), each classification may have some classifications errors and needs to satisfy product demand as much as possible. This study tries to make the following contributions for both academic researchers and real-world policymakers regarding production planning in a GSC:

- Presentation of theoretical analysis to evaluate the profitability obtained by combining the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit so as to satisfy product demand when used products are classified into a small number *N* of quality categories.
- Presentation of theoretical analysis to evaluate how quality classification errors affect the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of a manufacturer.
- Presentation of theoretical analysis to evaluate how both quality classification errors and quality distribution of used products affect the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of the manufacturer.

This paper focuses on the optimal production planning of a manufacturer in a GSC to operate remanufacturing of parts used in products such as mobile phone, personal computer and semiconductor and electronic component. This paper proposes an optimal production planning for the manufacturer's remanufacturing of parts extracted from used products under situations where there is uncertainty in the quality of used products and the potential for quality classification errors of used products. The optimal decisions regarding a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit so as to maximize the manufacturer's expected profit are discussed. If a manufacturer detects quality classification errors, corresponding to overestimated quality classification, the manufacturer reclassifies used products procured from a collection trader. In this case, the manufacturer incurs the unit reclassification cost to reclassify the misclassified (overestimated) used products correctly into their intrinsic quality levels and considers this cost as an additional cost for the manufacturer's expected profit. Concretely, the operational flows of GSC in this paper are considered as follows: a collection trader collects used products from customers and classifies used products according to their quality. In this case, the classification errors in quality grading of used products might occur because of a lack of classification equipment or expert knowledge of the collection trader. When a manufacturer accepts an order for parts from a retailer and procures used products from the collection trader, the higher the quality of used products is, the higher the procurement cost is, but the remanufacturing cost is lower. After procurement of used products, the manufacturer inspects if there is any quality classification errors. If the quality classified errors are detected, the manufacturer reclassifies the misclassified used products and incurs the unit reclassify cost of the misclassified used products. Accordingly the manufacturer decides to remanufacture from the higher-quality used products based on a remanufacturing ratio or produce parts from new materials. The manufacturer also determines a remanufacturing ratio for used products with the lower limit of procurement quality in order to produce parts in just proportion.

This paper develops a mathematical model and conducts the theoretical analysis in order to find how quality classification errors and their ratio affect not only the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit, but also the expected profit of a manufacturer.

The numerical analysis illustrates the results of the optimal production planning for the GSC proposed in this paper by providing numerical examples. The numerical analysis verifies the following topics for both academic researchers and real-world policymakers regarding production planning in a green supply chain:

- Profitability obtained from the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio under a small number of the total grade of quality classification for used products *I* to a manufacturer.
- Effect of quality classification errors on the optimal production planning regarding a lower limit of procurement quality of used products and a remanufac-

turing ratio under the lower limit and the expected profit of a manufacturer.

 Effect of both quality classification errors and quality distribution of used products on the optimal production planning regarding a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of a manufacturer.

Concretely, the numerical analysis investigates how the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit of procurement quality affect the manufacturer's expected profit under a small number of *I*. Also, the numerical analysis investigates how the following factors: i) quality classification errors, ii) quality distribution of used products, iii) the remanufacturing cost of used products, iv) ratio of quality classification errors, and v) the reclassification cost of misclassified used products—affect the optimal production planning and the expected profit of a manufacturer.

This paper focuses on the following situations which a manufacturer may face: 1) the uncertainty in quality of used products, 2) some quality classification errors after purchasing used products from a collection trader, 3) a small number of quality categories and 4) production of a single products so as to satisfy the product demand as much as possible. The contribution of this paper is to provide the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to both academic researchers and real-world policymakers regarding production planning in a green supply chain: the combination of the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit can bring more profit to a manufacturer who may face above situations.

The rest of this paper is organized as follows: in Section 2, notation used in our model is defined. In Section 3, model descriptions regarding operational flows of a GSC and model assumptions of a GSC are described. Section 4 formulates two cases of the expected profit of a manufacturer: one is the case without quality classification errors of used products and the other is the case with quality classification errors of used products. Section 5 proposes the optimal production planning regarding a lower limit of procurement quantity of used products and a remanufacturing ratio of the recyclable parts extracted from used products under the lower limit in cases without and with the quality classification errors. Section 6 shows the results of numerical analysis to illustrate managerial insights for the optimal production planning of a manufacturer in a GSC proposed in this paper. In Section 7, conclusions and future researches for this paper are summarized.

2. NOTATION

I : total grade of quality classification for used products

- ℓ : quality level classified for used products $(1 \le \ell \le I)$
- *R*: quantity of used products collected by a collection trader
- $f(\ell/I)$: the probability density function (pdf) of the normalized quality level ℓ/I of used products under quality level ℓ and total grade of quality classification I ($0 \le \ell/I \le 1$, $1 \le \ell \le I$)
- $F(\ell/I)$: ratio of used products in the interval between the normalized quality level $(\ell-1)/I$ and the normalized quality level ℓ/I $(1 \le \ell \le I)$ when no classification errors in quality grading of used products occurs
- $G(\ell/I)$: ratio of used products in the interval between the normalized quality level $(\ell-1)/I$ and the normalized quality level ℓ/I $(1 \le \ell \le I)$ when classification errors in quality grading of used products occurs
- $a(\ell)$: the unit procurement cost of used products classified into quality level $\ell(1 \le \ell \le I)$
- $c(\ell)$: the unit remanufacturing cost of parts extracted from used products classified into quality level $\ell(1 \le \ell \le I)$
- *D* : a retailer's order quantity of parts, corresponding to demand of parts
- *v* : the unit sales price of parts
- *h* : inventory holding cost per unit of excess remanufactured parts for demand *D* of parts per unit time
- c_m : the unit production cost of new parts produced by a manufacturer in order to cover the unsatisfied quantity for demand *D* of parts
- c_{rc} : the unit reclassification cost to reclassify the misclassified used products correctly into their intrinsic quality levels
- *L* : a lower limit of procurement quality of used products procured from a collection trader $(1 \le L \le I)$, referred to lower limit of procurement quality
- λ : a remanufacturing ratio for used products with the lower limit of procurement quality *L* to produce parts in just proportion $(0 \le \lambda \le 1)$
- $E[\pi(L, \lambda)]$: the manufacturer's expected profit for L and λ when no classification error in quality grading of used products occurs
- $E[\pi_e(L, \lambda)]$: the manufacturer's expected profit for L and λ when classification errors in quality grading of used products occurs
- L^* : the optimal lower limit of procurement quality when no classification error in quality grading of used products occurs $(1 \le L \le I)$
- L_e^* : the optimal lower limit of procurement quality when classification errors in quality grading occur $(1 \le L \le I)$
- λ^* : the optimal remanufacturing ratio when no classification errors in quality grading of used products occur
- λ_e^* : the optimal remanufacturing ratio when classification errors in quality grading of used products occur

3. MODEL DESCRIPTIONS

3.1 Operational Flows of a Green Supply Chain

In this paper, a GSC with a manufacturer, a retailer and a supplier is considered. Here, the GSC has operational flows where a single of used products such as consumer electronics (mobile phone, personal computer), semiconductor, and electronic component are collected from the market, are remanufacture as the recycled parts and are sold in a market. Concretely, the operational flows of GSC in this paper are addressed as follows:

- 1) A collection trader collects the quantity of used products *R* from consumers. Here, it is assumed that it is possible to collect the more quantity of used products *R* than the demand *D* of parts. The collection trader classifies used products according to a quality classification list provided by a manufacture. Concretely, used products are classified within [1, *I*] quality grades according to their quality levels $\ell(1 \le \ell \le I)$. The grade 1 indicates the worst quality, meanwhile the grade 1 indicates the best quality. In this case, any classification errors in quality grading might occurs due to a lack of classification equipment or expert knowledge of the collection trader.
- 2) When a manufacturer accepts an order of parts from a retailer, the manufacturer procures the required quantity of used products in order of high quality from the collection trader in order to satisfy the retailer's order quantity D of parts, corresponding to the demand D of parts. After procurement of used products, the manufacturer inspects all quality levels of used products and reclassifies the misclassified used products correctly into their intrinsic quality levels according to the manufacturer's quality classification list. When the quality classification errors are inspected, the manufacturer incurs the unit reclassification cost to reclassify the misclassified used products correctly into their intrinsic quality levels. In this case, it is assumed that it is impossible for the manufacturer to recognize any quality classification errors before the manufacturer inspects used products procured from the collection trader.
- 3) The manufacturer decides to remanufacture from the higher-quality used products based on a remanufacturing ratio or produce from new materials. The manufacturer also determines a remanufacturing ratio for used products with the lower limit of procurement quality in order to produce parts in just proportion. Here, the parts remanufactured from used products have as same quality as new parts produced from new materials.
- 4) The manufacturer incurs the inventory holding cost *h* per unit of the excess remanufactured parts for the demand *D* of parts per unit time.
- 5) When the quantity of the remanufactured parts does not satisfy the demand D of parts, the manufacturer produces new parts at the unit production cost c_m

by procuring new materials from a supplier in order to cover the unsatisfied quantity for demand D of parts.

6) The manufacturer sells parts at the unit price v to the retailer. Here, the remanufactured parts have as same quality as the new parts. So, the unit sales price of the remanufactured parts is as same as that of new parts.

As the optimal production planning in this paper, the manufacturer first determines a lower limit of procurement quality of used products procured from a collection trader and next a remanufacturing ratio for used products with the determined lower limit of procurement quality to not only to produce demand D of parts in just proportion, but also maximize the manufacture's expected profit.

3.2 Model Assumptions

- (1) The normalized quality level ℓ/I of used products under quality level ℓ and the total grade of quality classification $I(0 \le \ell/I \le 1, 1 \le \ell \le I)$ follows a probability distribution with the pdf $f(\ell/I)(0 \le \ell/I \le 1, 1 \le \ell \le I)$. Here, ℓ is an integer number.
- (2) It is assumed that a collection trader misclassifies the intrinsic quality level $\ell 1$ of used products at most by one grade equal to the quality level ℓ . In this case, a correct quality classification ratio which the collection trader classifies used products correctly into the intrinsic quality level ℓ is *p* (Van Wassenhove and Zikopoulos, 2010). Thus, when the quality classification errors occur, the quantity of used products for the normalized quality level ℓ/I under the quality level ℓ and the total grade of quality classification $I(0 \le \ell/I \le 1, 1 \le \ell \le I)$ is obtained as

$$R(1-p) \int_{\ell}^{\ell+1} f(\ell/I) d\ell + Rp \int_{\ell-1}^{\ell} f(\ell/I) d\ell \quad (\ell = 1, 2, \cdots, I-1),$$
(1)

$$Rp \int_{\ell-1}^{\ell} f(\ell/I) d\ell \quad (\ell=I).$$
⁽²⁾

Here, the first term in Eq. (1) indicates the quantity of used products which are classified into one more higher level $\ell + 1$ than the intrinsic quality levels ℓ . The second term in Eqs. (1) and (2) indicate the quantity of used products which are classified correctly into the intrinsic quality levels ℓ .

- (3) The manufacturer incurs the unit procurement cost of used products from the collection trader, the unit remanufacturing cost of parts extracted from used products under a lower limit of procurement quality *L* of used products and the unit reclassification cost when quality classification errors occur.
- (4) It is natural that the higher the quality level of used products is, the higher the procurement cost is, but

the lower the remanufacturing cost is. Concretely, the higher the quality level ℓ of used products a manufacturer purchased from a collection trader, the higher the procurement cost $a(\ell)$ the manufacturer incurred. Meanwhile, the lower the quality level ℓ of used products a manufacturer remanufactures, the higher the remanufacturing cost $c(\ell)$ the manufacturer incurs. Based on above situations, we assume the following relations between the cost functions of the procurement cost $a(\ell)$ and the remanufacturing cost $c(\ell)$ for the quality level

$$\ell(1 \le \ell \le I):$$

$$da(\ell)/d\ell > 0 \qquad (3)$$

$$dc(\ell)/d\ell < 0. \qquad (4)$$

4. EXPECTED PROFIT OF A MANUFAC-TURER

4.1 Case without Quality Classification Errors of Used Products

According to Section 3, the manufacturer's expected profit is obtained from the sales of the remanufactured parts, the procurement cost of used products, the remanufacturing cost of the remanufactured parts from used products, the inventory holding cost of the remanufactured parts and the production cost of new parts. Concretely, the manufacturer's expected profit $E[\pi(L, \lambda)]$ for a lower limit *L* of procurement quality of used products and a remanufacturing ratio λ for used products with the lower limit *L* when no quality classification error of used products occurs is formulated as

$$E\left[\pi(L,\lambda)\right](1 \le L \le I)$$

$$= vD - \left(R\left\{\sum_{\ell=L}^{I} a(\ell)F\left(\ell/I\right) - (1-\lambda)a(L)F(L/I)\right\}\right)$$

$$- \left(R\left\{\sum_{\ell=L}^{I} c(\ell)F(\ell/I) - (1-\lambda)c(L)F(L/I)\right\}\right)$$

$$-h\left\{\left[R\left\{\sum_{\ell=L}^{I} F(\ell/I) - (1-\lambda)F(L/I)\right\}\right] - D\right\}^{+}$$

$$-c_{m}\left\{D - \left[R\left\{\sum_{\ell=L}^{I} F(\ell/I) - (1-\lambda)F(L/I)\right\}\right]\right\}^{+}, \quad (5)$$

$$F(\ell/I) = \int_{\ell-1}^{\ell} f(\ell/I) d\ell \quad (1 \le \ell \le I).$$
(6)

4.2 Case with Quality Classification Errors of Used Products

According to Section 3, the manufacturer's expected profit is obtained from the sales of the remanufactured parts, the procurement cost of used products, the reclassification cost of used products, the remanufacturing cost of the remanufactured parts from used products, the inventory holding cost of the remanufactured parts and the production cost of new parts. Concretely, the manufacturer's expected profit $E[\pi_e(L, \lambda)]$ for a lower limit L of procurement quality of used products and a remanufacturing ratio λ for used products with the lower limit L when quality classification errors of used products occur is formulated as

$$E\left[\pi_{e}(L,\lambda)\right](1 \leq L \leq I)$$

$$= vD - \left(R\left\{\sum_{\ell=L}^{I} a(\ell)F(\ell/I) - (1-\lambda)a(L)F(L/I)\right\}\right)$$

$$-c_{rc}(1-p)R\sum_{\ell=L}^{I}F(\ell/I)$$

$$-\left(R\left\{\sum_{\ell=L}^{I} c(\ell)G(\ell/I) - (1-\lambda)c(L)G(L/I)\right\}\right)$$

$$-h\left\{\left[R\left\{\sum_{\ell=L}^{I}F(\ell/I) - (1-\lambda)F(L/I)\right\}\right] - D\right\}^{+}$$

$$-c_{m}\left\{D - \left[R\left\{\sum_{\ell=L}^{I}F(\ell/I) - (1-\lambda)F(L/I)\right\}\right]\right\}^{+}, \quad (7)$$

$$F(\ell/I) = \int_{\ell-1}^{\ell} f(\ell/I) d\ell$$
(8)

$$G(\ell/I) = \begin{cases} (1-p) \int_{\ell}^{\ell+1} f(\ell/I) d\ell + p \int_{\ell-1}^{\ell} f(\ell/I) d\ell \\ (\ell=1,2,\cdots,I-1). \\ p \int_{\ell-1}^{\ell} f(\ell/I) d\ell \\ (\ell=I) \end{cases}$$
(9)

Here, the third term in Eq. (7), $c_{rc}(1-p)\sum_{\ell=L}^{I} F(\ell/I)(1 \le 1)$

 $\ell \leq I$), indicates the reclassification cost to classify the misclassified used products correctly into their intrinsic quality levels ℓ . Also, the remanufacturing cost of parts extracted from used products is reformulated by using the quantity of used products for the normalized quality level when the quality classification errors occur by using Eqs. (1), (2), (8), and (9).

5. OPTIMAL PRODUCTION PLANNING

5.1 Case without Quality Classification Errors of Used Products

5.1.1 Optimal decision for a lower limit of procurement quality of used products

The optimal decision for a lower limit of procurement quality of used products L^* without quality classification errors is discussed. It is necessary to investigate if the expected profit of the manufacture without quality classification errors for used products in Eq. (5) is concave function in terms of a lower limit L of procurement quality under a remanufacturing ratio λ of the recyclable parts in order to determine the optimal lower limit of procurement quantity of used products in case without quality classification errors of used products. Here, it is complicated to derive analytically the first-order and second-order differential equations between a lower limit L of procurement quality and Eq. (5) under a remanufacturing ratio λ of recyclable parts. So, in this paper, the optimal lower limit L^* of procurement quality is determined as the lower limit L of procurement quality which maximizes the manufacturer's expected profit without quality classification errors $E[\pi(L|\lambda)]$ in Eq. (5) under a remanufacturing ratio λ of recyclable parts within the range where $1 \le L \le I$ by using the numerical search.

5.1.2 Optimal decision for a remanufacturing ratio of the recyclable parts under *L*^{*}

The optimal decision for a remanufacturing ratio λ^* for used products under L^* without quality classification errors is discussed. After substituting L^* into the manufacturer's expected profit without quality classification errors in Eq. (5), the optimal combination of the optimal lower limit L^* of procurement quality and the optimal remanufacturing ratio λ is determined as the remanufacturing ratio λ of recyclable parts which maximizes the manufacturer's expected profit $E[(\lambda | L^*)]$ without quality classification errors in Eq. (5) under the optimal lower limit L^* within the range where $0 \le \lambda \le 1$ by using the numerical search.

5.2 Case with Quality Classification Errors of Used Products

5.2.1 Optimal decision for lower limit of procurement quality of used products

The optimal decision for a lower limit of procurement quality of used products L_e^* with quality classification errors is discussed. It is necessary to investigate if the expected profit of the manufacture with quality classification errors for used products in Eq. (7) is concave function in terms of a lower limit L of procurement quality under a remanufacturing ratio λ of the recyclable parts in order to determine the optimal lower limit of procurement quantity of used products in case with quality classification errors of used products. Here, it is complicated to derive analytically the first-order and secondorder differential equations between a lower limit L of procurement quality and Eq. (7) under a remanufacturing ratio λ of recyclable parts. In a similar way to Section 5.1.1, the optimal lower limit L_e^* of procurement quality is determined is determined as the lower limit L of procurement quality which maximizes the manufacturer's expected profit $E \left| \pi_e(L|\lambda) \right|$ with quality classification errors in Eq. (7) under a remanufacturing ratio λ of recyclable parts within the range where $1 \le L \le I$ by using the numerical search.

5.2.2 Optimal decision for remanufacturing ratio of the recyclable parts under the lower limit of procurement quality under L_e^*

The optimal decision for a remanufacturing ratio λ_e^* for used products the optimal lower limit L_e^* with quality classification errors is discussed. In a similar way to Section 5.1.2, after substituting L_e^* into the manufacturer's expected profit in Eq. (7), the optimal combination of the optimal lower limit L_e^* of procurement quality and the optimal remanufacturing ratio λ_e^* is determined as the remanufacturing ratio λ of recyclable parts which maximizes the manufacturer's expected profit $E[(\lambda | L_e^*)]$ with quality classification errors in Eq. (7) under the optimal lower limit L_e^* within the range where $0 \le \lambda \le 1$ by using the numerical search.

The details of the decisions procedures for the optimal production planning in both cases without and with quality classification errors are shown in Section 6.2.

6. NUMERICAL ANALYSIS

This section illustrates results of the optimal production planning for the GSC proposed in Section 5 by providing numerical examples. The numerical analysis verifies the following topics for both academic researchers and real-world policymakers regarding production planning in a GSC:

- Profitability obtained from the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio under a small number of the total grade of quality classification for used products *I* to a manufacturer.
- Effect of quality classification errors on the optimal production planning regarding a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of a manufacturer.
- Effect of both quality classification errors and quality distribution of used products on the optimal production planning regarding a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit and the expected profit of a manufacturer.

Concretely, the numerical analysis investigates how the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit of procurement quality affects the manufacturer expected profit under a small number of *I*. Also, the numerical analysis investigates how the following factors: i) quality classification errors, ii) quality distribution of used products, iii) the remanufacturing cost of used products, iv) ratio of quality classification errors, and v) the reclassification cost of misclassified used products affect the optimal production planning and the expected profit of a manufacturer.

6.1 Numerical Examples

Data sources of numerical examples to operate a GSC addressed in this paper are provided as follows: suppose that used products were collected by a collection trader, and then they were classified into a small number *I* of quality categories according to a quality classification list provided by a manufacturer which remanufacturers parts. Each quality level $\ell(1 \le \ell \le I)$ of used products was classified from *I* to 1 in order of high quality grades. Here, *I* was set at 5 and 10, and ℓ is an integer number. Also, in numerical examples of this paper, it was assumed that the collection quantity *R* of used products was larger than the retailer's order quantity *D* of parts, corresponding to the demand *D* of parts. Here, *R* and *D* were set at R = 2000, D = 800.

From Section 3.2 model assumptions (1), some shapes of the distribution of the quality level of used products are simulated. Concretely, each shape of the distribution of the normalized quality level ℓ/I of used products is modeled under the quality level ℓ and the total grade of quality classification I by using the pdf of the beta distribution. Here, 'the distribution of the normalized quality level ℓ/I of used products' is referred to simply 'the quality distribution of used products'. This is the reason why the beta distribution is possible to express various shapes of distribution of reusable parts in used products, such as the uniform distribution-type shape, the normal distribution-type shape, the exponential distribution-type shape, the left-biased distribution shape, the right-biased distribution shape, by using the following pdf with the shape parameter and scale parameter (m, n):

$$f(x \mid m, n) = (\Gamma(m+n) / \Gamma(m) \Gamma(n)) x^{m-1} (1-x)^{n-1}, \quad (10)$$

where $x = \ell/I$ and $\Gamma(\cdot)$ denotes the gamma function. As shown in Figure 1, six cases of the quality distribution of used products are provided by changing parame-

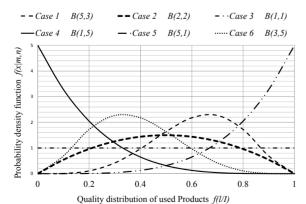


Figure 1. Five cases of the quality distribution of used products $x(= \ell/I)$ modeled as the probability density function f(x | m, n) of beta distribution B(x | m, n).

ters (m, n) of the beta distribution in Eq. (10) as follows:

- Case 1 B(x|5,3): situation including the more used products with the relatively high quality,
- Case 2 B(x|2, 2): a situation including the more used products with the middle quality,
- Case 3 B(x | 1, 1): a situation including used products whose the quality is uniformly distributed,
- Case 4 B(x|1, 5): a situation including the more used products with the low quality,
- Case 5 B(x | 5, 1): a situation including the more used products with the high quality,
- Case 6 B(x | 3, 5): a situation including the more used products with the relatively low quality.

By changing two shape parameters (m, n) of the pdf of the beta distribution, we can see how the results of the optimal production planning in the GSC change.

The unit procurement cost function for used products classified into the quality level ℓ is designed as

$$a(\ell) = 0.1\ell^2 + 0.1\ell \tag{11}$$

satisfying the relation between the unit procurement cost function and the quality level $\ell(1 \le \ell \le I)$ in Eq. (3) of Section 3.2 model assumptions (4) where the higher the quality level ℓ of used products a manufacturer purchased from a collection trader, the higher the procurement cost $a(\ell)$ the manufacturer incurred.

The unit remanufacturing cost function of parts from used products classified into quality level ℓ is set as two cases:

Case 1 :
$$c(\ell) = -0.115\ell^2 + 12.545$$
, (12)

Case 2:
$$c(\ell) = -1.15\ell + 12.545$$
, (13)

satisfying the relation between the unit remanufacturing cost function of parts extracted from used products and the quality level $\ell(1 \le \ell \le I)$ in Eq. (4) of Section 3.2

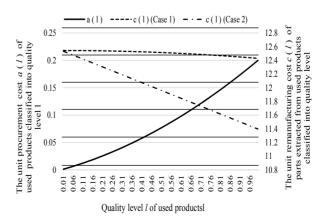


Figure 2. Relation between the unit procurement cost $a(\ell)$ and cases 1 and 2 of the unit remanufacturing cost $c(\ell)$ for quality level $\ell(1 \le \ell \le I)$ of used products.

model assumptions (4) where the lower the quality level ℓ of used products a manufacturer remanufactures, the higher the remanufacturing cost $c(\ell)$ the manufacturer incurs.

Figure 2 shows the relation between the unit procurement cost $a(\ell)$ and cases 1 and 2 of the unit remanufacturing cost $c(\ell)$ for quality level $\ell(1 \le \ell \le I)$ of used products.

Also, the relation between the unit sales price v, the unit production cost of new parts c_m , and the inventory holding cost h per unit of excess remanufactured parts per unit time were designed to satisfy the following conditions: in case without quality classification error: $v > (c_m + h)$, in the case with quality classification error: $v > (c_m + h + pc_{rc})$. Here, v, c_m, h, c_{rc} , and p were designed at v = 50, $c_m = 20$, h = 20, $c_{rc} = 10$, p = 0.7.

Here, in the case without quality classification error of used products, it is profitable for a manufacturer to remanufacturer of parts extracted from used product classified into quality level $\ell(1 \le \ell \le I)$ if the total cost regarding recycling of used products, corresponding to the sum of the procurement cost of used products, the remanufacturing cost of the remanufactured parts from used products and the inventory holding cost of the remanufactured parts is lower than the production cost of new parts. That is, from Eqs. (5) and (6), the relation between the total cost regarding recycling of used products and the procurement cost of new parts to obtain the profitability from the recycling of used products should satisfy the following condition:

$$\left(R\left\{\sum_{\ell=L}^{I} a(\ell)F\left(\ell/I\right) - (1-\lambda)a(L)F(L/I)\right\}\right) - \left(R\left\{\sum_{\ell=L}^{I} c(\ell)F(\ell/I) - (1-\lambda)c(L)F(L/I)\right\}\right) - h\left\{\left[R\left\{\sum_{\ell=L}^{I} F(\ell/I) - (1-\lambda)F(L/I)\right\}\right] - D\right\}^{+} < c_m R\left\{\sum_{\ell=L}^{I} F(\ell/I) - (1-\lambda)F(L/I)\right\} (1 \le \ell \le I).$$
(14)

Further, in the case with quality classification error of used products, it is profitable for a manufacturer to remanufacturer of parts extracted from used product classified into quality level $\ell (1 \le \ell \le I)$ if the total cost regarding recycling of used products, corresponding to the sum of the procurement cost of used products, the reclassification cost of used products, the remanufacturing cost of the remanufactured parts from used products and the inventory holding cost of the remanufactured parts is lower than the production cost of new parts. That is, from Eqs. (7)–(9), the relation between the total cost regarding recycling of used products and the procurement cost of new parts to obtain the profitability from the recycling of used products should satisfy the following condition:

$$\left(R\left\{\sum_{\ell=L}^{I} a(\ell)F(\ell/I) - (1-\lambda)a(L)F(L/I)\right\}\right) - c_{rc}(1-p)R\sum_{\ell=L}^{I}F(\ell/I) - (1-\lambda)c(L)G(L/I)\right\}\right) - \left(R\left\{\sum_{\ell=L}^{I}c(\ell)G(\ell/I) - (1-\lambda)c(L)G(L/I)\right\}\right) - h\left\{\left[R\left\{\sum_{\ell=L}^{I}F(\ell/I) - (1-\lambda)F(L/I)\right\}\right] - D\right\}^{+} < c_{m}R\left\{\sum_{\ell=L}^{I}F(\ell/I) - (1-\lambda)F(L/I)\right\}.$$
(15)

Otherwise, It is profitable for a manufacturer to produce new parts rather than to remanufacturer of parts extracted from used product classified into quality level ℓ ($1 \le \ell \le I$).

Here, all data sources set as the numerical examples in Section 6.1 are modifiable in order to conduct sensitive analysis, satisfying Section 3.2 model assumptions of a GSC.

6.2 Decisions Procedures for Optimal Production Planning using Numerical Computation and Numerical Search

Both the optimal production planning without and with quality classification errors of used products are obtained by numerical computation and numerical search. A computer programming to obtain each of the optimal solutions was developed by using the Visual C++ in Microsoft Visual Studio Express 2013 for Windows Desktop which was the integrated development environments. The decision procedures for each of the optimal production planning regarding a lower limit of procurement quality of used products and a remanufacturing ratio of parts under the lower limits of procurement quality are shown as follows:

- [Step 1] Set the parameter values described in Section 6.1.
- [Step 2] Choose a case of the quality distribution of used products among cases 1-6 by changing parameters (m, n) of the beta distribution in Eq. (10).
- [Step 3] Set the value of the lower limit of procurement quality at L = I.
- [Step 4] Set the value of remanufacturing value at $\lambda = \lambda = 0.01$.
- [Step 5] Calculate the expected profit of a manufacturer for a lower limit of procurement quality *L* under a remanufacturing ratio λ without quality classification errors of used products, $E[\pi(L|\lambda)]$, by substituting the system parameters from [Step 1] to [Step 4] into Eqs. (5) and (6). Meanwhile, calculate the expected profit of a manufacturer for a lower limit of procurement qual-

ity *L* under a remanufacturing ratio λ with quality classification errors of used products, *E* $\left[\pi_e(L|\lambda)\right]$, by substituting the system parameters from [Step 1] to [Step 4] into Eqs. (7)–(9).

- [Step 6] Update the value of the remanufacturing ratio λ under the lower limit of procurement quality L to maximize the expected profit of the manufacturer without quality classification errors of used products, $E[\pi(\lambda|L)]$. Meanwhile, the value of the remanufacturing ratio λ under the lower limit of procurement quality L to maximize the expected profit of the manufacturer with quality classification errors of used products, $E[\pi_e(\lambda|L)]$.
- [Step 7] Vary the remanufacturing ratio λ within the range $0.01 \le \lambda \le 1.0$ at the step size 0.01. Repeat the decision procedures from [Step 5] to [Step 7]. Find the tentatively optimal production planning without quality classification errors (λ^*, L) to maximize the expected profit of the manufacturer $E | \pi(\lambda | L) |$ for the remanufacturing ratio λ under the lower limit L in the case without quality classification errors in Eq. (5) in the chosen case of the quality distribution of used products. Meanwhile, find the tentatively optimal production planning with quality classification errors (λ_{e}^{*}, L) to maximize the expected profit of the manufacturer $E\left[\pi_{e}(\lambda|L)\right]$ for the remanufacturing ratio λ under the lower limit L in the case with quality classification errors in Eq. (7) in the chosen case of the quality distribution of used products.
- [Step 8] Vary the lower limit of procurement quality L within the range $I \ge L \ge 1$ at the step size 1. Repeat the decision procedures from [Step 5] to [Step 8]. Find the optimal production planning without quality classification errors (L^*, λ^*) to maximize the expected profit of the manufacturer in the case without quality classification errors in Eq. (5) in the chosen case of the quality distribution of used products. Substituting the optimal production planning without quality classification errors (L^*, λ^*) into Eq. (5), the maximal expected profit of the manufacturer in case without quality classification errors $E \left| \pi(L^*, \lambda^*) \right|$ can be obtained. Meanwhile, find the optimal production planning with quality classification errors (L_e^*, λ_e^*) to maximize the expected profit of the manufacturer in the case with quality classification errors in Eq. (7) in the chosen case of the quality distribution of used products. Substituting the optimal production planning with quality classification errors (L_e^*, λ_e^*) into Eq. (7), the maximal expected profit of the manufacturer in case with quality classification errors $E\left[\pi_e(L_e^*, \lambda_e^*)\right]$ can be obtained.

[Step 9] Both the difference of the manufacturer's ex-

pected profit and the reduction ratio of the manufacturer's expected profit for the optimal remanufacturing ratio λ^* under the optimal the lower limit of procurement quality L^* and that for 100(%) of the remanufacturing ratio, $\lambda = 1.0$, under L^* in the case without quality classification error of used products are calculated as the following equations:

Difference of the manufacturer's expected profit

$$= E\left[\pi(L^*, \lambda^*)\right] - E\left[\pi(L^*, 1.0)\right].$$
(16)

Reduction ratio of the expected profit (%)

$$=\frac{\left(E\left[\pi(L^{*},\lambda^{*})\right]-E\left[\pi(L^{*},1.0)\right]\right)}{E\left[\pi(L^{*},\lambda^{*})\right]}\times100(\%). (17)$$

Meanwhile, both the difference of the manufacturer's expected profit and the reduction ratio of the manufacturer's expected profit for the optimal remanufacturing ratio λ_e^* under the optimal the lower limit of procurement quality L_e^* and that for 100(%) of the remanufacturing ratio, $\lambda = 1.0$, under L_e^* in the case with quality classification error of used products are calculated as the following equations:

Difference of the manufacturer's expected profit

$$= E \Big[\pi_e(L_e^*, \lambda_e^*) \Big] - E \Big[\pi_e(L_e^*, 1.0) \Big].$$
(18)

Reduction ratio of the expected profit (%)

$$=\frac{\left(E\left[\pi_{e}(L_{e}^{*},\lambda_{e}^{*})\right]-E\left[\pi_{e}(L_{e}^{*},1.0)\right]\right)}{E\left[\pi_{e}(L_{e}^{*},\lambda_{e}^{*})\right]}\times100(\%).$$
 (19)

[Step 10] Both the difference of the manufacturer's expected profit and the reduction ratio of the manufacturer's expected profit without quality classification error and that with quality classification errors are calculated using the following equations:

Difference of the manufacturer's expected profit

$$= E\Big[\pi(L^*,\lambda^*)\Big] - E\Big[\pi_e(L^*_e,\lambda^*_e)\Big].$$
(20)

Reduction ratio of the expected profit (%)

$$=\frac{\left(E\left[\pi(L^*,\lambda^*)\right]-E\left[\pi_e(L^*_e,\lambda^*_e)\right]\right)}{E\left[\pi(L^*,\lambda^*)\right]}\times100(\%). (21)$$

[Step 11] Choose another case of the quality distribution of used products among cases 1-6 by changing parameters (m, n) of the beta distribution in Eq. (10). Repeat the decision procedures from [Step 3] to [Step 11] until all cases of the quality distribution of used products are chosen.

Through the decision procedures from [Step 1] to [Step 11], the optimal production planning in both cases of with and without quality classification errors under a given case of the quality distribution of used products among cases 1–6 can be determined by using the numerical computation and the numerical search in this paper.

6.3 Results of Numerical Analysis and Managerial Insights

6.3.1 Profitability of optimal combination

It is discussed how the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio can bring the more profit to a manufacturer under the total grade of quality classification for used products *I*. Table 1 shows the profitability of the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio under the total grade of quality classification for used products *I* on the manufacturer's expected profit (I = 5, I = 10 and case 1 of the optimal combination of the optimal combination of the optimal combination for used products *I* on the manufacturer's expected profit (I = 5, I = 10 and case 1 of the remanufacturing cost). Table 2 shows the profitability of the optimal combination.

Table 1. Profitability of the optimal combination of lower limit of procurement quality of used products and the remanufacturing ratio under *I* on the manufacturer's expected profit (I = 5, I = 10, and case 1 of the remanufacturing cost)

| Remanufacturing cost: Case 1 | | | - | = 1.0 | | = 0.7 | Expecte | ed profit | Reduction ratio of expected profit | |
|---|----------------------------|--|----------------------------------|--|--|---|---|---|------------------------------------|--------------------------|
| Ca | se i | | L^{*} | $\lambda^*(\%)$ | L_e^* | $\lambda_{e}^{*}(\%)$ | <i>p</i> = 1.0 | p = 0.7 | <i>p</i> = 1.0 | p = 0.7 |
| | Case 1 | $\lambda^*(\%)$ | 4 | 58 | 4 | 58 | 29826 | 28263 | 39.43 | 38.90 |
| | Case I | $\lambda = 100(\%)$ | 4 | 100 | 4 | 100 | 18066 | 17269 | 39.43 | 38.90 |
| Quality distribution | Case 2 | $\lambda^*(\%)$ | 3 | 16 | 4 | 100 | 29833 | 28559 | 54.35 | 0.00 |
| | Case 2 | $\lambda = 100(\%)$ | 3 | 100 | 4 | 100 | 13619 | 28559 | 54.55 | 0.00 |
| of used products | Case 3 | $\lambda^*(\%)$ | 4 | 100 | 4 | 100 | 29850 | 28610 | 0.00 | 0.00 |
| (Total grade of quality classification for used products: I = 5) | Case 3 | $\lambda = 100(\%)$ | 4 | 100 | 4 | 100 | 29850 | 28610 | 0.00 | 0.00 |
| | Case 4 | $\lambda^*(\%)$ | 1 | 10 | 2 | 100 | 29781 | 28608 | 2.99 | 0.00 |
| | Case 4 | $\lambda = 100(\%)$ | 1 | 100 | 2 | 100 | 28889 | 28608 | 2.99 | 0.00 |
| | Case 5 | $\lambda^*(\%)$ | 5 | 59 | 5 | 59 | 29815 | 28083 | 59.52 | 57.49 |
| | Case J | $\lambda = 100(\%)$ | 5 | 100 | 5 | 100 | 12071 | 11937 | 39.32 | |
| | Case 6 | $\lambda^*(\%)$ | 3 | 94 | 3 | 94 | 29802 | 29374 | 4.26 | 4.00 |
| | Case 0 | $\lambda = 100(\%)$ | 3 | 100 | 3 | 100 | 28531 | 28200 | 4.20 | |
| Remanufac | | st: | <i>p</i> = 1.0 | | <i>p</i> = 0.7 | | Expecte | ed profit | | on ratio ted profit |
| Ca | se 1 | - | L^* | $\lambda^*(\%)$ | L_e^* | $\lambda_{e}^{*}(\%)$ | <i>p</i> = 1.0 | p = 0.7 | p = 1.0 | p = 0.7 |
| | | * | | | | | | - | 1 | p 0.7 |
| | Casa 1 | $\lambda^*(\%)$ | 7 | 20 | 8 | 100 | 30111 | 27773 | 1 | 1 |
| | Case 1 | $\frac{\lambda^*(\%)}{\lambda = 100(\%)}$ | 7 7 | 20 100 | 8 8 | 100 100 | 30111 18426 | 27773 27773 | 38.81 | 0.00 |
| | | $\lambda = 100(\%)$ | | | - | | | | 38.81 | 0.00 |
| Quality distribution | Case 1 Case 2 | $\lambda = 100(\%)$ | 7 | 100 | 8 | 100 | 18426 | 27773 | 1 | 1 |
| Quality distribution of used products | Case 2 | $\lambda = 100(\%)$ $\lambda^{*}(\%)$ $\lambda = 100(\%)$ $\lambda^{*}(\%)$ | 7 6 | 100 32 | 8 7 | 100 100 | 18426 30070 | 27773 27580 | 38.81 | 0.00 |
| of used products | | $\lambda = 100(\%)$ $\lambda^{*}(\%)$ $\lambda = 100(\%)$ $\lambda^{*}(\%)$ | 7 6 6 | 100 32 100 | 8 7 7 | 100 100 100 | 18426 30070 23558 | 27773 27580 27580 | 38.81 | 0.00 |
| of used products (Total grade of quality · | Case 2 Case 3 | $\lambda = 100(\%)$ $\lambda^{*}(\%)$ $\lambda = 100(\%)$ $\lambda^{*}(\%)$ | 7 6 6 7 | 100 32 100 100 | 8 7 7 7 7 | 100 100 100 100 | 18426 30070 23558 30166 | 27773 27580 27580 27829 | 38.81 21.65 0.00 | 0.00 0.00 0.00 |
| of used products (Total grade of quality classification for used products: | Case 2 | λ = 100(%) λ*(%) λ = 100(%) λ*(%) λ = 100(%) | 7 6 6 7 7 7 | 100 32 100 100 100 | 8 7 7 7 7 7 7 | 100 100 100 100 100 | 18426 30070 23558 30166 30166 | 27773 27580 27580 27829 27829 | 38.81 | 0.00 |
| of used products (Total grade of quality classification | Case 2 Case 3 Case 4 | $\lambda = 100(\%) \\ \lambda^{*}(\%) \\ \lambda = 100(\%) \\ \lambda^{*}(\%) $ | 7 6 6 7 7 7 2 | 100 32 100 100 100 27 | 8 7 7 7 7 7 7 3 | 100 100 100 100 100 100 | 18426 30070 23558 30166 30166 29830 | 27773 27580 27580 27829 27829 27830 | 38.81 21.65 0.00 41.68 | 0.00 0.00 0.00 0.00 0.00 |
| of used products (Total grade of quality classification for used products: | Case 2 Case 3 | $\lambda = 100(\%) \\ \lambda^{*}(\%) \\ \lambda = 100(\%) \\ \lambda^{*}(\%) $ | 7 6 7 7 2 2 | 100 32 100 100 27 100 | 8 7 7 7 7 7 7 3 3 3 | 100 100 100 100 100 100 100 | 18426 30070 23558 30166 30166 29830 17397 | 27773 27580 27580 27829 27829 27829 27830 27830 | 38.81 21.65 0.00 | 0.00 0.00 0.00 |
| of used products (Total grade of quality classification for used products: | Case 2 Case 3 Case 4 | $\lambda = 100(\%) \\ \lambda^{*}(\%) \\ \lambda = 100(\%) \\ \lambda^{*}(\%) \\ \lambda^{*}(\%) $ | 7 6 7 7 2 2 10 | 100 32 100 100 27 100 97 | 8 7 7 7 7 7 7 3 3 3 10 | 100 100 100 100 100 100 100 97 | 18426 30070 23558 30166 30166 29830 17397 30320 | 27773 27580 27580 27829 27829 27830 27830 27830 28112 | 38.81 21.65 0.00 41.68 | 0.00 0.00 0.00 0.00 0.00 |

| Remanufa | • | st: | F | = 1.0 | 1 | = 0.7 | Expecte | ed profit | Reduction ratio of expected profit | |
|---|------------------|---|----------------|-----------------|----------------|---------------------|----------------|----------------|------------------------------------|------------------------|
| Ca | ise 2 | _ | L^{*} | $\lambda^*(\%)$ | L_e^* | $\lambda^*_{e}(\%)$ | <i>p</i> = 1.0 | p = 0.7 | <i>p</i> = 1.0 | p = 0.7 |
| | Case 1 - | λ^* (%) | 4 | 58 | 4 | 58 | 32061 | 29969 | 33.56 | 24.00 |
| | | $\lambda = 100(\%)$ | 4 | 100 | 4 | 100 | 21302 | 19779 | 55.50 | 34.00 |
| Quality distribution | Case 2 - | $\lambda^{*}(\%)$ | 3 | 16 | 4 | 100 | 32029 | 30108 | 46.88 | 0.00 |
| | Case 2 | $\lambda = 100(\%)$ | 3 | 100 | 4 | 100 | 17015 | 30108 | 40.88 | 0.00 |
| of used products | Case 3 - | $\lambda^{*}(\%)$ | 4 | 100 | 4 | 100 | 32104 | 30519 | 0.00 | 0.00 |
| (Total grade of quality classification for used products: I=5) | Case 5 | $\lambda = 100(\%)$ | 4 | 100 | 4 | 100 | 32104 | 30519 | 0.00 | 0.00 |
| | Case 4 - | $\lambda^{*}(\%)$ | 1 | 10 | 2 | 100 | 31223 | 29609 | 3.26 | 0.00 |
| | Case 4 | $\lambda = 100(\%)$ | 1 | 1 | 2 | 100 | 30206 | 29609 | 3.20 | 0.00 |
| | Case 5 - | $\lambda^{*}(\%)$ | 5 | 59 | 5 | 59 | 32096 | 29679 | 50.35 50.66 | |
| | Case 5 | $\lambda = 100(\%)$ | 5 | 100 | 5 | 100 | 15936 | 14644 | 50.55 | 50.00 |
| | Case 6 - | λ^* (%) | 3 | 94 | 3 | 94 | 31803 | 30908 | 3.70 | 3.56 |
| | Case 0 | $\lambda = 100(\%)$ | 3 | 100 | 3 | 100 | 30627 | 29807 | 5.70 | |
| Remanufa | | st: | <i>p</i> = 1.0 | | <i>p</i> = 0.7 | | Expecte | ed profit | Reducti of expec | on ratio ted profit |
| | ase 2 | | L^{*} | $\lambda^*(\%)$ | L_e^* | $\lambda^*_{e}(\%)$ | <i>p</i> = 1.0 | p = 0.7 | <i>p</i> = 1.0 | p = 0.7 |
| | Case 1 | λ^* (%) | 7 | 20 | 8 | 100 | 31338 | 28629 | 34.49 | 0.00 |
| | Case I | $\lambda = 100(\%)$ | 7 | 100 | 8 | 100 | 20530 | 28629 | 54.49 | 0.00 |
| - | Case 2 | λ^* (%) | 6 | 32 | 6 | 32 | 31558 | 28789 | 18.87 | 20.59 |
| Quality distribution - | Case 2 | $\lambda = 100(\%)$ | 6 | 100 | 6 | 100 | 25602 | 22860 | 10.07 | 20.39 |
| of used products | Casa 2 | λ^* (%) | 7 | 100 | 7 | 100 | 31224 | 28887 | 0.00 | 0.00 |
| | Case 3 | $\lambda = 100(\%)$ | 7 | 100 | 7 | 100 | 31224 | 28887 | 0.00 | 0.00 |
| (Total grade of quality - | | | | | | | | 29279 | | |
| classification | C 4 | λ*(%) | 2 | 27 | 3 | 100 | 31790 | 29279 | 26.90 | 0.00 |
| classification for used products: | Case 4 | × / | 2 2 | 27 100 | 3 | 100 100 | 31790 20063 | 29279 29279 | 36.89 | 0.00 |
| classification | | λ*(%) | | | | | | | | |
| classification for used products: | Case 4 Case 5 | $\frac{\lambda^*(\%)}{\lambda = 100(\%)}$ | 2 | 100 | 3 | 100 | 20063 | 29279 | 36.89 1.86 | 0.00 |
| classification for used products: | | $\frac{\lambda^*(\%)}{\lambda = 100(\%)}$ $\frac{\lambda^*(\%)}{\lambda^*(\%)}$ | 2 10 | 100 97 | 3 10 | 100 97 | 20063 30320 | 29279 28112 | | |

| Table 2. Profitability of the optimal combination of lower limit of procurement quality of used products and the remanufac- |
|---|
| turing ratio under I on the manufacturer's expected profit ($I = 5$, $I = 10$ and Case 2 of the remanufacturing cost) |

tion of lower limit of procurement quality of used products and the remanufacturing ratio under I on the manufacturer's expected profit (I = 5, I = 10 and case 2 of the remanufacturing cost).

Concretely, results of the optimal lower limit of procurement quality and the optimal remanufacturing ratio when p = 1.0 are compared with results of the optimal lower limit of procurement quality and 100(%) of the remanufacturing ratio, $\lambda = 1.0$, when p = 1.0 under both cases without and with quality classification errors of used products as to the unit remanufacturing cost $c(\ell)$.

From Tables 1 and 2, the following results can be seen: Under cases 1 and 2 of the remanufacturing cost $c(\ell)$, the manufacturer's expected profits for the optimal combinations of production planning (L^*, λ^*) and (L_e^*, λ_e^*) in cases without and with quality classification error of used products are compared with those for the production planning with 100(%) of the remanufacturing ratio under the optimal lower limit of procurement quality, (L^*, λ^*) and (L_e^*, λ_e^*) , in cases without and with quality classification error of used products as to the total grade of quality classification for used products I(I = 5, I = 10). The reduction ratio of the manufacturer's expected profit for a small number of I(I = 5) in cases 1, 2, 4, and 5 of the quality distribution of used products tends to be higher than that for a relative large number of I(I = 10)in both cases without and with quality classification error of used products. These results imply that the smaller the total grade I of quality classification for used products is in case 1, 2, 4, and 5 of the quality distribution of used products, the more only the optimal decision for the lower limit of procurement quality of used products can bring the reduction of profit to a manufacturer. Therefore, the results of Tables 1 and 2 can verify that the optimal combination of a lower limit of procurement quality of used products and a remanufacturing ratio can bring the more profit to a manufacturer as the total grade I of quality classification for used products becomes smaller under cases 1, 2, 4, and 5 of the quality distribution of used products.

6.3.2 Effect of quality classification errors on optimal production planning and manufacturer's expected profit

The effect of quality classification errors on the optimal production planning is discussed. Concretely, results of the optimal lower limit of procurement quality and the optimal remanufacturing ratio without quality classification errors when p = 1.0 are compared with those with it when $p \neq 1.0$. Concretely, results of the optimal production planning regarding a lower limit of procurement quality and a remanufacturing ratio without quality classification errors where p = 1.0 are compared with that with quality classification errors where $p \neq 1.0$ and p = 0.7. From the comparison of results, the effect of the quality classification errors on the optimal production planning and the manufacturer's expected profit are investigated. Table 3 shows the effect of quality classification errors of used products on the optimal production planning and the manufacturer's expected profit under cases 1–2 of the remanufacturing cost of recyclable parts extracted from used products under I = 10. In Table 3, p = 1.0 means a situation where no quality classification error of used products occurs, meanwhile p = 0.7 means a situation where quality classification errors of used products occur indicating that the correct quality classification ratio is p = 0.7 and the quality classification errors ratio is (1-p) = 0.3.

From Table 3, the following results for cases 1 and 2 of the remanufacturing cost $c(\ell)$ can be seen: results of the optimal lower limit of procurement quality and the optimal remanufacturing ratio with quality classification errors are different from those without it. In the situations where the remanufacturing cost are case 1 and p =0.7, the quality classification errors affects the optimal production planning under cases 1, 2, and 4 of the quality distribution of used products. In the situations where the remanufacturing cost is case 2 of the remanufacturing cost and p = 0.7, the quality classification errors affects the optimal production planning under cases 1 and 4 of the quality distribution of used products. Concretely, the optimal production planning with quality classification errors is determined as follows: The optimal lower limit of procurement quality L_e^* with quality classification errors is determined as one higher quality level than L^* without it. The optimal remanufacturing ratio with quality classification errors λ_e^* under L_e^* is determined as a higher ratio than λ^* without it, in this numerical examples $\lambda_e^* = 100(\%)$. This implies that the production quantity of the remanufactured parts de-

| | Remanufacturing cost : | | = 1.0 | P | = 0.7 | Expecte | ed profit | Difference | Reduction ratio | |
|---|------------------------|---------|-----------------|---------|---------------------|----------------|----------------|--------------------|--------------------|--|
| Case 1 I = 10 | | L^{*} | λ*(%) | L_e^* | λ^*_{e} (%) | <i>p</i> = 1.0 | <i>p</i> = 0.7 | of expected profit | of expected profit | |
| | Case 1 | 7 | 20 | 8 | 100 | 30111 | 27773 | 2338 | 7.76 | |
| | Case 2 | 6 | 32 | 7 | 100 | 30070 | 27580 | 2490 | 8.28 | |
| Quality distribution of used products | Case 3 | 7 | 100 | 7 | 100 | 30166 | 27829 | 2337 | 7.75 | |
| | Case 4 | 2 | 27 | 3 | 100 | 29830 | 27830 | 2000 | 6.70 | |
| | Case 5 | 10 | 97 | 10 | 97 | 30320 | 28112 | 2208 | 7.28 | |
| - | Case 6 | 5 | 89 | 5 | 89 | 29900 | 28273 | 1628 | 5.44 | |
| Remanufactur | | p | <i>p</i> = 1.0 | | p = 0.7 | | ed profit | Difference | Reduction ratio | |
| Case 2 I = 10 | | L^{*} | $\lambda^*(\%)$ | L_e^* | $\lambda^*_{e}(\%)$ | <i>p</i> = 1.0 | <i>p</i> = 0.7 | of expected profit | of expected profit | |
| | Case 1 | 7 | 20 | 8 | 100 | 31338 | 28629 | 2709 | 8.64 | |
| _ | Case 2 | 6 | 32 | 6 | 32 | 31558 | 28789 | 2769 | 8.78 | |
| Quality distribution of - | Case 3 | 7 | 100 | 7 | 100 | 31224 | 28887 | 2337 | 7.49 | |
| used products | Case 4 | 2 | 27 | 3 | 100 | 31790 | 29279 | 2510 | 7.90 | |
| ± – | Case 5 | 10 | 97 | 10 | 97 | 30320 | 28112 | 2208 | 7.28 | |
| | Case 6 | 5 | 89 | 5 | 89 | 32033 | 30117 | 1916 | 5.98 | |

Table 3. Effect of quality classification errors of used products on optimal production planning and the manufacturer's expected profit

creases, meanwhile the production quantity of new parts increases. Also, in the situations where the remanufacturing cost is case 1 of the remanufacturing cost and p = 0.7, the quality classification errors, the quality classification errors do not affect the optimal production planning under cases 3, 5, and 6 of the quality distribution of used products. In the situations where the remanufacturing cost is case 2 of the remanufacturing cost and p = 0.7, the quality classification errors do not impact the optimal production planning under cases 2, 3, 5, and 6 of the quality classification errors do not impact the optimal production planning under cases 2, 3, 5, and 6 of the quality distribution of used products.

Next, the reasons why the results of the optimal production planning with the classification errors bring to the different results from that without the classification errors as to the quality distribution of used products are discussed. From Eqs. (1) and (7), the error quantity of

quality classification of used products, $R(1-p)\sum_{\ell=L}^{I} F(\ell/I)$,

depends on the quality distribution of used products. Also, the error quantity of quality classification of used products affect the reclassification cost of used products and the remanufacturing cost of parts extracted from used products from Eqs. (1), (8), and (9). It can be seen that the amount of change of used products around L_e^* is large under cases 1, 2, and 4 of the quality distribution of used products in case 1 of the remanufacturing cost, meanwhile the amount of change of used products around L_e^* is large under cases 1 and 4 of the quality distribution of used products in case 2 of the remanufacturing cost. Therefore, it is verified that the effect degree of the quality classification errors on the optimal production planning depends on the quality distribution of used products without and with the quality classification errors in Eq. (1), (8), and (9).

Next, the impact of the quality classification errors of used products on the manufacturer's expected profit is investigated as to the quality distribution of used products. From Table 3, the following results for cases 1 and 2 of the remanufacturing cost $c(\ell)$ under each case of the quality distribution of used products can be seen: case 2 and case 1 of the quality distribution of used products bring to the first and second largest values of

Table 4. Results of the manufacturer's expected profit, the reclassification cost of used products (*CRC*), the remanufacturing cost (CRE) of parts and the production cost of new parts (*CN*) for the optimal production planning without and with the quality classification errors as to the quality distribution of used products

| 1 5 | | | - | - | | | | | |
|---|--------|--|---------|-------------------|-----------------|-------------------------------------|----------|------|-------------------------------|
| Remanufact cost: Case I = 10 | e 1 | Correct quality classification ratio (p = 1.0) | L^{*} | λ^* (%) | Expected profit | (i) <i>CRC</i> ($c_{rc} = 10$) | (ii) CRE | CN | Total cost of (i) and (ii) |
| | Case 1 | | 7 | 20 | 30111 | 0 | 3597 | 65 | 3597 |
| | Case 2 | Optimal | 6 | 32 | 30070 | 0 | 4387 | 26 | 4387 |
| Quality distribution of used products | Case 3 | production | 7 | 100 | 30166 | 0 | 3274 | 0 | 3274 |
| | Case 4 | planning without quality | 2 | 27 | 29830 | 0 | 8707 | 54 | 8707 |
| | Case 5 | classification errors | 10 | 97 | 30320 | 0 | 830 | 111 | 830 |
| | Case 6 | | 5 | 89 | 29900 | 0 | 6730 | 55 | 6730 |
| Remanufacturing cost: Case 1 | | Correct quality classification ratio $p = 0.7$ | L_e^* | λ_e^* (%) | Expected profit | (i) <i>CRC</i> ($c_{rc} = 10$) | (ii) CRE | CN | Total cost of (i) and (ii) |
| | Case 1 | | 8 | 100 | 27773 | 2118 | 2509 | 1883 | 4627 |
| | Case 2 | Optimal production | 7 | 100 | 27580 | 2112 | 3268 | 1920 | 5380 |
| Quality distribution of | Case 3 | planning with | 7 | 100 | 27829 | 2400 | 3211 | 0 | 5611 |
| used products | Case 4 | consideration | 3 | 100 | 27830 | 1966 | 5988 | 2893 | 7954 |
| Ĩ | Case 5 | of quality classification errors | 10 | 97 | 28112 | 2457 | 581 | 111 | 3038 |
| | Case 6 | | 5 | 89 | 28273 | 2519 | 5839 | 55 | 8358 |
| | Case 1 | | 7 | 20 | 27109 | 3481 | 3118 | 65 | 6599 |
| | Case 2 | Production | 6 | 32 | 27411 | 3000 | 4045 | 26 | 7045 |
| Quality | Case 3 | planning without | 7 | 100 | 27829 | 2400 | 3211 | 0 | 5611 |
| distribution of used products | Case 4 | consideration - of quality | 2 | 27 | 27493 | 3543 | 7501 | 54 | 11044 |
| 1 | Case 5 | classification errors | 10 | 97 | 28112 | 2457 | 581 | 111 | 3038 |
| | Case 6 | | 5 | 89 | 28273 | 2519 | 5839 | 55 | 8358 |
| | | | | | | | | | |

the reduction ratio to the manufacturer's expected profit, meanwhile case 6 of the distribution of the quality level of used products brings to the smallest value of the reduction ratio to the manufacturer's expected profit.

Next, the reasons why the quality distribution of used products impacts the reduction ratio of the manufacturer's expected profit when the quality classification errors of used products occur is discussed. Here, the reclassification cost of used products, the remanufacturing cost of parts and the production cost of new parts are investigated in the cases without and with the quality classification errors of used products. Also, the manufacturer's expected profit, the reclassification cost of used products, the remanufacturing cost of parts and the production cost of new parts for the optimal production planning with consideration of quality classification errors when they occur are compared with those without consideration of it even if they occur. Table 4 shows the results of the manufacturer's expected profit, the reclassification cost of used products, the remanufacturing cost of parts and the production cost of new parts for the optimal production planning without and with the quality classification errors as to the quality distribution of used products under case 1 of the remanufacturing cost under I = 10.

From Figure 1 and Table 4, the following results can be seen: from Figure 1 and Eqs. (1), (8), and (9), the amount of change of used products around L_e^* is large under cases 1, 2, and 4 of the quality distribution of used products in case 1 of the remanufacturing cost. Therefore, in cases 1, 2, and 4 of the quality distribution of used products, the total cost of the reclassification cost and the remanufacturing cost for the optimal production planning with quality classification errors (p = 0.7) are higher than that for the optimal production planning without it. Also, the total cost of the reclassification cost and the remanufacturing cost for the optimal production planning with consideration of quality classification errors when they occur are lower than that for the production planning without consideration of quality classification errors when they occur. Here, the production cost of new parts for the optimal production planning

with consideration of quality classification errors when they occur are higher than that for the production planning without consideration of quality classification errors when they occur. This indicates that it is better to choose production of new parts than does remanufacturing of parts from used products from the aspect of a manufacturer's profit. From the results, the effect of the quality classification errors on the increments of the reclassification cost and the remanufacturing cost and the production cost of new parts depends on quality distribution of used products. Therefore, the optimal production planning with consideration of quality classification errors when they occur can recommended from the aspect of the manufacturer's profit.

Meanwhile, in cases 3, 5, and 6, the reclassification cost, the remanufacturing cost and the production cost of new parts has no change even if the quality classification errors occur. The reason is considered as follows: from Figure 1 and Eqs. (1), (8), and (9), the amount of change of used products around L_{e}^{*} is small under cases 3, 5, and 6 of the quality distribution of used products in case 1 of the remanufacturing cost. Therefore, under those quality distributions of used products, it can been seen that the effect of the quality classification errors on the increments of the reclassification cost, the remanufacturing cost and the production cost of new parts are small. So, under those quality distributions of used products, the optimal production planning with quality classification errors is same as the optimal production planning without quality classification errors. The results for case 2 of the remanufacturing cost are same as the results for case 1 of the remanufacturing cost $c(\ell)$.

6.3.3 Effect of change of ratio of quality classification errors on optimal production planning

The impact of change of ratio of the quality classification errors on the optimal production planning is discussed. Under I = 10, Table 5 shows the effect of change of the correct quality classification ratio p on the optimal production planning with the quality classification errors of used products in case 1 of the remanufacturing cost,

Table 5. Effect of change of correct quality classification ratio p on optimal production planning with quality classification errors of used products

| Remanufacturi | Correct quality classification ratio p | | | | | | | | | | | |
|-------------------------|--|---------|-----------------|---------|-------------------|---------|---------|-----------------|---------|-------------------|---------|--|
| Case 1 | | | 1 | (|).8 | 0 | .7 | 0. | 0.6 | | 0.5 | |
| I = 10 | - | L^{*} | $\lambda^*(\%)$ | L_e^* | λ_e^* (%) | L_e^* | L^{*} | λ^* (%) | L_e^* | λ_e^* (%) | L_e^* | |
| | Case 1 | 7 | 21 | 7 | 100 | 8 | 100 | 8 | 100 | 8 | 100 | |
| | Case 2 | 6 | 32 | 6 | 32 | 7 | 100 | 7 | 100 | 7 | 100 | |
| Quality distribution of | Case 3 | 7 | 100 | 7 | 100 | 7 | 100 | 7 | 100 | 7 | 100 | |
| used products | Case 4 | 2 | 27 | 2 | 27 | 3 | 100 | 3 | 100 | 3 | 100 | |
| ł | Case 5 | 10 | 97 | 10 | 97 | 10 | 97 | 10 | 97 | 10 | 97 | |
| | Case 6 | 5 | 89 | 5 | 89 | 5 | 89 | 5 | 89 | 5 | 89 | |

and then Table 6 shows the effect of change of correct quality classification ratio p on the reclassification cost (CRC), the remanufacturing cost (CRE), and the production cost of new parts (CN) for this optimal production planning. Here, when p = 1.0, a manufacturer incurs no reclassification cost. From Tables 5 and 6, the following results can be seen: as the correct quality classification ratio p is lower, the CRC and the CN tends to increases, meanwhile the CRE tends to decrease. Under the situation, the manufacturer tends to change the optimal production planning so as to be gradually negative to remanufacture of recyclable parts extracted from used products and to lean to shift production of new parts using new materials. Also, from Figure 1, Eqs. (1), (7)-(9), it is verified that not only the increment of the CRC, CRE, and CN, but also the change of the optimal production planning due to reduction of p depends on quality distribution of used products.

6.3.4 Effect of reclassification cost on optimal production planning

The impact of change of the reclassification cost on the optimal production planning is discussed. Table 7 shows the effect of change of the reclassification cost on optimal production planning with quality classification errors of used products in case 1 of the remanufacturing cost $c(\ell)$ under I = 10. From Table 7, the following results can be seen: As the reclassification cost c_{re} is lower, the optimal lower limit of procurement quality L_e^* is determined as one lower value than that when $c_{rc} = 10$, meanwhile the optimal remanufacturing ratio λ_{a}^{*} under L_e^* is determined as a lower value than when $c_{rc} = 10$. The results imply that the manufacturer tends to be positive to remanufacture the recyclable parts extracted from used products as c_{rc} is lower. Also, from Figure 1, Eqs. (1), (7)-(9), the change of the optimal production planning, which is either the more positive remanufacturing or the more negative remanufacturing, due to increment of p depends on quality distribution of used products.

7. CONCLUSIONS AND FUTURE RESEARCHES

This paper focused on the optimal production planning of a manufacturer in a GSC to operate remanufac-

| Table 6. Effect of change of correct quality classification ratio p on reclassification cost (CRC), remanufacturing cost (CRE) |
|--|
| and production cost of new parts (CN) optimal production planning with quality classification errors of used products |

| Remanufacturing cost : Case 1 I = 10 | | Correct quality classification ratio p | | | | | | | | | | | | | |
|--|-----|--|-----|------|------|------|------|------|------|------|------|------|------|------|------|
| Quality | | 1 | | | 0.8 | | | 0.7 | | | 0.6 | | 0.5 | | |
| distribution of used products | CRC | CRE | CN | CRC | CRE | CN | CRC | CRE | CN | CRC | CRE | CN | CRC | CRE | CN |
| Case 1 | 0 | 3597 | 65 | 1412 | 2662 | 1883 | 2118 | 2509 | 1883 | 2823 | 2355 | 1883 | 3529 | 2202 | 1883 |
| Case 2 | 0 | 4387 | 26 | 2000 | 4159 | 26 | 2112 | 3268 | 1920 | 2816 | 3161 | 1920 | 3520 | 3054 | 1920 |
| Case 3 | 0 | 3274 | 0 | 1600 | 3232 | 0 | 2400 | 3211 | 0 | 3200 | 3190 | 0 | 4000 | 3170 | 0 |
| Case 4 | 0 | 8707 | 54 | 2362 | 7903 | 54 | 1966 | 5988 | 2893 | 2621 | 5653 | 2893 | 3277 | 5319 | 2893 |
| Case 5 | 0 | 830 | 111 | 1638 | 664 | 111 | 2457 | 581 | 111 | 3276 | 498 | 111 | 4095 | 415 | 111 |
| Case 6 | 0 | 6730 | 55 | 1680 | 6136 | 55 | 2519 | 5839 | 55 | 3359 | 5541 | 55 | 4199 | 5244 | 55 |

Table 7. Effect of change of reclassification cost on optimal production planning with quality classification errors of used products (remanufacturing cost: case 1 and I = 10)

| Distribution of quality level | Reclassification cost c_{rc} | | | | | | | | | | | |
|-------------------------------|--------------------------------|---------------------|---------|-------------------|-------------|-------------------|-------------|-------------------|-------------|---------------------|--|--|
| | | 10 | 9 | | | 8 | | 7 | 6 | | | |
| of used products | L_{e}^{*} | $\lambda_e^{*}(\%)$ | L_e^* | λ_e^* (%) | L_{e}^{*} | λ_e^* (%) | L_{e}^{*} | λ_e^* (%) | L_{e}^{*} | $\lambda_e^{*}(\%)$ | | |
| Case 1 | 8 | 100 | 8 | 100 | 8 | 100 | 8 | 100 | 7 | 20 | | |
| Case 2 | 6 | 32 | 6 | 32 | 6 | 32 | 6 | 32 | 6 | 32 | | |
| Case 3 | 7 | 100 | 7 | 100 | 7 | 100 | 7 | 100 | 7 | 100 | | |
| Case 4 | 3 | 100 | 2 | 27 | 2 | 27 | 2 | 27 | 2 | 27 | | |
| Case 5 | 10 | 97 | 10 | 97 | 10 | 97 | 10 | 97 | 10 | 97 | | |
| Case 6 | 5 | 89 | 5 | 89 | 5 | 89 | 5 | 89 | 5 | 89 | | |

turing of parts used in products such as mobile phone, personal computer and semiconductor and electronic component. This paper proposed an optimal production planning for the manufacturer's remanufacturing of parts in used products under situations where there was uncertainty in the quality of used products and the potential for quality classification errors of used products. The optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit so as to maximize the manufacturer's expected profit were discussed. If a manufacturer detects quality classification errors, corresponding to overestimated quality classification, the manufacturer reclassifies used products from a collection trader. In this case, the manufacturer incurred the unit reclassification cost to reclassify the misclassified (overestimated) used products correctly into their intrinsic quality levels and considered this cost as an additional cost for the manufacturer's expected profit. Under the situation, mathematical models were developed and theoretical analysis were conducted in order to find the effect of quality classification errors and their ratio on not only the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit, but also the expected profit of a manufacturer. Also, numerical analysis investigated how the following factors: (i) quality classification errors, (ii) quality distribution of used products, (iii) the remanufacturing cost of used products, (iv) ratio of quality classification errors, and (v) the reclassification cost of misclassified used products-affected the optimal production planning and the expected profit of a manufacturer.

The contribution of this paper is to provide the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to both academic researchers and real-world policymakers regarding production planning in a GSC:

- (1) Effect of the optimal production planning on the manufacturer's expected profit
- The combination of the optimal decisions for a lower limit of procurement quality of used products and a remanufacturing ratio under the lower limit proposed in this paper brought more profit to a manufacturer who faced not only the uncertainty in quality of used products, but also quality classification errors after purchasing used products, which were classified into a small number of quality categories, from a collection trader and needed to satisfy product demand as much as possible.
- (2) Effect of quality classification errors on optimal production planning and manufacturer's expected profit
- Quality classification errors affected the optimal production planning for a lower limit of procurement quality of used products and a remanufacturing ratio according to the quality distribution of used products which affected the quantity of quality classification

errors of used products.

- When quality classification errors occurred, the optimal production planning with quality classification errors was determined as follows: the optimal lower limit of procurement quality was determined as one higher quality level than that without it. The optimal remanufacturing ratio under optimal lower limit was determined as a higher ratio than that without it. This implied that the production quantity of the remanufactured parts decreased and the production quantity of new parts increased.
- Quality classification errors caused the reduction of the manufacturer's expected profit. The degree of the reduction of the manufacturer's expected profit was affected by the degree of increment of the reclassification cost and the remanufacturing cost according to the quantity of quality classification errors of used products, which was affected by the quality distribution of used products.
- (3) Effect of ratio of quality classification errors of used products on optimal production planning
- As the ratio of quality classification errors was higher, a manufacturer tended to change the optimal production planning so as to be gradually negative to remanufacture of recyclable parts extracted from used products and to lean to shift production of new parts using new materials.
- When the ratio of quality classification errors changed, not only the increment of the reclassification cost, the remanufacturing cost and the production cost of new parts, but also the change of the optimal production planning depended on quality distribution of used products.
- (4) Effect of reclassification cost on optimal production planning
- As the reclassification cost is lower, the optimal lower limit of procurement quality was determined as a lower value, meanwhile the optimal remanufacturing ratio under the optimal lower limit was determined a lower value. The results implied that a manufacturer tended to be positive to remanufacture the recyclable parts extracted from used products as the reclassification cost was lower.
- When the ratio of quality classification errors of used products changed, the change of the optimal production planning, which was either more positive remanufacturing or more negative remanufacturing, depended on quality distribution of used products.

Therefore, it is highly expected that research outcomes in this paper are capable of providing not only the optimal solution and its practices to construct a green supply chain with both the uncertainty in quality of used products and the quality classification errors made by firms, but also informative motivations for researchers and policymakers regarding inventory management in a green supply chain.

However, in this paper, the following topics are not discussed:

- Uncertainty in ratio of quality classification errors of used products,
- No penalty for quality classification errors is imposed on a collection trader who makes them,
- No effort of a collection trader who makes quality classification errors in order to improve them.

For future researches, it is necessary to incorporate the following extendable topics into the GSC model proposed in this paper in addition to above topics:

- Acceptable range regarding quality classification errors of used products in aspect of profit of a manufacturer,
- Optimal number of choice among multiple collection traders with different quality classification errors who sell used products and optimal procurement quantity of used products from each collection trader chosen optimally,
- Effect of uncertainty in quality classification ratio for each quality level of used products on the optimal decisions,
- Effect of the limited information regarding quality distribution of used products/reusable parts on the optimal production planning for GSC.

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