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# 평활 잔차 오류 정규화를 통한 자연 영상의 압축센싱 복원

## ( Compressive Sensing Recovery of Natural Images Using Smooth Residual Error Regularization )

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### 요 약

압축센싱은 성긴 (sparse) 신호에 대해 Nyquist rate 미만의 샘플링으로도 신호 획득이 가능하다는 것을 수학적으로 증명한 새로운 개념이다. 그동안 영상분야 압축센싱을 위한 수많은 복원 알고리즘들이 제안되어 왔으나, 낮은 측정률 하에서는 복원 화질 측면에서 아직 개선할 점이 많다. 일례로, 자연 영상의 압축센싱 복원 화질 향상을 위해, 영상과 관련한 사전 정보들로부터 정규화 식을 도출하여 복원에 적용해 볼 수 있을 것이다. 따라서, 본 논문에서는 Dantzig selector 및 평활 필터(가우시안 필터 및 nonlocal 평균 필터)기반의 평활 잔차 오류 정규화 방법을 제안한다. 또한, 복원 영상의 객체 및 배경에서 발생하는 edge 정보를 우수하게 보존하는 것으로 알려진 Total variation 기반 최소화 알고리즘에 적용하여 복원 영상의 화질을 향상시키는 방법을 제안한다. 제안하는 구조는 잔차신호의 평활화를 활용한다는 측면에서 새로운 압축센싱 복원 방식이라고 할 수 있다. 실험 결과, 제안방법은 기존 방법들에 비해 객관적 및 주관적 화질 측면에서 더 높은 성능 향상을 보여주었으며, 특히 기존 Bayesian 압축센싱 복원 방식과 비교 시 최대 9.14 dB 성능이 향상되었다.

### Abstract

Compressive Sensing (CS) is a new signal acquisition paradigm which enables sampling under Nyquist rate for a special kind of signal called sparse signal. There are plenty of CS recovery methods but their performance are still challenging, especially at a low sub-rate. For CS recovery of natural images, regularizations exploiting some prior information can be used in order to enhance CS performance. In this context, this paper addresses improving quality of reconstructed natural images based on Dantzig selector and smooth filters (i.e., Gaussian filter and nonlocal means filter) to generate a new regularization called smooth residual error regularization. Moreover, total variation has been proved for its success in preserving edge objects and boundary of reconstructed images. Therefore, effectiveness of the proposed regularization is verified by experimenting it using augmented Lagrangian total variation minimization. This framework is considered as a new CS recovery seeking smoothness in residual images. Experimental results demonstrate significant improvement of the proposed framework over some other CS recoveries both in subjective and objective qualities. In the best case, our algorithm gains up to 9.14 dB compared with the CS recovery using Bayesian framework.

**Keywords :** Compressive sensing, Total variation, Augmented Lagrangian method, Dantzig selector, Smooth filter

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## I. INTRODUCTION

Compressive sensing (CS) is an emerging framework which can reduce sampling cost for sparse or more generally for compressible signals. D. L. Donoho theoretically proved that a sparse signal which is sensed by a proper sensing matrix (e.g.,

i.i.d. Gaussian sensing matrix) can be perfectly reconstructed<sup>[1-3]</sup>. Natural images themselves are not sparse, but sparse in some transform domain such as Discrete Cosine Transform (DCT) or Discrete Wavelet Transform (DWT). The measurement vector  $b$  is sensed from an original image  $u$  using a sensing matrix  $A$  by:

$$b = Au \quad (1)$$

The reduction in number of samples to be sensed makes its recovery problem ill-posed. Accordingly, the signal  $u$  can be exactly recovered only if the original signal is sufficiently sparse, for example,  $K$ -sparse. Here,  $K$  refers to the number of maximum possible non-zero elements in the signal vector  $u$ . Often, a signal is  $K$ -term approximated before compressively sensed, that is, the signal  $u$  is transformed into a proper transform domain  $\Psi$ , and only the  $K$  largest transformed coefficients are retained in the approximated signal<sup>[4]</sup>.

In general, huge efforts have been made on developing CS reconstruction method such as,  $\ell_0$ <sup>[4]</sup>,  $\ell_1$ <sup>[4]</sup>, Bayesian Compressive Sensing<sup>[6]</sup>, Smooth Projected Landweber (SPL)<sup>[7-8]</sup>, and Total Variation (TV)<sup>[9-13]</sup>. However, quality of reconstructed images still poses a major challenge to CS. Some key factors to affect the overall quality of CS recovery are sparsity of signal, sub-rate, mutual incoherence between the sparsifying transform and the measurement matrix, and the effectiveness of reconstructed algorithms, etc.<sup>[1]</sup> Until now, most existing recovery algorithms use the measurement error as a regularization term based on the constraint  $Au = b$ , or using  $\ell_p$  norm to minimize cost  $\|Au - b\|_p \leq \epsilon_1$ , for example, with  $\ell_2$  norm, the minimization<sup>[4]</sup> is as follows:

$$\operatorname{argmin}_u \|u\|_1 \text{ s.t. } \|Au - b\|_2 \leq \epsilon_1 \quad (2)$$

The regularization in eq.(2) requires that measurements of the reconstructed image should be as similar as possible to the received measurement vector.

By the way, Candes and Tao<sup>[13]</sup> proposed a new regularization-term called the Dantzig selector (i.e.,  $\|A^T(Au - b)\|_\infty \leq \epsilon_1$ ) which is found to attain also good reconstructed images by solving the constrained problem:

$$\operatorname{argmin}_u \|u\|_1 \text{ s.t. } \|A^T(Au - b)\|_\infty \leq \epsilon_1 \quad (3)$$

In CS recovery, the reconstructed image desires to be close to the original image, so some related prior information is preferred to be used [5-12]. The pseudo inversed measurement error of  $(Au - b)$ , that is,  $(A^T A)^{-1} A^T(Au - b)$ , still composes of some detailed information of images, so we can make use of it to enhance performance of CS recoveries. However, this is not practical<sup>[13]</sup> due to its extremely high cost for (pseudo) inversing matrix of  $(A^T A)^{-1}$ . In [13], a substituted version  $A^T(Au - b)$  which can be roughly considered as a residual version of a reconstructed image which also contains image information. Hereby, a smooth filter is used to reduce noise and artifacts. As results, image information in  $A^T(Au - b)$  is clearer to see. This regularization also works well for CS recovery. The modification from the Dantzig selector is named as the smoothed residual error (SRE) regularization. Obviously, the proposed regularization makes measurement of the reconstructed image close to the received measurement as well as reduces noise in the reconstructed image. The effectiveness of our regularization is evaluated by one of the CS recovery algorithms.

TV is popularly used since it makes the reconstructed image quality sharper by well preserving the edge information and boundary compared with other recoveries<sup>[11-12]</sup>. Therefore, in this paper, we will use the TV framework to confirm our proposed method. More clearly, the proposed regularization is added to the TV optimization function, and then the augmented Lagrangian method<sup>[11-12]</sup> is brought in to minimize it. Experimental results show superior improvements of our proposed method compared with

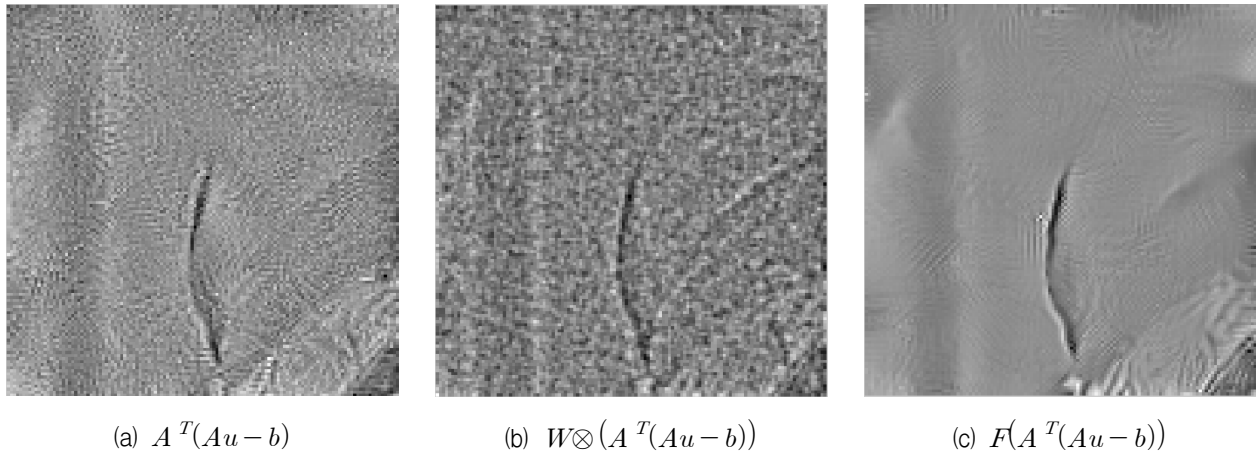


그림 1. SRE 정규화 비교 (a) Dantzig selector, (b) 가우시안 필터, (c) NLM 필터

Fig. 1. Comparison of SRE regularization (a) Dantzig selector, (b) Gaussian filter, (c) NLM filter.

the traditional regularization in eq.(2). In a nutshell, our contributions are summarized as:

- Based on the Dantzig selector, we propose a new regularization term using a smooth filter like Gaussian filter<sup>[8, 14, 19]</sup> or nonlocal means (NLM) filter<sup>[18]</sup>, called smooth residual error (SRE) regularization. This regularization not only suppresses noise and artifacts in reconstructed images, but also makes the reconstructed image closer to the original image.

- Motivated by successes of the augmented Lagrangian method<sup>[17]</sup> and TV<sup>[10-12]</sup> in solving inverse problems for CS reconstruction, we introduce a new CS recovery called augmented Lagrangian total variation using the SRE regularization (TVSRE). For general assesment, TVSRE employs two candidates including a classical filter (i.e., Gaussian filter) and a state-of-the-art filter (i.e., NLM filter).

The rest of this paper has the following structure. Section II briefly reviews the Dantzig selector as well as presents in detail the proposed SRE regularization. Implementation of the proposed SRE regularization to TV reconstruction is given in section III. Experimental results are discussed in section IV, and section V draws some conclusions.

## II. SMOOTH RESIDUAL ERROR BASED ON DANZIG SELECTOR

In this section, we first review the related work in [13] which provides a mathematical model of the Dantzig selector. After that, the proposed SRE regularization is presented in detail for CS recovery of natural images.

### 2. 1. Dantzig selector

In order to make an estimated image  $\hat{u}$  be as close to the original  $u$  as possible with a very overwhelming probability using eq.(4):

$$\| \hat{u} - u \|_2 \leq 2C^2 \log N \left( \sigma^2 + \sum_{i=1}^N \min(u_i^2, \sigma^2) \right) \quad (4)$$

where  $C$  is a constant and  $\sigma$  denotes the standard deviation of i.i.d. Gaussian noise, Candes and Tao [13] proposed a new regularization term called the Dantzig selector bounded by:

$$\| A^T(Au - b) \|_\infty \leq (1 + t^{-1})\sigma \sqrt{2 \log N} \quad (5)$$

where  $t$  is a positive scalar. In fact, eq.(5) expresses a regularization term based on the  $\ell_\infty$  norm, but in the discussion section [13], the authors also confirmed that eq.(5) is corrected for the  $\ell_1$  norm. Moreover, the Dantzig selector can be extended

to the  $\ell_2$  norm by the relationship of  $\ell_1, \ell_2$ , and  $\ell_\infty$  norms shown in Lemma 1.2<sup>[4]</sup> for  $K$ -sparse vector  $X$  (i.e.  $X \in R^K$ ):

$$\frac{\|X\|_1}{\sqrt{K}} \leq \|X\|_2 \leq \sqrt{K} \|X\|_\infty \quad (6)$$

Using the same proof in [4], it is easy to extend eq.(6) to a natural signal image  $u \in R^N$  as follows:

$$\frac{\|u\|_1}{\sqrt{N}} \leq \|u\|_2 \leq \sqrt{N} \|u\|_\infty \quad (7)$$

Exploiting eq.(7), eq.(5) is converted to  $\ell_2$  norm as:

$$\|A^T(b - Au)\|_2 \leq (1 + t^{-1})\sigma \sqrt{2N \log N} \quad (8)$$

Thanks to relationship of the  $\ell_p$  norms, eq. (8) expresses the Dantzig regularization via  $\ell_2$  norm which is easy in solving convex optimization. In this paper, eq.(8) will be further extended to create the SRE term as shown in the following subsection.

## 2. 2. The proposed SRE regularization

It is obvious that the term  $(Au - b)$  is considered as a measurement error that is highly random if the sensing matrix is a Gaussian matrix. On the contrary,  $A^T(Au - b)$  turns out to be a residual error version<sup>[13]</sup> still containing some features of the reconstructed image as illustrated in Fig. 1(a). Because  $A^T(Au - b)$  is only a roughly estimated version<sup>[13]</sup>, it includes lots of noise and artifacts (see Fig. 1 (a)). Therefore, by utilizing a proper lowpass filter for  $A^T(Au - b)$  which is expected to reduce noise, the optimization solution after the filtering is expected to be more accurate. As a result, the new regularization is modified from the Dantzig selector:

$$\|F(A^T(Au - b))\|_2 \leq \epsilon \quad (9)$$

Here,  $F$  stands for a filtering operator. Some filters as a Gaussian filter or an average filter have its own kernel, then eq.(9) is expressed by:

$$\|W \otimes (A^T(Au - b))\|_2 \leq \epsilon \quad (10)$$

where  $W$  stands for a filtering kernel and  $\otimes$  denotes the convolution operator.

Actually, incorporating a filter to a regularization is difficult to find the convergence rate and error bound<sup>[20]</sup>. Therefore, in this paper, we only experimentally indicate improvements of our proposed schemes.

Additionally, we propose a new CS reconstructed scheme based on the augmented Lagrangian TV<sup>[11~12]</sup> to evaluate the effectiveness of the proposed SRE regularization with both classical filter and state-of-the-art filter.

## III. TV RECOVERY WITH SRE REGULARIZATION (TVSRE)

This section first describes two popular smooth filters - NLM filter and Gaussian filter which will be used with our algorithm. Subsequently, implementation of the SRE regularization to TV (hereafter, TVSRE) is presented in detail.

### 3. 1. Smoothing filter selection for TVSRE

Gaussian filter is a classical filter typically used for reducing noise and artifacts<sup>[8, 12, 19]</sup>. The Gaussian filter is widely used due to its simplicity and effectiveness for images suffering much from noise. Moreover, it is also a local smoothing filter<sup>[19]</sup>. In fact, it is based on the assumption of piecewise smoothness<sup>[8, 12, 19]</sup>. In a natural image, this type of assumption is quite valid in smooth regions, but not quite so in a non-stationary regions near edge objects. In comparison to the state-of-the-art filter (i.e., NLM filter), the Gaussian filter takes much less computational complexity.

By the way, the NLM filter is deeply investigated by Buades et al.<sup>[18]</sup> and is well-known for its ability in preserving edge information of images. Because of considering similar patches, the NLM filter can preserve texture information of images. However, the

main demerit of the NLM filter is its higher computational cost than classical filters.

Fig. 1(a) shows much noise and artifacts in the Dantzig selector with the cropped image Lena. Thanks to smooth filters, Fig. 1(b) & (c) confirm reduction of noise and artifacts in the residual image compared with the Dantzig selector [13].

### 3. 2. Implementation of the proposed SRE regularization for augmented Lagrangian Total variation

In CS framework, the reconstructed images usually suffer much from noise and artifacts because they are sensed by sub-Nyquist sampling. Therefore, application of a smooth filter is necessary<sup>[7-8, 10-12]</sup>. Based on the Dantzig selector, we propose a new regularization (SRE) which reduces noise and artifacts in residual error images. In this sub-section we depict ways to apply SRE to the augmented Lagrangian TV with both Gaussian filter and NLM filter.

#### *TV reconstruction based on the SRE regularization using NLM filter (TVSRE\_N):*

The smooth residual error regularization is integrated to optimization problem as:

$$\begin{aligned} \operatorname{argmin}_u \|Du\|_p \\ \text{s.t. } \|F(A^T(Au-b))\|_2 \leq \epsilon \end{aligned} \quad (11)$$

where  $D \in (D_x, D_y)$  denotes the gradient operator composing vertical and horizontal directions, respectively. In this paper, we concentrate on a kind of Lagrange method called augmented Lagrangian and alternated direction algorithms (TVAL3)<sup>[11]</sup> that is popularly used due to its simplicity in solving problems as well as low decoding time compared with other algorithms. Additionally, the authors in [9] shows that TVAL3 also works well with block-based recovery. Due to non-differentiability of  $\ell_p$  norm<sup>[11-12]</sup>, let  $Du = w$  according to the splitting technique<sup>[9, 11]</sup>, the augmented Lagrangian method<sup>[11-12]</sup> is employed to

convert eq.(11) to the unconstrained optimization function for an-isotropic TV with  $p = 2$  as follows:

$$\operatorname{argmin}_{u,w} \left\{ \begin{aligned} &\|w\|_2 - \nu^T(Du-w) \\ &+ \frac{\beta}{2} \|Du-w\|_2^2 \\ &- \lambda^T F(A^T(Au-b)) \\ &+ \frac{\mu}{2} \|F(A^T(Au-b))\|_2^2 \end{aligned} \right\} \quad (12)$$

where  $\beta$  and  $\mu$  are positive penalty parameters, while Lagrangian multipliers  $\nu$  and  $\lambda$  are defined by:

$$\begin{aligned} \lambda &= \lambda - \mu F(A^T(Au-b)) \\ \nu &= \nu - \beta(Du-w) \end{aligned} \quad (13)$$

It is difficult to directly minimize the cost function of eq.(12) with both  $w$  and  $u$  at the same time. Thanks to the splitting technique [9, 11], eq.(12) can be treated by alternatively solving two subproblems of  $w$  and  $u$ . As efficient solution for each separate sub-problem is achieved, the whole algorithm becomes more efficient.

**w subproblem:** Given  $u$  sub-problem,  $w$  sub-problem is further minimized by the function:

$$\operatorname{argmin}_w \left\{ \begin{aligned} &\|w\|_2 - \nu^T(Du-w) \\ &+ \frac{\beta}{2} \|Du-w\|_2^2 \end{aligned} \right\} \quad (14)$$

eq.(14) is solved with the Shrinkage formula<sup>[10]</sup> with the element-wise product  $\odot$  as follows:

$$w = \max \left\{ \left\| Du - \frac{\nu}{\beta} \right\|_2 - \frac{1}{\beta}, 0 \right\} \odot \frac{(Du - \nu/\beta)}{\|Du - \nu/\beta\|_2} \quad (15)$$

**u sub-problem:** Given the  $w$  sub-problem and the cost function of  $u$  sub-problem is expressed by:

$$\operatorname{argmin}_u \left\{ \begin{aligned} &-\nu^T(Du-w) + \frac{\beta}{2} \|Du-w\|_2^2 \\ &- \lambda^T F(A^T(Au-b)) \\ &+ \frac{\mu}{2} \|F(A^T(Au-b))\|_2^2 \end{aligned} \right\} \quad (16)$$

eq.(16) is a quadratic function. Therefore, its minimization can be sought by taking the first derivative of the  $u$  sub-problem. In the minimization, calculation of the inverse matrix usually takes high cost. Similar to [11], the  $u$  sub-problem will be

solved by using steepest descent method:

$$\hat{u} = u - \eta d \tag{17}$$

where  $\eta$  denotes the Barzilai-Borwein step<sup>[11~12]</sup>, the gradient direction  $d$  is estimated by:

$$d = \left\{ \begin{array}{l} D^T(Du - w - \nu) - \kappa\lambda \\ + \mu\kappa F(A^T(Au - b)) \end{array} \right\} \tag{18}$$

Here,  $\kappa = \delta(F(A^T(Au - b)))/\delta u$ . Evaluation of  $\kappa$  is quite complicated, so eq.(18) is simplified to:

$$d \approx \left\{ \begin{array}{l} D^T(Du - w - \nu) - \kappa\lambda \\ + \mu\gamma\kappa F(A^T(Au - b)) \end{array} \right\} \tag{19}$$

where  $\gamma$  is a scaling factor making eq.(19) close to eq.(18). So far, all subproblems are handled and alternatively solved until the stopping criterion is satisfied. In a nutshell, the complete depiction of TV algorithm based on the SRE regularization using a Gaussian filter (TVSRE\_G) is stated in Table 2. For the stopping criterion, we use the ratio:

$$\frac{\|u_k - u_{k-1}\|_2^2}{\|u_k\|_2^2} \leq \zeta \tag{20}$$

The positive value  $\zeta$  is set to be close to zeros. Moreover, eq.(20) will be used for stopping criterion of both the inner loop and the outer loop as shown in Tables 1 and 2, but the stopping value of the inner loop should be larger than that of the outer loop.

표 1. NLM필터를 사용한 TVSRE 알고리즘 (TVSRE\_N)

Table 1. TVSRE algorithm using NLM filter.

---

**Input:** Measurement matrix  $A$ , measurement vector  $b$ , Lagrangian multipliers and penalty parameters.

**While** Outer stopping criteria unsatisfied **do**

**While** Inner stopping criteria unsatisfied **do**

Solve  $w$  sub-problem by computing eq.(15)

Solve  $u$  sub-problem by computing eq.(17) via calculating gradient direction by eq.(23)

**end**

Update Lagrangian multipliers  $\lambda$  and  $\nu$  by eq.(24)

**end**

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**Output:** The final CS reconstructed image

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표 2. 가우시안 필터 이용한 TVSRE 알고리즘 (TVSRE\_G)

Table 2. TVSRE algorithm using Gaussian filter. (TVSRE\_G)

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**Input:** Measurement matrix  $A$ , measurement vector  $b$ , Lagrangian multipliers and penalty parameters,

**While** Outer stopping criteria unsatisfied **do**

**While** Inner stopping criteria unsatisfied **do**

Solve  $w$  sub-problem by computing eq.(15)

Solve  $u$  sub-problem by computing eq.(17) via calculating gradient direction by eq.(18)

**end**

Update Lagrangian multipliers  $\lambda$  and  $\nu$  by eq.(13)

**end**

---

**Output:** The final CS reconstructed image

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**TV reconstruction based on the SRE regularization using Gaussian filter (TVSRE\_G):**

The Gaussian filter itself is a linear filter and it is incorporated to TV algorithms by the constrained problem:

$$\begin{array}{l} \operatorname{argmin}_u \|Du\|_p \\ \text{s.t. } \|W \otimes (A^T(Au - b))\|_2 \leq \epsilon \end{array} \tag{21}$$

Similar to TVSRE\_N, let  $Du = w$ , then the constrained optimization in eq.(21) is changed to unconstrained problem as:

$$\operatorname{argmin}_{u,w} \left\{ \begin{array}{l} \|w\|_2 - \nu^T(Du - w) \\ + \frac{\beta}{2} \|Du - w\|_2^2 \\ - \lambda^T W \otimes (A^T(Au - b)) \\ + \frac{\mu}{2} \|W \otimes (A^T(Au - b))\|_2^2 \end{array} \right\} \tag{22}$$

From eq.(22), because the  $w$  sub-problem does not depend on the filtering operator, so it can be still solved by eq.(23). The  $u$  sub-problem is solved by gradient descend as shown by eq.(17) with gradient direction calculated by:

$$d \approx \left\{ \begin{array}{l} D^T(Du - w - \nu) - \kappa\lambda \\ + \mu\gamma\kappa W \otimes (A^T(Au - b)) \end{array} \right\} \tag{23}$$

The  $u$  and  $w$  subproblems are solved to satisfy the inner stopping criteria, the Lagrangian multipliers are then updated by:

$$\begin{aligned} \lambda &= \lambda - \mu W \otimes (A^T(Au - b)) \\ \nu &= \nu - \beta(Du - w) \end{aligned} \quad (24)$$

We also apply the stopping criterion as shown by eq.(20) for TVSRE\_G which is summarized in Table 2.

## IV. EXPERIMENTAL RESULTS

### 4. 1. Test Condition

The effectiveness of the proposed algorithm is evaluated using nine natural images of size 256x256 including Barbara, Lena, House, Monarch, Boat, Parrot, Peppers, Cameraman, and Leaves as shown in Fig. 2. The penalty parameters are empirically selected to attain the best quality of reconstructed image in terms of PSNR ( $\mu = 512, \beta = 32$ ). The standard deviation of Gaussian kernel is set 0.5 and its window size is 3x3. The searching window of the NLM filter is 13x13 while its neighborhood window is 7x7. The NLM filter's decay parameter is equal to 0.15. Moreover, in our experiments, CS measurements are acquired by using a Gaussian random matrix. The value of stopping criterion in eq.(20) are set to  $10^{-4}$  for the inner loop, and  $10^{-5}$  for the outer loop.



그림 2. 원본 자연 영상  
Fig. 2. Original natural images.

### 4. 2. Objective Quality Evaluation

Table 3 compares conventional method of TVAL3<sup>[11]</sup>, and our two proposed methods, namely, TVSRE\_G and TVSRE\_N. It is worth emphasizing that TVAL3<sup>[11]</sup> is one of state-of-the-art algorithms giving quite good recovered images. Obviously, our two proposed algorithms outperform TVAL3<sup>[11]</sup> in terms of PSNR. In the best case, TVSRE\_G gains PSNR of 0.90 dB over TVAL3<sup>[11]</sup> with image Monarch at subrate 0.3, whilst the maximum PSNR gain of TVSRE\_N is up to 2.49 dB for fine textured image Barbara at subrate 0.3 compared with TVAL3<sup>[11]</sup>. For smooth images like House or Monarch in Table 3, the performance of Gaussian filter and NLM filter are similar to each other. However, for finer detailed image like Barbara or image with details of low contrast like Parrot, the NLM filter gains much in PSNR over the Gaussian filter (i.e., for image Parrot at subrate 0.1, PSNR of TVSRE\_N is 27.75dB while it is only 26.55dB if using TVSRE\_G).

Table 4 compares two proposed methods with four other non-TV algorithms including tree-structured CS with variational Bayesian analysis using Discrete Cosine Transform (TSDCT)<sup>[6]</sup>, Smooth Projected Landweber using Discrete Cosine Transform (SPLDCT<sup>[7]</sup>), Smooth Projected Landweber using Discrete Wavelet Transform (SPLDWT<sup>[7]</sup>), and Smooth Projected Landweber using Dual-tree Discrete Wavelet Transform (SPLDDWT<sup>[7]</sup>). Once again, the two proposed methods demonstrate superiority of performance over other non-TV CS recoveries for all tested images and sub-rates. On average of six tested images, TVSRE\_N gains better PSNR than TSDCT<sup>[6]</sup> by about 4.11 dB. Of course, thanks to the effectiveness of the NLM filter, TVSRE\_N averagely outperforms TSDCT<sup>[6]</sup> by up to 4.53dB. TSRE\_G and TSRE\_N are not only better than TSDCT<sup>[6]</sup>, but also show better performance than the family of Smoothed Landweber algorithms<sup>[11]</sup> (i.e., SPLDCT, SPLDWT, and SPLDDWT). Compared with SPLDCT<sup>[7]</sup>, TVSRE\_G

표 3. 여러 Total variation기반 압축센싱 복원 방법 비교 (PSNR: dB)

Table 3. Comparison of various compressive sensing total variation recoveries (PSNR: dB).

Image	Recovery	Sub-rate				
		0.1	0.15	0.2	0.25	0.3
Barbara	TVAL3[11]	22.51	23.36	24.23	25.01	26.03
	TVSRE_G	22.55	23.51	24.58	25.46	26.59
	TVSRE_N	<b>22.88</b>	<b>24.46</b>	<b>25.84</b>	<b>27.14</b>	<b>28.52</b>
Lena	TVAL3[11]	26.06	27.56	29.02	30.17	31.34
	TVSRE_G	26.35	28.08	29.46	30.65	31.85
	TVSRE_N	<b>26.38</b>	<b>28.15</b>	<b>29.71</b>	<b>31.08</b>	<b>32.09</b>
House	TVAL3[11]	29.93	31.91	33.21	34.22	35.19
	TVSRE_G	30.53	32.32	33.70	34.77	35.61
	TVSRE_N	<b>30.76</b>	<b>32.53</b>	<b>33.83</b>	<b>34.92</b>	<b>35.79</b>
Monarch	TVAL3[11]	23.81	26.39	28.20	30.04	31.62
	TVSRE_G	24.58	27.27	29.22	<b>31.17</b>	<b>32.52</b>
	TVSRE_N	<b>24.77</b>	<b>27.43</b>	<b>29.53</b>	31.16	32.49
Boat	TVAL3[11]	25.09	26.89	28.48	29.87	31.02
	TVSRE_G	<b>25.81</b>	<b>27.63</b>	<b>29.32</b>	<b>30.75</b>	31.89
	TVSRE_N	25.49	27.43	29.24	30.63	<b>31.96</b>
Parrot	TVAL3[11]	26.02	28.49	30.40	31.90	33.58
	TVSRE_G	26.55	29.05	31.08	32.61	33.82
	TVSRE_N	<b>27.75</b>	<b>30.00</b>	<b>31.82</b>	<b>33.40</b>	<b>34.49</b>
Peppers	TVAL3[11]	24.70	27.18	29.18	30.91	32.28
	TVSRE_G	25.47	28.01	30.03	31.65	33.03
	TVSRE_N	<b>26.08</b>	<b>28.69</b>	<b>30.52</b>	<b>32.00</b>	<b>34.49</b>
Cameraman	TVAL3[11]	24.72	26.53	27.92	29.06	30.06
	TVSRE_G	24.84	26.72	28.02	29.31	30.28
	TVSRE_N	<b>25.22</b>	<b>27.35</b>	<b>28.56</b>	<b>29.58</b>	<b>30.48</b>
Leaves	TVAL3[11]	18.93	21.15	23.19	24.95	26.80
	TVSRE_G	<b>19.86</b>	<b>22.35</b>	24.58	26.51	28.20
	TVSRE_N	19.56	22.27	<b>24.66</b>	<b>26.68</b>	<b>28.43</b>

표 4. 여러 압축센싱 복원 방법 비교 (PSNR: dB)

Table 4. Comparison of various compressive sensing recovery methods (PSNR: dB).

Image	Subrate	Recovery					
		TSDCT[6]	SPLDCT[7]	SPLDWT[7]	SPLDDWT[7]	TVSRE_G	TVSRE_N
Barbara	0.1	19.77	22.71	22.52	22.84	22.55	<b>22.88</b>
	0.15	22.78	23.55	23.35	23.64	23.51	<b>24.46</b>
	0.2	23.58	24.44	23.96	24.33	24.58	<b>25.84</b>
	0.25	24.64	25.09	24.74	25.00	25.46	<b>27.14</b>
	0.3	25.45	26.16	25.42	25.67	26.59	<b>28.52</b>
Lena	0.1	17.24	24.35	24.94	25.31	26.35	<b>26.38</b>
	0.15	24.71	25.77	26.43	26.87	28.08	<b>28.15</b>
	0.2	26.48	26.89	27.62	28.11	29.46	<b>29.71</b>
	0.25	27.71	27.83	28.55	28.99	30.65	<b>31.08</b>
	0.3	28.76	28.80	29.57	30.08	31.85	<b>32.09</b>
House	0.1	23.86	26.35	26.90	26.95	30.53	<b>30.76</b>
	0.15	27.45	28.34	29.01	29.17	32.32	<b>32.53</b>
	0.2	29.28	29.77	30.37	30.54	33.70	<b>33.83</b>



	0.25	30.79	30.88	31.47	31.74	34.77	<b>34.92</b>
	0.3	31.96	32.03	32.52	32.81	35.61	<b>35.79</b>
Monarch	0.1	19.54	21.13	21.45	21.80	24.58	<b>24.77</b>
	0.15	21.83	22.72	23.11	23.68	27.27	<b>27.43</b>
	0.2	23.36	24.25	24.72	25.26	29.22	<b>29.53</b>
	0.25	24.53	25.36	25.90	26.41	<b>31.17</b>	31.16
	0.3	25.62	26.72	27.19	27.80	<b>32.52</b>	32.49
Boat	0.1	22.32	24.02	24.41	24.58	<b>25.81</b>	25.49
	0.15	24.80	25.39	25.73	25.99	<b>27.63</b>	27.43
	0.2	26.43	26.52	26.79	27.02	<b>29.32</b>	29.24
	0.25	27.59	27.70	27.84	28.05	<b>30.75</b>	30.63
	0.3	28.97	28.67	28.81	29.02	31.89	<b>31.96</b>
Parrot	0.1	22.39	23.78	23.59	23.67	26.55	<b>27.75</b>
	0.15	24.37	24.80	24.41	24.78	29.05	<b>30.00</b>
	0.2	25.66	26.25	26.04	26.37	31.08	<b>31.82</b>
	0.25	26.55	26.93	26.71	27.23	32.61	<b>33.82</b>
	0.3	27.52	28.13	28.34	28.88	33.82	<b>34.49</b>
Peppers	0.1	21.01	23.70	24.45	24.58	25.47	<b>26.08</b>
	0.15	23.14	23.81	26.18	26.28	28.01	<b>28.69</b>
	0.2	24.57	25.10	27.79	27.84	30.03	<b>30.52</b>
	0.25	26.84	27.43	28.73	28.71	31.65	<b>32.00</b>
	0.3	28.38	26.61	29.89	29.79	33.03	<b>34.49</b>
Camera	0.1	20.18	21.59	21.77	21.64	24.84	<b>25.22</b>
	0.15	21.95	23.05	23.01	23.42	26.72	<b>27.35</b>
	0.2	23.02	24.27	24.57	24.79	28.02	<b>28.56</b>
	0.25	23.87	25.05	25.78	25.89	29.31	<b>29.58</b>
	0.3	24.78	26.01	26.77	27.02	30.28	<b>30.48</b>
Leaves	0.1	16.69	17.94	18.35	18.66	<b>19.86</b>	19.56
	0.15	18.98	19.32	19.56	19.98	<b>22.35</b>	22.27
	0.2	20.75	20.72	20.99	21.37	24.58	<b>24.66</b>
	0.25	22.09	21.66	21.99	22.36	26.51	<b>26.68</b>
	0.3	23.40	22.62	22.99	23.30	28.20	<b>28.43</b>

and TVSRE\_N are better by up to 5.81 dB and 6.47 dB, respectively. Finally, for the best Smoothed Landweber method<sup>[7]</sup> (SPLDDWT), TVSRE\_N gains by up to 6.17 dB with image Parrot at subrate 0.25. In case of TVSRE\_G, it gains PSNR better than SPLDDWT<sup>[7]</sup> by up to 5.38 dB.

#### 4. 3. Subjective Quality Evaluation

Visual quality of the recovered images (Boat and Barbara) for seven algorithms are compared in Fig. 3 and Fig. 4, respectively. They verify that the two proposed algorithms (i.e., TVSRE\_G and TVSRE\_N) attain good subjective quality. Moreover, a new image quality assessment model, Feature Similarity

(FSIM) is further used to evaluate the visual quality<sup>[15]</sup>. The higher FSIM value is, the better the visual quality is. TVSRE\_N achieves the highest FSIM score for all images, which again demonstrates the effectiveness of the proposed smooth residual error regularization. Because of over-smoothed problem, the Gaussian filter does not achieve good subjective quality compared with some SPL algorithms<sup>[7]</sup> such as SPLDCT and SPLDDWT. However, TVSRE\_G still achieves better visual quality than SPLDWT<sup>[7]</sup> and TSDCT<sup>[7]</sup>. Of course, thanks to Gaussian filter, noise and artifacts are reduced much in reconstructed image Barbara, so TVSRE\_G shows a better visual quality than TVAL3<sup>[11]</sup>.



그림 3. 여러 압축센싱 복원방법에서의 복원 영상(Boat) 화질 비교 (측정율 : 0.3)  
 Fig. 3 Visual quality comparison of a smooth image (image Boat) at subrate 0.3.



그림 4. 여러 압축센싱 복원방법에서의 복원 영상 (Barbara) 화질 비교 (측정율 : 0.3)  
 Fig. 4. Visual quality comparison of fine detail image (image Barbara) at subrate 0.3.

### V. CONCLUSION

In this paper, a new regularization called the smooth residual error (SRE) regularization is

proposed for CS recovery, which employs the Dantzig selector and a smooth filter. In per iteration, a smooth filter is exploited to reduce noise of the regularization  $A^T(Au - b)$ . The proposed

regularization takes two advantages at the same time: i) it makes the measurements of the reconstructed image closer to the received measurement vector  $b$ , and ii) it reduces noise in the reconstructed image. The SRE regularization is manifested effective by a proposed augmented Lagrangian algorithm, namely, total variation based on SRE regularization using Gaussian filter and NLM filter. Experimental results showed significant improvement of the proposed method both in subjective and objective qualities over other existing algorithms.

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