A Novel Spectrum Access Strategy with α-Retry Policy in Cognitive Radio Networks: A Queueing-Based Analysis

Yuan Zhao, Shunfu Jin, and Wuyi Yue

Abstract: **In cognitive radio networks, the packet transmissions of the secondary users (SUs) can be interrupted randomly by the primary users (PUs). That is to say, the PU packets have preemptive priority over the SU packets. In order to enhance the quality of service (QoS) for the SUs, we propose a spectrum access strategy with an α-Retry policy. A buffer is deployed for the SU packets. An interrupted SU packet will return to the buffer with probability α for later retrial, or leave the system with probability $(1-\alpha)$. For mathematical analysis, we build a preemptive priority queue and model the spectrum access strategy with an α-Retry policy as a two-dimensional discrete-time Markov chain (DTMC). We give the transition probability matrix of the Markov chain and obtain the steady-state distribution. Accordingly, we derive the formulas for the blocked rate, the forced dropping rate, the throughput and the average delay of the SU packets. With numerical results, we show the influence of the retrial probability for the strategy proposed in this paper on different performance measures. Finally, based on the trade-off between different performance measures, we construct a cost function and optimize the retrial probabilities with respect to different system parameters by employing an iterative algorithm.**

Index Terms: **α-Retry policy, cognitive radio networks, discrete-time Markov chain (DTMC), priority queue, spectrum access strategy.**

I. INTRODUCTION

Spectrum demand has greatly increased in the last two decades due to the proliferation of emerging wireless services and products. However, many research studies have shown that the utility of the spectrum is very low under conventional static spectrum access strategies [1]. For example, some of the spectrum utilization is no more than 6% [2]. In order to better utilize the spectrum, the Federal Communication Community (FCC) has proposed an opportunistic spectrum access strategy, which is the basis of cognitive radio networks [3].

There are two kinds of users in cognitive radio networks, namely, primary users (PUs) and secondary users (SUs). The SUs can use the licensed spectrum temporally when the spectrum is not being used by the PUs [4].

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Y. Zhao and S. Jin are with School of Information Science and Engineering, Yanshan University, Qinhuangdao, China and The Key Laboratory for Computer Virtual Technology and System Integration of Hebei Province, Yanshan University, Qinhuangdao, China, emails: yuanzh85@163.com, jsf@ysu.edu.cn.

W. Yue is with Department of Intelligence and Informatics, Konan University, Kobe, Japan, email: yue@konan-u.ac.jp.

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Generally speaking, cognitive radio networks can be classified into two categories: A slotted structure and an unslotted structure. In the slotted structure, the time axis is divided into time slots. All PUs and SUs are synchronized at the slot boundaries. The arrivals and departures of the system users can occur simultaneously, so the mathematical analysis for a slotted network is more complex. However, wireless networks are more often digital, and slotted models are more accurate and efficient when designing digital transmitting systems. In [5], the authors considered a slotted cognitive radio network, in which the SUs could be selected to send packets by carrier sense multiple access/collision avoidance (CSMA/CA) with a request to send/clear to send (RTS/CTS) mechanism. They built a three-dimensional Markov chain and obtained some performance measures, such as the throughput and the packet delay. In [6], the authors considered a slotted cognitive radio network, in which multiple SUs contended to access wireless channels. They assumed that each SU stochastically determined whether or not to access a wireless channel with a fixed probability.

Also, in the literature related to the performance evaluation of cognitive radio networks, queueing theory is widely used. In [7], the authors built an M/M/1 queue and an $\mathrm{M/G}/S/N$ queue to evaluate the spectrum access latency of the SUs. In [8], the authors studied the impact of multiple channels on the latency of the SUs by using a fluid queueing model.

As stated earlier, for the two types of users in cognitive radio networks, the PUs have higher priority over the SUs. The priority queueing systems which enable the modeling of non-identical behaviors of different types of customers are suitable for analyzing the performance of users in cognitive radio networks [9]. In [10], the author proposed a new priority discipline called the T-preemptive priority discipline in cognitive radio networks, and derived the waiting-time distribution of the SUs with an M/G/1 priority queue. In [11], the authors investigated the expected system delay for the SUs in a multi-channel cognitive radio network with an M/G/1 preemptive repeat priority queuing model. In [12], the authors proposed a dynamic channel bonding strategy for cognitive radio networks. By building a discrete-time priority queueing model, they derived the blocking ratio, the throughput, the average latency of the SUs and the closed channel ratio.

Due to the preemptive priority of PUs, the transmission of an SU packet is possible to be interrupted randomly by a newly arriving PU packet. Some existing studies assumed that the interrupted SU packets would definitely return to the queue of the SU packets and would wait for a future transmission [12], [13]. Obviously, this kind of return with probability 1, i.e., 1 persistent retrial transmission, may cause a greater delay for the SU packets. On the other hand, some studies assumed that the interrupted SU packets were forced to leave the system forever when there were no idle channels [14], [15]. Clearly, in this way, the dropping rate for the SU packets is higher. Considering the trade-off between the delay and the dropping rate of the SU packets, the interrupted SU packets will return to the buffer of the SU packets with the probability α $(0 < \alpha < 1)$. This is one motivation for our work.

In this paper, we propose a spectrum access strategy with α-Retry policy in cognitive radio networks. We set a finite buffer for the SU packets. We further suppose that an interrupted SU packet would return to the buffer with retrial probability α for later transmission, and leave the system with probability $\bar{\alpha} = 1 - \alpha$. To describe the working principle of the proposed spectrum access strategy and to capture the digital nature of modern networks, we build a discrete-time priority queue. By constructing a two-dimensional discrete-time Markov chain (DTMC), we obtain the steady-state distribution of the system. Then, we derive the formulas for different performance measures. Accordingly, with numerical results, we show the influence of the retrial probability on different performance measures, and verify the effectiveness of the proposed spectrum access strategy with α-Retry policy. Finally, we optimize the retrial probability with an iterative algorithm.

The remainder of this paper is organized as follows. A novel spectrum access strategy with α-Retry policy is proposed in Section II. The system model and performance analysis are given in Section III. In Section IV, the formulas for the blocked rate, the forced dropping rate, the throughput and the average delay of the SU packets are obtained. Moreover, numerical results are provided to show the influence of the retrial probability on performance measures. In Section V, optimization of the retrial probability is carried out by employing an iterative algorithm. Finally, conclusions are drawn in Section VI.

II. A NOVEL SPECTRUM ACCESS STRATEGY WITH AN α-RETRY POLICY

We focus on one spectrum in cognitive radio networks, and suppose this spectrum to be licensed to one PU. It means that only one PU packet or one SU packet can be transmitted on the spectrum at one time. The PU packets can use the spectrum with preemptive priority, and the SU packets can opportunistically use the spectrum when the spectrum is not occupied.

Firstly, it is worth mentioning the following assumption: The spectrum sensing of the SUs is perfect, and the SU packets will not interfere the transmissions of the PU packets in any case. This assumption is the same as that in [7]. On the arrival instant of an SU packet, if the spectrum is idle and no PU packet arrives at the system, the newly arriving SU packet will occupy this idle spectrum. Otherwise, this newly arriving SU packet will queue in the buffer and wait for later transmission. Obviously, if the buffer is full, this newly arriving SU packet will be blocked by the system.

On the arrival instant of a PU packet, if the spectrum is idle, the newly arriving PU packet will occupy this idle spectrum directly. If the spectrum is occupied by another PU packet, this newly arriving PU packet will be blocked by the system. Also, if the spectrum is occupied by an SU packet, this newly arriving PU packet will interrupt the transmission of the SU packet and occupy the spectrum.

When the transmission of an SU packet is interrupted by a PU packet, the interrupted SU packet will either return to the buffer with probability α for later retrial, or give up its transmission and leave the system with probability $\bar{\alpha} = 1 - \alpha$. We call α as the retrial probability in this paper.

When an interrupted SU packet decides to return to the buffer, if the buffer is full, the interrupted SU packet will be forced to leave the system. Otherwise, we assume the interrupted SU packet will queue at the end in the buffer. The reason for putting the interrupted SU packet at the end of the buffer is to guarantee the QoS of delay for the SU packets already waiting in the system.

Moreover, we assume the interrupted SU packets have priority over the newly arriving SU packets. If a new arrival of an SU packet and an interruption of an SU packet occur simultaneously, the interrupted SU packet choosing to return to the buffer will queue ahead of the newly arriving SU packet in the buffer. Specially, if there is only one vacancy in the buffer, the interrupted SU packet choosing to return to the buffer will occupy the only vacancy in the buffer, while the newly arriving SU packet will be blocked by the system. That is to say, the PU packets have the highest priority and the newly arriving SU packets have the lowest priority.

We call the spectrum access strategy mentioned above a spectrum access strategy with α-Retry policy.

In order to show this spectrum access strategy more specifically, a slotted time structure is considered. The time axis is divided into fixed length intervals called slots, and the slot boundaries are numbered by $u = 1, 2, \cdots$. We claim that the arrivals of packets can only occur between the interval (u, u^+), and the departures of packets can only occur between the interval (u^-, u).

We further assume the arriving intervals and transmission times for the packets are independent and identically distributed (i.i.d.) random variables. The arriving intervals of the PU packets and the SU packets are supposed to follow geometrical distributions with arrival rates λ_1 $(0 < \lambda_1 < 1, \bar{\lambda}_1 = 1 - \lambda_1)$ and λ_2 $(0 < \lambda_2 < 1, \bar{\lambda}_2 = 1 - \lambda_2)$, respectively. The transmission times for the PU packets and the SU packets are assumed to follow geometrical distributions with transmission rates μ_1 $(0 < \mu_1 < 1, \bar{\mu}_1 = 1 - \mu_1)$ and μ_2 $(0 < \mu_2 < 1, \bar{\mu}_2 = 1 - \mu_2)$, respectively. Without loss of generality, we impose the constraint for a feasible retrial probability as $0 \leq \alpha \leq 1$. Moreover, the traffic intensities for the PU packets and the SU packets are defined as ρ_1 $(\rho_1 = \lambda_1/\mu_1)$ and ρ_2 $(\rho_2 = \lambda_2/\mu_2)$, respectively.

Based on the above assumptions, we depict the spectrum access strategy with α-Retry policy in Fig. 1.

III. SYSTEM MODEL AND PERFORMANCE ANALYSIS

In the spectrum access strategy with α-Retry policy proposed in this paper, we regard the spectrum as a server, and the PU packets and the SU packets as two classes of customers in queueing theory. The transmission process can be seen as the service for the customers. Then, we can build a preemptive priority queueing model.

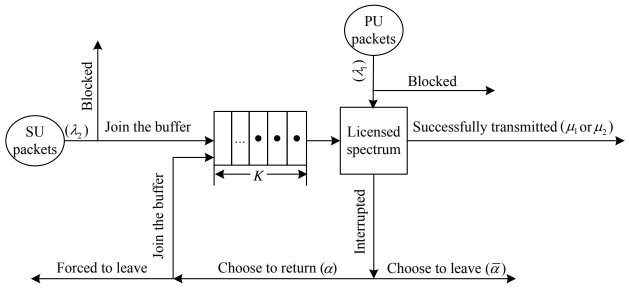

Fig. 1. Spectrum access strategy with α-Retry policy.

Let $L_u = i$ ($i = 0, 1, 2, \cdots, K+1$) be the total number of packets, including SU packets and PU packets, in the system at the instant $t = u^+$. Let $R_u = j$ ($j = 0, 1$) be the spectrum status at the instant $t = u^+$. R_u is given as follows

$$R_u = \begin{cases} 1, & \text{The spectrum is occupied by} \\ & \text{a PU packet at the instant } t = u^+, \\ 0, & \text{Otherwise.} \end{cases}$$

With the assumptions above, we conclude that $\{L_u, R_u\}$ constitutes a two-dimensional Markov chain. The state space of this Markov chain is given as follows

$$\mathbf{\Omega} = (0,0) \cup \{(i,j) : 1 \le i \le K+1, j = 0, 1\}$$

where state $(0,0)$ indicates that there is no packet in the system; state $(i,0)$ indicates that the spectrum is occupied by an SU packet and there are $(i-1)$ SU packets in the buffer; state $(i,1)$ indicates that the spectrum is occupied by a PU packet and there are $(i-1)$ SU packets in the buffer.

We define $\boldsymbol{P}$ as the state transition probability matrix for the two-dimensional Markov chain $\{L_u, R_u\}$. By ordering the elements of the state space $\mathbf{\Omega}$ for the Markov chain, we give $\boldsymbol{P}$ as a $(K+2) \times (K+2)$ block-structured matrix as follows

$$\boldsymbol{P} = \begin{pmatrix} \boldsymbol{C}_0 & \boldsymbol{B}_0 & \boldsymbol{A}_0 & & & \\ \boldsymbol{D}_1 & \boldsymbol{C}_1 & \boldsymbol{B}_1 & \boldsymbol{A}_1 & & \\ & \boldsymbol{D}_2 & \boldsymbol{C}_2 & \boldsymbol{B}_2 & \boldsymbol{A}_2 & \\ & & \ddots & \ddots & \ddots & \ddots \\ & & \boldsymbol{D}_{K-1} & \boldsymbol{C}_{K-1} & \boldsymbol{B}_{K-1} & \boldsymbol{A}_{K-1} \\ & & & \boldsymbol{D}_K & \boldsymbol{C}_K & \boldsymbol{E}_K \\ & & & & \boldsymbol{D}_{K+1} & \boldsymbol{F}_{K+1} \end{pmatrix}. \quad (1)$$

Each block in $\boldsymbol{P}$ will be discussed as follows.

(1) $\boldsymbol{C}_0$ is the one-step transition probability for the total number of packets being fixed at 0. That is to say, neither an SU packet nor a PU packet arrives at the system in a slot. So, $\boldsymbol{C}_0$ can be given as follows

$$\boldsymbol{C}_0 = \bar{\lambda}_1 \bar{\lambda}_2.$$

(2) $\boldsymbol{B}_0$ is the one-step transition probability vector for the total number of packets changing from 0 to 1. That is to say, either an SU packet or a PU packet arrives at the system in a slot. So, $\boldsymbol{B}_0$ can be given as follows

$$\boldsymbol{B}_0 = (\bar{\lambda}_1 \lambda_2, \lambda_1 \bar{\lambda}_2).$$

(3) $\boldsymbol{A}_0$ is the one-step transition probability vector for the total number of packets changing from 0 to 2. That is to say, an SU packet and a PU packet arrive at the system simultaneously in a slot. So, $\boldsymbol{A}_0$ can be given as follows

$$\boldsymbol{A}_0 = (0, \lambda_1 \lambda_2).$$

(4) $\boldsymbol{D}_i$ $(1 \le i \le K+1)$ is the one-step transition probability sub-matrix for the total number of packets decreasing from i to $(i-1)$. That is to say, a packet departs from and no packet arrives at the system.

When $i = 1$, $\boldsymbol{D}_1$ is a column vector given as follows

$$\boldsymbol{D}_1 = (\bar{\lambda}_1 \bar{\lambda}_2 \mu_2, \bar{\lambda}_1 \bar{\lambda}_2 \mu_1)^{\mathsf{T}}$$

where T describes the transpose operator.

When $1 < i \le K+1$, $\boldsymbol{D}_i$ is a 2×2 square matrix given by

$$\boldsymbol{D}_i = \begin{pmatrix} \bar{\lambda}_1 \bar{\lambda}_2 \mu_2 & 0 \\ \bar{\lambda}_1 \bar{\lambda}_2 \mu_1 & 0 \end{pmatrix}.$$

(5) $\boldsymbol{C}_i$ $(1 \le i \le K)$ is the one-step transition probability sub-matrix for the total number of packets being fixed at i. There are four cases to be discussed as follows.

(i) Given that the spectrum is occupied by an SU packet in the previous slot and there is no PU packet arrival in the current slot, the SU packet occupying the spectrum is successfully transmitted and one new SU packet arrives at the system; or the SU packet occupying the spectrum is not transmitted completely and there is no new SU packet arrival.

(ii) Given that the spectrum is occupied by an SU packet in the previous slot and there is one PU packet (but no SU packet) arrival in the current slot, the SU packet occupying the spectrum is successfully transmitted; or the SU packet occupying the spectrum is interrupted by the newly arriving PU packet and decides to leave the system.

(iii) Given that the spectrum is occupied by a PU packet in the previous slot and there is one SU packet arrival in the current slot, the PU packet occupying the spectrum is successfully transmitted and there is no PU packet arrival.

(iv) Given that the spectrum is occupied by a PU packet in the previous slot and there is no SU packet arrival in the current slot, the PU packet occupying the spectrum is successfully transmitted and there is one PU packet arrival; or the PU packet occupying the spectrum is not transmitted completely.

So, $\boldsymbol{C}_i$ can be given as follows

$$\boldsymbol{C}_i = \begin{pmatrix} \bar{\lambda}_1 (\bar{\lambda}_2 \bar{\mu}_2 + \lambda_2 \mu_2) & \lambda_1 (\bar{\lambda}_2 \mu_2 + \bar{\lambda}_2 \bar{\mu}_2 \bar{\alpha}) \\ \bar{\lambda}_1 \lambda_2 \mu_1 & \bar{\lambda}_2 (\lambda_1 \mu_1 + \bar{\mu}_1) \end{pmatrix}.$$

(6) $\boldsymbol{B}_i$ $(1 \le i \le K-1)$ is the one-step transition probability sub-matrix for the total number of packets increasing from i to $(i+1)$. There are three cases to be discussed as follows.

(i) Given that the spectrum is occupied by an SU packet in the previous slot and there is one SU packet (but no PU packet) arrival in the current slot, the SU packet occupying the spectrum is not transmitted completely.

(ii) Given that the spectrum is occupied by an SU packet in the previous slot and there is one PU packet arrival in the current slot, the SU packet occupying the spectrum is successfully transmitted and one new SU packet arrives at the system; or the SU packet occupying the spectrum is interrupted by the newly arriving PU packet and decides to leave the system. Moreover, there is one new SU packet arrival; or the SU packet occupying the spectrum is interrupted by the newly arriving PU packet and decides to return to the buffer. At the same time, there is no new SU packet arrival.

(iii) Given that the spectrum is occupied by a PU packet in the previous slot and there is one SU packet arrival in the current slot, the PU packet occupying the spectrum is successfully transmitted and there is one PU packet arrival; or the PU packet occupying the spectrum is not transmitted completely.

So, $\boldsymbol{B}_i$ can be given as follows

$$\boldsymbol{B}_i = \begin{pmatrix} \bar{\lambda}_1\lambda_2\bar{\mu}_2 & \lambda_1(\bar{\lambda}_2\bar{\mu}_2\alpha + \lambda_2\mu_2 + \lambda_2\bar{\mu}_2\bar{\alpha}) \\ 0 & \lambda_2(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix}.$$

(7) $\boldsymbol{A}_i$ $(1 \le i \le K-1)$ is the one-step transition probability sub-matrix for the total number of packets increasing from i to $(i+2)$. That is to say, an SU packet occupying the spectrum is interrupted by a newly arriving PU packet, and the interrupted SU packet decides to return to the buffer. Meanwhile, one new SU packet arrives at the system. So, $\boldsymbol{A}_i$ can be given as follows

$$\boldsymbol{A}_i = \begin{pmatrix} 0 & \lambda_1\lambda_2\bar{\mu}_2\alpha \\ 0 & 0 \end{pmatrix}.$$

(8) $\boldsymbol{E}_K$ is the one-step transition probability sub-matrix for the total number of packets increasing from K to $(K+1)$. The explanation for this case is similar to that of (6) and (7). So, $\boldsymbol{E}_K$ can be given as follows

$$\boldsymbol{E}_K = \begin{pmatrix} \bar{\lambda}_1\lambda_2\bar{\mu}_2 & \lambda_1(\lambda_2 + \bar{\lambda}_2\bar{\mu}_2\alpha) \\ 0 & \lambda_2(\lambda_1\mu_1 + \bar{\mu}_1) \end{pmatrix}.$$

(9) $\boldsymbol{F}_{K+1}$ is the one-step transition probability sub-matrix for the total number of packets being fixed at $(K+1)$. The explanation for this case is similar to that of (5)–(7). So, $\boldsymbol{F}_{K+1}$ can be given as follows

$$\boldsymbol{F}_{K+1} = \begin{pmatrix} \bar{\lambda}_1(\lambda_2\mu_2 + \bar{\mu}_2) & \lambda_1 \\ \bar{\lambda}_1\mu_1\lambda_2 & \lambda_1\mu_1 + \bar{\mu}_1 \end{pmatrix}.$$

Up to this point, all of the elements in $\boldsymbol{P}$ have been given. The structure of the transition probability matrix $\boldsymbol{P}$ indicates that the two-dimensional Markov chain $\{L_u, R_u\}$ is non-periodic, irreducible and positive recurrent [16]. We define the steady-state distribution $\pi_{i,j}$ of the two-dimensional Markov chain as follows

$$\pi_{i,j} = \lim_{u\to\infty} P\{L_u = i, R_u = j\}. \tag{2}$$

Let $\boldsymbol{\Pi} = (\pi_{\mathbf{0},\mathbf{0}}, \pi_{\mathbf{1},\mathbf{0}}, \pi_{\mathbf{1},\mathbf{1}}, \cdots, \pi_{\mathbf{K},\mathbf{0}}, \pi_{\mathbf{K},\mathbf{1}}, \pi_{\mathbf{K+1},\mathbf{0}}, \pi_{\mathbf{K+1},\mathbf{1}})$ be the steady-state probability vector. Based on the equilibrium equation and the normalization condition, we have

$$\begin{cases} \boldsymbol{\Pi P} = \boldsymbol{\Pi}, \\ \boldsymbol{\Pi e} = \mathbf{1} \end{cases} \tag{3}$$

where $\boldsymbol{e}$ is a column vector with $(2K+3)$ elements, all of which are equal to 1.

As the dimension of (3) is finite, we can obtain the steady-state probability vector $\boldsymbol{\Pi}$ by using a Gaussian elimination method with numerical results.

IV. PERFORMANCE MEASURES AND NUMERICAL RESULTS

We note that the transmissions of the PU packets are independent of the SU packets. The transmission process of the PU packets can be regarded as a simple pure losing queueing model with single server. By referencing the analysis results for the classical queueing model given in [16], we can evaluate the system performance of the PU packets. For example, the average delay of the PU packets is equal to the reciprocal of the transmission rate of the PU packets.

On the other hand, the transmissions of the SU packets will be affected by the PU packets. The performance measures of the SU packets will be influenced by different factors, such as the interruptions from the PU packets, the retrial probability of the interrupted SU packets, etc. In this section, we will derive and evaluate some important performance measures of the SU packets mathematically with numerical results.

A. Performance Measures

We define the blocked rate β of the SU packets as the number of newly arriving SU packets that are blocked by the system per slot. A newly arriving SU packet will be blocked by the system in the following two cases.

Given that the buffer is full, a newly arriving SU packet will be blocked by the system with two possibilities: (i) There is a PU packet arrival. (ii) There is no PU packet arrival, but the packet occupying the spectrum is not transmitted completely. The blocked rate β_1 of the SU packets for this case can be given as follows

$$\beta_1 = \lambda_2((\lambda_1 + \bar{\mu}_2\bar{\lambda}_1)\pi_{K+1,0} + (\lambda_1 + \bar{\mu}_1\bar{\lambda}_1)\pi_{K+1,1}). \tag{4}$$

Given that the spectrum is occupied by an SU packet and there is only one vacancy in the buffer, a newly arriving SU packet will possibly be blocked by the system if the transmission of that SU packet occupying the spectrum is interrupted by a newly arriving PU packet. This interrupted SU packet then decides to return to the buffer and occupies the only remaining vacancy. The blocked rate β_2 of the SU packets for this case can be given as follows

$$\beta_2 = \lambda_2\bar{\mu}_2\lambda_1\pi_{K,0}\alpha. \tag{5}$$

Combining (4) and (5), we give the blocked rate β (packets/slot) of the SU packets as follows

$$\beta = \beta_1 + \beta_2. \tag{6}$$

We define the forced dropping rate γ of the SU packets as the number of interrupted SU packets that are forced to leave the system per slot. These interrupted SU packets decide to return to the buffer, but find the buffer is full, so they have to leave the system. Then, γ (packets/slot) can be given as follows

$$\gamma = \pi_{K+1,0}\lambda_1\bar{\mu}_2\alpha. \tag{7}$$

On the other hand, in the spectrum access strategy with α-Retry policy, some interrupted SU packets will give up their transmissions and leave the system voluntarily. In this instance, we define the number of such SU packets per slot by κ. Then, κ (packets/slot) can be given as follows

$$\kappa = \sum_{i=1}^{K+1} \pi_{i,0}\lambda_1\bar{\mu}_2(1-\alpha). \tag{8}$$

We define the throughput θ of the SU packets as the number of SU packets that are successfully transmitted over the spectrum per slot. An SU packet can be successfully transmitted if and only if that SU packet is neither blocked by the system at the arrival instant, nor leaves the system (voluntary or enforced) before its transmission is completed successfully. Based on (6)–(8), the throughput θ (packets/slot) of the SU packets is therefore given as follows

$$\theta = \lambda_2 - \beta - \gamma - \kappa. \tag{9}$$

We define the delay of an SU packet as the time period from the instant that an SU packet arrives at the system to the instant when the transmission of that SU packet is completed successfully. In fact, the delay of an SU packet is the sojourn time of that SU packet.

Let S_u be the number of SU packets in the system at the instant $t = u^+$. Let $S = \lim_{u\to\infty} S_u$ be the number of SU packets in the system under the steady-state condition. We can get the average value $E[S]$ of S as follows

$$\begin{aligned} E[S] &= \sum_{i=0}^{K+1} iP\{S=i\} \\ &= (K+1)\pi_{K+1,0} + \sum_{i=0}^{K} i(\pi_{i,0}+\pi_{i+1,1}). \end{aligned} \tag{10}$$

By using Little's formula [16], the average delay δ (slots) of the SU packets can be given as follows

$$\delta = \frac{E[S]}{\theta}. \tag{11}$$

B. Numerical Results

In numerical results, the parameters are set as follows unless otherwise stated. The arrival rate λ_2 and the traffic intensities ρ_2 of the SU packets are set as $\lambda_2 = 0.12$ and $\rho_2 = \{0.6, 0.8\}$ to compute the different performance measures of the SU packets. In order to show the influence of the Retry policy on the system performances, the data set for the retrial probability is supposed as $\alpha = \{0.0, 0.1, 0.2, \cdots, 0.9, 1.0\}$. In order to evaluate how the PU affects the performance of the SUs, the transmission rate of the PU packets is set as $\mu_1 = 0.2$, and the data set for the traffic intensity ρ_1 of the PU packets is assumed to be $\rho_1 = \{0, 6, 0.8\}$. Moreover, aiming to ascertain the influence of the buffer capacity of the SUs on the system performance, we set the buffer capacity as $K = 5$ and $K = 7$, respectively.

We also provide simulation results in order to validate our analysis results. We develop simulations with MATLAB. The simulation results are obtained by averaging over 8 independent runs, and each run is conducted for 2×10^6 slots. In the following Figs. 2–6, we will find that the analysis results match well with those by simulations.

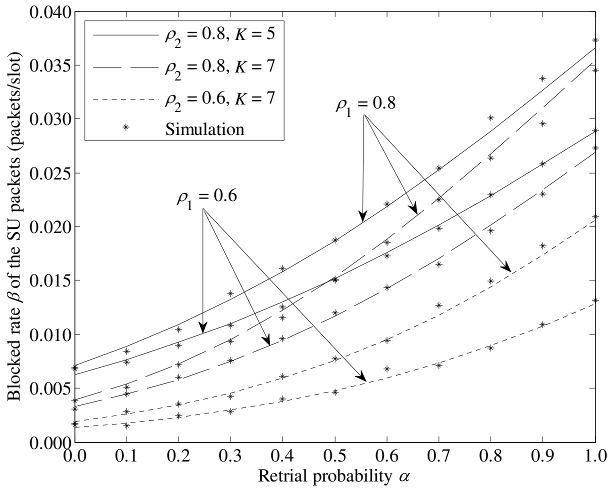

Fig. 2. Blocked rate β of the SU packets vs. retrial probability α.

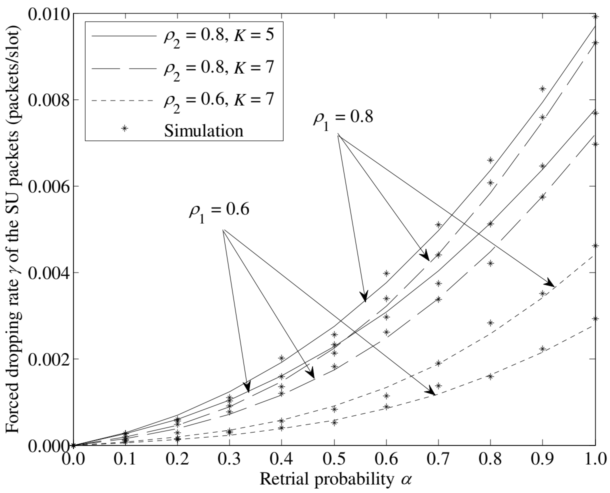

Fig. 3. Forced dropping rate γ of the SU packets vs. retrial probability α.

Figs. 2–5 demonstrate how the blocked rate β, the forced dropping rate γ, the throughput θ and the average delay δ of the SU packets change with respect to the retrial probability α.

From Figs. 2–5, we find that when other parameters are fixed, as the retrial probability α increases, the blocked rate β, the forced dropping rate γ, the throughput θ and the average delay δ of the SU packets will increase. The reason is that the larger the retrial probability is, the more SU packets will queue in the buffer, and the more the SU packets will be blocked by the system. Also, more interrupted SU packets returning to the buffer will be forced to leave the system. This will result in a higher blocked rate and a higher forced dropping rate of the SU packets. On the other hand, more SU packets waiting in the buffer will also inevitably make the average delay of the SU packets longer. Moreover, the larger the retrial probability is, the greater the number of interrupted SU packets returning to the buffer will be. This will result in more SU packets to be transmitted, so the

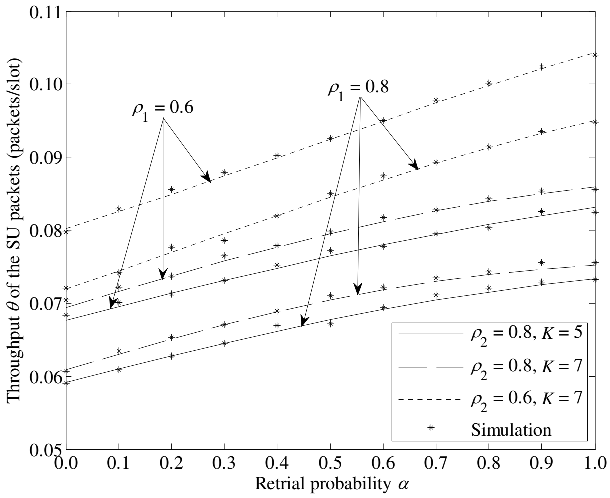

Fig. 4. Throughput θ of the SU packets vs. retrial probability α.

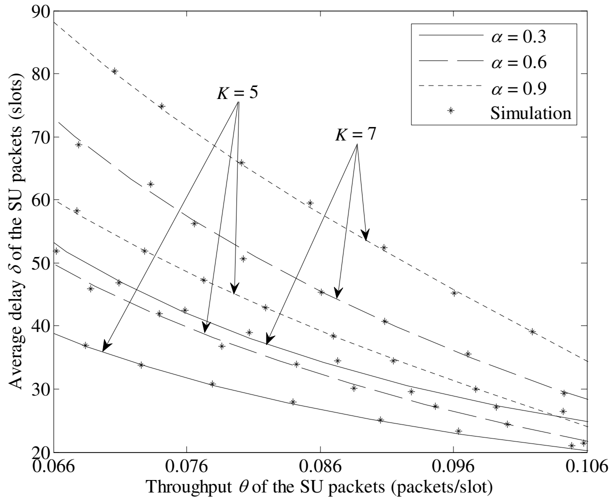

Fig. 6. Average delay δ vs. throughput θ.

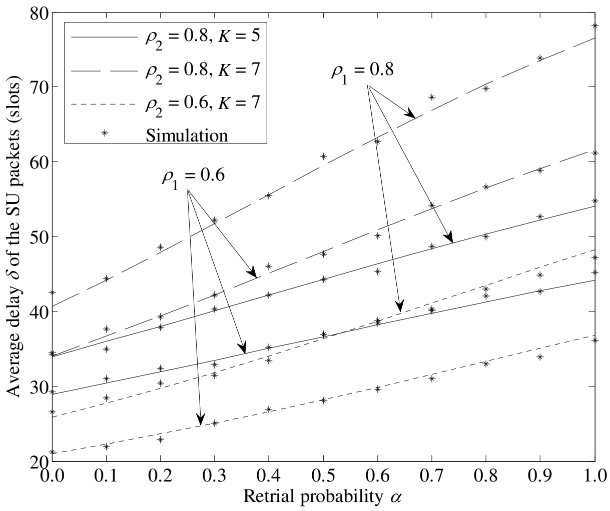

Fig. 5. Average delay δ of the SU packets vs. retrial probability α.

greater the throughput of the SU packets will be.

We also find that when other parameters are fixed, as the buffer capacity K of the SUs increases, the blocked rate β and the forced dropping rate γ of the SU packets will decrease. The intuitive reason is that when the buffer of the SUs has a larger capacity, it can accommodate a greater number of SU packets. This will induce a decrease in the blocked rate and the forced dropping rate of the SU packets. On the other hand, as the buffer capacity K of the SUs increases, both of the throughput θ and the average delay δ will increase. This is because as the buffer capacity of the SUs increases, there will be more SU packets joining the system to be transmitted. This will certainly induce an increase in the throughput of the SU packets. However, more SU packets joining and waiting in the system will increase the average delay of the SU packets.

Moreover, when other parameters are fixed, as the traffic intensity ρ_1 of the PU packets increases, the blocked rate β, the forced dropping rate γ and the average delay δ of the SU packets will increase, while the throughput θ of the SU packets will decrease. The reason for the change trends mentioned above is that as the traffic intensity of the PU packets increases, it is more likely that the spectrum is occupied by a PU packet, then more SU packets will have to wait in the buffer. As a result, the blocked rate, the forced dropping rate and the average delay of the SU packets will increase. At the same time, the possibility for the SU packets being transmitted in a slot is lower. Therefore, the throughput of the SU packets will decrease.

Additionally, when other parameters are fixed, as the traffic intensity ρ_2 of the SU packets increases, the blocked rate β, the forced dropping rate γ and the average delay δ of the SU packets will increase, while the throughput of the SU packets will decrease. The reason is that when the arrival rate of the SU packets is given, a higher traffic intensity of the SU packets will result in a lower transmission speed for the SU packets. Therefore, more SU packets will have to wait in the buffer, which will induce an increase in the blocked rate, the forced dropping rate and the average delay of the SU packets. Contrarily, the throughput of the SU packets will be lower.

The numerical results presented in Figs. 2-5 clearly show that the performance measures such as the blocked rate β, the forced dropping rate γ, the throughput θ and the average delay δ of the SU packets are heavily dependent on the retrial probability α in our proposed spectrum access strategy.

We also notice that two special cases can be obtained when the retrial probabilities α are set to $\alpha = 0.0$ and $\alpha = 1.0$, respectively.

For the special case of $\alpha = 0.0$, the spectrum access strategy without any retrial transmission for the interrupted SU packets can be evaluated. Compared with this kind of spectrum access strategy, we see that in the spectrum access strategy with α-Retry policy, the throughput of the SU packets improves significantly. However, the blocked rate, the forced dropping rate and the average delay of the SU packets increase correspondingly.

For the special case of $\alpha = 1.0$, the spectrum access strategy with 1 persistent retrial transmission for the interrupted SU packets can be evaluated. Compared with this kind of spectrum access strategy, we find that in the spectrum access strategy with α-Retry policy, the blocked rate, the forced dropping rate and the average delay of the SU packets are significantly reduced. However, the throughput of the SU packets correspondingly decreases.

The behaviors demonstrated in Figs. 4 and 5 are also observed in Fig. 6, which shows the relationship between the throughput θ and the average delay δ of the SU packets. In this figure, the traffic intensity ρ_2 of the SU packets is set as $\rho_2 = 0.8$. With the throughput growth direction, the traffic intensity ρ_1 is declining.

From Fig. 6, we find that for the lowest traffic intensity ρ_1 of the PU packets, the average delay δ is the shortest and the throughput θ is the greatest. On the other hand, when the traffic intensity ρ_1 of the PU packets is at its highest, the average delay δ is the longest and the throughput θ is the smallest. Moreover, we see that the increases in the buffer capacity K or the retrial probability α will make the throughput θ and the average delay δ increase simultaneously. Additionally, we can also observe that, for the same buffer capacity K and the same throughput θ of the SU packets, as the traffic intensity ρ_1 of the PU packets decreases, the gap between the average delays δ of the SU packets for different retrial probabilities α will also decrease. It is assumed the reason for this that when the traffic intensity of the PU packets is smaller, the possibility for the SU packets being interrupted by the PU packets will be lower, the influence of the retrial probability on the average delay will also be weaker, so the gap between the average delays for different retrial probabilities will be smaller.

V. OPTIMIZATION OF THE RETRIAL PROBABILITY

Generally speaking, in cognitive radio networks, the throughput and the average delay of the SU packets are the most important considerations when designing and optimizing the network. As shown in the numerical results, with an increase of the retrial probability, there is an increase in the throughput of the SU packets. This is what we hope to see. On the other hand, we also find that as the retrial probability increases, the average delay of the SU packets will become longer. Obviously, this is what we do not want to see. We conclude that the optimal retrial probability can be achieved by balancing the throughput and the average delay of the SU packets.

Taking into account the throughput θ and the average delay δ of the SU packets, we design a cost function $F(\alpha)$ to obtain the optimal retrial probability. $F(\alpha)$ can be given as follows

$$F(\alpha) = \frac{C_1}{\theta} + C_2\delta \tag{12}$$

where C_1 and C_2 are assumed to be the factors to the system cost for the throughput of the SU packets and the average delay of the SU packets, respectively. C_1 and C_2 can be set as needed in practice. For example, in the networks with throughput sensitive application, the impact factor C_1 will be set relatively higher. On the other hand, in the networks with lower tolerance for average delay, the impact factor C_2 will be set greater.

It is worth mentioning the following assumption: The spectrum sensing of the SUs is assumed to be perfect. So, we do not take into account the interference to the PU in the cost function.

From (12), the optimal retrial probability α^* can be given as follows

$$\alpha^* = \arg\min_{[0,1]}\{F(\alpha)\} \tag{13}$$

where "arg min" stands for the argument of the minimum.

Due to the complexity of the cost function $F(\alpha)$, it is not easy to obtain the exact solution for the optimal retrial probability α^*. For this, we estimate the optimal value of α^* iteratively. Considering the constraint of $\alpha \in [0, 1]$, we construct a penalty function $B(\alpha)$ as follows

$$B(\alpha) = F(\alpha) + \eta\omega(\alpha) \tag{14}$$

where $\eta > 0$ is the penalty factor, $\omega(\alpha) = 1/\alpha + 1/(1-\alpha)$ is called a penalty term. By referencing a steepest descent method [17], we give an iterative algorithm to obtain the optimal retrial probability as follows.

Step 1: Set the initial trial solution for $\alpha_n \in [0, 1]$ with $n = 0$, (for example, $\alpha_0 = 0.5$).

Step 2: Calculate the new solution $\alpha_{n+1} = \alpha_n - \left.\frac{\phi\partial B(\alpha)}{\partial\alpha}\right|_{\alpha=\alpha_n}$, where ϕ is the step size (for example, $\phi = 0.01$).

Step 3: Set $n = n + 1$ and repeat Step 2 if $|B(\alpha_{n+1}) - B(\alpha_n)| > \epsilon$ or $|\alpha_{n+1} - \alpha_n| > \epsilon$, where ϵ is tolerance (for example, $\epsilon = 10^{-6}$); otherwise, go to Step 4.

Step 4: Calculate $\eta\omega(\alpha_n)$. If $\eta\omega(\alpha_n) > \epsilon$, go back to Step 1 by setting $\alpha_0 = \alpha_n$ and $\eta = \eta D$, where D is the decline coefficient of the penalty factor η (for example, $D = 0.1$); otherwise, go to Step 5.

Step 5: Obtain the optimal value $\alpha^* = \alpha_n$, and compute the corresponding $F(\alpha^*)$.

In the iterative algorithm mentioned above, the differential operation for $B(\alpha)$ can be approximated numerically as follows

$$\frac{\partial B(\alpha)}{\partial\alpha} \approx \frac{B(\alpha+\Theta) - B(\alpha)}{\Theta} \tag{15}$$

where Θ is an arbitrary small number (for example, $\Theta = 10^{-6}$).

For discussing the convergence of the iterative algorithm, by referencing [18], we present a theorem as follows.

Theorem The proposed iterative algorithm is convergent if and only if point set $G_0 = \{\alpha | B(\alpha) \leq B(\alpha_0)\}$ is bounded. The iterative algorithm will stop after limited steps of iterative; or for a sequence point set $\{\alpha_n, n = 0, 1, 2, \cdots\}$ obtained by using the iterative algorithm, any limit point of $\{\alpha_n, n = 0, 1, 2, \cdots\}$ is the stagnation point of $B(\alpha)$.

Proof: Let α^* be the limit point of $\{\alpha_n, n = 0, 1, 2, \cdots\}$, which is obtained by using the iterative algorithm.

It is obvious that $\{B(\alpha_n), n = 0, 1, 2, \cdots\}$ monotonously decreases, and $\alpha_n \in G_0$.

For clearly, we denote that

$$g(\alpha) = \frac{\partial B(\alpha)}{\partial\alpha},\ g^n = g(\alpha_n) = \left.\frac{\partial B(\alpha)}{\partial\alpha}\right|_{\alpha=\alpha_n}.$$

In the following, with a proof by contradiction, we validate

$$g^* = \left.\frac{\partial B(\alpha)}{\partial\alpha}\right|_{\alpha=\alpha^*} = 0.$$

We firstly suppose $g^* \neq 0$, and then assume that there is a point set $\{\alpha_{n_v}\}$ converging to α^*. We also suppose that there is

Table 1. Optimal retrial probability α^* and minimum cost $F(\alpha^*)$.

System parameters			Optimal retrial probability	Minimum cost
λ_2	ρ_1	K	α^*	$F(\alpha^*)$
$\lambda_2 = 0.11$	$\rho_1 = 0.6$	$K = 5$	0.8889	243.6791
$\lambda_2 = 0.11$	$\rho_1 = 0.6$	$K = 7$	0.3542	257.1942
$\lambda_2 = 0.11$	$\rho_1 = 0.8$	$K = 5$	0.7248	281.1829
$\lambda_2 = 0.11$	$\rho_1 = 0.8$	$K = 7$	0.2700	297.0879
$\lambda_2 = 0.12$	$\rho_1 = 0.6$	$K = 5$	0.6663	242.6840
$\lambda_2 = 0.12$	$\rho_1 = 0.6$	$K = 7$	0.1126	254.9052
$\lambda_2 = 0.12$	$\rho_1 = 0.8$	$K = 5$	0.5377	280.1174
$\lambda_2 = 0.12$	$\rho_1 = 0.8$	$K = 7$	0.0646	294.6459

another point set $\{\alpha_{n_v+1}\}$, and $\{\alpha_{n_v+1}\}$ converges to α^{**}, too. That is to say,

$$\alpha^{n_v} \to \alpha^*, \;\; \alpha^{n_v+1} \to \alpha^{**}, \;\; (v \to \infty).$$

Because $\{B(\alpha_n), n = 0, 1, 2, \cdots\}$ monotonously decreases, moreover, both α^* and α^{**} are limit points, we have

$$B(\alpha^*) = B(\alpha^{**}).$$

Note that $g^* \neq 0$, then

$$B(\alpha^* - \phi g^*) < B(\alpha^*).$$

Considering $B(\alpha_{n_v+1}) \leq B(\alpha_{n_v} - \phi g^{n_v})$ and letting $v \to \infty$, we have

$$B(\alpha^{**}) \leq B(\alpha^* - \phi g^*) < B(\alpha^*).$$

This conclusion is contradictory to $B(\alpha^*) = B(\alpha^{**})$. So we have $g^* = 0$, i.e., α^* is the stagnation point of $B(\alpha)$.

This completes the proof. □

In order to show the efficiency of the iterative algorithm, by setting $\mu_1 = 0.20$, $\mu_2 = 0.15$, $C_1 = 13$, and $C_2 = 2$ as an example, we demonstrate the optimal retrial probability α^* and the corresponding minimum cost $F(\alpha^*)$ in Table 1.

In Table 1, the estimates for the optimal retrial probability and the minimum cost are accurate to four decimal places.

From Table 1, we see that for the same arrival rate λ_2 of the SU packets and the same traffic intensity ρ_1 of the PU packets, as the buffer capacity K of the SUs increases, the optimal retrial probability α^* will decrease. The dominate reason for this change trend is that as the buffer capacity of the SUs increases, the average delay of the SU packets will increase, and this will increase the system cost. In order to reduce the average delay of the SU packets, the optimal retrial probability of the SU packets will be set lower.

On the other hand, we conclude that for the same arrival rate λ_2 of the SU packets and the same buffer capacity K of the SUs, as the traffic intensity ρ_1 of the PU packets increases, the optimal retrial probability α^* will decrease. It is because that as the traffic intensity of the PU packets increases, the possibility for the spectrum being occupied by the PU packets will increase, and more SU packets will wait in the system. Then, it is more likely that the interrupted SU packets returning back to the buffer will be refused, so the optimal retrial probability of the SU packets will be set lower.

Moreover, we find that for the same traffic intensity ρ_1 of the PU packets and the same buffer capacity K of the SUs, as the arrival rate λ_2 of the SU packets increases, the optimal retrial probability α^* will decrease. The reason is that as the arrival rate of the SU packets increases, the possibility for the system being overflow is higher, and it is more likely that the interrupted SU packets returning back to the buffer will be refused, hence the optimal retrial probability of the SU packets will be set lower.

VI. CONCLUSIONS

In this paper, we focused on the interrupted SU packets and proposed a spectrum access strategy with α-Retry policy in cognitive radio networks. We assumed the interrupted SU packets would return to the buffer with retrial probability α. Considering the priority of the PU packets in the proposed spectrum access strategy, we built a preemptive priority retrial queueing model. We analyzed the steady-state distribution of the system model with a two-dimensional discrete-time Markov chain. Then, we derived the formulas for the blocked rate, the forced dropping rate, the throughput and the average delay of the SU packets.

With numerical results, we illustrated that the throughput of the SU packets in the spectrum access strategy with α-Retry policy proposed in this paper is greater than that in the conventional spectrum access strategy without any retrial transmission. On the other hand, we also showed that the blocked rate, the forced dropping rate and the average delay of the SU packets in the spectrum access strategy with α-Retry policy proposed in this paper are smaller than that in the conventional spectrum access strategy with 1 persistent retrial transmission. Moreover, we built a cost function and presented an iterative algorithm to optimize the retrial probability.

In this paper, we assumed that the SU packets would not interfere the transmissions of the PU packets in any case, and considered one spectrum, which was licensed to one PU. As a future work, we will consider the influence of the SU packets on the PU packets by releasing the assumption of perfect spectrum sensing of SUs. We will also consider the system with multiple spectrums and also the collisions among multiple PUs to evaluate the system performance.

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Yuan Zhao received the B.Eng. degree in Mathematics and Applied Mathematics from Tangshan Normal College, Tangshan, China and the M.Eng. degree in Operation Science and Control Theory from Yanshan University, Qinhuangdao, China. She is currently a Ph.D. candidate in School of Information Science and Engineering, Yanshan University, Qinhuangdao, China. Her research interests include spectrum allocation in cognitive radio networks, stochastic modeling and system optimization for communication networks, queueing theory, and its application.

Shunfu Jin received the B.Eng. degree in Computer and Application from North East Heavy Machinery College, Qiqihaer, China and the M.Eng. degree in Computer Science and Dr.Eng. degree in Circuit and System from Yanshan University, Qinhuangdao, China. Now Dr. Jin is a Professor at School of Information Science and Engineering, Yanshan University, Qinhuangdao, China. Her research interests include stochastic modeling for telecommunication, performance evaluation for computer system and network, and application for queueing system.

Wuyi Yue received the B.Eng. degree in Electronic Engineering from Tsinghua University, China and the M.Eng. and Dr.Eng. degrees in Applied Mathematics and Physics from Kyoto University, Japan. She was a Researcher and a Chief Researcher of ASTEM RI, an Associate Professor of Wakayama University, an Associate Professor and a Professor at the Department of Applied Mathematics, a Professor at Department of Information Science and Systems Engineering, Konan University, Japan. She is currently a Professor at Department of Intelligence and Informatics, Konan University, Japan. She is also the Dean of the faculty of Intelligence and Informatics, Konan University, Japan, now. She is a Senior Member of IEICE of Japan, a Fellow of the Operations Research Society of Japan, a Member of the IEEE, the System Engineers Society of China, and the Operations Research Society of China. Her research interests include queueing theory, stochastic processes and optimal methods as applied to system modeling, performance analysis and evaluation, and optimal resource allocation of wired and wireless/mobile communication networks (including mobile cellular, multi-hop, multi-traffic mobile communication networks), multimedia communication networks, traffic systems, stochastic systems, information systems, systems engineering, and operations research.