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Intuitionistic Smooth Bitopological Spaces and Continuity

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Abstract

In this paper, we introduce intuitionistic smooth bitopological spaces and the notions of intuitionistic fuzzy semiinterior and semiclosure. Based on these concepts, the characterizations for the intuitionistic fuzzy pairwise semicontinuous mappings are obtained.

Keywords: Intuitionistic, Smooth bitopology

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1. Introduction and Preliminaries

Chang [1] introduced the notion of fuzzy topology. Chang's fuzzy topology is a crisp subfamily of fuzzy sets. However, in his study, Chang did not consider the notion of openness of a fuzzy set, which seems to be a drawback in the process of fuzzification of topological spaces. To overcome this drawback, Šostak [2, 3], based on the idea of degree of openness, introduced a new definition of fuzzy topology as an extension of Chang's fuzzy topology. This generalization of fuzzy topological spaces was later rephrased as smooth topology by Ramadan [4].

Çoker and his colleague [5, 6] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets which were introduced by Atanassov [7]. Mondal and Samanta [8] introduced the concept of an intuitionistic gradation of openness as a generalization of a smooth topology.

On the other hand, Kandil [9] introduced the concept of fuzzy bitopological spaces as a natural generalization of Chang's fuzzy topological spaces. Lee and his colleagues [10, 11] introduced the notion of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

Lim et al. [12] defined the term "intuitionistic smooth topology," which is a slight modification of the intuitionistic gradation of openness of Mondal and Samanta, therefore, it is different from ours.

In this paper, we introduce intuitionistic smooth bitopological spaces and the notions of intuitionistic fuzzy $(\mathcal{T}_i, \mathcal{T}_j)$-$(r, s)$-semiinterior and semiclosure. Based on these concepts, the characterizations for the intuitionistic fuzzy pairwise (r, s)-semicontinuous mappings are obtained.

I denotes the unit interval $[0, 1]$ of the real line and $I_0 = (0, 1]$. A member μ of I^X is called a *fuzzy set* in X. For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value of 0 and 1, respectively.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq 1$. Obviously, every fuzzy set μ in X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$. $I(X)$ denotes a family of all intuitionistic fuzzy sets in X and "IF" stands for intuitionistic fuzzy.

Definition 1.1. ([4]) A *smooth topology* on X is a mapping $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.

(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.

(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth topological space*.

Definition 1.2. ([11]) A system (X, T_1, T_2) consisting of a set X with two smooth topologies T_1 and T_2 on X is called a *smooth bitopological space*.

Definition 1.3. ([5]) An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $\underline{0}$, $\underline{1} \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for each i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

2. Intuitionistic Smooth Bitopological Spaces

Now, we define the notions of intuitionistic smooth topological spaces and intuitionistic smooth bitopological spaces.

Definition 2.1. An *intuitionistic smooth topology* on X is a mapping $\mathcal{T} : I(X) \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\underline{0}) = \mathcal{T}(\underline{1}) = 1$.

(2) $\mathcal{T}(A \cap B) \geq \mathcal{T}(A) \wedge \mathcal{T}(B)$.

(3) $\mathcal{T}(\bigvee A_i) \geq \bigwedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called an *intuitionistic smooth topological space*.

Let $(X, \mathcal{T})$ be an intuitionistic smooth topological space. For each $r \in I_0$, an r-cut

$$\mathcal{T}_r = \{A \in I(X) \mid \mathcal{T}(A) \geq r\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $r \in I_0$. Then the mapping $T^r : I(X) \to I$ defined by

$$T^r(A) = \begin{cases} 1 & \text{if } \mu = \underline{0}, \underline{1}, \\ r & \text{if } A \in T - \{\underline{0}, \underline{1}\}, \\ 0 & \text{otherwise} \end{cases}$$

becomes an intuitionistic smooth topology on X.

Definition 2.2. Let A be an intuitionistic fuzzy set in intuitionistic smooth topological space $(X, \mathcal{T})$ and $r \in I_0$. Then A is said to be

(1) *IF $\mathcal{T}$-r-open* if $\mathcal{T}(A) \geq r$,

(2) *IF $\mathcal{T}$-r-closed* if $\mathcal{T}(A^c) \geq r$.

Definition 2.3. Let $(X, \mathcal{T})$ be an intuitionistic smooth topological space. For $r \in I_0$ and for each $A \in I(X)$, the *IF $\mathcal{T}$-r-interior* is defined by

$$\mathcal{T}\text{-int}(A, r) = \bigcup\{B \mid B \subseteq A, \mathcal{T}(B) \geq r\}$$

and the *IF $\mathcal{T}$-r-closure* is defined by

$$\mathcal{T}\text{-cl}(A, r) = \bigcap\{B \mid A \subseteq B, \mathcal{T}(B^c) \geq r\}.$$

Theorem 2.4. Let A be an intuitionistic fuzzy set in an intuitionistic smooth topological space $(X, \mathcal{T})$ and $r \in I_0$. Then

(1) $\mathcal{T}\text{-int}(A, r)^c = \mathcal{T}\text{-cl}(A^c, r)$.

(2) $\mathcal{T}\text{-cl}(A, r)^c = \mathcal{T}\text{-int}(A^c, r)$.

Proof. It follows from Lemma 2.5 in [13].

Definition 2.5. A system $(X, \mathcal{T}_1, \mathcal{T}_2)$ consisting of a set X with two intuitionistic smooth topologies $\mathcal{T}_1$ and $\mathcal{T}_2$ on X is called a *intuitionistic smooth bitopological space*(ISBTS for short). Throughout this paper the indices i, j take the value in $\{1, 2\}$ and $i \neq j$.

Definition 2.6. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then A is said to be

(1) an *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-*semiopen* set if there exist an IF $\mathcal{T}_i$-r-open set B in X such that $B \subseteq A \subseteq \mathcal{T}_j$-cl$(B, s)$,

(2) an *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-*semiclosed* set if there exist an IF $\mathcal{T}_i$-r-closed set B in X such that $\mathcal{T}_j$-int$(B, s) \subseteq A \subseteq B$.

Theorem 2.7. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then the following statements are equivalent:

(1) A is an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set.

(2) A^c is an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed set.

(3) $\mathcal{T}_j$-cl$(\mathcal{T}_i$-int$(A, r), s) \supseteq A$.

(4) $\mathcal{T}_j$-int$(\mathcal{T}_i$-cl$(A^c, r), s) \subseteq A^c$.

Proof. (1) $\Rightarrow$ (2) Let A be an $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set. Then there is an IF $\mathcal{T}_i$-r-open set B in X such that $B \subseteq A \subseteq \mathcal{T}_j$-cl$(B, s)$. Thus $\mathcal{T}_j$-int$(B^c, s) \subseteq A^c \subseteq B^c$. Since B^c is IF $\mathcal{T}_i$-r-closed in X, A^c is a IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed set in X.

(2) $\Rightarrow$ (1) Let A^c be an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed set. Then there is an IF $\mathcal{T}_i$-r-closed set B in X such that $\mathcal{T}_j$-int(B, s) $\subseteq A^c \subseteq B$. Hence $B^c \subseteq A \subseteq \mathcal{T}_j$-cl$(B^c, s)$. Because B^c is IF $\mathcal{T}_i$-r-open in X, A is an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set in X.

(1) $\Rightarrow$ (3) Let A be an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set in X. Then there exist an IF $\mathcal{T}_i$-r-open set B in X such that $B \subseteq A \subseteq \mathcal{T}_j$-cl$(B, s)$. Since B is IF $\mathcal{T}_i$-r-open, we have $B = \mathcal{T}_i$-int$(B, r) \subseteq \mathcal{T}_i$-int$(A, r)$. Thus

$$\mathcal{T}_j\text{-cl}(\mathcal{T}_i\text{-int}(A, r), s) \supseteq \mathcal{T}_j\text{-cl}(B, s) \supseteq A.$$

(3) $\Rightarrow$ (1) Let $\mathcal{T}_j$-cl$(\mathcal{T}_i$-int$(A, r), s) \supseteq A$ and take $B = \mathcal{T}_i$-int(A, r). Then B is an IF $\mathcal{T}_i$-r-open set and

$$\begin{aligned} B = \mathcal{T}_i\text{-int}(A, r) &\subseteq A \\ &\subseteq \mathcal{T}_j\text{-cl}(\mathcal{T}_i\text{-int}(A, r), s) \\ &= \mathcal{T}_j\text{-cl}(B, s). \end{aligned}$$

Hence A is an IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set.

(3) $\Leftrightarrow$ (4) It follows from Theorem 2.4.

Theorem 2.8. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then

(1) If A is IF $\mathcal{T}_1$-r-open in $(X, \mathcal{T}_1)$, then A is an IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r,s)$-semiopen set in $(X, \mathcal{T}_1, \mathcal{T}_2)$.

(2) If A is IF $\mathcal{T}_2$-s-open in $(X, \mathcal{T}_2)$, then A is an IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(s,r)$-semiopen set in $(X, \mathcal{T}_1, \mathcal{T}_2)$.

Proof. (1) Let A be an IF $\mathcal{T}_1$-r-open set in $(X, \mathcal{T}_1)$. Then $A = \mathcal{T}_1$-int(A, r). Thus we have

$$\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A, r), s) = \mathcal{T}_2\text{-cl}(A, s) \supseteq A.$$

Hence A is IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r,s)$-semiopen in $(X, \mathcal{T}_1, \mathcal{T}_2)$.

(2) Similar to (1).

The following example shows that the converses of the above theorem need not be true.

Example 2.9. Let $X = \{x, y\}$ and let A_1, A_2, A_3, and A_4 be intuitionistic fuzzy sets in X defined as

$$\begin{aligned} A_1(x) &= (0.1, 0.7), \quad A_1(y) = (0.7, 0.2); \\ A_2(x) &= (0.6, 0.2), \quad A_2(y) = (0.3, 0.6); \\ A_3(x) &= (0.1, 0.7), \quad A_3(y) = (0.9, 0.1); \end{aligned}$$

and

$$A_4(x) = (0.7, 0.1), \quad A_4(y) = (0.3, 0.6).$$

Define $\mathcal{T}_1 : I(X) \to I$ and $\mathcal{T}_2 : I(X) \to I$ by

$$\mathcal{T}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{2} & \text{if } A = A_1, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{T}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_2, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{T}_1, \mathcal{T}_2)$ is an ISBT on X. Note that

$$\mathcal{T}_2\text{-cl}(\mathcal{T}_1\text{-int}(A_3, \frac{1}{2}), \frac{1}{3}) = \mathcal{T}_2\text{-cl}(A_1, \frac{1}{3}) = \underline{1} \supseteq A_3$$

and

$$\mathcal{T}_1\text{-cl}(\mathcal{T}_2\text{-int}(A_4, \frac{1}{3}), \frac{1}{2}) = \mathcal{T}_1\text{-cl}(A_2, \frac{1}{2}) = \underline{1} \supseteq A_4.$$

Hence A_3 is IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(\frac{1}{2}, \frac{1}{3})$-semiopen and A_4 is IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(\frac{1}{3}, \frac{1}{2})$-semiopen in $(X, \mathcal{T}_1, \mathcal{T}_2)$. But A_3 is not an IF $\mathcal{T}_1$-$\frac{1}{2}$-open set in $(X, \mathcal{T}_1)$ and A_4 is not an IF $\mathcal{T}_2$-$\frac{1}{3}$-open set in $(X, \mathcal{T}_2)$.

Theorem 2.10. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS and $r, s \in I_0$. Then the following statements are true:

(1) If $\{A_k\}$ is a family of IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen sets in X, then $\bigcup A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen.

(2) If $\{A_k\}$ is a family of IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed sets in X, then $\bigcap A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed.

Proof. (1) Let $\{A_k\}$ be a collection of IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen sets in X. Then for each k,

$$A_k \subseteq \mathcal{T}_j\text{-cl}(\mathcal{T}_i\text{-int}(A_k, r), s).$$

So we have

$$\begin{aligned}\bigcup A_k &\subseteq \bigcup \mathcal{T}_j\text{-cl}(\mathcal{T}_i\text{-int}(A_k, r), s)\\ &\subseteq \mathcal{T}_j\text{-cl}(\mathcal{T}_i\text{-int}(\bigcup A_k, r), s).\end{aligned}$$

Thus $\bigcup A_k$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen.

(2) It follows from (1) using Theorem 2.7 .

Definition 2.11. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS and $r, s \in I_0$. For each $A \in I(X)$, the *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-*semiinterior* is defined by

$$\begin{aligned}&(\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)\\ &= \bigcup\{B \in I(X) \mid\\ &\qquad B \subseteq A,\ B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r,s)\text{-semiopen}\}\end{aligned}$$

and the *IF* $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-*semiclosure* is defined by

$$\begin{aligned}&(\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A, r, s)\\ &= \bigcap\{B \in I(X) \mid\\ &\qquad A \subseteq B,\ B \text{ is IF } (\mathcal{T}_i, \mathcal{T}_j)\text{-}(r,s)\text{-semiclosed}\}.\end{aligned}$$

Obviously, $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s)$ is the smallest IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed set which contains A and $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s)$ is the greatest IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set which is contained in A. Also, $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s) = A$ for any IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed set A and $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s) = A$ for any IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen set A.

Moreover, we have

$$\begin{aligned}\mathcal{T}_i\text{-int}(A, r) &\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)\\ &\subseteq A\\ &\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A, r, s)\\ &\subseteq \mathcal{T}_i\text{-cl}(A, r).\end{aligned}$$

Also, we have the following results:

(1) $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(\underline{0}, r, s) = \underline{0}$, $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(\underline{1}, r, s) = \underline{1}$.

(2) $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s) \supseteq A$.

(3) $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s) \cup (\mathcal{T}_i, \mathcal{T}_j)$-scl$(B, r, s)$ $\subseteq (\mathcal{T}_i, \mathcal{T}_j)$-scl$(A \cup B, r, s)$.

(4) $(\mathcal{T}_i, \mathcal{T}_j)$-scl$((\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s), r, s)$ $= (\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s)$.

(5) $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(\underline{0}, r, s) = \underline{0}$, $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(\underline{1}, r, s) = \underline{1}$.

(6) $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s) \subseteq A$.

(7) $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s) \cap (\mathcal{T}_i, \mathcal{T}_j)$-sint$(B, r, s)$ $\supseteq (\mathcal{T}_i, \mathcal{T}_j)$-sint$(A \cap B, r, s)$.

(8) $(\mathcal{T}_i, \mathcal{T}_j)$-sint$((\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s), r, s)$ $= (\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s)$.

Theorem 2.12. Let A be an intuitionistic fuzzy set in an ISBTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $r, s \in I_0$. Then we have

(1) $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)$-scl$(A^c, r, s)$.

(2) $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)$-sint$(A^c, r, s)$.

Proof. (1) Since

$$(\mathcal{T}_i, \mathcal{T}_j) - \text{sint}(A, r, s) \subseteq A \text{ and } (\mathcal{T}_i, \mathcal{T}_j) - \text{sint}(A, r, s)$$

is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen in X, $A^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s)^c$ and $(\mathcal{T}_i, \mathcal{T}_j)$-sint$(A, r, s)^c$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed. Thus

$$\begin{aligned}&(\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A^c, r, s)\\ &\quad\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-scl}((\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)^c, r, s)\\ &\quad= (\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)^c.\end{aligned}$$

From that $A^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)$-scl$(A^c, r, s)$ and $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A^c, r, s)$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiclosed, $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A^c, r, s)^c \subseteq A$ and $(\mathcal{T}_i, \mathcal{T}_j)$-scl$(A^c, r, s)^c$ is IF $(\mathcal{T}_i, \mathcal{T}_j)$-$(r,s)$-semiopen. Thus we have

$$\begin{aligned}&(\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A^c, r, s)^c\\ &\quad= (\mathcal{T}_i, \mathcal{T}_j)\text{-sint}((\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A^c, r, s)^c, r, s)\\ &\quad\subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s).\end{aligned}$$

Hence

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)^c \subseteq (\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A^c, r, s).$$

Therefore

$$(\mathcal{T}_i, \mathcal{T}_j)\text{-sint}(A, r, s)^c = (\mathcal{T}_i, \mathcal{T}_j)\text{-scl}(A^c, r, s).$$

(2) Similar to (1).

3. Continuity in Intuitionistic Smooth Bitopology

We define the notions of IF pairwise (r, s)-semicontinuous mappings in intuitionistic smooth bitopological spaces, and investigate their characteristic properties.

Definition 3.1. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a mapping from an intuitionistic smooth topological spaces X to an intuitionistic smooth topological spaces Y and $r \in I_0$. Then f is called an *IF r-continuous* mapping if $f^{-1}(B)$ is IF $\mathcal{T}$-r-open in X for each IF $\mathcal{U}$-r-open set B in Y.

Definition 3.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s)-continuous* if the induced mapping $f : (X, \mathcal{T}_1) \to (Y, \mathcal{U}_1)$ is an IF r-continuous mapping and the induced mapping $f : (X, \mathcal{T}_2) \to (Y, \mathcal{U}_2)$ is an IF s-continuous mapping.

Definition 3.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then f is said to be *IF pairwise (r, s)-semicontinuous* if $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r, s)$-semiopen set in X for each IF $\mathcal{U}_1$-r-open set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(s, r)$-semiopen set in X for each IF $\mathcal{U}_2$-s-open set B in Y.

Remark 3.4. It is obvious that every IF pairwise (r, s)-continuous mapping is IF pairwise (r, s)-semicontinuous. But the following example shows that the converse need not be true.

Example 3.5. Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be an ISBTS as described in Example 2.9. Define $\mathcal{U}_1 : I(X) \to I$ and $\mathcal{U}_2 : I(X) \to I$ by

$$\mathcal{U}_1(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ 0 & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}_2(A) = \begin{cases} 1 & \text{if } A = \underline{0}, \underline{1}, \\ \frac{1}{3} & \text{if } A = A_4, \\ 0 & \text{otherwise.} \end{cases}$$

Then $(\mathcal{U}_1, \mathcal{U}_2)$ is an ISBT on X. Consider a mapping $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (X, \mathcal{U}_1, \mathcal{U}_2)$ defined by $f(x) = x$ and $f(y) = y$. Then f is IF pairwise $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. But f is not an IF pairwise $(\frac{1}{2}, \frac{1}{3})$-continuous mapping.

Theorem 3.6. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r, s \in I_0$. Then the following statements are equivalent:

(1) f is IF pairwise (r, s)-semicontinuous.

(2) $f^{-1}(A)$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r, s)$-semiclosed set in X for each IF $\mathcal{U}_1$-r-closed set A in Y and $f^{-1}(B)$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(s, r)$-semiclosed set in X for each IF $\mathcal{U}_2$-s-closed set B in Y.

(3) For each intuitionistic fuzzy set B in Y,

$$\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(B), r), s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r))$$

and

$$\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(B), s), r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)).$$

(4) For each intuitionistic fuzzy set A in X,

$$f(\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A, r), s)) \subseteq \mathcal{U}_1\text{-cl}(f(A), r)$$

and

$$f(\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A, s), r)) \subseteq \mathcal{U}_2\text{-cl}(f(A), s).$$

Proof. (1) $\Leftrightarrow$ (2) Trivial.

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y. Then $\mathcal{U}_1$-cl(B, r) is IF $\mathcal{U}_1$-r-closed and $\mathcal{U}_2$-cl(B, s) is IF $\mathcal{U}_2$-s-closed in Y. Hence by (2), $f^{-1}(\mathcal{U}_1\text{-cl}(B, r))$ is an IF $(\mathcal{T}_1, \mathcal{T}_2)$-$(r, s)$-semiclosed set and $f^{-1}(\mathcal{U}_2\text{-cl}(B, s))$ is an IF $(\mathcal{T}_2, \mathcal{T}_1)$-$(s, r)$-semiclosed set in X. Thus we obtain

$$\begin{aligned}
&\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(B), r), s) \\
&\subseteq \mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(\mathcal{U}_1\text{-cl}(B, r)), r), s) \\
&\subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B, r))
\end{aligned}$$

and

$$\begin{aligned}
&\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(B), s), r) \\
&\subseteq \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(\mathcal{U}_2\text{-cl}(B, s)), s), r) \\
&\subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B, s)).
\end{aligned}$$

(3) $\Rightarrow$ (4) Let A be an intuitionistic fuzzy set in X. Then by (3), we have

$$\begin{aligned}
\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A, r), s) &\subseteq \mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(f(A)), r), s) \\
&\subseteq f^{-1}(\mathcal{U}_1\text{-cl}(f(A), r))
\end{aligned}$$

and

$$\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A,s),r) \subseteq \mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(f(A)),s),r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(f(A),s)).$$

Hence

$$f(\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(A,r),s)) \subseteq \mathcal{U}_1\text{-cl}(f(A),r)$$

and

$$f(\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(A,s),r)) \subseteq \mathcal{U}_2\text{-cl}(f(A),s).$$

(4) $\Rightarrow$ (2) Let A be any IF $\mathcal{U}_1$-r-closed set and B any IF $\mathcal{U}_2$ -s-closed set in Y. By (4), we obtain

$$\begin{aligned} f(\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(A),r),s)) &\subseteq \mathcal{U}_1\text{-cl}(f(f^{-1}(A)),r) \\ &\subseteq \mathcal{U}_1\text{-cl}(A,r) = A \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(B),s),r)) &\subseteq \mathcal{U}_2\text{-cl}(f(f^{-1}(B)),s) \\ &\subseteq \mathcal{U}_2\text{-cl}(B,s) = B. \end{aligned}$$

Hence

$$\mathcal{T}_2\text{-int}(\mathcal{T}_1\text{-cl}(f^{-1}(A),r),s) \subseteq f^{-1}(A)$$

and

$$\mathcal{T}_1\text{-int}(\mathcal{T}_2\text{-cl}(f^{-1}(B),s),r) \subseteq f^{-1}(B).$$

Therefore $f^{-1}(A)$ is an IF $(\mathcal{T}_1,\mathcal{T}_2)$-$(r,s)$-semiclosed set and $f^{-1}(B)$ is an IF $(\mathcal{T}_2,\mathcal{T}_1)$-$(s,r)$-semiclosed set in X.

Theorem 3.7. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from an ISBTS X to an ISBTS Y and $r,s \in I_0$. Then the following statements are equivalent:

(1) f is IF pairwise (r,s)-semicontinuous.

(2) For each intuitionistic fuzzy set A in X,

$$f((\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(A,r,s)) \subseteq \mathcal{U}_1\text{-cl}(f(A),r)$$

and

$$f((\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(A,s,r)) \subseteq \mathcal{U}_2\text{-cl}(f(A),s).$$

(3) For each intuitionistic fuzzy set B in Y,

$$(\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B,r))$$

and

$$(\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(B),s,r) \subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B,s)).$$

(4) For each intuitionistic fuzzy set B in Y,

$$f^{-1}(\mathcal{U}_1\text{-int}(B,r)) \subseteq (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(B),r,s)$$

and

$$f^{-1}(\mathcal{U}_2\text{-int}(B,s)) \subseteq (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r).$$

Proof. (1) $\Rightarrow$ (2) Let A be an intuitionistic fuzzy set in X. Then $\mathcal{U}_1$-cl$(f(A),r)$ is IF $\mathcal{U}_1$-r-closed and $\mathcal{U}_2$-cl$(f(A),s)$ is IF $\mathcal{U}_2$-s-closed in Y. Since f is IF pairwise (r,s)-semicontinuous, $f^{-1}(\mathcal{U}_1\text{-cl}(f(A),r))$ is an IF $(\mathcal{T}_1,\mathcal{T}_2)$-$(r,s)$-semiclosed set and $f^{-1}(\mathcal{U}_2\text{-cl}(f(A),s))$ is an IF $(\mathcal{T}_2,\mathcal{T}_1)$-$(s,r)$-semiclosed set in X. Hence

$$\begin{aligned} &(\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(A,r,s) \\ &\subseteq (\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(\mathcal{U}_1\text{-cl}(f(A),r)),r,s) \\ &= f^{-1}(\mathcal{U}_1\text{-cl}(f(A),r)) \end{aligned}$$

and

$$\begin{aligned} &(\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(A,s,r) \\ &\subseteq (\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(\mathcal{U}_2\text{-cl}(f(A),s)),s,r) \\ &= f^{-1}(\mathcal{U}_2\text{-cl}(f(A),s)). \end{aligned}$$

Therefore

$$f((\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(A,r,s)) \subseteq \mathcal{U}_1\text{-cl}(f(A),r)$$

and

$$f((\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(A,s,r)) \subseteq \mathcal{U}_2\text{-cl}(f(A),s).$$

(2) $\Rightarrow$ (3) Let B be an intuitionistic fuzzy set in Y. Then by (2), we obtain

$$\begin{aligned} f((\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(B),r,s)) &\subseteq \mathcal{U}_1\text{-cl}(f(f^{-1}(B)),r) \\ &\subseteq \mathcal{U}_1\text{-cl}(B,r) \end{aligned}$$

and

$$\begin{aligned} f((\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(B),s,r)) &\subseteq \mathcal{U}_2\text{-cl}(f(f^{-1}(B)),s) \\ &\subseteq \mathcal{U}_2\text{-cl}(B,s). \end{aligned}$$

Hence

$$(\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(B),r,s) \subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B,r))$$

and

$$(\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(B),s,r)\subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B,s)).$$

(3) $\Rightarrow$ (4) Let B be an intuitionistic fuzzy set in Y. Then by (3), we have

$$(\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(B^c),r,s)\subseteq f^{-1}(\mathcal{U}_1\text{-cl}(B^c,r))$$

and

$$(\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(B^c),s,r)\subseteq f^{-1}(\mathcal{U}_2\text{-cl}(B^c,s)).$$

Hence

$$\begin{aligned}f^{-1}(\mathcal{U}_1\text{-int}(B,r)) &= (f^{-1}(\mathcal{U}_1\text{-cl}(B^c,r)))^c\\ &\subseteq (\mathcal{T}_1,\mathcal{T}_2)\text{-scl}(f^{-1}(B^c),r,s)^c\\ &= (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(B),r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\mathcal{U}_2\text{-int}(B,s)) &= (f^{-1}(\mathcal{U}_2\text{-cl}(B^c,s)))^c\\ &\subseteq (\mathcal{T}_2,\mathcal{T}_1)\text{-scl}(f^{-1}(B^c),s,r)^c\\ &= (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r).\end{aligned}$$

(4) $\Rightarrow$ (1) Let A be any IF $\mathcal{U}_1$-r-open set and B any IF $\mathcal{U}_2$-s-open set in Y. Then $\mathcal{U}_1\text{-int}(A,r)=A$ and $\mathcal{U}_2\text{-int}(B,s)=B$. Hence

$$\begin{aligned}f^{-1}(A) &= f^{-1}(\mathcal{U}_1\text{-int}(A,r))\\ &\subseteq (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(A),r,s)\\ &\subseteq f^{-1}(A)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(B) &= f^{-1}(\mathcal{U}_2\text{-int}(B,s))\\ &\subseteq (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r)\\ &\subseteq f^{-1}(B).\end{aligned}$$

Thus

$$f^{-1}(A)=(\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(A),r,s)$$

and

$$f^{-1}(B)=(\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r).$$

Hence $f^{-1}(A)$ is an IF $(\mathcal{T}_1,\mathcal{T}_2)$-$(r,s)$-semiopen set and $f^{-1}(B)$ is an IF $(\mathcal{T}_2,\mathcal{T}_1)$-$(s,r)$-semiopen set in X. Therefore f is IF pairwise (r,s)-semicontinuous.

Theorem 3.8. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2)\to(Y,\mathcal{U}_1,\mathcal{U}_2)$ be a bijective mapping from an ISBTS X to an ISBTS Y and $r,s\in I_0$. Then f is IF pairwise (r,s)-semicontinuous if and only if

$$\mathcal{U}_1\text{-int}(f(A),r)\subseteq f((\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(A,r,s))$$

and

$$\mathcal{U}_2\text{-int}(f(A),s)\subseteq f((\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(A,s,r))$$

for each intuitionistic fuzzy set A in X.

Proof. Let A be an intuitionistic fuzzy set in X. Since f is one-to-one, by Theorem 3.7, we have

$$\begin{aligned}f^{-1}(\mathcal{U}_1\text{-int}(f(A),r)) &\subseteq (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(f(A)),r,s)\\ &= (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(A,r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\mathcal{U}_2\text{-int}(f(A),s)) &\subseteq (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(f(A)),s,r)\\ &= (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(A,s,r).\end{aligned}$$

Because f is onto, we obtain

$$\begin{aligned}\mathcal{U}_1\text{-int}(f(A),r) &= f(f^{-1}(\mathcal{U}_1\text{-int}(f(A),r)))\\ &\subseteq f((\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(A,r,s))\end{aligned}$$

and

$$\begin{aligned}\mathcal{U}_2\text{-int}(f(A),s) &= f(f^{-1}(\mathcal{U}_2\text{-int}(f(A),s)))\\ &\subseteq f((\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(A,s,r)).\end{aligned}$$

Conversely, let B be an intuitionistic fuzzy set in Y. Since f is onto, we obtain

$$\begin{aligned}\mathcal{U}_1\text{-int}(B,r) &= \mathcal{U}_1\text{-int}(f(f^{-1}(B)),r)\\ &\subseteq f((\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(B),r,s))\end{aligned}$$

and

$$\begin{aligned}\mathcal{U}_2\text{-int}(B,s) &= \mathcal{U}_2\text{-int}(f(f^{-1}(B)),s)\\ &\subseteq f((\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r)).\end{aligned}$$

Because f is one-to-one, we have

$$\begin{aligned}f^{-1}(\mathcal{U}_1\text{-int}(B,r)) &\subseteq f^{-1}(f((\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(B),r,s)))\\ &= (\mathcal{T}_1,\mathcal{T}_2)\text{-sint}(f^{-1}(B),r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\mathcal{U}_2\text{-int}(B,s)) &\subseteq f^{-1}(f((\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r)))\\ &= (\mathcal{T}_2,\mathcal{T}_1)\text{-sint}(f^{-1}(B),s,r).\end{aligned}$$

Therefore by Theorem 3.7, f is an intuitionistic fuzzy pairwise (r, s)-semicontinuous mapping.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

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