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Double-framed Soft Filters in CI-algebras

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Abstract. The notion of double-framed soft filters of a CI-algebra is introduced, and related properties are investigated. Further characterization of a double-framed soft filter is considered, and conditions for a double-framed soft set to be a double-framed soft filter are provided. Finally a new double-framed soft filter from old one is established.

1. Introduction

In 1966, Imai and Iséki [2] and Iséki [3] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. As a generalization of a BCK-algebra, Kim and Kim [8] introduced the notion of a BE-algebra, and investigated several properties. In [1], Ahn and So introduced the notion of ideals in BE-algebras. They gave several descriptions of ideals in BE-algebras. The notion of CI-algebras is introduced by Meng [11] as a generalization of BE-algebras. Filter theory and properties in CI-algebras are studied by Kim [7], Meng [12] and Piekart et al. [14]. Molodtsov [13] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Worldwide, there has been a rapid growth in interest in soft set theory and its applications in recent years. Evidence of this can be found in the increasing number of high-quality articles on soft sets and related topics that have been published in a variety of international journals, symposia, workshops, and international conferences in recent years. Maji et al. [10] described the application of soft set theory to a decision making problem. Maji et al. [9] also studied several operations on the theory of soft sets. Jun and Park [5] studied applications of soft sets in ideal theory of BCK/BCI-algebras. In [4], Jun et al. introduced the notion of double-framed soft sets (briefly, DFS-sets), and applied it to BCK/BCI-algebras. They discussed double-framed soft algebras (briefly, DFS-algebras) and

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investigated related properties. The present author together with Y. B. Jun and G. Muhiuddin introduced the notion of a (closed) double-framed soft ideal (briefly, a (closed) DFS-ideal) in BCK/BCI-algebras [6]. We discussed the relation between a DFS-algebra and a DFS-ideal, and established characterizations of a (closed) DFS-ideal. We also shown that the int-uni DFS-set of two DFS-ideals is a DFS-ideal, and provided conditions for a DFS-ideal to be closed (see [6]).

In this paper, we introduced the notion of double-framed soft filters of a CI-algebra, and investigate related properties. We consider characterization of a double-framed soft filter, and provide conditions for a double-framed soft set to be a double-framed soft filter. We make a new double-framed soft filter from old one.

2. Preliminaries

An algebra $(X; *, 1)$ of type $(2, 0)$ is called a *CI-algebra* if it satisfies the following properties:

(CI1) $x * x = 1$,

(CI2) $1 * x = x$,

(CI3) $x * (y * z) = y * (x * z)$,

for all $x, y, z \in X$.

Let $(X; *, 1)$ be a CI-algebra, A subset F of X is called a *filter* (see [8]) of X if

(F1) $1 \in F$,

(F2) $(\forall x, y \in X)(x * y,\ x \in F \Rightarrow y \in F)$.

Molodtsov [13] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. We say that the pair (U, E) is a *soft universe*. Let $P(U)$ denotes the power set of U and $A, B, C, \cdots \subseteq E$.

Definition 2.1 [13]. A pair (α, A) is called a *soft set* over U, where α is a mapping given by

$$\alpha : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\alpha(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (α, A). Clearly, a soft set is not a set. For illustration, Molodtsov considered several examples in [13].

In what follows, we take $E = X$, as a set of parameters, which is a CI-algebra and $A, B, C, \cdots$ be subalgebras of E unless otherwise specified.

Definition 2.2 [4]. A double-framed soft pair $\langle(\alpha, \beta); A\rangle$ is called a *double-framed soft set* of A over U (briefly, DFS-set of A), where α and β are mappings from A to $P(U)$.

For a double-framed soft set $\langle(\alpha,\beta);A\rangle$ of A over U and two subsets γ and δ of U, the γ-*inclusive set* and the δ-*exclusive set* of $\langle(\alpha,\beta);A\rangle$, denoted by $i_A(\alpha;\gamma)$ and $e_A(\beta;\delta)$, respectively, are defined as follows:

$$i_A(\alpha;\gamma) := \{x \in A \mid \gamma \subseteq \alpha(x)\}$$

and

$$e_A(\beta;\delta) := \{x \in A \mid \delta \supseteq \beta(x)\},$$

respectively. The set

$$DF_A\left(\alpha,\beta\right)_{(\gamma,\delta)} := \{x \in A \mid \gamma \subseteq \alpha(x),\ \delta \supseteq \beta(x)\}$$

is called a *double-framed including set* of $\langle(\alpha,\beta);A\rangle$. It is clear that

$$DF_A\left(\alpha,\beta\right)_{(\gamma,\delta)} = i_A(\alpha;\gamma) \cap e_A(\beta;\delta).$$

3. Double-framed Soft Filters

Firstly, we establish the notion of double-framed soft filter with an example:

Definition 3.1. A DFS-set $\langle(\alpha,\beta);A\rangle$ of A is called a *double-framed soft filter* of A over U (briefly, DFS-filter of A) if it satisfies :

$$(\forall x \in A)\left(\alpha(1) \supseteq \alpha(x),\ \beta(1) \subseteq \beta(x)\right), \tag{1.1}$$

$$(\forall x,y \in A)\begin{pmatrix} \alpha(y) \supseteq \alpha(x*y) \cap \alpha(x), \\ \beta(y) \subseteq \beta(x*y) \cup \beta(x) \end{pmatrix}. \tag{1.2}$$

Example 3.1. Suppose that there are five houses in the initial universe set U given by

$$U = \{h_1, h_2, h_3, h_4, h_5\}.$$

Let a set of parameters $E = \{e_0, e_1, e_2, e_3\}$ be a set of status of houses in which

e_0 stands for the parameter “beautiful”,

e_1 stands for the parameter “cheap”,

e_2 stands for the parameter “in good location”,

e_3 stands for the parameter “in green surroundings, with the following binary operation:

$*$	e_0	e_1	e_2	e_3
e_0	e_0	e_1	e_2	e_3
e_1	e_0	e_0	e_0	e_3
e_2	e_0	e_0	e_0	e_3
e_3	e_3	e_3	e_3	e_0

Then $(E, *, e_0)$ is a CI-algebra. Consider a double-framed soft set $\langle(\alpha, \beta); E\rangle$ of E over U as follows:

$$\alpha : E \to P(U), \quad x \mapsto \begin{cases} \{h_1, h_2, h_3, h_4, h_5\} & \text{if } x = e_0, \\ \{h_1, h_3, h_4\} & \text{if } x \in \{e_1, e_2\}, \\ \{h_1, h_4\} & \text{if } x = e_3, \end{cases}$$

and

$$\beta : E \to P(U), \quad x \mapsto \begin{cases} \{h_1, h_3\} & \text{if } x \in \{e_0, e_1, e_2\}, \\ \{h_1, h_2, h_3, h_5\} & \text{if } x = e_3 \end{cases}$$

It is routine to verify that $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E.

We first provide a characterization of a DFS-filter.

Theorem 3.1. For a double-framed soft set $\langle(\alpha, \beta); E\rangle$ of E over U, the following are equivalent:

(1) $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E.

(2) For every subsets γ and τ of U with $\gamma \in Im(\alpha)$ and $\tau \in Im(\beta)$, the γ-inclusive set and the τ-exclusive set of $\langle(\alpha, \beta); E\rangle$ are filters of E.

Proof. Assume that $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E. Let $x, y \in E$ be such that $x * y,\ x \in i_E(\alpha; \gamma)$ and $x * y,\ x \in e_E(\beta; \tau)$ for every subsets γ and τ of U with $\gamma \in Im(\alpha)$ and $\tau \in Im(\beta)$. Then $\gamma \subseteq \alpha(x)$, $\gamma \subseteq \alpha(x*y)$, $\tau \supseteq \beta(x)$ and $\tau \supseteq \beta(x*y)$. It follows from (1.1) and (1.2) that $\alpha(1) \supseteq \alpha(x) \supseteq \gamma$, $\alpha(y) \supseteq \alpha(x*y) \cap \alpha(x) \supseteq \gamma$, $\tau \supseteq \beta(x) \supseteq \beta(1)$ and $\tau \supseteq \beta(x*y) \cup \beta(x) \supseteq \beta(y)$ for all $x, y \in E$. Hence $1 \in i_E(\alpha; \gamma)$, $y \in i_E(\alpha; \gamma)$, $1 \in e_E(\beta; \tau)$ and $y \in e_E(\beta; \tau)$. Thus $i_E(\alpha; \gamma)$ and $e_E(\beta, \tau)$ are filters of E.

Conversely, suppose that $i_E(\alpha; \gamma)$ and $e_E(\beta; \tau)$ are filters of E for all $\gamma, \tau \in P(U)$ with $i_E(\alpha; \gamma) \neq \emptyset$ and $e_E(\beta; \tau) \neq \emptyset$. If we let $\alpha(x) = \gamma$ for any $x \in X$, then $x \in i_E(\alpha; \gamma)$. Since $i_E(\alpha; \gamma)$ is a filter of E, we have $1 \in i_E(\alpha; \gamma)$ and so $\alpha(x) = \gamma \subseteq \alpha(1)$. For any $x, y \in X$, let $\alpha(x*y) = \gamma_{x*y}$ and $\alpha(x) = \gamma_x$. Take $\gamma = \gamma_{x*y} \cap \gamma_x$. Then $x * y \in i_E(\alpha; \gamma)$ and $x \in i_E(\alpha; \gamma)$ which imply that $y \in i_E(\alpha; \gamma)$. Hence

$$\alpha(y) \supseteq \gamma = \gamma_{x*y} \cap \gamma_x = \alpha(x*y) \cap \alpha(x).$$

For any $x \in X$, let $\beta(x) = \tau$. Then $x \in e_E(\beta;\tau)$. Since $e_E(\beta;\tau)$ is a filter of E, we have $1 \in e_E(\beta;\tau)$ and so $\beta(x) = \tau \supseteq \beta(1)$. For any $x, y \in X$, let $\beta(x * y) = \tau_{x*y}$ and $\beta(x) = \tau_x$. Take $\tau = \tau_{x*y} \cup \tau_x$. Then $x * y \in e_E(\beta;\tau)$ and $x \in e_E(\beta;\tau)$ which imply that $y \in e_E(\beta;\tau)$. Hence

$$\beta(y) \subseteq \tau = \tau_{x*y} \cup \tau_x = \beta(x * y) \cup \beta(x).$$

Therefore $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E. □

Corollary 3.1. If $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E over U, then the double-framed including set $DF_E\,(\alpha, \beta)_{(\gamma,\delta)}$ is a filter of E.

Proof. Straightforward. □

Proposition 3.1. Every DFS-filter $\langle(\alpha, \beta); E\rangle$ of E over U satisfies:

$$(\forall x, y \in E)\,(x * y = 1 \;\Rightarrow\; \alpha(x) \subseteq \alpha(y),\; \beta(x) \supseteq \beta(y))\,. \tag{1.3}$$

□

If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies the condition (1.3), then we say that $\langle(\alpha, \beta); E\rangle$ is *order preserving.*

Proof. Assume that $x * y = 1$ for all $x, y \in E$. Then

$$\alpha(x) = \alpha(1) \cap \alpha(x) = \alpha(x * y) \cap \alpha(x) \subseteq \alpha(y)$$

and

$$\beta(x) = \beta(1) \cup \beta(x) = \beta(x * y) \cup \beta(x) \supseteq \beta(y)$$

for all $x, y \in E$ by using (1.1) and (1.2). □

Proposition 3.2. Let $\langle(\alpha, \beta); E\rangle$ be a DFS-set of E over U which satisfies the condition (1.1) and

$$(\forall x, y, z \in E) \begin{pmatrix} \alpha(x * y) \supseteq \alpha(x * (y * z)) \cap \alpha(y), \\ \beta(x * y) \subseteq \beta(x * (y * z)) \cup \beta(y) \end{pmatrix}. \tag{1.4}$$

Then $\langle(\alpha, \beta); E\rangle$ is order preserving.

Proof. Let $x, y \in E$ be such that $x * y = 1$. It follows from (CI2), (1.4), (CI1) and (1.1) that

$$\alpha(y) = \alpha(1 * y) \supseteq \alpha(1 * (x * y)) \cap \alpha(x) = \alpha(1 * 1) \cap \alpha(x) = \alpha(x)$$

and

$$\beta(y) = \beta(1 * y) \subseteq \beta(1 * (x * y)) \cup \beta(x) = \beta(1 * 1) \cup \beta(x) = \beta(x).$$

Therefore $\langle(\alpha, \beta); E\rangle$ is order reversing. □

Proposition 3.3. Every DFS-filter $\langle(\alpha, \beta); E\rangle$ of E over U satisfies:

$$(\forall x, a, b \in E)\left(a * (b * x) = 1 \ \Rightarrow \begin{cases} \alpha(x) \supseteq \alpha(a) \cap \alpha(b) \\ \beta(x) \subseteq \beta(a) \cup \beta(b) \end{cases}\right). \tag{1.5}$$

Proof. Let $a, b, x \in E$ be such that $a * (b * x) = 1$. Using (1.1) and (1.2), we have

$$\begin{aligned} \alpha(x) &\supseteq \alpha(b * x) \cap \alpha(b) \supseteq \alpha(a * (b * x)) \cap \alpha(a) \cap \alpha(b) \\ &= \alpha(1) \cap \alpha(a) \cap \alpha(b) = \alpha(a) \cap \alpha(b) \end{aligned}$$

and

$$\begin{aligned} \beta(x) &\subseteq \beta(b * x) \cup \beta(b) \subseteq \beta(a * (b * x)) \cup \beta(a) \cup \beta(b) \\ &= \beta(1) \cup \beta(a) \cup \beta(b) = \beta(a) \cup \beta(b). \end{aligned}$$

This completes the proof. □

As a generalization of Proposition , we have the following result.

Proposition 3.4. If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U is a DFS-filter of E, then

$$\prod_{i=1}^{n} a_i * x = 1 \ \Rightarrow \ \alpha(x) \supseteq \bigcap_{i=1}^{n} \alpha(a_i), \ \beta(x) \subseteq \bigcup_{i=1}^{n} \beta(a_i) \tag{1.6}$$

for all $x, a_1, \cdots, a_n \in E$, where

$$\prod_{i=1}^{n} a_i * x = a_n * (a_{n-1} * (\cdots (a_1 * x) \cdots)).$$

Proof. The proof is by induction on n. Let $\langle(\alpha, \beta); E\rangle$ be a DFS-filter of E over U. By Propositions and , we know that the condition (1.6) is valid for $n = 1, 2$. Assume that $\langle(\alpha, \beta); E\rangle$ satisfies the condition (1.6) for $n = k$, that is,

$$\prod_{i=1}^{k} a_i * x = 1 \ \Rightarrow \ \alpha(x) \supseteq \bigcap_{i=1}^{k} \alpha(a_i), \ \beta(x) \subseteq \bigcup_{i=1}^{k} \beta(a_i)$$

for all $x, a_1, \cdots, a_k \in E$. Suppose that $\prod_{i=1}^{k+1} a_i * x = 1$ for all $x, a_1, \cdots, a_k, a_{k+1} \in E$. Then

$$\alpha(a_1 * x) \supseteq \bigcap_{i=2}^{k+1} \alpha(a_i), \ \beta(a_1 * x) \subseteq \bigcup_{i=2}^{k+1} \beta(a_i).$$

Since $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E, it follows from (1.2) that

$$\alpha(x) \supseteq \alpha(a_1 * x) \cap \alpha(a_1) \supseteq \left(\bigcap_{i=2}^{k+1} \alpha(a_i)\right) \cap \alpha(a_1) = \bigcap_{i=1}^{k+1} \alpha(a_i)$$

and

$$\beta(x) \subseteq \beta(a_1 * x) \cup \beta(a_1) \subseteq \left(\bigcup_{i=2}^{k+1} \beta(a_i)\right) \cup \beta(a_1) = \bigcup_{i=1}^{k+1} \beta(a_i).$$

This completes the proof. □

Lemma 3.1 [11]. Every CI-algebra E satisfies:

(1.7) $$(\forall x, y \in E)\,(x * ((x * y) * y) = 1)\,.$$

We provide conditions for a DFS-set to be a DFS-filter.

Theorem 3.2. If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies two conditions (1.1) and (1.5), then $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E.

Proof. Using Lemma , (1.1) and (1.5), we have

$$\alpha(y) \supseteq \alpha(x * y) \cap \alpha(x), \ \ \beta(y) \subseteq \beta(x * y) \cup \beta(x)$$

for all $x, y \in E$. Hence $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E. □

Proposition 3.5 If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies the condition (1.2), then it satisfies the following condition:

(1.8) $$(\forall x, y, z \in E)\begin{pmatrix} \alpha(x * z) \supseteq \alpha(x * (y * z)) \cap \alpha(y), \\ \beta(x * z) \subseteq \beta(x * (y * z)) \cup \beta(y) \end{pmatrix}.$$

Proof. Using (1.2) and (CI3), we have

$$\alpha(x * z) \supseteq \alpha(y * (x * z)) \cap \alpha(y) = \alpha(x * (y * z)) \cap \beta(y)$$

and

$$\beta(x*z) \subseteq \beta(y*(x*z)) \cup \beta(y) = \beta(x*(y*z)) \cup \beta(y)$$

for all $x, y, z \in E$. □

Corollary 3.2. Every DFS-filter $\langle(\alpha,\beta);E\rangle$ of E over U satisfies the condition (1.8).

Theorem3.3. If a DFS-set $\langle(\alpha,\beta);E\rangle$ of E over U satisfies two conditions (1.1) and (1.8), then $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E.

Proof. If we take $x = 1$ in (1.8) and use (CI2), then

$$\alpha(z) = \alpha(1*z) \supseteq \alpha(1*(y*z)) \cap \alpha(y) = \alpha(y*z) \cap \alpha(y)$$

and

$$\beta(z) = \beta(1*z) \subseteq \beta(1*(y*z)) \cup \beta(y) = \beta(y*z) \cup \beta(y)$$

for all $y, z \in E$. Therefore $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E. □

Lemma 3.2. Every DFS-filter $\langle(\alpha,\beta);E\rangle$ of E over U satisfies:

$$(1.9) \qquad (\forall x, y \in E)\,(\alpha(x) \subseteq \alpha((x*y)*y),\ \beta(x) \supseteq \beta((x*y)*y))\,.$$

Proof. If we take $y = (x*y)*y$ in (1.2), then

$$\begin{aligned}\alpha((x*y)*y) &\supseteq \alpha(x*((x*y)*y)) \cap \alpha(x)\\ &= \alpha((x*y)*(x*y)) \cap \alpha(x)\\ &= \alpha(1) \cap \alpha(x) = \alpha(x)\end{aligned}$$

and

$$\begin{aligned}\beta((x*y)*y) &\subseteq \beta(x*((x*y)*y)) \cup \beta(x)\\ &= \beta((x*y)*(x*y)) \cup \beta(x)\\ &= \beta(1) \cup \beta(x) = \beta(x)\end{aligned}$$

by using (CI3), (CI1) and (1.1). □

Proposition 3.6. Every DFS-filter $\langle(\alpha,\beta);E\rangle$ of E over U satisfies:

$$(1.10) \qquad (\forall x, a, b \in E)\begin{pmatrix}\alpha((a*(b*x))*x) \supseteq \alpha(a) \cap \alpha(b),\\ \beta((a*(b*x))*x) \subseteq \beta(a) \cup \beta(b)\end{pmatrix}.$$

Proof. Using Proposition and Lemma , we have

$$\alpha((a*(b*x))*x) \supseteq \alpha((a*(b*x))*(b*x)) \cap \alpha(b) \supseteq \alpha(a) \cap \alpha(b)$$

and

$$\beta((a*(b*x))*x) \subseteq \beta((a*(b*x))*(b*x)) \cup \beta(b) \subseteq \beta(a) \cup \beta(b)$$

for all $a, b, x \in E$. □

Theorem 3.4. If a DFS-set $\langle(\alpha,\beta);E\rangle$ of E over U satisfies the condition (1.10) and

$$(\forall x, y \in E)\,(\alpha(y*x) \supseteq \alpha(x),\ \beta(y*x) \subseteq \beta(x))\,, \tag{1.11}$$

then $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E.

Proof. If we take $y = x$ in (1.11), then $\alpha(1) = \alpha(x*x) \supseteq \alpha(x)$ and $\beta(1) = \beta(x*x) \subseteq \beta(x)$ for all $x \in E$. Using (CI1), (CI2) and (1.10), we have

$$\alpha(y) = \alpha(1*y) = \alpha(((x*y)*(x*y))*y) \supseteq \alpha(x*y) \cap \alpha(x)$$

and

$$\beta(y) = \beta(1*y) = \beta(((x*y)*(x*y))*y) \subseteq \beta(x*y) \cup \beta(x)$$

for all $x, y \in E$. Therefore $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E. □

We make a new DFS-filter from old one.

Theorem 3.5. Let $\langle(\alpha,\beta);E\rangle$ be a DFS-set of E over U and define a DFS-set $\langle(\alpha^*,\beta^*);E\rangle$ of E over U by

$$\alpha^* : E \to P(U),\ x \mapsto \begin{cases} \alpha(x) & \text{if } x \in i_E(\alpha;\gamma), \\ \eta & \text{otherwise} \end{cases}$$
$$\beta^* : E \to P(U),\ x \mapsto \begin{cases} \beta(x) & \text{if } x \in e_E(\beta;\tau), \\ \delta & \text{otherwise} \end{cases}$$

where $\gamma, \tau,\ \eta$ and δ are subsets of U satisfying $\eta \subsetneq \bigcap_{x \notin i_E(\alpha;\gamma)} \alpha(x)$ and $\delta \supsetneq \bigcup_{x \notin e_E(\beta;\tau)} \beta(x)$. If $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E, then so is $\langle(\alpha^*,\beta^*);E\rangle$.

Proof. Assume that $\langle(\alpha,\beta);E\rangle$ is a DFS-filter of E. Then $i_E(\alpha;\gamma)(\neq \emptyset)$ and $e_E(\beta;\tau)(\neq \emptyset)$ are filters of E for all $\gamma, \tau \in P(U)$ by Theorem . Hence $1 \in i_E(\alpha;\gamma)$

and $1 \in e_E(\beta;\tau)$, and so $\alpha^*(1) = \alpha(1) \supseteq \alpha(x) \supseteq \alpha^*(x)$ and $\beta^*(1) = \beta(1) \subseteq \beta(x) \subseteq \beta^*(x)$ for all $x \in E$. Let $x, y \in E$. If $x * y \in i_E(\alpha;\gamma)$ and $x \in i_E(\alpha;\gamma)$, then $y \in i_E(\alpha;\gamma)$. Hence

$$\alpha^*(y) = \alpha(y) \supseteq \alpha(x * y) \cap \alpha(x) = \alpha^*(x * y) \cap \alpha^*(x).$$

If $x * y \notin i_E(\alpha;\gamma)$ or $x \notin i_E(\alpha;\gamma)$, then $\alpha^*(x * y) = \eta$ or $\alpha^*(x) = \eta$. Thus

$$\alpha^*(y) \supseteq \eta = \alpha^*(x * y) \cap \alpha^*(x).$$

If $x * y \in e_E(\beta;\tau)$ and $x \in e_E(\beta;\tau)$, then $y \in e_E(\beta;\tau)$. Hence

$$\beta^*(y) = \beta(y) \subseteq \beta(x * y) \cup \beta(x) = \beta^*(x * y) \cup \beta^*(x).$$

If $x * y \notin e_E(\beta;\tau)$ or $x \notin e_E(\beta;\tau)$, then $\beta^*(x * y) = \delta$ or $\beta^*(x) = \delta$. Thus

$$\beta^*(y) \subseteq \delta = \beta^*(x * y) \cup \beta^*(x).$$

Therefore $\langle(\alpha^*, \beta^*); E\rangle$ is a DFS-filter of E. □

Theorem 3.6. If $\langle(\alpha, \beta); E\rangle$ is a DFS-filter of E, then the set

$$E_{(\alpha,\beta)} := \{x \in X \mid \alpha(x) = \alpha(1),\ \beta(x) = \beta(1)\}$$

is a filter of E.

Proof. Obviously $1 \in E_{(\alpha,\beta)}$. Let $x, y \in E$ be such that $x \in E_{(\alpha,\beta)}$ and $x * y \in E_{(\alpha,\beta)}$. Then $\alpha(x) = \alpha(1) = \alpha(x * y)$ and $\beta(x) = \beta(1) = \beta(x * y)$. It follows that $\alpha(y) \supseteq \alpha(x * y) \cap \alpha(x) = \alpha(1)$ and $\beta(y) \subseteq \beta(x * y) \cup \beta(x) = \beta(1)$. The condition (1.1) implies that $\alpha(y) = \alpha(1)$ and $\beta(y) = \beta(1)$. Hence $y \in E_{(\alpha,\beta)}$, and therefore $E_{(\alpha,\beta)}$ is a filter of E. □

Corollary 3.3. If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies two conditions (1.1) and (1.5), then the set

$$E_{(\alpha,\beta)} := \{x \in X \mid \alpha(x) = \alpha(1),\ \beta(x) = \beta(1)\}$$

is a filter of E.

Corollary 3.4. If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies two conditions (1.1) and (1.8), then the set

$$E_{(\alpha,\beta)} := \{x \in X \mid \alpha(x) = \alpha(1),\ \beta(x) = \beta(1)\}$$

is a filter of E.

Corollary 3.5. If a DFS-set $\langle(\alpha, \beta); E\rangle$ of E over U satisfies two conditions (1.10) and (1.11), then the set

$$E_{(\alpha,\beta)} := \{x \in X \mid \alpha(x) = \alpha(1),\ \beta(x) = \beta(1)\}$$

is a filter of E.

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