# Numerical Analysis of Si-based Photovoltaic Modules with Different Interconnection Methods 

Chihong Park<br>LG Innotek, Ansan 426-791, Korea<br>Nari Yoon<br>Hyundai Heavy Industry, Yongin 446-912, Korea<br>Yong-Ki Min, Jae-Woo Ko, Jong-Rok Lim, Dong-Sik Jang, Jae-Hyun Ahn, and Hyungkeun $\mathrm{Ahn}^{\dagger}$<br>Department of Electrical Engineering, Konkuk University, Seoul 143-701, Korea

Received January 8, 2014; Revised February 27, 2014; Accepted March 6, 2014


#### Abstract

This paper investigates the output powers of PV modules by predicting three unknown parameters: reverse saturation current, and series and shunt resistances. A theoretical model using the non-uniform physical parameters of solar cells, including the temperature coefficients, voltage, current, series and shunt resistances, is proposed to obtain the I-V characteristics of PV modules. The solar irradiation effect is included in the model to improve the accuracy of the output power. Analytical and Newton methods are implemented in MATLAB to calculate a module output. Experimental data of the non-uniform solar cells for both serial and parallel connections are used to extend the implementation of the model based on the I-V equation of the equivalent circuit of the cells and to extend the application of the model to $m$ by $n$ modules configuration. Moreover, the theoretical model incorporates, for the first time, the variations of series and shunt resistances, reverse saturation current and irradiation for easy implementation in real power generation. Finally, this model can be useful in predicting the degradation of a PV system because of evaluating the variations of series and shunt resistances, which are critical in the reliability analysis of PV power generation.


Keywords: Photovoltaic module, Solar cell connection, Newton method, Compensation of series resistance, Lambert W Function

## 1. INTRODUCTION

The solar cells are connected in series or parallel as a module to generate stable and durable power from the external environment. A power drop, compared with the sum of each solar cell

[^0]Copyright ©2014 KIEEME. All rights reserved.
This is an open-access article distributed under the terms of the Creative Commons Attribution Non-Commercial
License (http://creativecommons.org/licenses/by-nc/3.0) which permits unrestricted noncommercial use License (http://creativecommons.org/licenses/by-nc/3.0) which permits unrestricted noncommercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

I-V equation. Maximum errors of $3.11 \%$ in current and $0.76 \%$ in voltage occur at the mismatched connections. A power drop of $15.13 \%$ is shown for the series mismatched connection, which is greater than that for the series normal connection, and a drop of $9.36 \%$ is shown for the parallel mismatched connection, which is greater than that for the parallel normal connection. Thus, it is useful to predict the output power in a mismatched connection when PV modules are manufactured.

## 2. PREDICTION OF PV MODULE OUTPUT CHARACTERISTICS

### 2.1 I-V equation of PV module

A PV module operates similar to a solar cell, so the I-V equation was derived from the I-V equation for a solar cell. The current equation, as well as the series resistance and shunt resistance, of a PV module is introduced in this chapter [3,4].

$$
\begin{equation*}
I=I_{L}-I_{S}\left[\exp \left(\frac{q\left(V+R_{s} I\right)}{n k T}\right)-1\right]-\frac{V+R_{s} I}{R_{s h}} \tag{2.1}
\end{equation*}
$$

A PV module has a number of solar cells connected in series or connected in parallel. A module parameter was subject to the parameters of solar cells in the module.

$$
\begin{align*}
& I_{M}=N_{P} I  \tag{2.2}\\
& I_{s c M}=N_{P} I_{s c} \\
& V_{M}=N_{S} V \\
& V_{o c M}=N_{S} V_{o c} \\
& R_{s M}=\frac{N_{S}}{N_{P}} R_{S} \tag{2.6}
\end{align*}
$$

where $I_{M}$ is the PV module current and $N_{p}, I_{s c M}, V_{M}, N_{S}, V_{\text {ocM }}, R_{s M}$, and $R_{\text {shm }}$ are the number of parallel strings or cells, short circuit current, voltage of the module, number of series-connected cells, open circuit voltage, series resistance, and shunt resistance, respectively, where the subscript M denotes "module". The equation of the PV module current is developed by using these values.

$$
\begin{equation*}
\frac{I_{M}}{N_{P}}=\frac{I_{s M}}{N_{P}}-\frac{I_{s M}}{N_{P}}\left[\exp \left(\frac{q\left(\frac{V_{M}}{N_{S}}+\frac{I_{M} N_{P}}{N_{P} N_{S}} R_{s M}\right)}{n k T}\right)-1\right]-\frac{\frac{V_{M}}{N_{S}}+\frac{I_{M} N_{P}}{N_{P} N_{S}} R_{s M}}{R_{s h M}} \tag{2.7}
\end{equation*}
$$

$$
\begin{equation*}
I_{M}=I_{s c M}-I_{s M}\left[\exp \left(\frac{q\left(V_{M}+R_{s M} I_{M}\right)}{N_{S} n k T}\right)-1\right]-\frac{N_{P}\left(V_{M}+R_{S M} I_{M}\right)}{N_{S} R_{s h M}} \tag{2.8}
\end{equation*}
$$

From equation (2.8), the undefined $I_{s M}, R_{s M}$ and $R_{s h M}$ were found by the following steps. First, if the shunt resistance is infinite, the equivalent circuit becomes a simple circuit that was represented by using Eq. (2.9), and it was further transformed into Eq. (2.10).


Fig. 2.1. solar cell equivalent circuit without a shunt resistance.

$$
\begin{align*}
& I=I_{L}-I_{S}\left[\exp \left(\frac{q\left(V+R_{s} I\right)}{n k T}\right)-1\right]  \tag{2.9}\\
& I_{M}=I_{s c M}-N_{P} I_{s}\left[\exp \left(\frac{q\left(\frac{V_{M}}{N_{S}}+\frac{I_{M}}{N_{S}} R_{s M}\right)}{n k T}\right)-1\right] \tag{2.10}
\end{align*}
$$

The equation of Is for a module is obtained by placing a zero into the equation instead of $I$ because $\mathrm{V}_{\mathrm{M}}=\mathrm{V}_{\text {ocM }}$ when $\mathrm{I}_{\mathrm{M}}=0$ in Eq. (2.10).

$$
\begin{equation*}
I_{s M}=\frac{I_{s c M}}{N_{P}\left[\exp \left(\frac{q V_{o c M}}{n k T N_{S}}\right)-1\right]} \tag{2.11}
\end{equation*}
$$

Next, the fill factor is used to obtain the PV module series resistance $\left(\mathrm{R}_{\mathrm{sM}}\right)$, and we assume that the unit fill factors of a PV module $\left(\mathrm{FF}_{\mathrm{oM}}\right)$ and solar cell $\left(\mathrm{FF}_{\mathrm{o}}\right)$ are equal $[5,6]$.

$$
\begin{equation*}
F F_{o M}=\frac{I_{m M} V_{m M}}{I_{s c M} V_{o c M}}=\frac{I_{m} V_{m}}{I_{s c} V_{o c}}=F F_{o} \tag{2.12}
\end{equation*}
$$

Eq. (2.13) shows a rearranged equation of Eq. (2.12) by using Green's theorem, and it is meaningful when the value is greater than 10.

$$
\begin{align*}
& F F_{o}=\frac{v_{o c}-\ln \left(v_{o c}+0.72\right)}{1+v_{o c}}  \tag{2.13}\\
& v_{o c}=\frac{q V_{o c M}}{N_{s} n k T}  \tag{2.14}\\
& F F_{M}=F F_{o M}-F F_{o M} \gamma_{S M}=F F_{o M}\left(1-\gamma_{S M}\right) \tag{2.15}
\end{align*}
$$

where $\gamma_{\mathrm{sM}}$ is the normalized resistance that leads to the decrease in the fill factor.

$$
\begin{equation*}
P_{\max } \cong\left(V_{\mathrm{mp}}^{\prime} '-I_{\mathrm{mp}}{ }^{\prime} R_{s}\right) I_{\mathrm{mp}}^{\prime}=P_{\max } \quad\left(1-\frac{I_{s c}}{V_{o c}} R_{s}\right) \tag{2.16}
\end{equation*}
$$

where $\mathrm{V}_{\mathrm{mp}}{ }^{\prime}$ is the ideal maximum power voltage and $\mathrm{I}_{\mathrm{mp}}{ }^{\prime}$ is the ideal maximum power current. Eq. (2.16) was expressed by Eq. (2.19) and applied to the fill factor.

$$
\begin{align*}
& R_{c h}=\frac{V_{o c}}{I_{s c}}  \tag{2.17}\\
& \gamma_{s}=\frac{R_{s}}{R_{c h}}  \tag{2.18}\\
& P_{\max }=P_{\max }^{\prime}\left(1-\gamma_{s}\right)  \tag{2.19}\\
& F F=F F_{o}\left(1-\gamma_{s}\right) \tag{2.20}
\end{align*}
$$

Eq. (2.15) is expanded to Eq. (2.20). Eq. (2.21) shows the maximum power of the PV module and from this equation, $R_{s M}$ is obtained.

$$
\begin{align*}
& P_{\max M}=F F_{M} V_{o c M} I_{s c M}=F F_{o M}\left(V_{o c M} I_{s c M}-I_{s c M}^{2} R_{S M}\right)  \tag{2.21}\\
& R_{s M}=\frac{V_{o c M}}{I_{s c M}}-\frac{P_{\max M}}{F F_{o M} I_{s c M}^{2}} \tag{2.22}
\end{align*}
$$

Finally, $R_{\text {shm }}$ was obtained by using the values of $P_{\text {maxm }}$. To obtain $R_{\text {shM }}$, Eq.(2.8) was calculated with changing value of $R_{\text {shm }}$ until calculated P_maxM is similar to measured $P_{\max }$ within $0.4 \%$ error. In the following chapter, this simulation is explained in detail. To prove the theory so far, MATLAB is utilized by using four factors from a real PV module and the Lambert W function [7].

$$
\begin{align*}
& I_{M}=\frac{\left[-q V_{M}+\left[-f_{\text {lam }}+f_{A}\right] N_{s} n k T\right]}{q R_{s M}}  \tag{2.23}\\
& f_{\text {lam }}=\text { lambert } w\left(\frac{q R_{s M} I_{s M} R_{s h M} \exp \left(f_{A}\right)}{\left.R_{s M} n k T+R_{s h M} n k T N_{s}\right)}\right)  \tag{2.24}\\
& f_{A}=\frac{R_{s h} q\left(V_{M}+R_{s M} I_{s M}+R_{s M} I_{s c M}\right)}{n k T\left(N_{S} R_{s h M}+R_{s M}\right)} \tag{2.25}
\end{align*}
$$

The PV module current is then calculated by changing the PV module voltage from 0 to $\mathrm{V}_{\mathrm{oc}}$. The diode ideality factor ( n ) is changed according to the solar cell characteristics [7]. In this simulation, the ideality factor is 1.4 when $\mathrm{V}_{\text {ocm }}$ divided by the number of solar cells in a module is greater than $0.6[\mathrm{~V}]$. Otherwise, when the voltage is less than $0.6[\mathrm{~V}]$, the ideality factor is 1.8.

## 3. ANALYSIS OF PV MODULE INTERCONNECTION CHARACTERISTICS

### 3.1 Mismatched connection in parallel

To simplify the equation of the mismatched connection in parallel, two cells are initially used in parallel. The normalized I-V equation of the cells is shown below and the subscript represents the number of cells.

$$
\begin{equation*}
I_{i}=I_{s c i}-I_{s i}\left[\exp \left(\frac{q\left(V_{i}+R_{s i} I_{i}\right)}{n k T}\right)-1\right]-\frac{V_{i}+R_{s i} I_{i}}{R_{s h i}} \tag{3.1}
\end{equation*}
$$

Eq. (3.2) is induced when $V$ is a variable, so we use $V_{1}=V_{2}$.

$$
\begin{align*}
I_{T}= & I_{1}+I_{2} \\
= & I_{s c 1}+I_{s c 2}-\exp \left(\frac{q V}{n k T}\right)\left[I_{s 1} \exp \left(\frac{q R_{s 1} I_{1}}{n k T}\right)+I_{s 2} \exp \left(\frac{q R_{s 2} I_{2}}{n k T}\right)\right] \\
& +I_{s 1}+I_{s 2}-\frac{V_{1}+R_{s 1} I_{1}}{R_{s h 1}}-\frac{V_{2}+R_{s 2} I_{2}}{R_{s h 2}} \tag{3.2}
\end{align*}
$$

Eq. (3.2) can be expanded to Eq. (3.3) when $\mathrm{N}_{\mathrm{P}}$ solar cells are connected in parallel.

$$
\begin{align*}
I_{T}= & \sum_{i=1}^{N_{p}} I_{s c i}-\exp \left(\frac{q V}{n k T}\right)\left[\sum_{i=1}^{N_{p}} I_{s i} \exp \left(\frac{q I_{i} R_{s i}}{n k T}\right)\right] \\
& +\sum_{i=1}^{N_{p}} I_{s i}-V \sum_{i=1}^{N_{p}}\left(\frac{1}{R_{s h i}}\right)-\sum_{i=1}^{N_{p}}\left(\frac{I_{i} R_{s i}}{R_{s h i}}\right) \tag{3.3}
\end{align*}
$$

### 3.2 Mismatched connection in series

The total current is easy to calculate when a deficient cell is connected in parallel, but difficult to count when a deficient cell is connected in series because the equation expands in terms of voltage. For this reason, Eq. (2.1) should be rearranged in terms of current as below.

$$
\begin{align*}
I= & I_{s c}-I_{s}\left[\exp \left(\frac{q\left(V+R_{s} I\right)}{n k T}\right)-1\right]-\frac{V+R_{s} I}{R_{s h}} \\
& \therefore V=\frac{n k T}{q} \ln \left(\frac{I_{s c}-I \frac{V+I R_{s}}{R_{s h}}}{I_{s}}+1\right)-I R_{s} \tag{3.4}
\end{align*}
$$

$V_{T}$, defined as $V_{1}+V_{2}$, is the total voltage drop in a series connection, obtained from Eq. (3.4) as follows.

$$
\begin{align*}
V_{T}= & \frac{n k T}{q}\left[\ln \left(\frac{I_{s c 1}-I_{1}-\frac{V_{1}+I_{1} R_{s 1}}{R_{s h 1}}}{I_{s 1}}+1\right)\right. \\
& \left.+\ln \left(\frac{I_{s c 2}-I_{2}-\frac{V_{2}+I_{2} R_{s 2}}{R_{s h 2}}}{I_{s 2}}+1\right)\right]-I_{1} R_{s 1}-I_{2} R_{s 2} \tag{3.5}
\end{align*}
$$

If the electrical characteristics used in the connection are identical, Eq. (3.5) can be rearranged as follows.

$$
\begin{equation*}
V_{T}=\frac{2 n k T}{q}\left[\ln \left(\frac{I_{s c}-I_{1}-\frac{\frac{V_{T}}{2}+I_{1} R_{s 1}}{R_{s h 1}}}{I_{s 1}}+1\right)\right]-2 I_{1} R_{s 1} \tag{3.6}
\end{equation*}
$$

Eq. (3.6) is expanded to Eq. (3.7) when $\mathrm{N}_{\mathrm{s}}$ solar cells are con-
nected in series under the assumption of same electrical characteristics.

$$
\begin{equation*}
V_{T}=\frac{N_{s} n k T}{q}\left[\ln \left(\frac{I_{s c 1}-I_{1}-\frac{\frac{V_{T}}{N_{S}}+I_{1} R_{s 1}}{R_{s h 1}}}{I_{s 1}}+1\right]-N_{S} I_{1} R_{s 1}\right. \tag{3.7}
\end{equation*}
$$

Eq. (3.7) is expressed in terms of current as

$$
\begin{equation*}
I=I_{s c 1}-I_{s 1}\left[\exp \left(\frac{q\left(V_{T}+N_{S} R_{s 1} I\right)}{N_{S} n k T}\right)-1\right]-\frac{\frac{V_{T}}{N_{S}}+R_{s 1} I}{R_{s h 1}} \tag{3.8}
\end{equation*}
$$

The comparison of Eq. (3.8) and Eq. (2.8) shows that the shunt resistance $\left(\mathrm{R}_{\text {shm }}\right)$ of the PV module is equal to that of the solar cell $\left(R_{\text {sh }}\right)$. $N_{s}$ cells and one deficient cell are connected in series to extend the model to a real application. The value of $\mathrm{I}_{\mathrm{b}}$ can be changed to $\mathrm{I}_{\mathrm{T}}$ when I is a variable in Eq. (3.9), so the I-V characteristics in this connection can be predicted. The new total voltage equation is $V_{T}^{\prime}=V_{T}+V_{b}$, where $\mathrm{V}_{\mathrm{T}}$ is from Eq. (3.7) and $\mathrm{V}_{\mathrm{b}}$ is from Eq. (3.9).

$$
\begin{equation*}
V_{b}=\frac{n k T}{q} \ln \left(\frac{I_{s c b}-I_{b}-\frac{V_{b}+I_{b} R_{s b}}{R_{s h b}}}{I_{s b}}+1\right)-I_{b} R_{b} \tag{3.9}
\end{equation*}
$$

where $I_{s c b}, I_{b}, R_{s b}, R_{\text {shb }}, I_{s b}$ and $R_{b}$ represent the short circuit current, total current, series resistance, shunt resistance, reverse saturation current and total resistance of a deficient cell, respectively.

### 3.3 Simulation of I-V characteristics of the cells

Simulation was carried out based on the theory introduced to confirm the effect of the electrical mismatch of solar cells in a PV module. The sample for the experiment consisted of three solar cells and their characteristics are provided in Table. 3.1. The parameters from Table 3.1 were obtained by using a solar cell tester, CT801. Also, the values $R_{s}, R_{s h}$ and $I_{s}$ in Table 3.2 were extracted from the results of the cells that were simulated for the introduced model by using the parameters from Table 3.1. The value of Pmax is used to distinguish normal and bad samples in Table 3.1.

### 3.3.1 Mismatched connection in parallel

Figure 3.1 shows the parallel connection of normal solar cells. Because the cells have the same characteristics, the equation of $I_{T}=I_{1}+I_{2}$ was used. Figure 3.2 shows the I-V characteristic of a normal cell and a PV module of three normal cells. As predicted, the output voltages are the same and the current value of the PV module is three times the current of the solar cell. The parameters from Table 3.2 are used for the simulation.

Figure 3.3 shows a parallel connection of 2 normal cells and 1 deficient cell, which has the same voltage characteristic as a normal cell but a poor current characteristic.

Table 3.1. Parameters of solar cell for simulation.

|  | $\mathrm{I}_{\mathrm{sc}}(\mathrm{A})$ | $\mathrm{I}_{\mathrm{mp}}(\mathrm{A})$ | $\mathrm{V}_{\mathrm{oc}}(\mathrm{V})$ | $\mathrm{V}_{\mathrm{mp}}(\mathrm{V})$ | $\mathrm{P}_{\max }(\mathrm{W})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Normal cells in parallel | 4.83 | 4.39 | 0.633 | 0.523 | 2.29 |
| Bad cells in parallel | 3.34 | 3.08 | 0.619 | 0.519 | 1.6 |
| Normal cells in series | 4.83 | 4.39 | 0.633 | 0.523 | 2.29 |
| Bad cells in series | 3.74 | 3.45 | 0.621 | 0.516 | 1.78 |

Table 3.2. Physical parameters of solar cell obtained from the simulation.

| Cell $\left(\mathrm{I}_{\mathrm{sc}}\right)$ | $\mathrm{R}_{\mathrm{s}}(\Omega)$ | $\mathrm{R}_{\mathrm{sh}}(\Omega)$ | $\mathrm{I}_{\mathrm{s}}(\mathrm{A})$ |
| :---: | :---: | :---: | :---: |
| 4.83 | 0.0022 | 59 | $0.848 \times 10^{-6}$ |
| 3.34 | 0.0006 | 51 | $0.586 \times 10^{-6}$ |
| 3.74 | 0.0005 | 45 | $0.657 \times 10^{-6}$ |



Fig. 3.1. Parallel connection of normal solar cells.


Fig. 3.2. I-V characteristics of a normal cell and a PV module with three normal cells in parallel connection.


Fig 3.3. Parallel connection of 2 normal cells and 1 bad cell.

Figure 3.4 shows the simulation result of the connection in Fig. 3.3. The total current value is lower than that shown in Fig. 3.2 because of the deficient cell.

### 3.3.2 Mismatched connection in series

Eq. (3.9) is rearranged in the following steps from Eq. (3.10) to (3.12) to simulate a series connection. The voltage value was calculated by using the current change starting from 0 . In the case of common I-V curve of a module, the current value is almost constant as $\mathrm{I}_{\mathrm{sc}}$ although the voltage value is changing at the low


Fig 3.4. I-V characteristics of a PV module with 2 normal cells and 1 bad cell in parallel connection.
voltage region. The general PV module equation expressed by the current equation does not have a problem because the voltage value is changed, and the current value is then obtained from the voltage change. However, the result from calculated voltage equation (3.11), a current variable, is an imaginary quantity after starting point which the current is saturated as $\mathrm{I}_{\mathrm{sc}}$. Therefore, the voltage value is calculated by Eq. 3.11 up to the imaginary quantity; subsequently, the general PV module current equation is used for simulation in this paper (Eq. (3.12)). In other words, two equations are used for this simulation; non-linear characteristics, therefore, exist in the secondary differential equation.

$$
\begin{aligned}
f\left(V_{T}^{\prime}\right)= & -V_{T}^{\prime}-\frac{N_{s} n k T}{q}\left[\ln \left[\frac{I_{s c 1}-I_{T}-\frac{\frac{V_{T}}{N_{S}}+I_{T} R_{s 1}}{R_{s h 1}}+I_{s 1}}{I_{s 1}}\right)\right]-N_{S} I_{T} R_{s}+ \\
& -N_{s} I_{T} R_{s 1}+\frac{n k T}{q}\left[\ln \left(\frac{I_{s c b}-I_{b}-\frac{V_{b}+I_{b} R_{s b}}{R_{s h b}}+I_{s b}}{I_{s b}}\right)\right] \\
& -I_{b} R_{s b}=0
\end{aligned}
$$

$$
\begin{equation*}
V_{T(i+1)}^{\prime}=V_{T(i)}{ }^{\prime}-\frac{f\left(V_{T(i)}{ }^{\prime}\right)}{f^{\prime}\left(V_{T(i)}^{\prime}\right)}(i=0,1,2,3,4,5) \tag{3.11}
\end{equation*}
$$

$$
\begin{equation*}
I=I_{s c s u m}-I_{s 1}\left(\exp \left(\frac{q\left(\mathrm{~V}-\mathrm{N}_{\mathrm{s}} \mathrm{M}\right)}{\left(N_{s}+1\right) \mathrm{nkT}}\right)\right)-\frac{V}{R_{s h}} \tag{3.12}
\end{equation*}
$$

where $N_{s}$ is the number of normal cells and $M$ is the correction factor that make the two equations give the same value at the interconnection point. M is approximately $0.05 \sim 1.5$, and this work uses 0.1. $\mathrm{I}_{\text {scsum }}$ is the total number of normal cells and deficient cells. The normalized voltage equation that ignores the resistance factor is shown below and the subscript $i$ represents the number of cells.

$$
\begin{equation*}
V_{i}=\frac{n k T}{q} \ln \left(\frac{-I_{i}+I_{s c i}+I_{s i}}{I_{s i}}\right) \tag{3.13}
\end{equation*}
$$



Fig. 3.5. I-V characteristics of a normal cell and a PV module with three normal cells in a series connection.


Fig. 3.6. Series connection of 2 normal cells and 1 bad cell.


Fig. 3.7. I-V characteristic of a PV module with a bad cell and normal cells in series connection.

$$
\begin{equation*}
V_{T}=V_{1}+V_{2}=\frac{n k T}{q} \ln \left[\left(\frac{-I_{1}+I_{s c 1}+I_{s 1}}{I_{s 1}}\right)\left(\frac{-I_{2}+I_{s c 2}+I_{s 2}}{I_{s 2}}\right)\right] \tag{3.14}
\end{equation*}
$$

The equation $\mathrm{I}=\mathrm{I}_{\mathrm{sc}}$ is valid when $V_{1}+V_{2}=0$ in Eq. (3.14), so $\mathrm{I}_{\text {scsum }}$ is defined as a current when $V_{T}$ is zero.

$$
\begin{align*}
& \left(\frac{-I_{s c s u m}+I_{s c 1}+I_{s 1}}{I_{s 1}}\right)\left(\frac{-I_{s c s u m}+I_{s c 2}+I_{s 2}}{I_{s 2}}\right)=1  \tag{3.15}\\
& I_{s c s u m}= \\
& \frac{1}{2}\left(I_{s 1}+I_{s c 1}+I_{s 2}+I_{s c 2}\right) \pm \\
& \frac{1}{2} \sqrt{\left(I_{s 2}^{2}+2 I_{s 2} I_{s c 2}+2 I_{s c 1} I_{s 2}-2 I_{s c 1} I_{s 2}+I_{s c 2}^{2}-2 I_{s 1} I_{s c 2}+I_{s 1}^{2}+2 I_{s c 1} I_{s 1}+I_{s c 1}^{2}\right)}
\end{align*}
$$

The valid value is that with the minus sign before the root in

Eq. (3.15); this result means that the total current value follows the lowest value, the current of the deficient cell. Fig. 3.5 shows the I-V characteristics of a normal cell and a PV module with three normal cells in series. The output current values are the same, but the output voltage value of the PV module is three times greater than that of the cell.

Figure 3.6 shows a series connection of 2 normal cells and 1 deficient cell that has the same voltage characteristics as the normal cells but poor current characteristics.

Figure 3.7 shows the simulation result of the connection in Fig. 3.6. The total current value is less than that of a PV module with three normal cells due to the one deficient cell.

## 4. RESULTS AND DISCUSSION

### 4.1 PV module output characteristic prediction

Seven types of PV modules are tested to prove the proposed equation and to compare their I-V characteristics with the theoretical predictions. Table 4.1 shows the parameters of the PV modules, the numbers of cells and the error between theoretical and experimental data for $1,000 \mathrm{~W} / \mathrm{m}^{2}$ irradiance.

The measured and simulated I-V characteristics of four samples ' A , ' B ', ' C ' and ' D ' are compared for $1,000\left[\mathrm{~W} / \mathrm{m}^{2}\right]$ irradiance. Current deviation is calculated at the voltage lower than $\mathrm{P}_{\text {max }}$, and voltage deviation is also counted at the voltage higher than $\mathrm{P}_{\text {max }}$.

Figure 4.1 shows the result of the ' A ' module, which has the maximum errors of $0.21 \%$ in current and $0.24 \%$ in voltage.

The result shows that the sun simulator data and theoretical prediction data at $1,000 \mathrm{~W} / \mathrm{m}^{2}$ irradiance are matched well in Fig. 4.1. The maximum errors of $1.27 \%$ in current and $0.74 \%$ in voltage are shown. To prove the proposed equation by irradiance variation, simulated and measured results are compared at irradiances of $1,000 \mathrm{~W} / \mathrm{m}^{2}, 800 \mathrm{~W} / \mathrm{m}^{2}, 600 \mathrm{~W} / \mathrm{m}^{2}$ and $400 \mathrm{~W} / \mathrm{m}^{2}$ with ' E ', ' F ' and ' G ' samples, and the temperature of the PV module is assumed as $25^{\circ} \mathrm{C}$. Figure 4.2 describes the result of the ' F ' module that has maximum errors of $1.32 \%$ in current and $0.48 \%$ in voltage.

The simulation by irradiance needs to consider series resistance, so it is assumed that series resistance decreases as the irradiance reduces. This assumption is applied to Eq. (4.1). This new equation is used for the experiments according to irradiance.

$$
\begin{equation*}
R_{s M}^{\prime}=R_{s M}-\alpha R_{s M}\left(1-r_{I r r}\right) \tag{4.1}
\end{equation*}
$$

where $r_{\text {Irr }}$ is the ratio of real irradiance to standard irradiance, $R_{\text {sM }}{ }^{\prime}$ is the new resistance according to irradiance, $\alpha$ is the correction factor that considers ribbon resistance, which is usually $0.8 \sim 1.2$ but in this work, is set to 1.0 . Moreover, the short circuit current is compensated in Eq. (4.2) because the short circuit current decreases non-linearly with irradiance reduction. Eq. (4.2) provides more accurate results and is good agreement with measured output of a module, shown former example.

$$
\begin{equation*}
I_{s c M}^{\prime}=I_{s c M} r_{I r r}-\beta I_{s c M}\left(1-r_{I r r}\right) \tag{4.2}
\end{equation*}
$$

where $I_{\mathrm{scm}}{ }^{\prime}$ is the new short circuit current that considers irradiance, $\beta$ is that correction factor that considers the absorbance coefficient and generation rate of irradiance, which is normally $0.02 \sim 0.03$. In this paper, the correction factor is 0.025 .

Table 4.1. parameter of PV module for comparison.

| Name | Cell <br> Type <br> (") | Number <br> Of Cell | $\begin{aligned} & V_{o c} \\ & (\mathrm{~V}) \end{aligned}$ | $\mathrm{V}_{\mathrm{mp}}$ <br> (V) | $\begin{aligned} & \mathrm{I}_{\mathrm{sc}} \\ & (\mathrm{~A}) \end{aligned}$ | $\mathrm{I}_{\mathrm{mp}}$ <br> (A) | $\mathrm{P}_{\text {max }}$ <br> (W) | Error in current (\%) | Error in voltage (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | Single(5) | 72 | 44.1 | 35.9 | 5.21 | 4.81 | 172.8 | 0.21 | 0.24 |
| B | Single(5) | 72 | 43.5 | 35.4 | 5.38 | 4.9 | 173.6 | 0.56 | 0.74 |
| C | Poly(6) | 54 | 33.0 | 25.8 | 8.29 | 7.84 | 221.4 | 0.68 | 0.67 |
| D | Poly(6) | 54 | 32.3 | 25.5 | 7.88 | 7.08 | 180.8 | 1.27 | 0.49 |
| E | Poly(6) | 54 | 32.3 | 25.5 | 7.93 | 7.11 | 181.3 | - | - |
| F | Single(5) | 72 | 44.2 | 35.9 | 5.24 | 4.89 | 174.7 | - | - |
| G | Poly(6) | 54 | 32.3 | 25.5 | 7.87 | 7.11 | 181.7 | - | - |



Fig. 4.1. Comparison of theoretical prediction and sun simulator data of 9045 s .


Fig. 4.2. Comparison of theoretical prediction and sun simulator data of $\mathrm{F}(9043 \mathrm{~s})$ by irradiance variation.

The irradiance variation affects the value, $\mathrm{V}_{\mathrm{ocm}}$, which is related to the short circuit current and reverse saturation current. The reverse saturation current, which is defined by Eq. (2.11), has an open circuit voltage value at the irradiance of $1 \mathrm{~kW} / \mathrm{m}^{2}$. The open circuit voltage should change with irradiance, so the new equation is therefore developed below.

$$
\begin{equation*}
V_{o c M I s}=V_{o c M}+\delta V_{o c M}\left(1-r_{I r r}\right) \tag{4.3}
\end{equation*}
$$

where $V_{\text {ocm }}$ is the open circuit voltage at the irradiance of $1 \mathrm{~kW} /$ $\mathrm{m}^{2}, \mathrm{~V}_{\mathrm{ocMIs}}$ is the imaginary open circuit voltage according to irradiance in the PV module, which is proposed to obtain the reverse


Fig. 4.3. Comparison of theoretical prediction and sun simulator data for PV module with 3 normal cells in series.


Fig. 4.4. Comparison of theoretical prediction and sun simulator data for PV module with 1 bad cell and 2 normal cells in series


Fig. 4.5. Comparison of theoretical prediction and sun simulator data for PV module with 3 normal cells in parallel.
saturation current, and $\delta$ is a correction factor that considers the variation of voltage according to irradiance, which is normally $0.02 \sim 0.05$ but is 0.033 in this work.

### 4.2 Comparison results between theoretical and measured data

The PV modules manufactured for the experiment were tested, and the measured results were compared with the simulated data from the previous chapter. One of the PV modules had three normal cells and the other one had two normal cells and one deficient cell. The modules were not laminated; only the solar cells


Fig. 4.6. Comparison of theoretical prediction and sun simulator data for PV module with 1 bad cell and 2 normal cells in parallel.


Fig. 4.7. Comparison of theoretical prediction and sun simulator data for PV module with 3 normal cells in parallel.


Fig. 4.8. Comparison of theoretical prediction and sun simulator data for PV module with 1 bad cell and 2 normal cells in parallel.

Table 4.2. Errors of different interconnections.

|  | Normal cells <br> in series | Bad cells <br> in series | Normal cells <br> in parallel | Bad cells <br> in parallel |
| :---: | :---: | :---: | :---: | :---: |
| Error in <br> current (\%) | 2.56 | 3.11 | 2.56 | 2.26 |
| Error in <br> voltage (\%) | 0.29 | 0.58 | 0.76 | 0.55 |

were connected onto the glass. The parameters from the solar cells were obtained in the simulation, but when the PV module was made, an electrode ribbon and interconnection ribbon were
used. Therefore, the resistance from the modulation process should be considered when the parameters are obtained only from the solar cells. The following equation represents the open circuit voltage of a PV module that has a smaller value than the total sum of the open circuit voltages of all cells due to contact resistance.

$$
\begin{equation*}
V_{o c M}^{\prime}=V_{o c M}-R_{c M} I_{s c M} \tag{4.4}
\end{equation*}
$$

Where $R_{c M}$ is the increased value of the contact resistance; $0.005 \Omega$ was used for series and parallel connections in this work. However, the contact resistance of the parallel connection is greater than that of the series connection because the ribbon is used more frequently in parallel connection. Eq. (4.5) shows this consideration.

$$
\begin{equation*}
R_{s M}^{\prime}=R_{s M}+R_{c L M} \tag{4.5}
\end{equation*}
$$

where $R_{s M}{ }^{\prime}$ is the resistance that considers the increased value of the contact resistance and $\mathrm{R}_{\mathrm{cLM}}$ is the increased value of contact resistance according to the number of parallel connections and the method of soldering. The value of $\mathrm{R}_{\mathrm{cLM}}$ is normally $0.01[\Omega] \sim$ $0.1[\Omega] ; 0.02[\Omega]$ was in this work. When the solar cells were connected in parallel, the series resistance of $0.02[\Omega]$ was used for simulation. Similar to the method shown in the former chapter, the current deviation is compared at the voltage lower than $\mathrm{p}_{\max }$, and the voltage deviation is compared at voltage higher than $\mathrm{p}_{\text {max }}$.

Figure 4.3 shows the I-V characteristics of the theoretical prediction and the sun simulator data of a PV module with three solar cells in series. The result shows the maximum errors of $2.56 \%$ in current and $0.29 \%$ in voltage. Also, the simulated value of $\mathrm{P}_{\operatorname{maxM}}$ is 6.61 W and the theoretical value is 6.52 [W], showing an error of $1.36 \%$. Figure 4.4 shows the I-V characteristics of the theoretical prediction and sun simulator data of a PV module with 1 deficient cell and 2 normal cells in series. The result shows the maximum errors of $3.11 \%$ in current and the $0.58 \%$ in voltage. Also, the simulated value of $\mathrm{P}_{\operatorname{maxM}}$ is $5.61[\mathrm{~W}]$ and the theoretical value is $5.55[\mathrm{~W}]$, showing an error of $1.07 \%$.

Figure 4.5 shows the I-V characteristics of the theoretical prediction and sun simulator data of a PV module with 3 solar cells in parallel. The result shows maximum errors of $2.56 \%$ in current and $15.10 \%$ in voltage. A significant error in voltage is shown because only the open circuit voltage with increase in contact resistance is considered. Therefore, the series resistance shows a significant difference, resulting in the large error in voltage.

Figure 4.6 shows the I-V characteristics of the theoretical prediction and sun simulator data of the PV module with 1 deficient cell and 2 normal cells in parallel. The result shows maximum errors of $2.56 \%$ in current and $14.04 \%$ in voltage. This result also shows a large error in voltage for the same reason as that in Fig. 4.5. These errors result from additional resistances in parallel connection due to the longer electrode and interconnection ribbons. Simulations were then performed by considering those additional resistances.

The I-V characteristics of Fig. 4.7, which include the increase in contact resistances of electrode ribbon and interconnection ribbon, are different from those of Fig. 4.5. While these results are more accurate accounting for the increase in contact resistance, they still show slight errors. The result shows maximum errors of $2.56 \%$ in current and $0.76 \%$ in voltage. Also, the simulated value of $\mathrm{P}_{\text {maxM }}$ is $5.45[\mathrm{~W}]$ and the theoretical value is $5.36[\mathrm{~W}]$, showing
an error of $1.65 \%$.
The I-V characteristics of Fig. 4.8, which also considers the increase in contact resistances of electrode ribbon and interconnection ribbon, are different from those of Fig. 4.6. The result shows maximum errors of $2.26 \%$ in current and $0.55 \%$ in voltage. Also, the simulated value of $\mathrm{P}_{\text {max }}$ is $4.94[\mathrm{~W}]$, and the theoretical value is 4.91 [W], showing an error of $0.61 \%$.
The difference in the output power between the PV module with 3 normal solar cells and the mismatched PV module is $9.36 \%$ for the sun simulator result and is $8.41 \%$ for the theoretical result. These values are slightly smaller than those of the PV modules with the series connection. The results are summarized in Table 4.2.

## 5. CONCLUSIONS

The parameters $\mathrm{V}_{\mathrm{oc}}, \mathrm{I}_{\mathrm{sc}}, \mathrm{V}_{\mathrm{mp}}$, and $\mathrm{I}_{\mathrm{mp}}$ and the temperature coefficient were used to obtain the series resistance, shunt resistance and reverse saturation current of a PV module, which are essential for predicting the output power of a PV module. The proposed equation worked well with the results according to the amount of irradiance. When the irradiance was $1,000 \mathrm{~W} /$ $\mathrm{m}^{2}$, maximum errors of $1.27 \%$ in current and $0.74 \%$ in voltage were obtained between the theoretical predicted output and experimental output. Moreover, the maximum errors changed to $1.88 \%$ in current and $1.71 \%$ in voltage when the irradiance was changed. The change of series resistance according to irradiance was considered and short circuit current according to irradiance was approached practically. Also, a new equation for the open circuit voltage to analyze the reverse saturation current was proposed.

The theory proposed, which can predict output characteristics by the interconnection method, was verified by comparing the measurements of the real PV module with the 3 solar cells in series and in parallel. Increasing contact resistance, which induced the decrease of the open circuit voltage, was considered when the solar cells were connected. Furthermore, the additional contact resistance in parallel connection due to a longer electrode ribbon and interconnection ribbon was considered. Therefore, the results from the new equation and the data from a real PV module were similar.
The standard of current differed between a cell tester that simulates a solar cell and a sun simulator that simulates a PV module. Current value comparisons are somewhat inaccurate, but maximum errors of $3.11 \%$ in current and $0.58 \%$ in voltage occurred. The maximum error of $2.55 \%$ in current was shown for a parallel connection when only the contact resistance was considered as a series connection, but a significant error of $15.10 \%$ in voltage occurred. Therefore, the additional series resistance for a parallel connection was included to obtain better results: maximum errors of $2.56 \%$ in current and $0.76 \%$ in voltage. An output power drop occurred when a deficient cell was connected in this work. The drop rate was $14.88 \%$ for a series connection and $8.40 \%$ for a parallel connection.

This work will be helpful in analyzing the characteristics of a degraded PV module and in predicting the output power, which is critical at PV power stations, when cells gradually degrade in PV modules.

## REFERENCES

[1] S. T. Kim, G. H., C. H. Park, H. G. Ahn, D. Y. Han, and Y. G. Jong, The Korean Institute of Electrical and Electronic Material Engineers, 21, 12 (2008) [DOI: http://dx.doi.org/10.4313/

JKEM.2008.21.1.012].
[2] T. H. Jung, J.W. Ko, G. H. Kang, H.K. Ahn, "Output Characteristics of PV Module considering partially reversed biased conditions", Solar Energy, 92, 214 (2013) [DOI: http://dx.doi. org/10.1016/j.solener.2013.03.015].
[3] S. M. Sze, Physics of Semiconductor Devices, 2nd ed, (Wileyinterscience Publication, 1981) p.790-838.
[4] S. M. Sze, Semiconductor Devices Physics and Technology, 2nd ed. (Wiley-interscience Publication, 2001) p. 318-330
[5] S. R. Wenham, M. A. Green, M. E. Watt, and R. Corkish, Applied Photovoltaics, 2nd ed. (Earthscould, 2007) p. 43-95.
[6] L. Castaner and S.Silvestre, Modelling photovoltaic systems Using PSpice (Wiley-interscience Publication, 2002) p. 19-101.
[7] G. Walker, Journal of Electrical \& Electronics Engineering, 21, 49 (2001).
[8] J. I. Rosell and M. Ibanez, Journal of Energy Conversion \& Management, 47, 2424 (2006) [DOI: http://dx.doi.org/10.1016/ j.enconman.2005.11.004].


[^0]:    ${ }^{\dagger}$ Author to whom all correspondence should be addressed: E-mail: hkahn@konkuk.ac.kr

