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Optimal designing of skip lot sampling plan of type SkSP-2 with double sampling plan as the reference plan under generalized exponential distribution

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Abstract. In this paper, a optimal designing methodology is proposed to determine the parameters for skip-lot sampling plan of type SkSP-2 plan with double sampling plan as reference plan, when the lifetime of the product follows generalized exponential distribution. The two points on the operating characteristic curve approach are used to find the optimal parameters for the proposed plan. The plan parameters are determined so as to minimize the average sample number subject to satisfying simultaneously both producer and consumer risks at the acceptable and limiting quality levels respectively.

Key Words: *Binomial sampling, consumer's risk, producer's risk, generalized exponential distribution, double sampling, skip-lot sampling*

1. INTRODUCTION

An acceptance sampling plan involves quality contracting on product orders between the producers and the consumers. In a time – truncated sampling plan, a random sample is selected from a lot of products and put on to test where the number of failures is recorded until the pre–specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling. The probability of rejecting a good lot is known as the producer's risk and the probability of accepting a bad lot is called the consumer's risk. An acceptance sampling plan should be designed so that both risks are smaller than the required values.

The skip lot sampling schemes are widely used to reduce inspection cost and widely used when the quality of the lot is good. In the skip-lot sampling operational procedure, only the fraction of submitted product is inspected for the acceptance or rejection of the submitted product. Skip lot sampling has much more applications in variety of fields.

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Dodge (1955) discussed the applications of the SkSP-1 sampling plan bulk material and product produced in successive lots. For more details about the skip-lot sampling one can refer Schilling (1982) and Balamurali et al. (2008). Perry (1973) has studied the properties of SkSP-2 plan with single sampling plan as the reference plan. He has also provided operating characteristics of SkSP-2 plan with some selected parameters. Parker and Kessler (1981) proposed modified skip lot sampling plan (MSkSP-1) and discussed its applications for the product that is produced completely out of control. Suresh (1993) has given for the selection of Skip-lot Sampling Plan of type SkSP-2 with reference plans SSP(c=0), SSP(c≠0) and DSP(0,1) using consumer and producer quality levels. Govindaraju (1994) has studied the properties of SkSP-2 plan using single sampling plan with zero acceptance number as the reference plan.

Suresh and Barathan (2012) have studied a procedure for the selection of skip-lot plans of type SkSP-2 which use chain sampling plan (ChSP-1) as reference plan through relative slopes at various points on the OC curve, which describe the degree of steepness of the OC curve.

Sudamani Ramaswamy (2012) has presented double acceptance sampling plans for truncated life tests are developed when the lifetimes of test items follows generalized exponential distribution. Probability of acceptance is calculated for different consumers confidence levels fixing the producer risk.

Gupta and Groll (1961) have studied application of Gamma distribution in acceptance sampling based on life tests, Aslam and Shabaz (2007) have studied Economic reliability test plans using the generalized exponential distribution, Baklizi and El Masri (2004) have studied acceptance sampling plans based on truncated life tests in the Birnbaum Saunders model, Muhammad Aslam et al (2010) have studied Time truncated acceptance sampling plans for generalized exponential distribution and also Muhammad Aslam (2007) have studied double acceptance sampling based on truncated life tests in Rayleigh distribution, Srinivasa Rao (2011) have studied double acceptance sampling plans based on truncated life tests for the Marshall – Olkin extended exponential distribution. All these authors developed the sampling plans for life tests using single acceptance sampling.

Skip-lot sampling is used for sampling chemical and physical processes in order to bring about substantial savings on inspection of products, which normally conform to specification. This particular sampling plan is useful when the lots are small or where inspection is slow and costly. SkSP-2 plan is considered as more reliable than the single sampling plan in that the required sample size to be inspected can be reduced. Another advantage of this plan is that we can obtain higher probability of acceptance at good quality levels than the single sampling plans. The SkSP-2 plan is operated as follows.

Step 1: Start with normal inspection (inspecting every lot), using the reference plan.

Step 2: When i consecutive lots are accepted on normal inspection, switch to skipping inspection. During the skipping inspection, only a fraction f of the lots are inspected.

Step 3: When a lot is rejected on skipping inspection, immediately revert to the normal inspection.

2. SkSP-2 PLAN WITH DOUBLE SAMPLING PLAN

An SkSP-2 plan is considered to be a good plan if it minimizes the average sample number (ASN) at the same time minimizing or at least maintaining the risks at the corresponding quality levels. An SkSP-2 plan with the double sampling plan (DSP) as the reference plan is characterized by six parameters namely i, f, n_1, n_2, c_1 and c_2 is as follows:

1) From a lot, select a random sample of size n1 and observe the number of nonconforming units, d_1.
2) If $d_1 \leq c_1$, then the lot is accepted. If $d_1 > c_2$ then the lot is rejected. If $c_1 < d_1 \leq c_2$, then go for second sampling. During second sampling, select a random sample of size n_2 and observe the number of nonconforming units, d_2.
3) If $d_1 + d_2 \leq c_2$, then the lot is accepted, otherwise the lot is rejected.

Thus the purpose of this paper is to find the parameters of a SkSP-2 plan using two-point approach by minimizing ASN and satisfying the producer's and consumer's risks when the lifetime of the product follows the GE distribution. A simulation experiment is performed to find the plan parameters such that both the risks are satisfied.

2.1. SkSP-2 Plan with DSP as the reference plan

The operating procedure of the SkDSP-2 plan is stated as follows:

1) At the outset, start with normal inspection (inspecting every lot) using DSP as the reference plan. During the normal inspection, lots are inspected one by one in the order of production or in the order of being submitted to inspection.
2) When i consecutive lots are accepted on normal inspection, discontinue the normal inspection and switch to skipping inspection.
3) During skipping inspection, inspect only a fraction f of the lots using the DSP as the reference plan selected at random. Skipping inspection is continued until a sampled lot is rejected.
4) When a lot is rejected on skipping inspection, then immediately revert to normal inspection as per (1).

The SkDSP-2 plan is characterized by six parameters, which are, i, f $(0 < f < 1)$, n_1, n_2, c_1 and c_2.

2.2. Designing of SkDSP-2 plan

In general any sampling plan can be designed for two specified points on the operating characteristic (OC) curve, namely, acceptable quality level (AQL) and limiting quality level (LQL), along with the corresponding producer's risk (α) and the consumer's risks (β). According to the American National Standards Institute/American Society for Quality (2008) (ANSI/ASQ Z1.4-2008), the AQL is defined as "the maximum proportion or percent defective (or the maximum number of defects per hundred units) that, for purposes of sampling inspection, can be considered satisfactory as a process average". AQL is usually defined as the worst-case quality level, in percentage or ratio, that is still

considered acceptable. As an AQL is an acceptable level, the probability of acceptance of a lot at the AQL should be high. The American National Standard Institute / American Society for Quality (2008) (ANSI / ASQ Z1.4-2008) defines the LQL as "the percentage or proportion of variant units in a batch or lot for which, for the purposes of sampling inspection, the consumer wishes the probability of acceptance to be restricted to a specified low value". LQL is used as an index for consumer protection for designing an acceptance sampling plan. AQL is denoted by p1 and the LQL is denoted by p2. Based on the principle of two points on the OC curve, the designing methodology of the proposed plan is explained below.
According to Perry (1973), the operating characteristic (OC) function of SkDSP-2 plan is given as,

$$P_a(p) = \frac{fP + (1-f)P^i}{f + (1-f)P^i} \tag{1}$$

where P is the probability of acceptance under the double sampling scheme and is given by

$$P = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{j=c_1+1}^{c_2} \sum_{i=0}^{c_2-j} \left[\binom{n_1}{j} p^j (1-p)^{n_1-j}\right]\left[\binom{n_2}{i} p^i (1-p)^{n_2-i}\right] \tag{2}$$

under AQL (=p1) and LQL (=p2) Conditions, Eq. (2) can be written as

$$P_1 = \sum_{i=0}^{c_1} \binom{n_1}{i} p_1^i (1-p_1)^{n_1-i} + \sum_{j=c_1+1}^{c_2} \sum_{i=0}^{c_2-j} \left[\binom{n_1}{j} p_1^j (1-p_1)^{n_1-j}\right]\left[\binom{n_2}{i} p_1^i (1-p_1)^{n_2-i}\right] \tag{3}$$

$$P_2 = \sum_{i=0}^{c_1} \binom{n_1}{i} p_2^i (1-p_2)^{n_1-i} + \sum_{j=c_1+1}^{c_2} \sum_{i=0}^{c_2-j} \left[\binom{n_1}{j} p_2^j (1-p_2)^{n_1-j}\right]\left[\binom{n_2}{i} p_2^i (1-p_2)^{n_2-i}\right] \tag{4}$$

The design parameters of the reference plan can be determined such that the following inequalities should be satisfied

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p_1^i (1-p_1)^{n_1-i} + \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p_1^x (1-p_1)^{n_1-x} \left[\sum_{i=0}^{c_2-x} \binom{n_2}{i} p_1^i (1-p_1)^{n_2-i}\right] \geq 1-\alpha \tag{5}$$

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p_2^i (1-p_2)^{n_1-i} + \sum_{x=c_1+1}^{c_2} \binom{n_1}{x} p_2^x (1-p_2)^{n_1-x} \left[\sum_{i=0}^{c_2-x} \binom{n_2}{i} p_2^i (1-p_2)^{n_2-i}\right] \leq \beta \tag{6}$$

Similarly, under the conditions of AQL and LQL, the parameters of the SkDSP-2 plan namely, i, f, n_1, n_2, c_1 and c_2 will be determined such that the following inequalities are satisfied.

$$\frac{fP_1 + (1-f)P_1^i}{f + (1-f)P_1^i} \geq 1-\alpha \tag{7}$$

$$\frac{fP_2 + (1-f)P_2^i}{f + (1-f)P_2^i} \leq \beta \tag{8}$$

where P_1 and P_2 are obtained by using equations (3) and (4) respectively.
The ASN of the SkSP-2 plan at the quality level of p is given by

$$ASN(p) = (ASN_{double}(p))\frac{f}{f+(1-f)P^{i}} \quad (9)$$

where $ASN_{double}(p)$ is the ASN of the DSP, which is given as

$$ASN_{double}(p) = n_1 + n_2 \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i} \quad (10)$$

Gupta and Kundu (2009) originally developed the generalized exponential (GE) distribution with δ as the shape parameter. The cdf of the GE distribution is given by

$$F(t;\delta,\lambda) = \left(1-\exp\left(-\frac{t}{\lambda}\right)\right)^{\delta}$$

where δ is the shape parameter and λ is the scale parameter.
The p-th percentile of GE distribution is given as

$$\theta_g = -\lambda \ln(1-p^{1/\delta})$$

Aslam et al. (2010) derived the equation of the probability of failure and is given as

$$p_g = \left[1 - e^{(-a\ln(1-p^{1/\delta})/(\theta_g/\theta^0))}\right]^{\delta}$$

3. NUMERICAL EXAMPLE

Suppose one wants to determine parameters of an SkSP-2 plan from Table 1 according to the conditions given that $p_1 = 0.05$, $p_2 = 0.082$, $\alpha = 0.05$ and $\beta = 0.10$. From this table, one can find the optimal parameters as i=5, f=0.005, $\theta_g/\theta^0 = 2$, n=128 and $c_1=1$, $c_2=6$ corresponding to the above mentioned AQL and LQL values. ASN of this plan at LQL is 140 and which is minimum. With simulation experiment, it is to be pointed out that as the value of i increases, the ASN decreases. At some particular values of i, there would be no effect on the ASN. The design parameters for the SkSP-2 plan and for the double sampling plan are presented in Tables 1 and 2, respectively, when $\alpha = 0.05$ and $\beta = 0.1$.
The optimal plan parameters of the proposed plan when the lifetime of the product follows the GE distribution can be determined for two values of the shape parameter (δ=2,3) and with other parameters are used in Table 1 and 2. The plan parameters along with the probability of acceptance values and the ASN values for the GE distribution are reported in Tables 1 and 2. The probability of acceptance values are computed at AQL and at the same the ASN values are calculated at the LQL. From Table 1, it is clear that, p_2 increases for fixed values of p_1, the clearance number of sampling inspection, i, the acceptance number, c and the ASN are decreased in general.

4. CONCLUSION

We proposed a skip lot sampling plan under Generalized Exponential distribution in reliability and acceptance sampling areas by considering the double sampling plan as the reference plan. It is recommended that industrial engineers and experimenters use the proposed plan when they have the facility to install more than one item in a single tester.

In this paper, we have developed tables and designing methodology for selecting the parameters of a system of skip-lot sampling plan of type SkSP-2 with double sampling plan as the reference plan, when the lifetime of the product follows the generalized exponential distribution under the conditions of AQL and LQL. The approach of two points on the OC curve is adopted to find the design parameters of the proposed plan. Sampling plans presented here will have minimum ASN while satisfying the AQL and LQL conditions at the same time.

Table 1. Optimal parameters for the SkDSP-2 Plan under the Generalized Exponential Distribution with Shape Parameter δ=2 and a=0.5

AQL (p_1)	LQL (p_2)		Optimal Parameters							
		(θ_g/θ^0)	i	f	$n_1=n_2=n$	c_1	c_2	ASN	$P_a(p_1)$%	$P_a(p_2)$%
0.001	0.007	2	2	0.765	880	1	2	928	95.32	5.55
	0.008	4	3	0.988	788	1	2	827	95.49	4.91
	0.009	6	2	0.997	785	1	2	807	95.49	2.77
	0.010	8	3	0.997	756	1	2	770	95.89	1.89
	0.011	10	4	0.987	751	1	2	759	95.99	1.09
0.005	0.020	2	3	0.876	402	1	4	440	95.26	9.52
	0.021	4	3	0.792	400	1	4	431	95.73	7.69
	0.022	6	4	0.945	382	1	4	411	95.76	7.65
	0.023	8	5	0.965	376	1	4	401	95.91	6.59
	0.024	10	5	0.988	374	1	4	394	95.91	5.37
0.01	0.030	2	6	0.375	310	1	5	340	95.16	9.52
	0.031	4	6	0.387	298	1	5	327	95.86	9.84
	0.033	6	4	0.678	279	1	5	307	95.37	9.99
	0.035	8	4	0.897	265	1	5	290	95.25	9.60
	0.036	10	4	0.878	259	1	5	283	95.74	9.33
0.05	0.082	2	5	0.005	128	1	6	140	95.50	9.34
	0.084	4	5	0.007	124	1	6	136	95.64	9.70
	0.086	6	5	0.009	121	1	6	133	95.74	9.71
	0.087	8	5	0.011	119	1	6	131	95.66	9.96
	0.092	10	6	0.013	113	1	6	124	95.51	9.61
0.10	0.165	2	7	0.004	69	1	7	76	95.83	9.89
	0.175	4	7	0.011	65	1	7	71	95.28	9.81
	0.183	6	7	0.022	62	1	7	68	95.00	9.85
	0.192	8	7	0.033	59	1	7	65	95.76	9.82
	0.209	10	8	0.083	54	1	7	59	95.25	9.81

Table 2. Optimal parameters for the SkDSP-2 Plan under the Generalized Exponential Distribution with Shape Parameter δ=3 and a=1.0

AQL (p_1)	LQL (p_2)		Optimal Parameters							
		(θ_g/θ^0)	I	f	$n_1=n_2=n$	c_1	c_2	ASN	$P_a(p_1)\%$	$P_a(p_2)\%$
0.001	0.007	2	2	0.898	791	1	2	859	95.83	8.61
	0.008	4	2	0.923	783	1	2	823	95.83	5.07
	0.009	6	3	0.911	780	1	2	802	95.90	2.87
	0.010	8	4	0.945	772	1	2	785	95.86	1.67
	0.011	10	4	0.948	768	1	2	775	95.90	0.94
0.005	0.011	2	3	0.020	679	1	3	720	95.39	6.89
	0.012	4	3	0.030	640	1	3	673	95.07	5.57
	0.013	6	4	0.033	584	1	3	616	95.03	5.48
	0.014	8	4	0.038	571	1	3	595	95.00	4.16
	0.015	10	4	0.042	560	1	3	578	95.06	3.14
0.01	0.017	2	5	0.002	490	1	4	529	95.06	8.20
	0.018	4	5	0.003	473	1	4	507	95.00	7.26
	0.020	6	5	0.012	410	1	4	446	95.06	8.69
	0.021	8	6	0.011	394	1	4	427	95.02	8.28
	0.022	10	6	0.018	373	1	4	405	95.08	8.61
0.05	0.073	2	3	0.001	148	1	5	154	95.23	8.30
	0.075	4	3	0.002	139	1	5	146	95.20	9.18
	0.078	6	4	0.003	122	1	5	132	95.16	9.11
	0.079	8	4	0.004	119	1	5	129	95.08	9.60
	0.086	10	5	0.007	106	1	5	116	95.12	9.96
0.10	0.160	2	5	0.005	64	1	6	70	95.42	9.70
	0.172	4	5	0.014	59	1	6	65	95.06	9.95
	0.191	6	5	0.041	53	1	6	58	95.08	9.79
	0.206	8	6	0.065	49	1	6	54	95.09	9.72
	0.255	10	6	0.388	39	1	6	43	95.29	9.86

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