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# Erratum : Two Variations of the Choquet Game

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**Erratum**

In this short note, we provide a correction to the proof of Theorem 2.1 appeared in "J. Cao and Z. Piotrowski, Two Variations of the Choquet Game, *Kyungpook Math. J.* **44** (2004), 495–504".

The following theorem was proved in [1] as Theorem 2.1.

**Theorem 2.1.**([1]) The following are equivalent for a regular space $X$.

(a) There is an open sieve $(\{U_i : i \in I_n\}, \pi_n)_{n<\omega}$ on $X$ such that for each $\pi$-chain $(i_n)_{n<\omega}$ with $\bigcap_{n<\omega} U_{i_n} \neq \varnothing$, $(U_{i_n})_{n<\omega}$ is a complete sequence.

(b) Player $\alpha$ has a stationary winning strategy in MP$(X)$.

(c) Player $\alpha$ has a winning strategy in MP$(X)$.

Recently, Professor Dikran Dikranjan and D. Hounkanli Komlan have pointed to us that the proof of (c) $\Rightarrow$ (a) provided in [1] is invalid. In this short note, we shall provide a corrected version for this part.

*Proof.* (c) $\Rightarrow$ (a). Suppose that $\alpha$ has a winning strategy $\sigma$ in MP$(X)$. Let $I_0 = \{i_0\}$ be an arbitrary singleton, and define $U_{i_0} = X$. For each $n > 0$, define $I_n$ as the set of all finite $\sigma$-sequences of length $n$. Let $U_i = \sigma(\langle x_0, B_0\rangle, \cdots, \langle x_{n-1}, B_{n-1}\rangle)$ for every $i = (\langle x_0, B_0\rangle, \cdots, \langle x_{n-1}, B_{n-1}\rangle) \in I_n$. Define $\pi_0 : I_1 \to I_0$ such that $\pi_0(i_1) = i_0$ for every $i_1 \in I_1$. Furthermore, for each $n \in \mathbb{N}$, we define a map $\pi_n : I_{n+1} \to I_n$ such that

$$\pi_n(\langle x_0, B_0\rangle, \cdots, \langle x_{n-1}, B_{n-1}\rangle, \langle x_n, B_n\rangle) = (\langle x_0, B_0\rangle, \cdots, \langle x_{n-1}, B_{n-1}\rangle)$$

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for every finite $\sigma$-sequence $(\langle x_0, B_0\rangle, \cdots, \langle x_{n-1}, B_{n-1}\rangle, \langle x_n, B_n\rangle)$ in $I_{n+1}$. It can be checked readily that $(\{U_i : i \in I_n\}, \pi_n)_{n<\omega}$ is an open sieve on $X$. We are to prove that this sieve satisfies $(a)$. Let $(i_n)_{n<\omega}$ be any $\pi$-chain with $\bigcap_{n<\omega} U_{i_n} \neq \varnothing$. By the construction above, there exists a $\sigma$-sequence $(\langle x_n, B_n\rangle)_{n<\omega}$ in $\mathrm{MP}(X)$ corresponding to $(i_n)_{n<\omega}$ such that $\bigcap_{n<\omega} B_n \neq \varnothing$. Let $\mathcal{F}$ be a filterbase controlled by $(U_{i_n})_{n<\omega}$. Then, $\mathcal{F}$ is also controlled by $(B_n)_{n<\omega}$. If $\mathcal{F}$ does not cluster at anywhere, then for each point $p \in \bigcap_{n<\omega} B_n$ there exist an open neighborhood $W_p$ of $p$ and an $F_p \in \mathcal{F}$ with $W_p \cap F_p = \varnothing$. Since $\bigcap_{n<\omega} B_n$ is compact, there are finitely many points $p_1, \cdots, p_k$ in $\bigcap_{n<\omega} B_n$ such that $W = \bigcup_{i=1}^{k} W_{p_i} \supseteq \bigcap_{n<\omega} B_n$. It follows that $W \cap (\bigcap_{i=1}^{k} F_{p_i}) = \varnothing$. Since $\sigma$ is a winning strategy for $\alpha$, then by condition (ii) in $\mathrm{MP}(X)$, we may select an $F \in \mathcal{F}$ such that $F \subseteq (\bigcap_{i=1}^{k} F_{p_i}) \cap W$. However, this is a contradiction. The proof is completed. □

## References

[1] Z. Piotrowski, *Two variations of the Choquet game*, Kyungpook Math. J., **44**(2004), 495-504.