Unified Analytic Calculation Method for Zoom Loci of Zoom Lens Systems with a Finite Object Distance

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The number of lens groups in modern zoom camera systems is increased above that of conventional systems in order to improve the speed of the auto focus with the high quality image. As a result, it is difficult to calculate zoom loci using the conventional analytic method, and even the recent one-step advanced numerical calculation method is not optimal because of the time-consuming problem generated by the iteration method. In this paper, in order to solve this problem, we suggest a new unified analytic method for zoom lens loci with finite object distance including infinite object distance. This method is induced by systematically analyzing various distances between the object and other groups including the first lens group, for various situations corresponding to zooming equations of the finite lens systems after using a spline interpolation for each lens group. And we confirm the justification of the new method by using various zoom lens examples. By using this method, we can easily and quickly obtain the zoom lens loci not only without any calculation process of iteration but also without any limit on the group number and the object distance in every zoom lens system.

Keywords : Lens design, Zoom lens system, Zoom locus, Gaussian bracket
OCIS codes : (110.0110) Imaging systems; (220.1010) Aberrations (global); (220.2740) Geometric optical design; (220.4830) Systems design

I. INTRODUCTION

An optical imaging system is generally classified by two types of finite and infinite object distances according to various applications [1]. The object distance is infinite for camera systems, while it is finite for microscopes and optical inspection systems. However, for certain camera systems, the optical systems have sometimes finite object distances, Such is the case for macro lenses [2]. Although a macro lens is classified as a single focus lens, the optical design method does not shows any difference compared to a conventional zoom lens design because two or more lens groups are generally used for focusing. When two or more lens groups are moved, it is called a floating system. Even in this case, the displacement of each lens group according to the focusing adjustment could be derived in the same way as the general zoom lens system. Of course, the displacement of each lens group determines the locus of the optical system. The result from this calculation is important in the manufacturing process for the cam components in zoom lens systems. In addition, there are optical systems with finite object distances that achieve a fixed magnification by moving the sub-groups along an optical axis, in spite of the variation in the object distance [3]. In particular, this type of system is called a multi-configurative optical system [3] because it does not follow the typical definition of a zoom lens [4]. Nevertheless, the multi-configurative optical system is also designed with from three to five discontinuous zoom configurations like an ordinary zoom lens system in order to improve the performance of the optical system. The displacement of each group needs to be precisely calculated for cam manufactures similar to other zoom lenses because the optical system consists of a variator, which prevents the variation of the lateral magnification, and a compensator, which compensates the variation of the image plane depending on the movement of the variator.

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Therefore, in an optical system with finite object distance, it is necessary to calculate the zoom loci, similar to the conventional zoom lens, when more than two lens groups move. In the calculation of zoom equations for the zoom loci, the values of these loci can be obtained analytically according to the given constraint of an optical system [1]. This method derives the values of these equations analytically after classifying the optical systems according to constraints. However, the analytic method corresponding to each constraint is difficult to use practically and universally when the number of the group of the lenses is large, because every situation must calculated one by one. A numerical analysis method for the locus calculation has already been proposed as a way to solve all the constraints [5, 6]. Nevertheless, this method iterates the calculations by a Jacobian matrix differential equation simultaneously with the zoom equations and constraints. Therefore, this method is time-consuming, and the programming for the numerical method becomes very complicated when solving the simultaneous nonlinear equations of the zoom equation and its inverse matrix. In addition, because the convergence rate is becoming small as the absolute value of the determinant of the Jacobian matrix is converging to 0, the precision of zoom loci is rapidly reduced. Recently, although the new analytic method has been proposed in order to solve the problem, it is the method obtained by applying to a zoom lens for an optical system with infinite object distance, such as a camera lens [7].

In this paper, a new method using a zoom lens as an optical system with finite object distance, which is more powerful and effective in expanding compared to using the results from Reference [7] is proposed. In other words, the effectiveness of a unified analytic method for calculating the zoom locus, which is the non-iterative analytic calculations of the locus, is discussed for the unified zoom system, regardless of the number of lens groups and constraints, the distance from zoom lens to object, and format of the optical system such as a floating and multi-configurative optical system [8].

II. THE UNIFIED ANALYTIC METHOD OF CALCULATING LENS LOCI

Figure 1 shows the optical layout of a finite object optical system with N groups in terms of the axial rays of
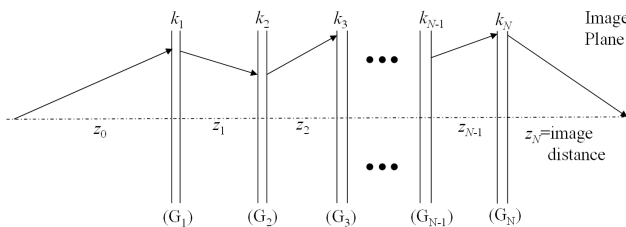

FIG. 1. Thin lens model of a finite object lens system with N groups.

the finite object. The zoom locus is derived by calculating the distance between groups in order to constrain image points to 0 on a paraxial image point at all times regardless of its location. Currently, the axial ray is needed to satisfy the given magnification and object distance, while the image height of the axial ray remains 0 on the paraxial image point.

Therefore, for the calculation of the zoom locus, the distance between groups must be obtained in order for the lateral magnification of the optical system to have the desired value with the image point of the axial ray fixed at 0 on the image plane. The distance between groups is virtually given by paraxial ray tracing about the axial ray. In such an optical system, paraxial ray tracing about an axial ray is expressed in terms of a Gaussian bracket [1].

$$[-z_0, k_1, -z_1, k_2, -z_2, k_3, \cdots, k_{N-1}, -z_{N-1}, k_N] = \frac{1}{m}$$
$$[-z_0, k_1, -z_1, k_2, -z_2, k_3, \cdots, k_{N-1}, -z_{N-1}, k_N, -z_N] = 0$$
(1)

where k_i is the refracting power of the i-th group, z_i ($i = 1, 2, \cdots, N$-1) is the distance between the principal points between the i-th group and the i+1-th group, z_0 is the distance between the 1st principal point of the 1st group and the object, z_N is the distance between the 2nd principal point of the last group and the image point, m is the lateral magnification of the optical system. Equation (1) is usually referred to as the zoom equation, and the lens locus of all optical systems can be derived using these equations.

Because the object distance is infinite regardless of the zoom configurations of the optical system, the object distance has an unknown variable value. However, the object distance is often given in optical systems with a finite object distance. The lens locus is the data that a mechanical designer of a camera is given from an optical system designer in order to design and manufacture CAM components after finishing the lens design. Meanwhile, the physical quantity that determines the field of view in an optical system with infinite object distance is the refracting power or focal length. However, magnification is the physical quantity that determines the optical system. Therefore, compared to the unknown parameter of the two types of optical systems in equal N groups, the number of the unknown parameters in an optical system with infinite object distance is N+1. This includes the focal length and number of intervals of each group. However, the total number of unknown parameters in an optical system with finite object distance is N+2 when the object distance is added. Therefore, the unknown parameters in equation (1) are $z_0 \sim z_N$ and m. However, the zoom equation can be easily calculated assuming that the object distance is altered because the object distance can be used as a compensator for the image plane position in an optical system with finite object distance. It is virtually impossible for a user (a photographer) to know the exact distance

from the object to the 1st surface of the optical system, especially in a floating optical system. In addition, when observing a subject with a desired magnification in a zoom system such as an optical inspection system in which the subject is fixed, the accuracy of the observed magnification is of primary, and the object distance is of secondary concern. Thus, utilizing the object distance as a compensator is acceptable. In this regard, when calculating equation (1), there are two cases: the displacement of the object distance with the 1st group and that with another group.

On the other hand, because the optical system is complicated, the constraints are not clearly given and only two variables can be unknown when calculating the two zoom equation. Figure 2 shows the optical layout when the i-th group is moved Δz_i in order to make the given the magnification of the optical system and axial ray height is 0 at the image point. The object distance z_0 is used as a compensator.

The i-th group in the N group zoom system moves as much as Δz_i, then equation (1) becomes

$$[-z_0, k_1, -z_1, \cdots, -z_{i-1}+\Delta z_i, k_i, -z_i-\Delta z_i, \cdots, k_{N-1}, -z_{N-1}, k_N] = \frac{1}{m}$$
$$[-z_0, k_1, -z_1, \cdots, -z_{i-1}+\Delta z_i, k_i, -z_i-\Delta z_i, \cdots, k_{N-1}, -z_{N-1}, k_N, -z_N] = 0 \quad (2)$$

The distances of each group, $z_1 \sim z_N$, including the object distance z_0 are unknown parameters in the initial locus calculation. However, when designing the optical system in general, at least one of the middle positions between a wide and tele position are unknown. In other words, less than three times optical zoom has three zoom positions and more than five times optical zoom has five zoom positions for the design of wide and tele systems. Thus, the value of the middle position is determined and fixed by the optical design. The value of the others points of the zoom locus is calculated using interpolation. Among the

various interpolation methods, the method to make a curve gradual and smooth should be used to prevent singular points or discontinuous locus of each group. If a spline interpolation [7] is used, a relatively smooth locus can be obtained because the interpolation guarantees that the 1st derivative and the evaluated zoom position are continuous. Figure 3 shows the locus of an arbitrary zoom optical system for any group. The x and y-axis are the zoom step and interval between groups, respectively. The points on the curve indicate the designed zoom positions. In Fig. 3, the five points indicate the five designed zoom positions.

As shown in Fig. 3, each section of the curve can be expressed as a third-order polynomial similar to equation (3). This polynomial is continuous at the zoom position. If the 1st derivative is continuous, the equation (3) is satisfied. Because x_i, z_i, z_i' are on the designed zoom position, these parameters are already given and a, b, c, d are the unknown parameters.

$$z_i = ax_i^3 + bx_i^2 + cx_i + d$$
$$z_i' = \frac{dz_i}{dx_i} = 3ax_i^2 + 2bx_i + c$$
$$z_{i+1} = ax_{i+1}^3 + bx_{i+1}^2 + cx_{i+1} + d \quad (3)$$
$$z_{i+1}' = \frac{dz_{i+1}}{dx_{i+1}} = 3ax_{i+1}^2 + 2bx_{i+1} + c$$

If equation (3) is expressed in the matrix form, then equations (4), and (5) can be determined by solving the inverse matrix of equation (4) with the unknown parameters a, b, c, d.

$$\begin{bmatrix} z_i \\ z_i' \\ z_{i+1} \\ z_{i+1}' \end{bmatrix} = \begin{bmatrix} x_i^3 & x_i^2 & x_i & 1 \\ 3x_i^2 & 2x_i & 1 & 0 \\ x_{i+1}^3 & x_{i+1}^2 & x_{i+1} & 1 \\ 3x_{i+1}^2 & 2x_{i+1} & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} x_i^3 & x_i^2 & x_i & 1 \\ 3x_i^2 & 2x_i & 1 & 0 \\ x_{i+1}^3 & x_{i+1}^2 & x_{i+1} & 1 \\ 3x_{i+1}^2 & 2x_{i+1} & 1 & 0 \end{bmatrix}^{-1} \cdot \begin{bmatrix} z_i \\ z_i' \\ z_{i+1} \\ z_{i+1}' \end{bmatrix}$$

$$= \frac{1}{(x_i - x_{i+1})^3} \begin{bmatrix} -2z_i - x_{i+1}z_i' + 2z_{i+1} - x_{i+1}z_{i+1}' + x_i(z_i' + z_{i+1}') \\ -x_i^2(z_i' + 2z_{i+1}') + x_i(3z_i - x_{i+1}z_i' - 3z_{i+1} + x_{i+1}z_{i+1}') + x_{i+1}(3z_i + 2x_{i+1}z_i' - 3z_{i+1} + x_{i+1}z_{i+1}') \\ -x_{i+1}^3 z_i' + x_i^3 z_{i+1}' + x_i^2 x_{i+1}(2z_i' + z_{i+1}') - x_i x_{i+1}(6z_i + x_{i+1}z_i' - 6z_{i+1} + 2x_{i+1}z_{i+1}') \\ -x_{i+1}^3 z_{i+1} + x_i x_{i+1}^2(3z_i + x_{i+1}z_i') + x_i^3(z_{i+1} - x_{i+1}z_{i+1}') + x_i^2 x_{i+1}\{-3z_{i+1} + x_{i+1}(-z_i' + z_{i+1}')\} \end{bmatrix} \quad (5)$$

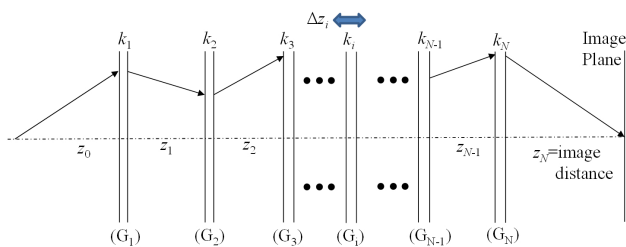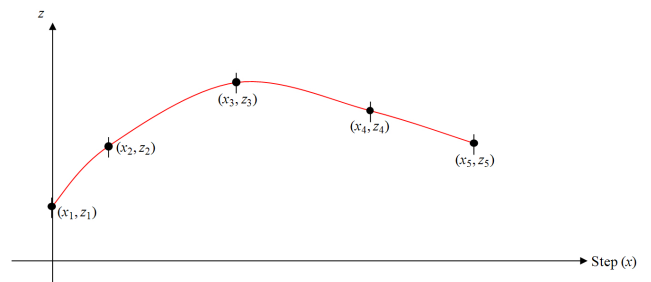FIG. 2. Optical layout for moving the i-th group of the lens system with N groups.

FIG. 3. Example of a locus for the lens group.

For equation (3) - (5), $z_1 \sim z_N$ are derived from a spline interpolation. Refracting power of each group is predetermined by the optical design. When the two equations in equation (2) are calculated, the zoom locus is obtained because only two unknown parameters, z_0 and Δz_i remain. In this regard, when the intervals between each group are determined, and a spline interpolation is used, the constraint of the distance between the groups is satisfied unconditionally. Therefore, considering the two cases of a finite object distance in the zoom system, the method to find the value of the unified analytic locus calculations, rather than that of the numerical analysis locus calculations, must be discussed with each case.

2.1. Case 1: The Case That the Object Distance Changes and the 1st Lens Group Moves

When the position of the 1st lens group and the object distance varies simultaneously, equation (2) can be expressed as

$$[-z_0, k_1, -z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N] = \frac{1}{m}$$
$$[-z_0, k_1, -z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N, -z_N] = 0 \tag{6}$$

Where unknown parameters are the object distance z_0 and compensating parameter Δz_1 of the 1st group.

When z_0 is eliminated, after being taken out from Gaussian bracket, the two equations in equation (6) become

$$z_0 = \frac{[-z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N] - \frac{1}{m}}{[k_1, -z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N]} = \frac{[-z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N, -z_N]}{[k_1, -z_1 - \Delta z_1, k_2, -z_2, \cdots, k_N, -z_N]} \tag{7}$$

The numerator and denominator of equation (7) can be rewritten in terms of Δz_1 as

$$z_0 = \frac{-A_1 \cdot \Delta z_1 + B_1}{-A_2 \cdot \Delta z_1 + B_2} = \frac{-C_1 \cdot \Delta z_1 + D_1}{-C_2 \cdot \Delta z_1 + D_2}$$
$$A_1 \equiv [k_2, -z_2, \cdots, k_N]$$
$$B_1 \equiv [-z_1, k_2, -z_2, \cdots, k_N] - \frac{1}{m}$$
$$A_2 \equiv k_1 \cdot A_1$$
$$B_2 \equiv [k_1, -z_1, k_2, -z_2, \cdots, k_N] \tag{8}$$
$$C_1 \equiv [k_2, -z_2, \cdots, k_N, -z_N]$$
$$D_1 \equiv [-z_1, k_2, -z_2, \cdots, k_N, -z_N]$$
$$C_2 \equiv k_1 \cdot C_1$$
$$D_2 \equiv [k_1, -z_1, k_2, -z_2, \cdots, k_N, -z_N]$$

When equation (8) is rewritten in terms of Δz_1, it becomes a quadratic equation:

$$(A_1 \cdot \Delta z_1 - B_1) \cdot (C_2 \cdot \Delta z_1 - D_2) = (A_2 \cdot \Delta z_1 - B_2) \cdot (C_1 \cdot \Delta z_1 - D_1)$$
$$\therefore (A_1 \cdot C_2 - A_2 \cdot C_1) \cdot (\Delta z_1)^2 + (A_1 \cdot D_2 - B_1 \cdot C_2 - A_2 \cdot D_1 + B_2 \cdot C_1) \tag{9}$$
$$\cdot \Delta z_1 + B_1 \cdot D_2 - B_2 \cdot D_1 = 0$$

Between the two solutions of Δz_1 from equation (9), the lower absolute value is preferred because it is more beneficial to manipulate in terms of both engineering and economic reasons.

2.2. Case 2: The Case That the Object Distance Changes and the Other Lens Group Moves

For Case 2, image distance (z_N) and magnification are compensated by the object distance and the other lens group in an optical system with finite object distance. When the object distance changes and i-th group moves, equation (2) can be written as

$$[-z_0, k_1, -z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N] = \frac{1}{m}$$
$$[-z_0, k_1, -z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N, -z_N] = 0 \tag{10}$$

where the unknown parameter is the compensating parameter Δz_i of the i-th group and the object distance z_0.

Solving equation (10) simultaneously for z_0 yields

$$z_0 = \frac{[-z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N] - \frac{1}{m}}{[k_1, -z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N]}$$
$$= \frac{[-z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N, -z_N]}{[k_1, -z_1, \cdots, -z_{i-1} + \Delta z_i, k_i, -z_i - \Delta z_i, \cdots, k_N, -z_N]} \tag{11}$$

Similarly, the numerator and denominator of equation (11) can be rewritten for Δz_i

$$\frac{A_1 \cdot C_1 \cdot k_i \cdot (\Delta z_i)^2 + (A_1 \cdot D_1 + B_1 \cdot C_1) \cdot k_i \cdot \Delta z_i + E_1 - \frac{1}{m}}{A_2 \cdot C_1 \cdot k_i \cdot (\Delta z_i)^2 + (A_2 \cdot D_1 + B_2 \cdot C_1) \cdot k_i \cdot \Delta z_i + E_2}$$
$$= \frac{A_1 \cdot C_2 \cdot k_i \cdot (\Delta z_i)^2 + (A_1 \cdot D_2 + B_1 \cdot C_2) \cdot k_i \cdot \Delta z_i + E_3}{A_2 \cdot C_2 \cdot k_i \cdot (\Delta z_i)^2 + (A_2 \cdot D_2 + B_2 \cdot C_2) \cdot k_i \cdot \Delta z_i + E_4}$$

$$\begin{aligned}
&A_1 \equiv [-z_1, \cdots, k_{i-1}] & &B_1 \equiv [-z_1, \cdots, k_{i-1}, -z_{i-1}] \\
&C_1 \equiv -[k_{i+1}, \cdots, k_N] & &D_1 \equiv [-z_i, k_{i+1}, \cdots, k_N] \\
&A_2 \equiv [k_1, -z_1, \cdots, k_{i-1}] & &B_2 \equiv [k_1, -z_1, \cdots, k_{i-1}, -z_{i-1}] \\
&C_2 \equiv -[k_{i+1}, \cdots, k_N, -z_N] & &D_2 \equiv [-z_i, k_{i+1}, \cdots, k_N, -z_N] \\
&E_1 \equiv [-z_1, \cdots, k_N] & &E_2 \equiv [k_1, -z_1, \cdots, k_N] \\
&E_3 \equiv [-z_1, \cdots, k_N, -z_N] & &E_4 \equiv [k_1, -z_1, \cdots, k_N, -z_N]
\end{aligned} \tag{12}$$

Even if equation (12) looks complicated, using the rules of algebraic equations yield a quadratic equation for Δz_i as

$$\left\{ A_1 \cdot C_1 \cdot E_4 + A_2 \cdot C_2 \cdot \left(E_1 - \frac{1}{m} \right) - A_1 \cdot C_2 \cdot E_2 - A_2 \cdot C_1 \cdot E_3 \right\} \cdot (\Delta z_i)^2 +$$
$$\left\{ (A_1 \cdot D_1 + B_1 \cdot C_1) \cdot E_4 + (A_2 \cdot D_2 + B_2 \cdot C_2) \cdot \left(E_1 - \frac{1}{m} \right) \right.$$
$$\left. - (A_1 \cdot D_2 + B_1 \cdot C_2) \cdot E_2 - (A_2 \cdot D_1 + B_2 \cdot C_1) \cdot E_3 \right\} \cdot (\Delta z_i) + \tag{13}$$
$$\left\{ \left(E_1 - \frac{1}{m} \right) \cdot E_4 - E_2 \cdot E_3 \right\} \cdot \frac{1}{k_i} = 0$$

As in Case 2, the lower absolute value for Δz_i is preferred because it is easier to manipulate in terms of both engineering and economic reasons.

III. EXAMPLES OF CALCULATING LENS LOCI

The effectiveness of the unified analytic method, which has been previously described in session II, is verified by calculating the zoom locus for some of the optical systems with a finite object distance.

The first example of JP2008-020656[8] will be used to examine the usability of equations (7) and (9) of Case 1. This example is a type of optical system that moves both the 1st and 2nd groups. This optical system, a double-gauss type, was mainly used for the macro lens in the initial stage. It is easier to move than fix the 2nd group in order to make the lens aperture smaller, considering the size of the mount for the interchangeable lens. Figure 4 shows the optical layout of the 1st example of JP2008-020656, and Table 1 represents the specifications, such as the focal length for each group, the location of the principal point, and the distance between groups. In Table

1, the locus is calculated, assuming that the cover glass of image sensor and the optical low pass filter (OLPF) are located at the back of the 2nd group. It also assumes the total thickness of both the cover glass and the OLPF is 3 mm.

Table 2 and Fig. 5 are the result of the zoom locus calculation for JP2008-020656. The zoom locus is calculated by assuming that the 2nd group moves linearly. This is practical because of its advantage of the linear displacement for CAM manufacture. Because the main purpose is to verify the given equation of the locus calculation in Case 1, the zoom locus is not linearly calculated as a compensator at this point although it is possible to linearly calculate the 1st group.

The groups in the 2nd example of US2010-0177407A1[9] are not fixed but movable. This is because it is designed to examine the convenience and accuracy of its usability for the zoom locus calculation by using equations (10) and (13) for Case 2. In this case, optical system must be brighter than JP2008-020656 due to the lower F-number, Sufficient optical performance can be obtained when it is possible to move the three groups, which are divided by the stop in its front and back. In this case, the group in the front of the stop is the 1st group, the one in the back of the stop is the 2nd group, and the one behind the 2nd
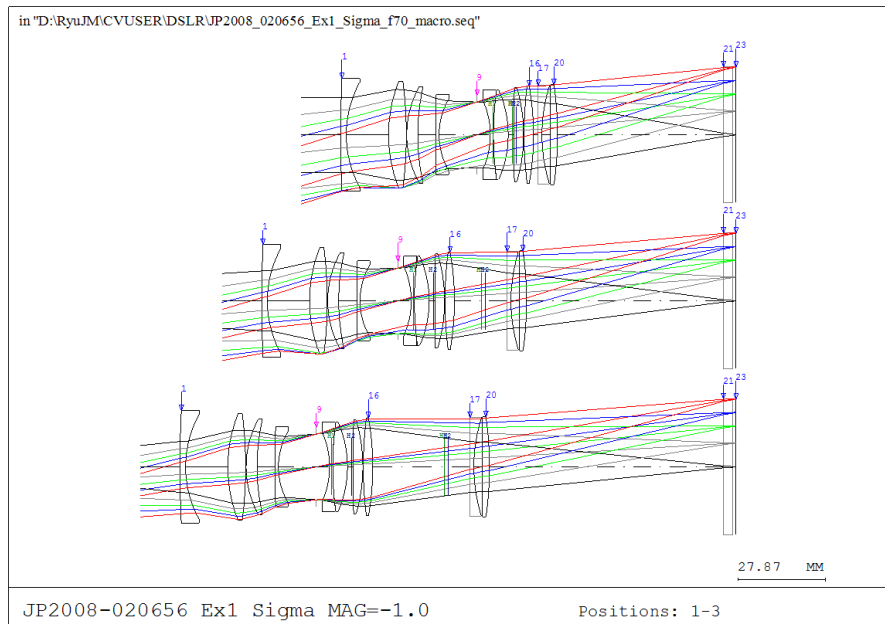

FIG. 4. Optical layout for JP2008-020656.

TABLE 1. The specifications for JP2008-020656

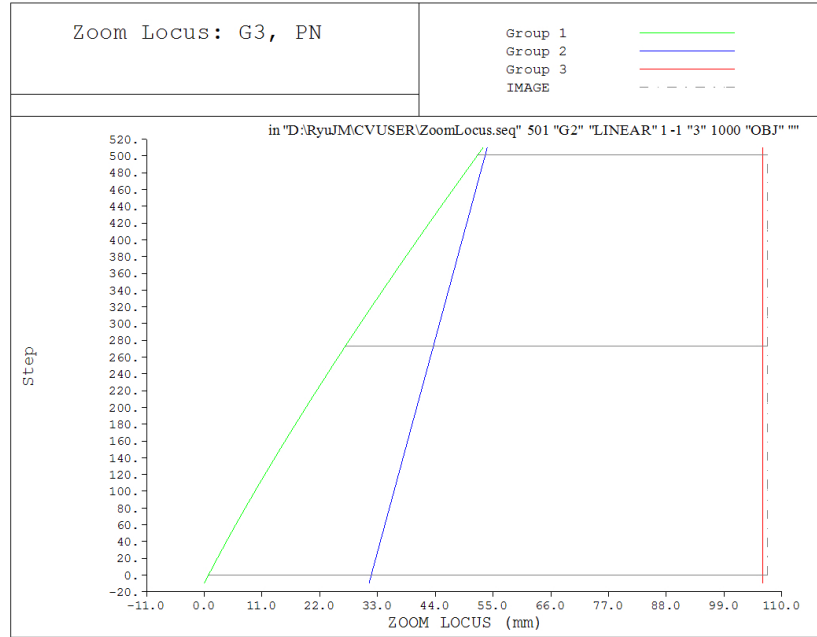|	EFL	H1	H2	Thickness		
				Infinity	Normal	Macro
Object				1.00e10	152.221017	94.225195
1st group	55.480912	48.491509	6.583202	1.5	16.8	31
2nd group	-285.296785	-8.106655	13.118133	52.817481	62.678070	74.483326
Filter				1	1	1

FIG. 5. The locus for US2010-0177407A1.

TABLE 2. The locus for US2010-0177407A1 determined by compensation of the 1st group for linearization of the 2nd group

Step	Mag	Obj	thi s16	thi s20	thi si-1
0	0	1.00e10	1.5	52.81748	1
50	-0.11174	627.2201	5.065854	54.98407	1
100	-0.22204	321.0123	8.525081	57.15065	1
150	-0.33061	219.1573	11.86726	59.31723	1
200	-0.43723	168.3867	15.08375	61.48382	1
250	-0.54163	138.0543	18.16625	63.6504	1
300	-0.64355	117.9524	21.10528	65.81699	1
350	-0.74262	103.7162	23.88791	67.98357	1
400	-0.83831	93.17802	26.49389	70.15016	1
450	-0.9297	85.15887	28.88758	72.31674	1
500	-1.01506	79	31	74.48333	1

group and the closest to the image plane is the 3rd group. Figure 6 displays the optical layout of US2010-0177407A1, and Table 3 shows the specifications, such as the focal lengths for each group, the location of the principal point, and the distance between groups.

Table 4 shows the displacement of the 2nd group and the object distance. This represents the locus value for a non-linear magnification that depends on the rotation of the CAM and the given image distance. The magnification of this system is changed by 2nd-order polynomial $y = ax^2 + bx$, where x is the rotation of the CAM, m is the magnification, a is 2.250136E-06 and b is -2.952377E-03. Figure 7 shows a graph of the locus points described Table 4. In Figure 7, there is an inflection point on the locus of the 1st group; however, it is advantageous not to

have any inflection point in the actual product. This inflection point is identified because it performs the spline interpolation on each lens group by dividing the zoom positions into three. Therefore, in order to design a product of a similar specification without the creation of an inflection point, it requires more zoom positions and checking to ensure that the locus is in the middle of the optical design.

For US2010-0177407A1, it is impossible to calculate the locus shown in Fig. 8, unlike Case 1. This is because when calculating the longitudinal sensitivity [10], the magnification of the 1st group becomes 1x in between the infinite zoom position and the middle zoom position. This results in a zero longitudinal sensitivity. Therefore, when it comes to calculating the zoom locus, the determination of whether the longitudinal sensitivity becomes 0 or not must
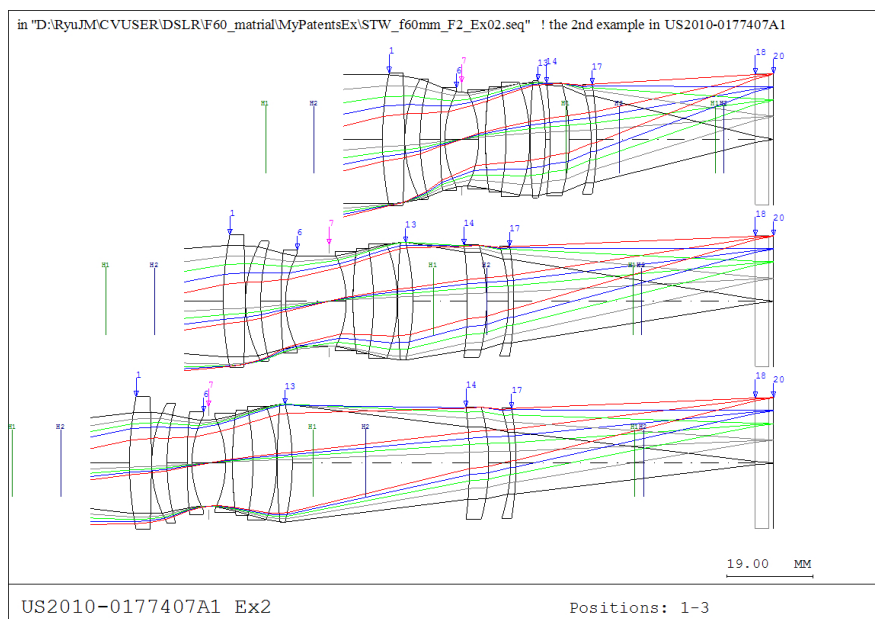
FIG. 6. Optical layout for US2010-0177407A1.

TABLE 3. The specifications for US2010-0177407A1

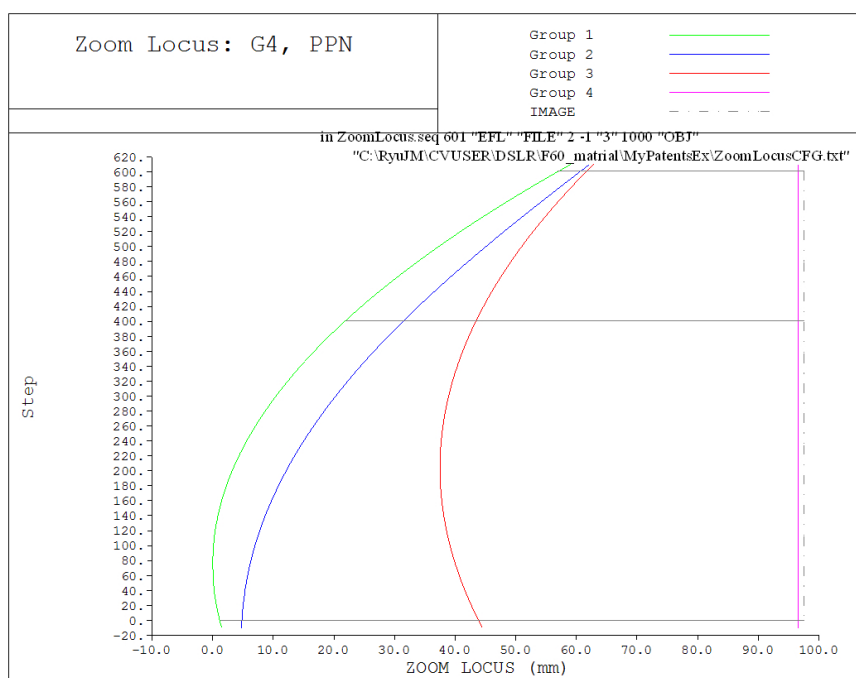|	EFL	H1	H2	Thickness		
				Infinity	Normal	Macro
Object				1.0e10	152.221017	94.225195
1st group	117.402038	-25.668378	28.756237	3.670927	9.597389	3.670000
2nd group	61.931373	23.065459	-16.314423	1.045288	12.020816	39.036917
3rd group	-1220.141927	36.440112	-28.107341			

FIG. 7. Locus for US2010-0177407A1.

TABLE 4. The locus for US2010-0177407A1 determined by compensation of the 1st group for magnification by given 2nd-order polynomial ($y=ax^2+bx$)

| Step	Mag	Obj	thi s6	thi s13	thi s17	thi si-1
0	0	1.0e10	3.670927	1.045289	34.997884	1.000256
50	-0.14199	456.67136	6.096701	3.242363	40.694960	1.000369
100	-0.27274	254.36449	7.795741	5.901271	45.624903	1.000477
150	-0.39223	186.32029	8.925521	8.864537	49.787713	1.000578
200	-0.50047	152.22101	9.597389	12.020816	53.183390	1.000674
250	-0.59746	131.95450	9.888120	15.293328	55.811933	1.000763
300	-0.68320	118.78974	9.849144	18.630647	57.673343	1.000847
350	-0.75769	109.82855	9.513183	22.000049	58.767620	1.000924
400	-0.82093	103.61674	8.898800	25.382972	59.094764	1.000996
450	-0.87292	99.346894	8.013367	28.772043	58.654774	1.001061
500	-0.91365	96.538842	6.854917	32.169230	57.447651	1.001121
550	-0.94314	94.894446	5.413176	35.584807	55.473394	1.001174
600	-0.96138	94.225195	3.670000	39.036917	52.732004	1.001222
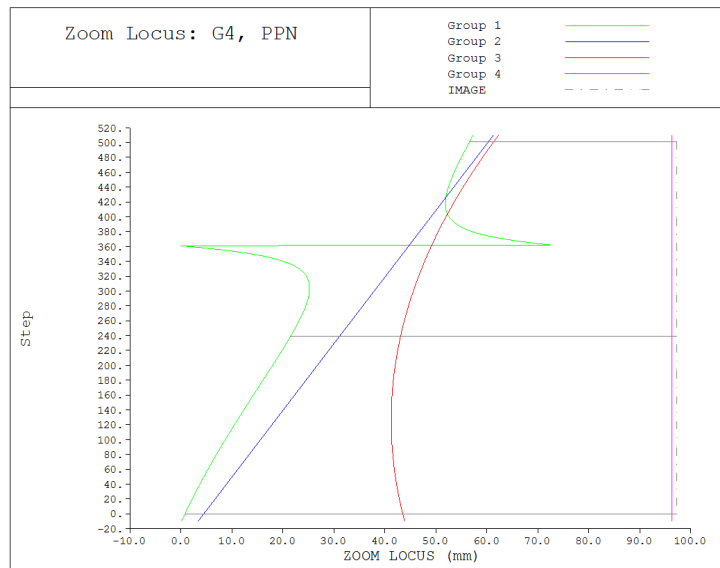
FIG. 8. Locus for US2010-0177407A1 by compensation of the 1st group.

be verified in advance. Similarly, the 3rd group is not adequate as a compensator because the maximum value is too low despite the longitudinal sensitivity of the 3rd group not being 0. Therefore, it cannot practically be used as a compensator.

The effectiveness of Case 2 can be more clearly examined when there are more groups of the zoom optical system. US2011-0096410A1 [11] is a macro lens. This optical system uses the inner focus system where the entire length of the optical system is fixed while adjusting the focus. Therefore, this optical system is more advantageous to take a picture of a moving subject such as insects. However, extraordinary technology is required due to the larger

sensitivity of the macro lens in the form of double-gauss. Figure 9 displays the optical layout of US2011-0096410A1, and Table 5 shows the specifications to calculate the locus.

Table 6 and Fig. 10 are the result of the zoom calculation of the compensations for the displacement of the 4th group and the object distance assuming a linear magnification for US2011-0096410A1. In effect, Table 6 verifies the linear magnification for US2011-0096410A1.

Lastly, the case of an optical system with a fixed magnification regardless of the changing object distance, which is classified into the multi-configurative optical system [7], is determined. Because the magnification for every zoom position is not zero, this optical system, as a device for
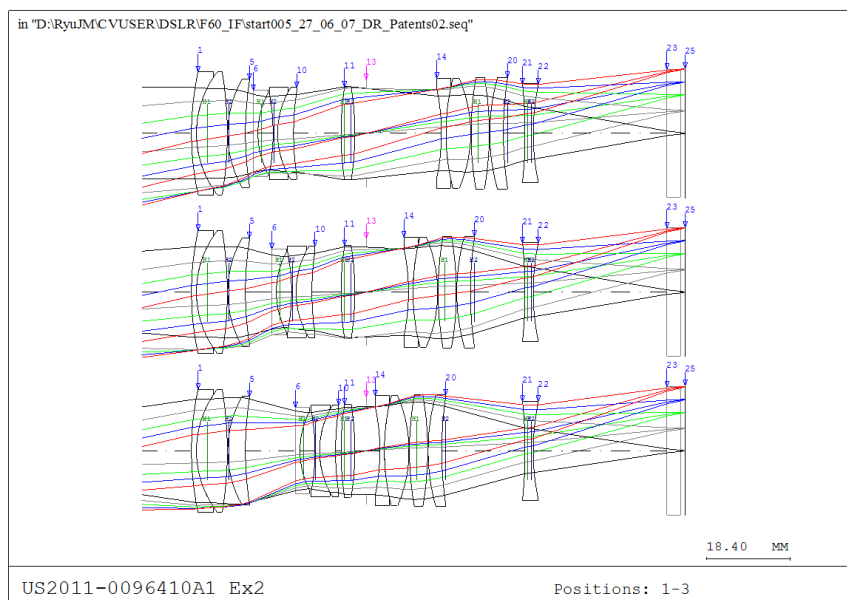
FIG. 9. Optical layout for US2011-0096410A1.

TABLE 5. The specifications for US2011-0096410A1

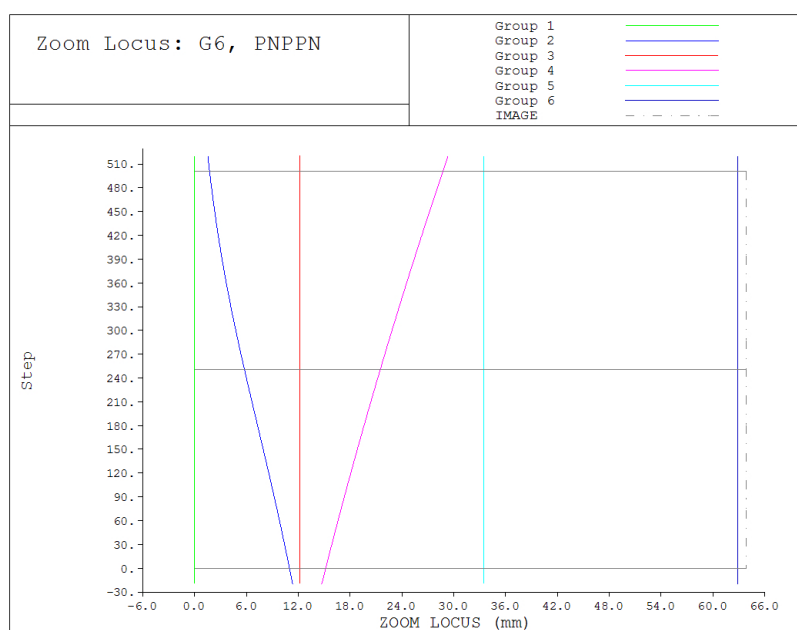|	EFL	H1	H2	Thickness		
				Wide	Normal	Tele
Object				1.00e10	119.337009	75.8
1st group	30.377352	3.611755	3.469398	1.752158	5.797891	10.984968
2nd group	-16.229263	1.798180	4.276165	10.434166	6.388433	1.201355
3rd group	39.655410	0.825277	3.325277	16.53990	9.296400	3.000682
4th group	35.455340	8.099158	-0.909789	4.756606	12.000103	18.295821
5th group	-42.951123	0.656680	0.331287			

FIG. 10. Locus for US2011-0096410A1.

TABLE 6. The locus for US2011-0096410A1 determined by compensation of the 4th group for linearization of magnification

| Step	Mag	Obj	thi s5	thi s10	thi s13	thi s20	thi s22	thi si-1	thi si
0	0	1.00e10	1.752158	10.43417	16.5399	4.756607	29.39117	0.915616	0.083864
50	-0.09999	578.4384	2.46883	9.717493	15.15317	6.143338	29.39117	0.955693	0.043722
100	-0.19998	288.9039	3.231416	8.954908	13.73066	7.565841	29.39117	0.988712	0.010636
150	-0.29997	193.3797	4.039914	8.146409	12.2744	9.022104	29.39117	1.014674	-0.01539
200	-0.39996	146.5568	4.894326	7.291998	10.79258	10.50392	29.39117	1.033579	-0.03436
250	-0.49995	119.4112	5.79465	6.391673	9.30162	11.99488	29.39117	1.045426	-0.04628
300	-0.59994	102.2909	6.740888	5.445436	7.82801	13.46849	29.39117	1.050216	-0.05114
350	-0.69993	91.06135	7.733038	4.453285	6.409384	14.88712	29.39117	1.047949	-0.04894
400	-0.79992	83.63452	8.771102	3.415222	5.093986	16.20252	29.39117	1.038624	-0.03968
450	-0.89991	78.80743	9.855078	2.331245	3.937981	17.35852	29.39117	1.022242	-0.02337
500	-0.9999	75.8	10.98497	1.201355	3.000682	18.29582	29.39117	0.998802	0
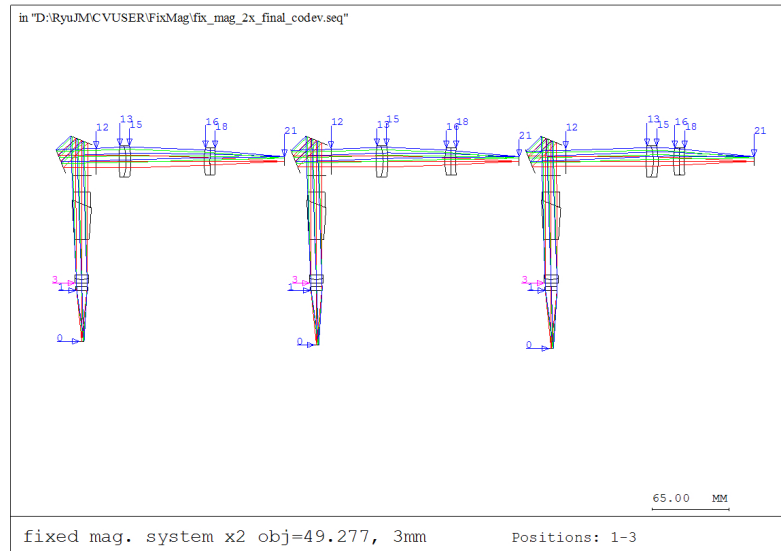
FIG. 11. Optical layout for the example in Reference [7].

TABLE 7. The specifications for the example in Reference [7]

	EFL	H1	H2	Thickness		
				Wide	Normal	Tele
Object				42.925	45.925	48.925
1st group	47.999642	2.074553	133.728761	19.192	38.143	67.954
2nd group	134.649060	5.346948	0.975389	61.704	48.015	12.941
3rd group	232.509536	-7.946896	13.015396	58.985463	53.722705	58.985331

lead frame inspection, has a finite object distance. The method introduced in section II can be used to calculate the locus for this optical system.

In the Reference [7], the specifications and the product design data are described in detail; thus, they are omitted at this point. Figure 11 displays the optical layout of this optical system, and Table 7 lists the focal length of each group, the location of principal point, and the distance between groups. The 1st group consists of a total of two lens groups. They are classified into 1-1 group that is comprised of one piece of a biconvex lens and 1-2 group that consists of two pieces of cemented lenses. The entire 1st group is fixed when the 2nd and 3rd groups move. Table 8 shows the locus table by the compensation of the
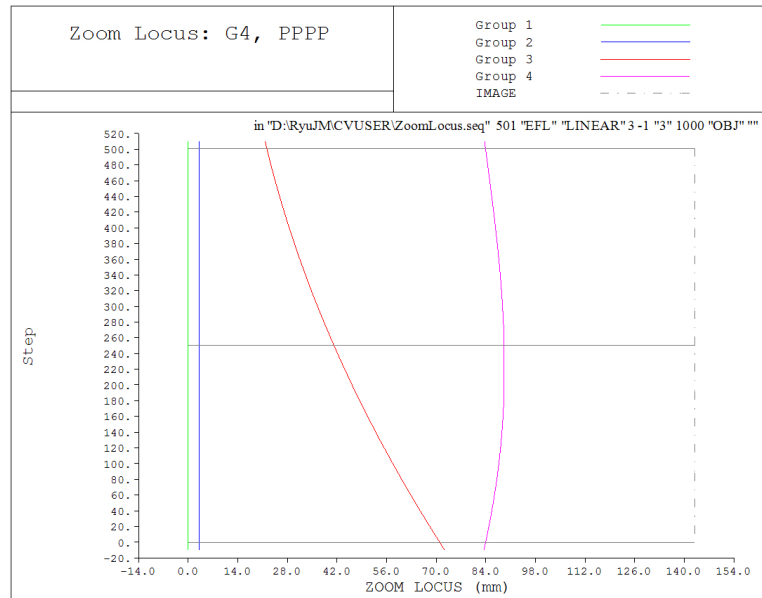
FIG. 12. Locus for the example in Reference [7].

TABLE 8. The locus for the example in Reference [7] determined by compensation of the 3rd group for fixed magnification

Step	Mag	Obj	thi s2	thi s12	thi s15	thi si-1
0	-2.00002	42.925	3	19.192	61.704	58.98546
50	-2.00002	43.45483	3	22.1134	60.23109	57.53679
100	-2.00002	44.03034	3	25.4692	58.22123	56.19069
150	-2.00002	44.64121	3	29.2594	55.57669	55.04487
200	-2.00002	45.27658	3	33.484	52.20342	54.19341
250	-2.00002	45.925	3	38.143	48.015	53.72271
300	-2.00002	46.57427	3	43.2364	42.93723	53.70697
350	-2.00002	47.21159	3	48.7642	36.91307	54.20324
400	-2.00002	47.82395	3	54.7264	29.90747	55.24657
450	-2.00002	48.39882	3	61.123	21.91094	56.84644
500	-2.00002	48.925	3	67.954	12.941	58.98533

3rd group for fixed magnification, and Fig. 12 presents the locus for the example described in Reference [7].

Similar to the procedures described above, various practical examples were verified using the unified analytic method of calculating a lens loci that adapts then Gaussian bracket method. This is derived from the examination of all zoom lens types and the multi-configurative optical system and a similar type of a zoom lens optical system for both case 1 and case 2. Consequently, the existing results correspond with those of Figs. 5, 7, 10, and 12 that are each calculated by adapting References 9 and 10 (about the general zoom lens), Reference 12 (about the macro lens with a number of groups), and Reference 7 (about multi-configurative optical system, a similar type of a zoom lens optical system).

IV. CONCLUSION

There is a high demand for zoom optical systems with finite object distance, because it is widely used with macro lenses as a sub-part of interchangeable lenses and many optical inspection systems. Even though this optical system does not fit within the definition of a standard zoom lens, many groups still attempt to use this system as an optical system. Therefore, the displacement of each group should be calculated for each CAM build. As a result, the designing method is very analogous to that of a general zoom lens. Thus, the locus calculation is required for this optical system with finite object distance, too. Although the existing analytic and numerical method is known, our new method for the locus calculation is seriously needed because of the problems such as the complex process of

obtaining the analytic value, waste of time for iterations, and calculation for the differential of the inverse matrix.

To achieve this, in this paper, an analytic method is proposed to show that the zoom loci can be obtained by moving the object distance and an added group, regardless of the number of groups. Imaging conditions need to be satisfied on the image plane for a given magnification and axial ray after interpolating the displacement of each group based on the data given in the initial optical design by a spline interpolation. This verifies that the desired locus can be easily obtained from various types of optical systems.

Various practical examples were verified by the unified analytic method of calculating lens loci that adapts the Gaussian bracket method, which is derived from the examination of all zoom lens types, multi-configurative optical systems, and a similar type of a zoom lens optical system in both Cases 1 and Cases 2. Consequently, the existing results corresponding with Figs. 5, 7, 10, and 12 are each calculated by adapting References 9 and 10 (about a general zoom lens), Reference 12 (about a macro lens with a number of groups), and Reference 7 (about a multi-configurative optical system, a similar type of a zoom lens optical system). Unlike the conventional zoom locus calculation method, which separately calculates the different kinds of groups for zoom lenses and the types of lenses, or adapting a numerical analysis from the results, this paper verifies the effectiveness of this new method by both deriving an analytic equation and adapting it to many cases successfully.

ACKNOWLEDGMENT

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