The Asymptotic Throughput and Connectivity of Cognitive Radio Networks with Directional Transmission

Zhiqing Wei, Zhiyong Feng, Qixun Zhang, Wei Li, and T. Aaron Gulliver

Abstract: **Throughput scaling laws for two coexisting ad hoc networks with m primary users (PUs) and n secondary users (SUs) randomly distributed in an unit area have been widely studied. Early work showed that the secondary network performs as well as stand-alone networks, namely, the per-node throughput of the secondary networks is $\Theta(1/\sqrt{n \log n})$. In this paper, we show that by exploiting directional spectrum opportunities in secondary network, the throughput of secondary network can be improved. If the beamwidth of secondary transmitter (TX)'s main lobe is $\delta = o(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n \log n})$ for directional transmission and omni reception (DTOR), which is $\Theta(\log n)$ times higher than the throughput without directional transmission. On the contrary, if $\delta = \omega(1/\log n)$, the throughput gain of SUs is $2\pi/\delta$ for DTOR compared with the throughput without directional antennas. Similarly, we have derived the throughput for other cases of directional transmission.**

The connectivity is another critical metric to evaluate the performance of random ad hoc networks. The relation between the number of SUs n and the number of PUs m is assumed to be $n = m^\beta$. We show that with the HDP-VDP routing scheme, which is widely employed in the analysis of throughput scaling laws of ad hoc networks, the connectivity of a single SU can be guaranteed when $\beta > 1$, and the connectivity of a single secondary path can be guaranteed when $\beta > 2$. While circumventing routing can improve the connectivity of cognitive radio ad hoc network, we verify that the connectivity of a single SU as well as a single secondary path can be guaranteed when $\beta > 1$. Thus, to achieve the connectivity of secondary networks, the density of SUs should be (asymptotically) bigger than that of PUs.

Index Terms: **Cognitive radio networks, connectivity, directional transmission, spectrum holes, throughput scaling laws.**

I. INTRODUCTION

Cognitive radio (CR) [1] is one of the most promising technologies for efficient spectrum utilization. It enables flexible and comprehensive use of the available spectrum [2], and allows for optimization of radio resource utilization by exploiting spectrum holes. The spectrum holes are the available spectrum for secondary users (SUs), and can exist in several dimensions such as frequency, time, geographical space, code, and angle [3]. The available spectrum holes (or spectrum opportunities) can be used to improve SU spectrum access and throughput. In this paper, we evaluate the performance of cognitive radio networks (CRNs) that exploiting directional spectrum opportunities in the view of throughput and connectivity. Throughput is the efficiency of a network while connectivity is the reliability of a network. Throughput together with connectivity can reveal more complete features of cognitive radio networks. Therefore, we study the throughput and connectivity.

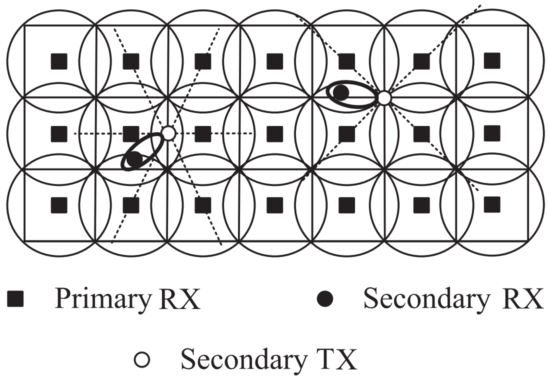

Fig. 1. SUs exploiting directional spectrum opportunities.

The throughput of large scale wireless networks has been widely explored since the seminal work of Gupta and Kumar [4]. For n ad hoc nodes randomly distributed in an unit area, they showed that the per-node throughput is $\Theta(1/\sqrt{n \log n})$. A recent study of throughput scaling laws for cognitive radio ad hoc networks (CRAHNs) produced similar results. In [5], Jeon *et al.* showed that in a heterogeneous environment with n SUs and m primary users (PUs) coexisting, the secondary network can achieve per-node throughput of $\Theta(1/\sqrt{n \log n})$. In [6], Yin *et al.* achieved the same results with a more practical assumption that SUs only know the (geographic) location of the PU transmitters (TX). This result was verified by Huang *et al.* [7] for a general model.

In the existing literature on CRAHN throughput scaling laws, only spectrum holes in space and time domains are exploited. The throughput scaling laws for CRAHNs that exploit directional spectrum holes have not yet been investigated. Yi *et al.* [8] and Li *et al.* [9] showed that a traditional ad hoc network with directional transmission can achieve a better per-node throughput higher than $\Theta(1/\sqrt{n \log n})$. In [10], Zhao *et al.* showed that by exploiting directional spectrum holes, both the probability of successful communication and the spectrum efficiency

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of the cognitive radio network are improved. Zhang *et al.* in [11] have analyzed the capacity of wireless mesh networks with omni and directional antennas. They showed that there is a capacity gain for wireless mesh networks when using directional antennas. Further, Dai *et al.* in [12] have investigated the capacity of multi-channel wireless networks using directional antennas. They have shown that deploying directional antennas to multi-channel networks can greatly improve the network capacity due to increased network connectivity and reduced interference. Wang *et al.* in [13] investigated spectrum holes in four dimensions, i.e., time, frequency, location and direction. All of these results show that directional transmission in secondary networks can increase the spectrum opportunities of SUs and improve the performance of secondary network. In this paper, we analyze the throughput gain of SUs with directional transmission. Consider the example illustrated in Fig. 1. The PU preservation regions (shown as circles), cover the entire area. Thus, according to the network protocols in [5] and [6], the SUs have no transmission opportunities. However, if the SUs transmit directionally, the interference to PUs can be mitigated and the throughput of secondary network is improved.

In this paper, we verify that if the beamwidth of secondary TX's main lobe is $\delta = o(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for directional transmission and omni reception (DTOR), which is $\Theta(\log n)$ times higher than the throughput without directional transmission. Similarly, if the beamwidth of secondary receiver (RX)'s main lobe is $\phi = o(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for omni transmission and directional reception (OTDR). If $\delta\phi = \omega(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for directional transmission and directional reception (DTDR). On the contrary, if $\delta = \omega(1/\log n)$, the throughput gain of SUs is $2\pi/\delta$ for DTOR compared with the throughput without directional transmission. If $\delta\phi = \omega(1/\log n)$, the throughput gain of SUs is $2\pi/\phi$ for OTDR. If $\delta\phi = \omega(1/\log n)$, throughput gain of SUs is $4\pi^2/\delta\phi$ for DTDR.

The connectivity is another critical metric to evaluate the performance of random ad hoc networks. The asymptotic connectivity of large-scale wireless networks has been widely explored since the seminal work of Gupta and Kumar [14]. Assuming n ad-hoc nodes are randomly distributed in a disc of unit area, they showed that if each node transmits at a power level so as to cover an area of $\pi r^2 = (\log n + c(n))/n$, then the resulting network can achieve 1-connectivity if and only if $c(n) \to \infty$ [4], i.e., there exists a path between any pair of nodes with high probability (*w.h.p.*). Zhang *et al.* in [15] considered the k-connectivity, i.e., at least k disjoint paths exist between any pair of nodes. Due to the interaction between PUs and SUs, the connectivity of a CRAHN differs from that of traditional homogeneous ad hoc networks. In cognitive radio network, the channels are dynamic because of the spatial and temporal dynamics of the primary traffic [16]. Ren *et al.* in [16] determined the delay and connectivity scaling behavior in ad hoc cognitive radio networks using the theories of continuum percolation and ergodicity. Ao *et al.* in [17] investigated the connectivity of cooperative cognitive radio network from a percolation-based perspective. In [18], Abbagnale *et al.* proposed a Laplacian matrix based method to measure the connectivity of CRAHNs.

Table 1. Key acronyms and parameters.

Symbol	Description
PU	Primary user.
SU	Secondary user.
HDP	Horizontal data path.
VDP	Vertical data path.
TX, RX	Transmitter and receiver.
m	The number of PUs.
n	The number of SUs.
a_s	The area of a secondary cell.
Δ, δ	The beamwidth of SU's TX beam.
Φ, ϕ	The beamwidth of SU's RX beam.
R_p, R_s	Data rate of PU and SU.
$\lambda_p(m)$, $\lambda_s(n)$	Per-node throughput of PUs and SUs.

However, most of the existing literature on CRAHN connectivity does not consider the routing scheme in the connectivity analysis. Investigating connectivity without considering the routing is not practical, since the routing schemes in most networks are relatively simple and can not find the complex path. In this paper, we show that the routing scheme has a significant influence on connectivity of CRAHN. Assuming two coexisting ad hoc networks with m PUs and n SUs randomly distributed in a unit area. And the relation between n and m is $n = m^\beta$. With the horizontal data paths-vercical data paths (HDP-VDP) routing scheme, which is widely adopted in the analysis of throughput scaling laws of ad hoc networks, when $\beta > 1$, we can guarantee the connectivity of a single SU, when $\beta > 2$, we can guarantee the connectivity of a single secondary path. While circumventing routing can improve the connectivity of CRAHN, we show that when $\beta > 1$, we can guarantee the connectivity of a single SU as well as the connectivity of a single secondary path. Thus, to achieve the connectivity in secondary networks, the density of SUs must be bigger than that of PUs asymptotically and a smart routing scheme is essential. Our research may guide the deployment and routing design of cognitive radio networks.

The rest of this paper is organized as follows. The network protocol and definitions are presented in Section II. The concept of directional spectrum holes and the corresponding statistics are provided in Section III. In Section IV, the throughput scaling laws network protocols (asymptotic throughput) are derived. In Section V, the connectivity of cognitive network is investigated. This paper is summarized in Section VI.

II. NETWORK PROTOCOL AND DEFINITIONS

We consider m PUs and n SUs uniformly distributed in an unit square, which share the same space and spectrum. The PUs act as if the SUs do not exist while the SUs must mitigate the interference to PUs. We assume that $n = m^\beta$, where β is a positive number. The channel power gain is $g(r) = 1/r^\alpha$, where r is the distance between TX and RX, and $\alpha > 2$ is the path loss exponent. Both the primary and secondary networks are random ad hoc networks. PUs act as if SUs do not exist, while SUs must

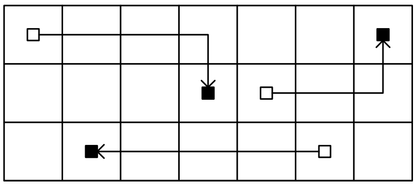

Fig. 2. The HDP and VDP.

mitigate the interference to PUs. The network protocols of SUs are as follows.

- Divide the unit square into small squares with area $a_s = K \log n/n, K > 1$. Such a small square is called a secondary cell. A *preservation region* is a square containing 9 secondary cells with a primary RX in the center cell. When a secondary TX falls into this region, it detects the directional spectrum opportunities. If there are no spectrum opportunities, it buffers the data and waits for other temporal or directional spectrum opportunities. The SUs employ the multi-hop routing protocol.
- SUs employ a 9-ime division multiple access (TDMA) transmission scheme. Similar to the approach in [6], the duration of a secondary frame is equal to that of one primary time slot (the reader is referred to Fig. 2 in [6] for details).
- SU's traffic is transmitted along the HDPs and VDPs. A secondary TX searches for directional spectrum holes using a DOA algorithm. If a spectrum hole exists, it transmits with power $P_1 a_s^{\alpha/2}$, where P_1 is a constant. Different from the apporaches in [5] and [6], *more than one* secondary TX can transmit in an active secondary cell as long as they can find the spectrum opportunities.

A. Achievable Rate and Throughput

The rate of the ith primary TX-RX pair is

$$R_p(i) = \log\left(1 + \frac{P_p(i) g\left(\|X_{p,\mathrm{tx}}(i) - X_{p,\mathrm{rx}}(i)\|\right)}{N_0 + I_p(i) + I_{sp}(i)}\right) \quad (1)$$

where $P_p(i)$ is the transmit power of the ith primary TX, and $X_{p,\mathrm{tx}}(i)$ and $X_{p,\mathrm{rx}}(i)$ are the positions of the ith primary TX and RX, respectively. $I_p(i)$ is the aggregate interference received by the ith primary RX from the primary network, $I_{sp}(i)$ is the aggregate interference from the SU network to the ith primary RX, and N_0 is the power spectral density of the additive white Gaussian noise (AWGN). In this paper, we normalize the bandwidth to 1. The data rate of the jth secondary TX-RX pair is

$$R_s(j) = \log\left(1 + \frac{P_s(j) g\left(\|X_{s,\mathrm{tx}} - X_{s,\mathrm{rx}}\|\right)}{N_0 + I_s(j) + I_{ps}(j)}\right) \quad (2)$$

where $P_s(j)$ is the transmit power of the jth secondary TX, and $X_{s,\mathrm{tx}}(j)$ and $X_{s,\mathrm{rx}}(j)$ are the positions of the jth secondary TX and RX, respectively. $I_s(j)$ is the aggregate interference received by jth secondary RX from the secondary network, and $I_{ps}(j)$ is the aggregate interference from the PU network to the jth secondary RX. We denote the PU and SU per-node throughput as $\lambda_p(m)$ and $\lambda_s(n)$, respectively [6].

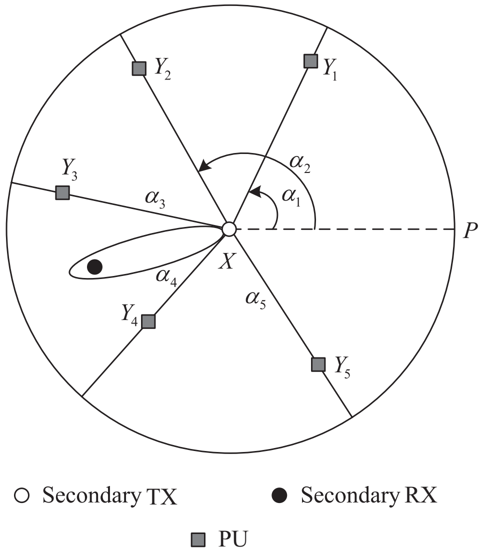

Fig. 3. Directional spectrum opportunities for SUs.

B. Definitions of Connectivity

As to the connectivity, because of the presence of PUs, a secondary cell may falls into the preservation region and result in outage for a secondary path. Suppose m PUs are randomly distributed in the unit area, where m can be regarded as the number of *active* PUs if we consider the dynamics of PUs' traffic in temporal dimension. We provide three definitions of connectivity.

1. Connectivity of a single node: For a specific secondary node, if it is not an isolated node *w.h.p.*, then we can guarantee the connectivity of the single node.
2. Connectivity of a single path: For a specific secondary source-destination pair, if there exists a path between them *w.h.p.*, then we can guarantee the connectivity of the single path.
3. Connectivity of network (connectivity of all nodes): If all the nodes is not isolated *w.h.p.*, then we can guarantee connectivity of all nodes.

III. DIRECTIONAL SPECTRUM OPPORTUNITIES

In this section, firstly the notion of directional spectrum opportunity (hole) is introduced. Then, the statistics of directional spectrum opportunities is addressed in Lemma 2. Finally, we investigate the number of PUs in the interference region of SU in Lemma 3.

Fig. 3 shows a circular area with SU at the center and several randomly distributed PUs. Denote the location of the SU as X, and the locations of the PUs as $Y_1, Y_2, \cdots, Y_L$, where L is the number of PUs surrounding this SU. For example, $L = 5$ in Fig. 3. The reference line with angle 0 is shown as the dashed line XP. Denote the angle between XP and XY_i as α_i. Then, the angle $\angle Y_i X Y_{i+1}$ for $i < L$ is

$$\angle Y_i X Y_{i+1} = \alpha_{i+1} - \alpha_i \triangleq \Delta_i. \quad (3)$$

Δ_i represents a directional spectrum opportunity (or spectrum hole) for the SU. If the SU is able to transmit within this angle to a secondary RX, then the interference to the PUs will be small (if the side lobes of the directional antenna are neglected). When $i = L$, the angle $\angle Y_L X Y_1$ is

$$\angle Y_L X Y_1 = \alpha_1 - \alpha_L + 2\pi \triangleq \Delta_L \quad (4)$$

which is another directional spectrum opportunity. Direction of arrival (DOA) estimation algorithms can be employed in SUs to find these directional spectrum opportunities. To determine the angles with multiple PUs, multiple signal classification (MUSIC) algorithms [24] can be used. In addition, an antenna array can be used in SUs to detect the directional spectrum opportunities and also for directional transmission.

For an SU with L PUs around it, there are L directional spectrum opportunities (L angles). As L increases, these angles will decrease and thus it will be more difficult to exploit the directional spectrum holes. The statistics of these directional spectrum opportunities are given by the following lemmas.

Lemma 1 ([25]) For an ascending sequence of random variable (r.v.)s $X_1, X_2, \cdots, X_L$ uniformly distributed in $[0, 1]$, the probability density function (PDF) of the range of X_r and X_s, with $r < s$, is

$$f_{W_{rs}}(w_{rs}) = \frac{1}{B(s-r, L-s+r+1)} w_{rs}^{s-r-1}(1-w_{rs})^{L-s+r} \quad (5)$$

where $0 \le w_{rs} \le 1$ and

$$B(a, b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt, \ a > 0, \ b > 0. \quad (6)$$

Lemma 2: The PDF of the angle of a directional spectrum hole (denoted as Δ) is

$$f_\Delta(\delta) = \frac{L\left(1 - \frac{\delta}{2\pi}\right)^{L-1}}{2\pi}, 0 \le \delta \le 2\pi \quad (7)$$

with expected value

$$E[\Delta] = \frac{2\pi}{L+1}. \quad (8)$$

Proof: The sequence of the angles $\Delta_1, \Delta_2, \cdots, \Delta_L$ are r.v.s. They are not independent since

$$\sum_{i=0}^{L} \Delta_i = 2\pi. \quad (9)$$

However, as shown in Fig. 3, the angle $\alpha_i (i = 1, 2, \cdots, L)$ is independent and identically distributed (i.i.d.) r.v. with uniform distribution between 0 and 2π. The PDF of α_i is

$$f(\alpha) = \frac{1}{2\pi}, 0 < \alpha \le 2\pi. \quad (10)$$

These angles are in ascending order, i.e., $\alpha_1 \le \alpha_2 \le \cdots \le \alpha_L$. In addition, $\delta_i = \alpha_{i+1} - \alpha_i, i < L$, is the range of the r.v.'s α_{i+1} and α_i. In the theory of order statistics [25], the PDF of the range of uniformly distributed r.v.'s is given by Lemma 1. Let $s = r + 1$ in this lemma so that $W_r = X_{r+1} - X_r$, which has a PDF given by

$$f_{W_r}(w_r) = \frac{1}{B(1, L)}(1 - w_r)^{L-1}, 0 \le w_r \le 1. \quad (11)$$

Let $X_i = x_i = \alpha_i / 2\pi \sim U(0, 1)$. If $W_r = X_{r+1} - X_r$ we have $\Delta_r = \delta_r = \alpha_{r+1} - \alpha_r = 2\pi W_r$, and from (11) the PDF is

$$f_{\Delta_r}(\delta_r) = \frac{L\left(1 - \frac{\delta_r}{2\pi}\right)^{L-1}}{2\pi}, 0 \le \delta_r \le 2\pi \quad (12)$$

where the relation $B(1, L) = 1/L$ is used. Although the Δ_i's are not independent, they are identically distributed. Omitting the index r, the PDF of the directional spectrum holes is

$$f_\Delta(\delta) = \frac{L\left(1 - \frac{\delta}{2\pi}\right)^{L-1}}{2\pi}, 0 \le \delta \le 2\pi \quad (13)$$

with expected value

$$E[\Delta] = \int_0^{2\pi} \delta f_\Delta(\delta) d\delta = \frac{2\pi}{L+1}. \quad (14)$$

□

When the angle of the main lobe Δ has a lower bound δ_{th}, according to Lemma 2, the probability that a directional spectrum hole is available is given by

$$\Pr\{\Delta > \delta_{\text{th}}\} = \left(1 - \frac{\delta_{\text{th}}}{2\pi}\right)^L \to 0 \, (\text{when } L \to \infty) \quad (15)$$

where L is the number of PUs in the interference region of SU. An *interference region* is a square containing 9 secondary cells with an SU in the center cell. Note that the L PUs in (15) are in a circle, while the interference region is a square. However, the difference between these shapes can be ignored, as the (tight) upper and lower bounds on $\Pr\{\Delta > \delta_{\text{th}}\}$ can be achieved by addressing the PUs in the inscribed circle or circumscribed circle of the interference region. The number of PUs L in an interference region is given in the following lemma.

Lemma 3: Denote the number of PUs in the interference region of an SU as L, which is a function of n (or m). Then, for different values of β, we have three cases as follows.

1. If $\beta > 1$, then for any positive number ε,

$$\lim_{n\to\infty} \Pr\{L \ge \varepsilon\} = 0 \quad (16)$$

namely, L converges to 0 in probability.

2. If $\beta = 1$, we have

$$9K\left(\frac{1}{2} - \frac{1}{e}\right)\log n \le L \le 9Ke\log n \quad (17)$$

w.h.p., namely, $l = \Theta(\log n)$.

3. If $\beta < 1$, we have

$$9K\left(\frac{1}{2} - \frac{1}{e}\right)\log n \le L \le 9Ken^{1/\beta - 1}\log n \quad (18)$$

w.h.p..

Proof: This lemma investigates the number of PUs in the interference region of SUs. For a particular secondary cell, a PU $X_i, i = 1, 2, \cdots, m$ falls in its interference region with probability $p_n = 9a_s = 9K\log n/n$, which is a Bernoulli event. Denote the number of PUs in the interference region as L, which is a r.v.

follows Bernoulli distribution with parameters (p_n, m). And the expectation of L is

$$E[L] = p_n m = 9Kn^{1/\beta-1}\log n. \tag{19}$$

Notice that when $\beta > 1$, $\lim_{n\to\infty} E[L] = 0$. We use the Markov inequality to determine the upper bound of L. For any positive number ε, we have

$$\Pr\{L > \varepsilon\} \le \frac{E[L]}{\varepsilon} \to 0. \tag{20}$$

Let $\varepsilon = 1/2$, then we have $\Pr\{L > 1/2\} \to 0$. As L is a positive integer, thus L converges to 0 in probability.

When $\beta \le 1$, we use the Chernoff bound to get the lower bound of L, which is

$$\begin{aligned}\Pr\{L \le a\} &\le \min_{t<0} \frac{E\left[e^{tL}\right]}{e^{ta}} \\ &\overset{(a)}{=} \min_{t<0} \frac{\left(1+(e^t-1)p_n\right)^m}{e^{ta}} \\ &\overset{(b)}{\le} \frac{\left(1+(e^{-\phi}-1)p_n\right)^m}{e^{-\phi a}} \\ &\overset{(c)}{\le} \frac{\exp\left(\frac{m\log n^{9K(e^{-\phi}-1)}}{n}\right)}{e^{-\phi a}}\end{aligned} \tag{21}$$

where $(21a)$ is derived by substituting the value of $E[e^{tL}]$, $(21b)$ is derived by replacing t with a negative constant $-\phi$. According to the inequality $1 + x \le e^x$, we have $\left(1+(e^{-\phi}-1)p_n\right)^m \le \exp\left(m\log n^{9K(e^{-\phi}-1)}/n\right)$ and $(21c)$ is derived. For $\beta \le 1$, substitute $\phi = 1$, $m = n^{1/\beta}$ and $a = 9K\left(1/2 - 1/e\right)\log n$ into (21), we have

$$\begin{aligned}\Pr\{L \le a\} &\le \frac{\exp\left(\frac{n^{1/\beta}\log n^{9K(e^{-1}-1)}}{n}\right)}{\exp\left(-9K(\frac{1}{2}-e^{-1})\log n\right)} \\ &\overset{(d)}{\le} \frac{\exp\left(\log n^{9K(e^{-1}-1)}\right)}{\exp\left(-9K(\frac{1}{2}-e^{-1})\log n\right)} \\ &= \frac{n^{9K(e^{-1}-1)}}{n^{-9K(\frac{1}{2}-e^{-1})}} = \frac{1}{n^{9K(\frac{1}{2}-e^{-1})}}\end{aligned} \tag{22}$$

where $(22d)$ is due to $n^{1/\beta} \ge n$. Since $9K(1/2 - e^{-1}) > 0$, we have $1/n^{9K(1/2-e^{-1})} \to 0$. Thus, for $\beta \le 1$, the lower bound of L is $9K(1/2 - e^{-1})\log n$ *w.h.p.*. Actually, we have $9K(1/2 - e^{-1}) > 1$ because $K > 1$. Use the union bound, the probability that the number of PUs in *any* secondary cell is more than $9K(1/2 - e^{-1})\log n$ PUs is smaller than $n/\left(K\log n \times n^{9K(1/2-e^{-1})}\right) \to 0$, which means the number of PUs in all SUs' interference regions are lower bounded by $9K(1/2 - e^{-1})\log n$.

To find the upper bound of L, we use the Chernoff bound.

$$\begin{aligned}\Pr\{L \ge a\} &\le \min_{t>0} \frac{E\left[e^{tL}\right]}{e^{ta}} \\ &= \min_{t>0} \frac{\left(1+(e^t-1)p_n\right)^m}{e^{ta}} \\ &\overset{(e)}{\le} \frac{\left(1+(e^{\phi}-1)p_n\right)^m}{e^{\phi a}} \\ &\overset{(f)}{\le} \frac{\exp\left(\frac{m\log n^{9K(e^{\phi}-1)}}{n}\right)}{e^{\phi a}}\end{aligned} \tag{23}$$

where $(23e)$ and $(23f)$ use the similar technique as $(21b)$ and $(21c)$ respectively. And ϕ is a positive number. We address two situations as follows.

- For the case $\beta = 1$, substitute $\phi = 1$, $m = n$ and $a = 9Ke\log n$ into (23), we have

$$\begin{aligned}\Pr\{L \ge a\} &\le \frac{\exp\left(\frac{n\log n^{9K(e-1)}}{n}\right)}{e^{9Ke\log n}} \\ &= \frac{n^{9K(e-1)}}{n^{9Ke}} = \frac{1}{n^{9K}}.\end{aligned} \tag{24}$$

  As $9K > 0$, we have $1/n^{9K} \to 0$. Thus, for $\beta = 1$, the upper bound of L is $9Ke\log n$ *w.h.p.*. Similar with previous discussion, as we have the relation $9K > 1$ because $K > 1$, use the union bound, the probability that the number of PUs in *any* secondary cell is more than $9Ke\log n$ is smaller than $n/(K\log n \times n^{9K}) \to 0$.
- For the case $\beta < 1$, substitute $m = n^{1/\beta}$ and $a = 9Ken^{1/\beta-1}\log n$ into (23), we have

$$\begin{aligned}\Pr\{L > a\} &\le \frac{\exp\left(\frac{n^{1/\beta}\log n^{9K(e-1)}}{n}\right)}{e^{9Ken^{1/\beta-1}\log n}} \\ &= \frac{1}{n^{9Kn^{1/\beta-1}}}.\end{aligned} \tag{25}$$

  As $9Kn^{1/\beta-1} > 0$, we have $1/n^{9Kn^{1/\beta-1}} \to 0$, which means $9Ken^{1/\beta-1}\log n$ is the upper bound of L *w.h.p.* Similar with previous discussion, we have $9Kn^{1/\beta-1} > 1$, thus $n/(K\log n \times n^{9Kn^{1/\beta-1}}) \to 0$. Namely, the probability that the number of PUs in *any* secondary cell is more than $9Ken^{1/\beta-1}\log n$ tends to 0.

□

According to Lemma 3 and Lemma 2, spectrum opportunities will exist when $\beta > 1$. Thus, when investigating the throughput with directional transmission, we only consider the situation when $\beta > 1$. In this paper, we investigate the cases of omnidirectional transmission and omnidirectional reception (OTOR), DTOR, OTDR, and DTDR.

IV. ASYMPTOTIC THROUGHPUT

In order to derive the per-node throughput of primary and secondary networks, we should investigate the data rate that a primary/secondary cell can support and the number of concurrent routings that pass through a primary/secondary cell. In the

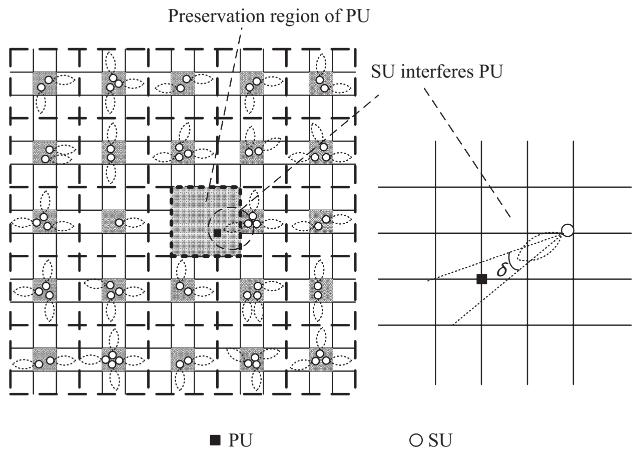

Fig. 4. The interference from a secondary TX to a primary RX. The dashed boxes denote the 9-TDMA cluster and the shaded squares are the active secondary cells.

derivation of per-node throughput of primary networks, Lemma 4 has addressed the data rate that a primary cell can support and Lemma 5 has investigated the number of primary paths that pass through a primary cell. In the derivation of per-node throughput of secondary networks, Lemma 6, Lemma 7, Lemma 8, and Lemma 9 have investigated the data rate that a secondary cell can support. Because the secondary and primary networks both adopt the HDP-VDP routing protocol, the number of secondary paths that pass through a secondary cell is similar to that in primary networks.

A. Throughput of Primary Network

To determine the PU throughput, we require the following two lemmas.

Lemma 4: If one primary RX receives interference from at most one secondary TX in a secondary cell, then each primary cell can support a data rate of K_1, where K_1 is a constant independent of m or n.

Proof: The data rate of the ith primary cell is

$$R_p(i) = \frac{1}{9}\log\left(1+\frac{P_p(i)g\left(\|X_{p,\mathrm{tx}}(i)-X_{p,\mathrm{rx}}(i)\|\right)}{N_0+I_p(i)+I_{sp}(i)}\right) \quad (26)$$

where $1/9$ denotes the rate loss due to 9-TDMA transmission, $I_p(i)$ is the interference suffered by primary RX i within primary network and $I_{sp}(i)$ is the interference suffered by primary RX i from secondary TXs. If one primary RX receives interference from at most one secondary TX in a secondary cell, both [5] and [6] have proved that $I_p(i)$ and $I_{sp}(i)$ are finite when SUs adopt the 9-TDMA (or 25-TDMA) protocol. Hence, the interference suffered by primary RX is bounded. Thus, we have

$$I_{sp}(i) \le I_{sp} < \infty, \quad (27)$$

$$I_p(i) \le I_p < \infty \quad (28)$$

so that

$$\begin{aligned} R_p(i) >& \frac{1}{9}\log\left(1+\frac{P_0\left(\sqrt{a_p}\right)^{\alpha}\left(\sqrt{5a_p}\right)^{-\alpha}}{N_0+I_p+I_{sp}}\right) \\ =& \frac{1}{9}\log\left(1+\frac{P_0\left(\sqrt{5}\right)^{-\alpha}}{N_0+I_p+I_{sp}}\right) \triangleq K_1 < \infty. \end{aligned} \quad (29)$$

□

As an example of Lemma 4, for the case of DTOR in Fig. 4, if the main lobes of the secondary TXs in a secondary cell do not overlap, each primary cell can support a constant data rate of K_1.

Lemma 5: With the primary network protocol in [5], the number of primary source-destination paths passing through a primary cell is upper bounded by $4\sqrt{2m\log m}$ *w.h.p.*.

The PU throughput is given in the following theorem.

Theorem 1: The primary network can achieve a per-node throughput of $\lambda_p(m) = \Theta(1/\sqrt{m\log m})$ *w.h.p.*.

Proof: If Lemmas 4 and 5 are satisfied, the results in [5], [6], and [21] show that there exists a TDMA transmission scheme such that a data rate of K_1 is shared by $4\sqrt{2m\log m}$ paths. We only need to divide the entire time slot into $\Theta(\sqrt{m\log m})$ small time slots and each transmission use one of them. Thus, the PU per-node throughput is $\Theta(1/\sqrt{m\log m})$. □

B. Throughput of Secondary Network: OTOR

The case of OTOR has been investigated in [5] and [6], and they have shown that the per-node throughput of secondary network is $\Theta(1/\sqrt{n\log n})$, which is the same as a stand-alone network.

C. Throughput of Secondary Network: DTOR

For SU TXs in secondary cells, we have the following lemma.

Lemma 6: If one primary RX receives interference from at most one secondary TX in a secondary cell, then each TX in the secondary cell can support a data rate of K_2, where K_2 is a constant independent of m or n.

Proof: The proof is similar to that of Lemma 4. □

For the case of DTOR, if the main lobes of the secondary TXs in a secondary cell do not overlap with each other, then each secondary cell can support a data rate of K_2. Denote the angle of the ith secondary TX's main lobe by Δ_i. To obtain the highest throughput in the secondary network, we adopt the most compact case where $\sum_{i=1}^{\xi}\Delta_i = 2\pi$, i.e., the directional opportunities are all occupied by secondary TXs. In this case, the data rate that a secondary cell can support will be ξK_2, which is ξ times bigger than for a secondary cell that does not use directional transmission. If the SU's main lobe is $\delta = o\left(1/\log n\right)$, we have the following lemma.

Lemma 7: If the angle of the secondary TX's main lobe is $\delta = o\left(1/\log n\right)$, then the data rate that each secondary cell can support is $\Theta(\log n)$ *w.h.p.*.

Proof: According to Lemma 8, the number of SUs in a secondary cell is $\Theta\left(\log n\right)$. When the angle of the secondary TX main lobe is $\delta = o\left(1/\log n\right)$, all secondary TX nodes in an active secondary cell can transmit with the constraint that the main lobes do not overlap. Since a secondary cell has $\Theta(\log n) = c_2 \log n$ TX nodes, from Lemma 6, the data rate that a secondary cell can support is $c_2 K_2 \log n = \Theta(\log n)$. □

Lemma 8: If $K(1/2 - 1/e) > 1$, then the number of SUs in a secondary cell is $\Theta(\log n)$.

Proof: According to [21, Lemma 5.7], an upper bound on the number of SUs in a secondary cell is $\Theta(\log n)$. Using an approach similar to the proof of Lemma 3, the condition $K(1/2 - 1/e) > 1$ gives that the lower bound is also $\Theta(\log n)$. □

Lemma 9: If the angle of the secondary TX's main lobe is $\delta = \omega(1/\log n)$, then the achievable data rate that each secondary cell can support is $K_2 2\pi/\delta$.

Proof: When $\beta > 1$, directional spectrum holes exists *w.h.p.* (Lemma 3), but the number of SUs in a secondary cell is $\Theta(\log n) \to \infty$. Thus, the number of main lobes (namely, the number of *active secondary TXs*) ξ is asymptotically smaller than the number of SUs. And the maximum value of ξ is $2\pi/\delta$. □

Similar to Lemma 5, the number of secondary source-destination paths passing through each secondary cell is upper bounded by $4\sqrt{2n \log n}$ *w.h.p.*. Using Lemmas 6, 7, and 9, the SU throughput is given in the following theorem.

Theorem 2: In the case of DTOR, with the given secondary network protocol, when the main lobe of secondary TX is $\delta = o(1/\log n)$, the secondary network can achieve a per-node throughput of $\lambda_s(n) = \Theta(\sqrt{\log n/n})$ *w.h.p.*. When $\delta = \omega(1/\log n)$, the throughput gain of secondary network is $2\pi/\delta$.

Proof: When $\delta = o(1/\log n)$, one secondary cell can support a data rate of $Kc_2 \log n$, which is shared by $4\sqrt{2n \log n}$ paths. Thus, the per-node throughput is $\Theta(1/\sqrt{n \log n})$. When $\delta = \omega(1/\log n)$, the gain of data rate in the secondary cell is $2\pi/\delta$ compared with the secondary networks without directional transmission, and the per-node throughput gain is also $2\pi/\delta$. When SUs have no transmission opportunities and need to buffer their data, the data rate should be multiplied by an opportunistic factor [6]. However, this factor is a constant that will not change the scaling results obtained. In fact, for $\beta > 1$, the opportunistic factor is 1 *w.h.p.* because directional spectrum opportunities always exist. □

D. Throughput of Secondary Network: OTDR

For the case of OTDR, denote the angle of the main lobe of secondary RX as ϕ. Then, if $\delta\phi = \omega(1/\log n)$, the achievable data rate that each secondary cell can support is $K_2 2\pi/\phi$. If $\phi = o(1/\log n)$, the achievable data rate that each secondary cell can support is $Kc_2 \log n$. Thus, we have the conclusion as follow.

Theorem 3: In the case of OTDR, with the given secondary network protocol, when the main lobe of secondary RX is $\phi = o(1/\log n)$, the secondary network can achieve a per-node throughput of $\lambda_s(n) = \Theta(\sqrt{\log n/n})$ *w.h.p.*. When $\delta\phi = \omega(1/\log n)$, the throughput gain of secondary network is $2\pi/\phi$.

E. Throughput of Secondary Network: DTDR

For the case of OTDR, if $\delta\phi = \omega(1/\log n)$, the achievable data rate that each secondary cell can support is $K_2 4\pi^2/\delta\phi$. If $\delta\phi = o(1/\log n)$, the achievable data rate that each secondary cell can support is $Kc_2 \log n$. Thus, we have the conclusion as follow.

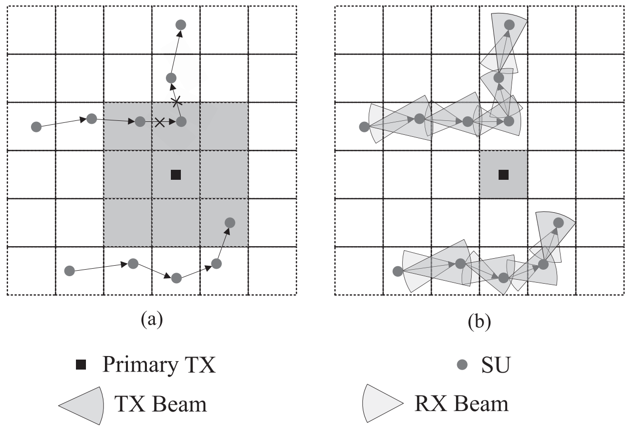

Fig. 5. SUs encounter the preservation region, where the shaded part is the preservation region of a primary RX and "×" means an outage: (a) The OTOR case and (b) the DTDR case.

Theorem 4: In the case of OTDR, with the given secondary network protocol, when the main lobe of secondary RX is $\delta\phi = o(1/\log n)$, the secondary network can achieve a per-node throughput of $\lambda_s(n) = \Theta(\sqrt{\log n/n})$ *w.h.p.*. When $\delta\phi = \omega(1/\log n)$, the throughput gain of secondary network is $4\pi^2/\delta\phi$.

V. ASYMPTOTIC CONNECTIVITY

SUs with the network protocol in Section II can guarantee connectivity if PUs do not exist, i.e., there are at least one SU in each secondary cell *w.h.p.*. However, as the presence of PUs, a secondary cell may falls into the preservation region and will result in outage for a secondary path, as illustrated in Fig. 5(a). However, SUs in the preservation region can still receive data, which is also illustrated in Fig. 5(a). In this paper, we suppose m PUs are randomly distributed in the unit area, where m can be regarded as the number of *active* PUs if we consider the dynamics of PUs' traffic in temporal dimension. It is noted that n and m are also the density of SUs and PUs respectively, since the SUs and PUs are deployed in a square with unit area. With directional transmission and direction reception, the preservation region is reduce and the probability of outage event is also reduced, which is illustrated in Fig. 5(b).

A. Connectivity of Cognitive Networks with HDP-VDP Routing

With the network protocol in Section II, an SU is not an isolated node if and only if it does not fall into the preservation region of any PU. Thus, the connectivity of SUs is related with PUs and we firstly investigate the number of PUs in the *interference region* of SUs, where interference region is a square containing 9 secondary cells with a secondary TX in the center cell. If a PU falls into the *interference region* of a SU, then the SU falls into the *preservation region* of the PU. The number of PUs L in an interference region of SU is given in Lemma 3.

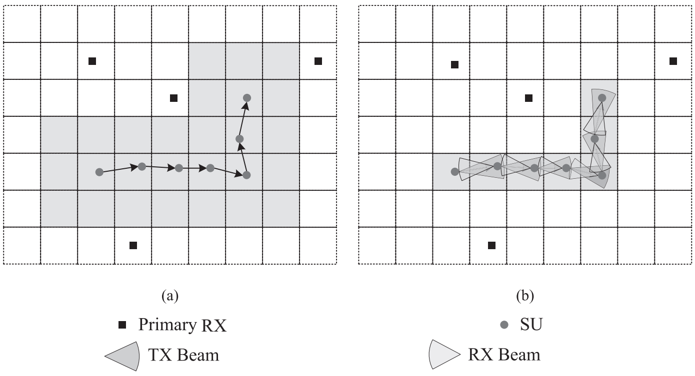

Fig. 6. The connectivity of a secondary path if there are no PUs in the shaded region, this path is connected: (a) The OTOR case and (b) the DTDR case.

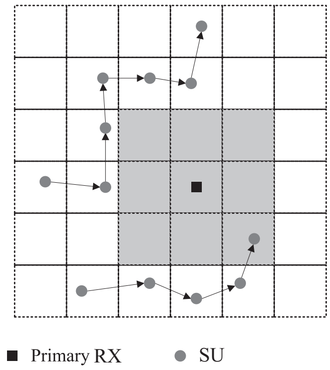

Fig. 7. Secondary path goes around if it is blocked by a preservation region.

A.1 Connectivity of a Secondary Node

Since we assume $n = m^{\beta}$, the lower and upper bounds expressed by n can also be expressed by m, Thus, we have another version of Lemma 3 using m. When $\beta > 1$, the number of PUs in the interference region of a specific SU tends to 0 in probability, thus we have a theorem.

Theorem 5: When $\beta > 1$, with the HDP-VDP routing, we can guarantee the connectivity of a single SU *w.h.p.*.

A.2 Connectivity of a Secondary Path

We define the interference region of a secondary path in this section. As illustrated in Fig. 6, the shaded region is the interference region of the secondary path, which is the union of the interference regions of all the SUs along this path. The PUs in the interference region of the secondary path will suffer the interference from secondary TX. We investigate the connectivity of a secondary path and have a theorem.

Theorem 6: When $\beta > 2$, with the HDP-VDP routing, we can guarantee the connectivity of a single secondary path *w.h.p.*.

Proof: As illustrated in Fig. 6, if there are no PUs in the interference regions of the secondary path, then the connectivity of this path can be guaranteed. As the distance between the source and destination of this path is $\Theta(1)$, thus the number of hops is

$$M = \Theta\left(\frac{1}{\sqrt{a_s}}\right) = \Theta\left(\sqrt{\frac{n}{\log n}}\right) \triangleq c_1\sqrt{\frac{n}{\log n}} \tag{30}$$

where c_1 is a constant, a_s is the area of a secondary cell. For the case of OTOR, the number of secondary cells along the secondary path is

$$\begin{aligned} N &= c_1\sqrt{\frac{n}{\log n}} + \left(c_1\sqrt{\frac{n}{\log n}} - 2\right) + \left(c_1\sqrt{\frac{n}{\log n}} + 2\right) \\ &= 3c_1\sqrt{\frac{n}{\log n}}. \end{aligned} \tag{31}$$

For the case of DTOR, OTDR, and DTDR, the number of secondary cells along the secondary path is less than $N = 3c_1\sqrt{n/\log n}$ in (31), which is illustrated in Fig. 6(b). We investigate the probability that there are no PUs in $N = 3c_1\sqrt{n/\log n}$ secondary cells. Denote the number of PUs in N secondary cells as Ω. Similar with the proof in Lemma 3, Ω is a r.v. follows Bernoulli distribution with parameters (p_n^*, m), where

$$p_n^* = 3c_1\sqrt{\frac{n}{\log n}}\frac{K\log n}{n} = 3Kc_1\sqrt{\frac{\log n}{n}}. \tag{32}$$

Thus, the expectation of Ω is

$$E[\Omega] = p_n^* m = 3Kc_1 n^{\frac{1}{\beta}-\frac{1}{2}}\sqrt{\log n}. \tag{33}$$

When $\beta > 2$, we have $E[\Omega] \to 0$. Use Markov inequality, for any positive number ε, we have

$$\Pr\{\Omega > \varepsilon\} \le \frac{E[\Omega]}{\varepsilon} \to 0. \tag{34}$$

Let $\varepsilon = 1/2$, then $\Pr\{\Omega > 1/2\} \to 0$. As Ω is an integer, thus there are no PUs along a secondary path *w.h.p.*.

When $1 < \beta < 2$, use a similar technique as Lemma 3, we can get the lower bound of Ω as $3Kc_1(1/2 - e^{-1})\sqrt{\log n}$, which does not converge to 0 when $n \to \infty$. Thus, $\beta > 2$ is the necessary and sufficient condition to guarantee the connectivity of a secondary path for HDP-VDP routing scheme. □

Note that for the case of DTOR, OTDR, and DTDR, the number of secondary cells along the secondary path is of order $\Theta\left(\sqrt{n/\log n}\right)$. Thus, Theorem 5 and 6 are also satisfied for the case of DTOR, OTDR, and DTDR.

A.3 Connectivity of All SUs

As PUs and SUs share the same geographic area, thus there must be an secondary cell that contains at least one PU, then the connectivity of the secondary path originated from this secondary cell cannot be guaranteed. Thus, the connectivity of all SUs is not feasible for SUs and PUs coexisted network.

B. Connectivity of Cognitive Networks with Circumventing Routing

According to Theorem 6, to guarantee the connectivity of a secondary path, we must have $\beta > 2$, namely, the SUs must be much denser than PUs. To achieve a better connectivity when the SUs is not that dense, the secondary path can go around if it is blocked by a preservation region of PU. As illustrated in Fig.

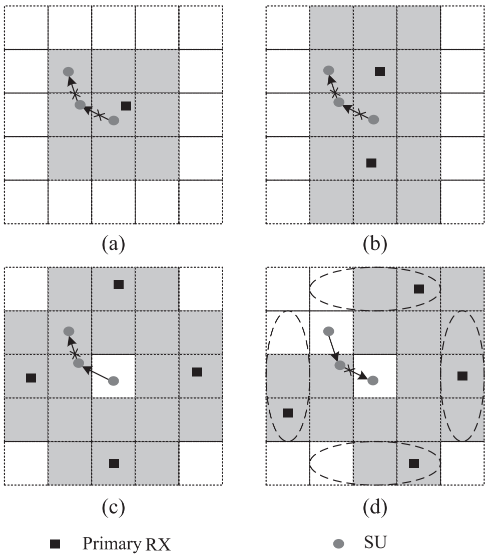

Fig. 8. PUs block the transmission of SU, where "×" means an outage.

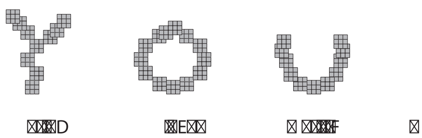

Fig. 9. The preservation region chain (PRC): (a) Open structure, (b) closed structure, and (c) open structure.

7, which is proposed in [5] but the connectivity is not fully analyzed therein. With this circumventing routing, we summarize the outage cases for SUs.

1. The source of a secondary path falls in a preservation region of PU, which is illustrated in Figs. 8(a) and (b), where the source of a secondary path is located in the preservation region, thus an outage will occur.
2. When the preservation regions enclose a secondary source or destination, this secondary path will be in outage, which is illustrated in Figs. 8(c) and (d).

To investigate the connectivity of a secondary path with circumventing routing, we firstly define the PRC as follow.

Definition 1: PRC is a set of preservation regions, that each preservation region in PRC has al least one adjacent neighbors (preservations regions).

Some of the PRCs are listed in Fig. 9, where Fig. 9(a) and 9(c) can not block a secondary path, while Fig. 9(b) can block the secondary path that has a source or destination inside it. Before investigating the PRC, we need a definition.

Definition 2: Primary cell: Divide the unit square into $m/2\log m$ small squares, which is defined as "primary cell". The area of a primary cell is $a_p = 2\log m/m$.

There is a lemma related with the primary cell, which is Lemma 5.7 in [21].

Lemma 10: Each primary cell holds at least one but no more than $2e\log m$ PUs w.h.p..

As to the PRC, we have a lemma as follow.

Lemma 11: If $\beta > 1$, then any preservation region chain (PRC) cannot pass through a primary cell w.h.p..

Proof: If $\beta > 1$, then a primary cell contains N_m secondary cells and we have

$$N_m = \Theta\left(\frac{\frac{2\log m}{m}}{\frac{K\log n}{n}}\right) = c_2 m^{\beta-1} \tag{35}$$

where c_2 is a constant. If a PRC pass through a primary cell, it needs at least $\sqrt{N_m}/3 = \sqrt{c_2}/3m^{\beta-1/2}$ PUs in this primary cell, where the preservation regions of this PRC form a line and are not overlapped. According to Lemma 10, the number of PUs in a primary cell is at most $2e\log m$ and we notice that $2e\log m < \sqrt{N_m}/3$ when m is sufficiently large. Thus, there are no sufficient PUs in *any* primary cell that can support a PRC that pass through this primary cell. □

As to the maximum number of preservation regions in a PRC, we have a lemma as follow.

Lemma 12: For $\beta > 1$, the number of preservation regions in any PRC is at most $18e\log m$ w.h.p.

Proof: We choose a preservation region in a PRC. When $\beta > 1$, according to Lemma 11, the PRC can not pass through a primary cell. Thus, the PRC is confined in a cluster of 9 primary cells where the center cell contains a preservation region of the PRC. According to Lemma 10, the number of PUs in cluster of 9 primary cells is at most $18e\log m$, thus the maximum number of preservation regions in a PRC is at most $18e\log m$ *w.h.p.*. □

If SUs adopt the circumventing routing, we have a lemma.

Lemma 13: For $\beta > 1$, the number of SUs that is blocked by preservation regions or PRCs is at most $\Theta(m(\log m)^2)$ w.h.p.

Proof: We denote the area of all preservation regions as A_1 and the area of the isolated area enclosed by PRCs as A_2. Thus, the area of the regions where SUs are blocked by PUs is $A = A_1 + A_2$. When the preservation regions are not overlapped, A_1 has an upper bound.

$$A_1 \le 9m\frac{K\log n}{n} = 9Kn^{1/\beta-1}\log n. \tag{36}$$

As illustrated in Fig. 10, the maximum isolated area that is environed by a PRC is a quarter circle. A PRC has at most $18e\log m$ preservation regions (Lemma 12), and each preservation region contribute a length of $6\sqrt{a_s}$ to the circumference of the quarter circle at most, thus the radius of the quarter circle is $(216e\sqrt{a_s}\log m)/\pi$ at most. We suppose that each PRC enclose a quarter circle and can find an upper bound of the isolated area surrounded by PRCs.

$$\begin{aligned} A_2 &\le \frac{m}{18e\log m}\frac{\pi}{4}\left(\frac{216e\sqrt{a_s}\log m}{\pi}\right)^2 \\ &= \frac{648e}{\pi}a_s m\log m. \end{aligned} \tag{37}$$

According to (36) and (37), we have

$$A \le 9\beta\left(K + \frac{72e}{\pi}K\log m\right)m^{1-\beta}\log m \triangleq A_u. \tag{38}$$

We investigate the number of SUs in the block region with area A, which is denoted as N_A and is a r.v. follows Bernoulli distribution with parameters (A_u, n). Use the Chernoff bound, we

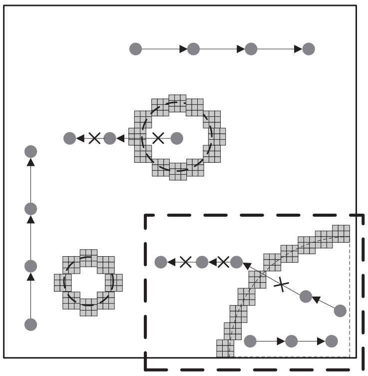

Fig. 10. The largest block area of PUs, where "×" means an outage. This figure is a modification of Fig. 8 in [5].

will show $N_A \leq A_u en$ *w.h.p.*

$$\begin{aligned}\Pr\{N_A > A_u en\} &\leq \min_{t>0} \frac{E\left[e^{tN_A}\right]}{e^{tA_u en}} \\ &\overset{(a)}{=} \min_{t>0} \frac{(1+(e^t-1)A_u)^n}{\exp(tA_u en)} \\ &\overset{(b)}{\leq} \frac{(1+(e-1)A_u)^n}{\exp(enA_u)} \\ &\overset{(c)}{\leq} \frac{\exp((e-1)nA_u)}{\exp(enA_u)} \\ &= \exp(-nA_u) \to 0\end{aligned} \tag{39}$$

where $(39a)$ is derived by substituting the value of $E[e^{tN_A}]$, $(39b)$ is derived by replacing t with 1. According to the inequality $1+x \leq e^x$, we can achieve $(39c)$. According to (39), we have the upper bound of N_A, which is $A_u en = \Theta(m(\log m)^2)$ □

According to Lemma 13, the fraction of SUs that is blocked by PUs is

$$\frac{1}{n}\Theta(m(\log m)^2) = \Theta(\frac{(\log m)^2}{m^{\beta-1}}) = \Theta(\frac{(\log n)^2}{n^{1-1/\beta}}) \to 0. \tag{40}$$

Thus, we have a theorem as follow.

Theorem 7: When $\beta > 1$, with the circumventing routing, we can guarantee the connectivity of a single SU *w.h.p.*

Thus, the connectivity of a single secondary node can be guaranteed with circumventing routing when $\beta > 1$. As to the connectivity of a single secondary path, we have a theorem.

Theorem 8: When $\beta > 1$, with the circumventing routing, we can guarantee the connectivity of a single secondary path *w.h.p.*.

Proof: The source and destination nodes are not blocked by PRCs *w.h.p.*. Besides, the PRCs cannot block the secondary path. As in Fig. 11, a secondary path arrives at a primary cell, to block this secondary path, the PRCs must enclose the primary cell (or at least an edge of the primary cell). However, according to Lemma 11, a PRC cannot pass through the primary cell, thus PRCs are impossible to enclose the primary cell and block the secondary path. The secondary path will circumvent the PRCs and reach the destination, as illustrated in Fig. 11. □

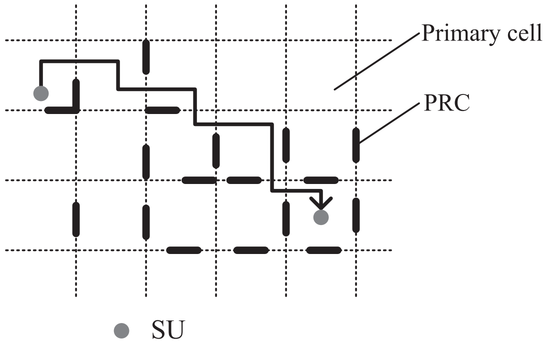

Fig. 11. The circumventing routing.

However, the circumventing routing still cannot guarantee the connectivity of the whole secondary networks (all SUs). Besides, when $\beta \leq 1$, according to Lemma 3, the number of PUs in the interference region of a SU is at least 1 *w.h.p.*, thus the preservation regions will always cover the SUs. Even adopting the circumventing routing, the connectivity of a single all SUs is still impossible. Finally, for the case of DTOR, OTDR, and DTDR, when $\beta > 1$, with the circumventing routing, we can guarantee the connectivity of a single secondary path *w.h.p.*, namely, the condition for the connectivity of OTOR is the sufficient condition of connectivity for DTOR, OTDR, and DTDR.

VI. CONCLUSION

In this paper, we show that the throughput of secondary network can be improved by exploiting directional spectrum opportunities. If the beamwidth of secondary TX's main lobe is $\delta = o(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for DTOR, which is $\Theta(\log n)$ times higher than the throughput without directional transmission. Similarly, if the beamwidth of secondary RX's main lobe is $\phi = o(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for OTDR. If $\delta\phi = \omega(1/\log n)$, SUs can achieve a per-node throughput of $\Theta(1/\sqrt{n\log n})$ for DTDR. On the contrary, if $\delta = \omega(1/\log n)$, the throughput gain of SUs is $2\pi/\delta$ for DTOR compared with the throughput without directional transmission. If $\delta\phi = \omega(1/\log n)$, the throughput gain of SUs is $2\pi/\phi$ for OTDR. If $\delta\phi = \omega(1/\log n)$, throughput gain of SUs is $4\pi^2/\delta\phi$ for DTDR.

We also investigate the connectivity of random cognitive radio networks with different routing schemes, namely, the HDP-VDP routing and circumventing routing. Assuming two coexisting ad hoc networks with m PUs and n SUs randomly distributed in a unit area and $n = m^\beta$, we have achieved some conclusions as follows.

1. If we adopt the HDP-VDP routing, then when $\beta > 1$, we can guarantee the connectivity of a single SU, when $\beta > 2$, we can guarantee the connectivity of a single secondary path.
2. If we use the circumventing routing, then when $\beta > 1$, we can guarantee the connectivity of a single SU as well as the connectivity of a single secondary path.
3. We can not guarantee the connectivity of the whole secondary network no matter what β is.
4. When $\beta \leq 1$, we can not guarantee the connectivity for both

the HDP-VDP routing and circumventing routing.

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