Maximum Diversity Achieving Decoders in MIMO Decode-and-Forward Relay Systems with Partial CSI

Xianglan Jin, Eun-Ji Kum, and Dae-Woon Lim

Abstract: **We consider multiple-input multiple-output decode-and-forward relay systems in Rayleigh fading channels under the partial channel state information (CSI) that the channel statistics of the source-relay (SR) link and the instantaneous CSI of the source-destination and relay-destination links are known at the destination. In this paper, we propose a new near maximum likelihood (near-ML) decoder with two-level pairwise error probability (near-ML-2PEP) which uses the average PEP instead of the exact PEP. Then, we theoretically prove that the near-ML and near-ML-2PEP decoders achieve the maximum diversity, which is confirmed by Monte Carlo simulations. Moreover, we show that the near-ML-2PEP decoder can also achieve the maximum diversity by substituting the average PEP with the values that represent the error performance of the SR link.**

Index Terms: **Decode-and-forward (DF), maximum likelihood (ML), multiple-input multiple-output (MIMO), relay.**

I. INTRODUCTION

Recently, cooperative communication systems have been extensively studied [1]–[4]. In cooperative communication systems, a cooperative diversity is obtained through assisting relays. According to different relay operations, amplify-and-forward (AF) and decode-and-forward (DF) relay systems were developed in [1]. For the DF relay systems, a maximum likelihood (ML) decoder was presented in [2] under the situation of a single antenna and binary phase shift keying (BPSK) being used. The authors in [3] derived a closed-form formula of the approximated bit error probability (BEP) for the ML decoder in the DF relay system using M-pulse amplitude modulation (PAM) and M-quadrature amplitude modulation (QAM).

Due to the benefit of the multiple-input multiple-output (MIMO) systems whose capacity can be increased dramatically without increasing the bandwidth or transmit power [5], [6], MIMO technique has been also applied in cooperative communication systems. In [4], the ML decoder at the destination in MIMO DF relay systems was presented under the assumption that the relay knows the channel state information (CSI) of the source-relay (SR) link and the destination knows all the instantaneous CSI of the source-destination (SD), SR, and relay-destination (RD) links, i.e., full CSI. Also, a full diversity achievable near-ML decoder was proposed to reduce the high complexity of the ML decoder.

To apply the above decoders, the CSI of the SR link is needed to be known at the destination, which is not easy in practice. Thus, the minimum distance (MD) decoder [4] which assumes that the relay always decodes correctly can be used at the destination. In [7] and [8], the cooperative partial detection strategy with perfect relays was also introduced. In addition, the authors in [9] used the same assumption that identical symbol estimates are obtained at each relay to present the transmit diversity and relay selection algorithms for multi-relay cooperative MIMO systems with the linear minimum mean square error, successive interference cancellation, and adaptive reception decoders. However, as shown in [4], decoders with the assumption of perfect relays increase error probabilities.

By considering the above problems, the ML and piecewise-linear (PL) decoders were presented by using the average error probability (AEP) of the SR link [10], [11]. The ML and PL decoders were also applied in differential modulation [12], M-QAM [13], and distributed Alamouti code [14]. In [15], the ML and PL decision rules for the orthogonal space-time block coded (OSTBCed) DF relay system were also presented by applying the AEP of the SR link. Moreover, in [16], the ML and PL decoders with OSTBCs utilizing arbitrary M-ary modulations were presented without the CSI of the SR link, and it was proved that the PL decoder achieves the maximum possible diversity. All of the works considered the cases of the single-antenna system or OSTBCs in which the AEP of the SR link is relatively easy to derive via the channel statistics due to the orthogonality of codes. However, single-symbol decoding in [17] is impossible in most of the MIMO systems, and then the AEP of the SR link is not easy to derive. High-performance decoders in such general MIMO DF relay systems without the CSI of the SR link have not been investigated yet.

In this paper, we consider the MIMO DF relay systems with single source, single relay, and single destination in Rayleigh fading channels under the partial CSI that means the instantaneous CSI of the SR link is known at the relay, and the instantaneous CSI of the SD and RD links, and the channel statistics of the SR link, are known at the destination. Clearly, learning the channel statistics of the SR link at the destination represents much less overhead than learning the real values of the channel coefficients. We present the ML and near-ML decoders in general MIMO DF relay systems under partial CSI, and based on the average pairwise error probability (PEP), we propose a new decoder called near-ML with two-level-PEP (near-ML-2PEP). We prove that the near-ML and near-ML-2PEP decoders under partial CSI can achieve the maximum diversity order. Furthermore, we show that not only the average PEP but also other values which represent the error performance of the SR link can

Manuscript received May 30, 2013; approved for publication by Ogose, Shigeaki, Division II Editor, November 8, 2013.

This work was supported by the National Research Foundation of Korea grant funded by the Korean Government (2013R1A1A2004381) and the research program of Dongguk University.

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Digital object identifier 10.1109/JCN.2014.000006

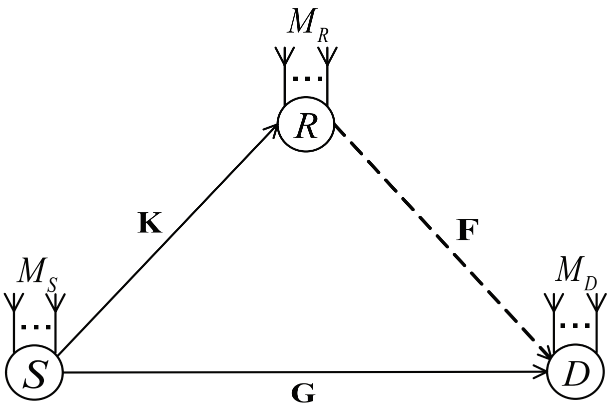

Fig. 1. A MIMO DF relay system with one source, one relay, and one destination. The solid line denotes the first phase transmission and the dashed line denotes the second phase transmission.

be applied to the near-ML-2PEP decoder with the maximum diversity. We call these decoders maximum diversity achieving decoders in this paper. Monte Carlo simulations confirm the analytically proved diversity and the characteristics of the maximum diversity achieving decoders.

The rest of the paper is organized as follows. The system model is introduced in Section II. Section III described the ML and near-ML decoders under full and partial CSIs and proposed a simple decoder, near-ML with two-level PEP. The diversity for the MIMO DF relay system under partial CSI is derived in Section IV. Discussion and simulation results are provided to confirm the analytical results in Section V. Finally, the conclusion is given in Section VI.

The following notations are used in this paper: Re$(\cdot)$ means the real part of a complex number; $E[\cdot]$ denotes the expectation; $\mathbf{I}_n$ denotes the $n \times n$ identity matrix; $\|\cdot\|$ and tr$(\cdot)$ represent the Frobenius norm and the trace of a matrix, respectively; $[\cdot]_i$ means the ith column vector of a matrix; the superscript $(\cdot)^\dagger$ denotes the complex conjugate transpose. $\mathbf{A} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{nm})$ denotes that the nm elements of an $n \times m$ random matrix $\mathbf{A}$ are independent and identically distributed (i.i.d.) circularly symmetric Gaussian random variables with zero mean and variance σ^2.

II. SYSTEM MODEL

A MIMO DF relay system of one source, one relay, and one destination with M_S, M_R, and M_D antennas, respectively, is shown in Fig. 1. Half duplex transmission and frequency-flat quasi-static Rayleigh fading are assumed. It is also assumed that the relay knows the instantaneous CSI of the SR link, and the destination knows the instantaneous CSI of the SD and RD links, and channel statistics of the SR link. Let T_1 and T_2 be the numbers of transmitted symbols at the source and relay during the first and second phases, respectively, and $\mathcal{A}$ be a set of message symbols from the M-ary signal constellation.

In the first phase, the source broadcasts the $M_S \times T_1$ codeword $\mathbf{X}_S(\mathbf{x})$ constructed from L data symbols $\mathbf{x} = (x_1, x_2, \cdots, x_L) \in \mathcal{A}^L$ to the relay and destination, and in the second phase, the relay sends the $M_R \times T_2$ codeword $\mathbf{X}_R(\mathbf{x}_R)$ constructed from $\mathbf{x}_R = (x_1^R, x_2^R, \cdots, x_L^R) \in \mathcal{A}^L$ which are the decoded symbols at the relay in the first phase. The received signals at the relay and destination in the first phase are given by

$$\begin{aligned}\mathbf{Y}_{SR} &= \sqrt{P_S}\mathbf{K}\mathbf{X}_S(\mathbf{x}) + \mathbf{N}_{SR},\\ \mathbf{Y}_{SD} &= \sqrt{P_S}\mathbf{G}\mathbf{X}_S(\mathbf{x}) + \mathbf{N}_{SD}\end{aligned} \tag{1}$$

where P_S is the average transmit power of each antenna at the source, $\mathbf{K} \sim \mathcal{CN}(0, \sigma_{SR}^2\mathbf{I}_{M_R M_S})$ and $\mathbf{G} \sim \mathcal{CN}(0, \sigma_{SD}^2\mathbf{I}_{M_D M_S})$ are the channel coefficient matrices of the SR and SD links, respectively. $\mathbf{N}_{SR} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{M_R T_1})$ and $\mathbf{N}_{SD} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{M_D T_1})$ represent the noise matrices at the relay and destination in the first phase.

In the second phase, the received signal at the destination is given by

$$\mathbf{Y}_{RD} = \sqrt{P_R}\mathbf{F}\mathbf{X}_R(\mathbf{x}_R) + \mathbf{N}_{RD} \tag{2}$$

where P_R is the average transmit power of each antenna at the relay. $\mathbf{F} \sim \mathcal{CN}(0, \sigma_{RD}^2\mathbf{I}_{M_D M_R})$ is the channel coefficient matrix of the RD channel and $\mathbf{N}_{RD} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_{M_D T_2})$ is the noise matrix at the destination in the second phase.

It is assumed that the total transmit powers at both the source and relay are equal to one. Then, the transmit SNRs in the SR, SD, and RD links are all equal to $SNR = 1/\sigma^2$.

III. DECODERS UNDER FULL AND PARTIAL CSI

A. ML and Near-ML Decoders under Full CSI

We consider the ML decoder of the MIMO DF relay system under full CSI. In the ML decoder at the destination, we choose $\hat{\mathbf{x}}$ to maximize the probability density function (pdf) of the received signals given a transmitted signal and the CSI of the SR, SD, and RD links, $p\left(\mathbf{Y}_{SD}, \mathbf{Y}_{RD}|\check{\mathbf{x}}, \mathbf{K}, \mathbf{G}, \mathbf{F}\right)$, i.e.,

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\max_{\check{\mathbf{x}}\in\mathcal{A}^L} p\left(\mathbf{Y}_{SD}, \mathbf{Y}_{RD}|\check{\mathbf{x}}, \mathbf{K}, \mathbf{G}, \mathbf{F}\right)\\ &= \arg\max_{\check{\mathbf{x}}\in\mathcal{A}^L} p\left(\mathbf{Y}_{SD}|\mathbf{X}_S(\check{\mathbf{x}}), \mathbf{G}\right)\\ &\qquad \sum_{\check{\mathbf{x}}_R\in\mathcal{A}^L} p\left(\mathbf{Y}_{RD}|\mathbf{X}_R(\check{\mathbf{x}}_R), \mathbf{F}\right) P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}}, \mathbf{K})\\ &= \arg\max_{\check{\mathbf{x}}\in\mathcal{A}^L}\Big[-\frac{\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\|^2}{\sigma^2}\\ &\quad +\ln\sum_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\exp\Big(\frac{-\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2+\sigma^2\ln P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}},\mathbf{K})}{\sigma^2}\Big)\Big]\end{aligned} \tag{3}$$

where $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}}, \mathbf{K})$ is the probability that the relay decodes the received signal to $\check{\mathbf{x}}_R$ when the source transmits data $\check{\mathbf{x}}$. Due to the difficulty deriving $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}}, \mathbf{K})$ in most multiple-antenna cases [18], the conditional PEP at the relay $P_{SR}(\check{\mathbf{x}} \to \check{\mathbf{x}}_R|\mathbf{K})$ is used instead of $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}}, \mathbf{K})$ in [4]. Since $P_{SR}(\check{\mathbf{x}} \to \check{\mathbf{x}}_R|\mathbf{K})$ only considers $\check{\mathbf{x}}$ and $\check{\mathbf{x}}_R$ but not the other signals, their decision boundaries are relatively simple and so is the derivation of $P_{SR}(\check{\mathbf{x}} \to \check{\mathbf{x}}_R|\mathbf{K})$ [4], [19]. Also, the widely-used max-log approximation [3], [20], [21], $\ln\sum_i e^{z_i} \approx \max_i z_i$ is used. Then, the near-ML decoder that achieves full diversity [4] is written as

$$\hat{\mathbf{x}} = \arg\min_{\check{\mathbf{x}}\in\mathcal{A}^L}\Big\{\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\|^2 + \min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2-\sigma^2\ln P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R|\mathbf{K})\Big]\Big\} \quad (4)$$

where the conditional PEP between $\check{\mathbf{x}}$ and $\check{\mathbf{x}}_R$ [22] is

$$P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R|\mathbf{K}) = Q\left(\sqrt{\frac{P_S}{2\sigma^2}\|\mathbf{K}(\mathbf{X}_S(\check{\mathbf{x}})-\mathbf{X}_S(\check{\mathbf{x}}_R))\|^2}\right). \quad (5)$$

The performance of the near-ML decoder approaches to that of the ML decoder as shown in [4].

B. Decoders under Partial CSI

Under partial CSI, the destination knows the instantaneous CSI of the SD and RD links and the channel statistics of the SR link. Therefore, the ML decoder can be written as

$$\begin{aligned}\hat{\mathbf{x}} &= \arg\max_{\check{\mathbf{x}}\in\mathcal{A}^L} p(\mathbf{Y}_{SD},\mathbf{Y}_{RD}|\check{\mathbf{x}},\mathbf{G},\mathbf{F})\\ &= \arg\max_{\check{\mathbf{x}}\in\mathcal{A}^L}\Big[-\frac{\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\|^2}{\sigma^2}\\ &\quad+\ln\sum_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\exp\Big(\frac{-\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2+\sigma^2\ln P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}})}{\sigma^2}\Big)\Big]\end{aligned} \quad (6)$$

where $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}}) = E_{\mathbf{K}}\left[P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}},\mathbf{K})\right]$.

Similar to the full CSI case, the probability $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}})$ is very difficult to derive, thus the PEP $P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R) = E_{\mathbf{K}}\left[P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R|\mathbf{K})\right]$ is used instead of $P_{SR}(\check{\mathbf{x}}_R|\check{\mathbf{x}})$, and then the near-ML decoder under partial CSI is presented as

$$\hat{\mathbf{x}} = \arg\min_{\check{\mathbf{x}}\in\mathcal{A}^L}\Big\{\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\|^2 + \min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2-\sigma^2\ln P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R)\Big]\Big\}. \quad (7)$$

Taking an expectation of the conditional PEP in (5), the PEP can be derived by using the channel statistics of the SR link.

Lemma 1: The PEP $P_{SR}(\mathbf{x}\to\mathbf{z})$ between a signal pair $\mathbf{x}$ and $\mathbf{z}\neq\mathbf{x}$ at the relay is derived as

$$P_{SR}(\mathbf{x}\to\mathbf{z}) = \frac{1}{2}\sum_{i=1}^{r}\sum_{k=1}^{M_R}A_{kl}\left[1-\sqrt{\frac{c_i}{1+c_i}}\sum_{j=0}^{k-1}\binom{2j}{j}[4(1+c_i)]^{-j}\right] \quad (8)$$

where $A_{kl} = \frac{\left\{\frac{d^{M_R-k}}{dz^{M_R-k}}\prod_{n=1,n\neq i}^{r}\left(\frac{1}{1+c_n z}\right)^{M_R}\right\}\big|_{z=-c_i^{-1}}}{(M_R-k)!c_i^{M_R-k}}$, $c_i = P_S\sigma_{SR}^2\lambda_i/4\sigma^2$, and $\lambda_1,\cdots,\lambda_r$ are nonzero eigenvalues of $(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))^\dagger$ whose rank is r. Considering the limitation of $\sigma^2\to 0$ $(SNR\to\infty)$, we have

$$\lim_{\sigma^2\to 0}\frac{\ln P_{SR}(\mathbf{x}\to\mathbf{z})}{\ln\sigma^2} = rM_R. \quad (9)$$

Let r_S be the minimum rank of $(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))^\dagger$ for all $\mathbf{z}\neq\mathbf{x}$. Then, the diversity d_{SR} at the relay is achieved as $d_{SR} = r_S M_R$.

Proof: See Appendix-A. □

As shown in Lemma 1, the PEPs $P_{SR}(\mathbf{x}\to\mathbf{z})$ for all the pairs of $\mathbf{x}$ and $\mathbf{z}$ do not depend on the instantaneous CSI and the received signal, thus these can be saved in memory. However, the memory requirement increases exponentially with the transmitted signal number L and the constellation size M.

To save memory, we only use two levels of PEPs, i.e., pairwise probability of correctly decoding $P_{SR}(\mathbf{x}\to\mathbf{x}) = 1/2$ for $\mathbf{z}=\mathbf{x}$ and the average PEP $\overline{P_{SR}} = \frac{1}{M^L(M^L-1)}\sum_{\mathbf{x},\mathbf{z}\neq\mathbf{x}}P_{SR}(\mathbf{x}\to\mathbf{z})$ for $\mathbf{z}\neq\mathbf{x}$ which is linearly proportional to $SNR^{-d_{SR}}$ in the high SNR. Then, the decoder in (7) becomes (10) in the top of next page. We call it near-ML with two-level-PEP (near-ML-2PEP). It is trivial that only one value $\overline{P_{SR}}$ is needed to save in memory. Next, we will analyze the diversity of the near-ML and near-ML-2PEP decoders for the MIMO DF relay systems under partial CSI.

Note that the near-ML and near-ML-2PEP decoders can easily be extended to multiple-relay systems.

IV. DIVERSITY ANALYSIS OF MIMO DF RELAY SYSTEMS UNDER PARTIAL CSI

To clearly analyze the near-ML and near-ML-2PEP decoders, we use a function $f(\mathbf{x},\mathbf{z})$ instead of $P_{SR}(\mathbf{x}\to\mathbf{z})$ and $\overline{P_{SR}}$ in (7) and (10), respectively. Then we can write both decoders in the same form as follows

$$\hat{\mathbf{x}} = \arg\min_{\check{\mathbf{x}}\in\mathcal{A}^L}\Big\{\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\|^2 + \min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2-\sigma^2\ln f(\check{\mathbf{x}},\check{\mathbf{x}}_R)\Big]\Big\} \quad (11)$$

where $f(\check{\mathbf{x}},\check{\mathbf{x}}_R) = P_{SR}(\check{\mathbf{x}}\to\check{\mathbf{x}}_R)$ for the near-ML decoder and $f(\check{\mathbf{x}},\check{\mathbf{x}}_R) = \begin{cases}1/2 & \text{if } \check{\mathbf{x}}_R=\check{\mathbf{x}}\\ \overline{P_{SR}} & \text{if } \check{\mathbf{x}}_R\neq\check{\mathbf{x}}\end{cases}$ for the near-ML-2PEP decoder. Thus, we have $\max_{\check{\mathbf{x}}_R} f(\check{\mathbf{x}},\check{\mathbf{x}}_R) = f(\check{\mathbf{x}},\check{\mathbf{x}}) = 1/2$. Now, we derive the PEP of the MIMO DF relay system with the decoder in (11).

Let $P(\mathbf{x}\to\tilde{\mathbf{x}})$ be the PEP between symbols $\mathbf{x}$ and $\tilde{\mathbf{x}}$ at the destination. The diversity d of the system is given as follows

$$d = \min_{\mathbf{x},\tilde{\mathbf{x}}\neq\mathbf{x}}\lim_{\sigma^2\to 0}\frac{\ln P(\mathbf{x}\to\tilde{\mathbf{x}})}{\ln\sigma^2}. \quad (12)$$

Considering all possible transmit symbols at the relay, the PEP at the destination should be written as

$$\begin{aligned}P(\mathbf{x}\to\tilde{\mathbf{x}}) &= E_{\mathbf{G},\mathbf{F}}[P(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{G},\mathbf{F})]\\ &= \sum_{\mathbf{x}_R\in\mathcal{A}^L}E_{\mathbf{G},\mathbf{F}}\Big[P(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R,\mathbf{G},\mathbf{F})\Big]P_{SR}(\mathbf{x}_R|\mathbf{x})\end{aligned} \quad (13)$$

where $P(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R,\mathbf{G},\mathbf{F})$ denotes the conditional PEP of deciding $\tilde{\mathbf{x}}$ at the destination when $\mathbf{x}$ and $\mathbf{x}_R$ are transmitted from the source and relay, respectively, for given $\mathbf{G}$ and $\mathbf{F}$. It can be

$$\hat{\mathbf{x}}=\arg\min_{\check{\mathbf{x}}}\left\{\left\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\check{\mathbf{x}})\right\|^2+\min\left[\left\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}})\right\|^2,\ \min_{\check{\mathbf{x}}_R}\left\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\right\|^2-\sigma^2\ln\left(2\overline{P_{SR}}\right)\right]\right\}. \quad (10)$$

written as

$$P(\mathbf{x}\rightarrow\tilde{\mathbf{x}}|\mathbf{x}_R,\mathbf{G},\mathbf{F})=P\Big(m(\mathbf{x})>m(\tilde{\mathbf{x}})\Big) \quad (14)$$

where the metric function $m(\mathbf{z})=\|\mathbf{Y}_{SD}-\sqrt{P_S}\mathbf{G}\mathbf{X}_S(\mathbf{z})\|^2+\min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\|^2-\sigma^2\ln f(\mathbf{z},\check{\mathbf{x}}_R)\Big]$. Putting (1) and (2) to the metric function, then, we have

$$m(\mathbf{x})=\|\mathbf{N}_{SD}\|^2+\min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\left\|\sqrt{P_R}\mathbf{F}(\mathbf{X}_R(\mathbf{x}_R)-\mathbf{X}_R(\check{\mathbf{x}}_R))+\mathbf{N}_{RD}\right\|^2 -\sigma^2\ln f(\mathbf{x},\check{\mathbf{x}}_R)\Big] \quad (15)$$

and

$$m(\tilde{\mathbf{x}})=\left\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\right\|^2 +\min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\left\|\sqrt{P_R}\mathbf{F}(\mathbf{X}_R(\mathbf{x}_R)-\mathbf{X}_R(\check{\mathbf{x}}_R))+\mathbf{N}_{RD}\right\|^2 -\sigma^2\ln f(\tilde{\mathbf{x}},\check{\mathbf{x}}_R)\Big]. \quad (16)$$

Before the derivation, we give the following useful theorem.

Theorem 1: Let $\mathbf{A}$ and $\mathbf{B}$ be complex matrices satisfying $\|\mathbf{B}\|^2>\|\mathbf{A}\|^2$ and $\mathbf{N}$ a random matrix of the statistically independent entries with complex Gaussian distribution $\mathcal{CN}(0,\sigma^2)$ [4]. Then, for $\sigma^2\rightarrow 0$, $\|\mathbf{B}+\mathbf{N}\|^2\geq\|\mathbf{A}+\mathbf{N}\|^2$ in probability, i.e.,

$$\lim_{\sigma^2\rightarrow 0}P\big(\|\mathbf{B}+\mathbf{N}\|^2\geq\|\mathbf{A}+\mathbf{N}\|^2\big)=1.$$

For simplicity, $P(a=b)$ in probability, i.e., $\lim_{\sigma^2\rightarrow 0}P(a=b)=1$, is denoted by $a\overset{P}{=}b$ and similarly the notations $\overset{P}{\leq}$ and $\overset{P}{\geq}$ are also used in this paper.

Next, we derive the achievable diversity via the following subsections.

A. *Simplification of the* min *Function in (15) and (16)*

Plugging (15) and (16) into (14), the conditional PEP can be derived. However, the min functions make it difficult to derive. Thus, similar to [4], we consider the above two metrics (15) and (16) by dividing the summand in (13) into two cases of $\mathbf{x}_R=\mathbf{x}$ and $\mathbf{x}_R\neq\mathbf{x}$ in the high SNR region.

The case of $\mathbf{x}_R=\mathbf{x}$:

From Theorem 1 and the fact that $\max_{\mathbf{z}}f(\mathbf{x},\mathbf{z})=f(\mathbf{x},\mathbf{x})=1/2$, we have

$$\begin{aligned}m(\mathbf{x})&=\|\mathbf{N}_{SD}\|^2+\min_{\check{\mathbf{x}}_R\in\mathcal{A}^L}\Big[\left\|\sqrt{P_R}\mathbf{F}(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\check{\mathbf{x}}_R))+\mathbf{N}_{RD}\right\|^2 - \sigma^2\ln f(\mathbf{x},\check{\mathbf{x}}_R)\Big]\\ &\overset{P}{=}\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln\frac{1}{2}.\end{aligned}$$

Similarly, for $m(\tilde{\mathbf{x}})$ in (16), since

$$\begin{aligned}&\left\|\sqrt{P_R}\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\check{\mathbf{x}}_R)\big)+\mathbf{N}_{RD}\right\|^2-\sigma^2\ln f(\tilde{\mathbf{x}},\check{\mathbf{x}}_R)\\ &\begin{cases}=\|\mathbf{N}_{RD}\|^2-\sigma^2\ln f(\tilde{\mathbf{x}},\mathbf{x}) & \text{for } \check{\mathbf{x}}_R=\mathbf{x}\\ \overset{P}{\geq}\left\|\sqrt{P_R}\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min})\big)+\mathbf{N}_{RD}\right\|^2-\sigma^2\ln\frac{1}{2} & \text{for } \check{\mathbf{x}}_R\neq\mathbf{x}\end{cases}\end{aligned}$$

where $\mathbf{x}_{\mathbf{F}}^{\min}=\arg\min_{\check{\mathbf{x}}_R\neq\mathbf{x}}\left\|\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\check{\mathbf{x}}_R)\big)\right\|^2$, we have

$$\begin{aligned}m(\tilde{\mathbf{x}})\overset{P}{\geq}&\left\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\right\|^2\\ &+\min\Big[\|\mathbf{N}_{RD}\|^2-\sigma^2\ln f(\tilde{\mathbf{x}},\mathbf{x}),\\ &\qquad\left\|\sqrt{P_R}\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min})\big)+\mathbf{N}_{RD}\right\|^2-\sigma^2\ln\frac{1}{2}\Big].\end{aligned}$$

Using $P_{SR}(\mathbf{x}_R=\mathbf{x}|\mathbf{x})\leq 1$, the summand in (13) for the case of $\mathbf{x}_R=\mathbf{x}$ can be upper bounded as (17) in the top of next page.

The case of $\mathbf{x}_R\neq\mathbf{x}$:

Since the value of the min function in (15) must be less than or equal to one of the elements including in the min function, an upper bound on $m(\mathbf{x})$ can be obtained as

$$m(\mathbf{x})\overset{P}{\leq}\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln f(\mathbf{x},\mathbf{x}_R) \quad (18)$$

by setting $\check{\mathbf{x}}_R=\mathbf{x}_R$. On the other hand, a lower bound on $m(\tilde{\mathbf{x}})$ occurs when $\check{\mathbf{x}}_R=\mathbf{x}_R=\tilde{\mathbf{x}}$ from Theorem 1, i.e.,

$$m(\tilde{\mathbf{x}})\overset{P}{\geq}\left\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\right\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln\frac{1}{2}. \quad (19)$$

Since $P_{SR}(\mathbf{x}_R|\mathbf{x})$ is equal to or less than $P_{SR}(\mathbf{x}\rightarrow\mathbf{x}_R)$ for $\mathbf{x}_R\neq\mathbf{x}$, by using (18) and (19), the summand in (13) for the case of $\mathbf{x}_R\neq\mathbf{x}$ can be upper bounded as

$$\begin{aligned}&E_{\mathbf{G},\mathbf{F}}\Big[P(\mathbf{x}\rightarrow\tilde{\mathbf{x}}|\mathbf{x}_R\neq\mathbf{x},\mathbf{G},\mathbf{F})\Big]P_{SR}(\mathbf{x}_R\neq\mathbf{x}|\mathbf{x})\\ &\overset{P}{\leq}E_{\mathbf{G},\mathbf{F}}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\mathbf{x},\mathbf{x}_R)\\ &>\left\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\right\|^2+\|\mathbf{N}_{RD}\|^2\Big)\Big]P_{SR}(\mathbf{x}\rightarrow\mathbf{x}_R).\end{aligned} \quad (20)$$

B. *Upper Bounds on the Summands in (13)*

In this subsection, we continue to derive the upper bounds on the summands in (13). We define an $M_D\times r_S$ matrix $\mathbf{G}'$ and an $M_D\times r_R$ matrix $\mathbf{F}'$ as

$$[\mathbf{G}']_i=[\mathbf{G}\mathbf{U}]_i \text{ for } i=1,\cdots,r_S$$

and

$$[\mathbf{F}']_i=[\mathbf{F}\mathbf{V}]_i \text{ for } i=1,\cdots,r_R$$

$$E_{\mathbf{G},\mathbf{F}}\Big[P\big(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R=\mathbf{x},\mathbf{G},\mathbf{F}\big)\Big]P_{SR}(\mathbf{x}_R=\mathbf{x}|\mathbf{x})$$
$$\overset{P}{\leq} E_{\mathbf{G},\mathbf{F}}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2>\big\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\big\|^2$$
$$+\min\Big[\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\tilde{\mathbf{x}},\mathbf{x}),\ \big\|\sqrt{P_R}\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min})\big)+\mathbf{N}_{RD}\big\|^2\Big]\Big)\Big]. \quad (17)$$

respectively, where $\mathbf{U}$ and $\mathbf{V}$ are the unitary matrices whose columns are the eigenvectors of $(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}}))(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}}))^\dagger$ and $(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min}))(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min}))^\dagger$, respectively. Let $\omega_{\min}$ and $\mu_{\min}$ be the minimum values among nonzero eigenvalues of $(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z}))^\dagger$ and $(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{z}))(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{z}))^\dagger$ for all $\mathbf{z}\neq\mathbf{x}$, respectively. Using Fact 1 in Appendix-B, we have $P_S\big\|\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)\big\|^2\geq P_S\omega_{\min}\|\mathbf{G}'\|^2$ and $P_R\big\|\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min})\big)\big\|^2\geq P_R\omega_{\min}\|\mathbf{F}'\|^2$. Also, from Theorem 1, we have $\big\|\sqrt{P_S}\mathbf{G}\big(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\tilde{\mathbf{x}})\big)+\mathbf{N}_{SD}\big\|^2\overset{P}{\geq}\|\sqrt{P_S\omega_{\min}}\mathbf{G}'+\mathbf{N}_{SD}\|^2$ and $\big\|\sqrt{P_R}\mathbf{F}\big(\mathbf{X}_R(\mathbf{x})-\mathbf{X}_R(\mathbf{x}_{\mathbf{F}}^{\min})\big)+\mathbf{N}_{RD}\big\|^2\overset{P}{\geq}\|\sqrt{P_R\omega_{\min}}\mathbf{F}'+\mathbf{N}_{RD}\|^2$. Then the upper bounds on the summands in (17) and (20) can be rewritten as

$$E_{\mathbf{G},\mathbf{F}}\Big[P\big(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R=\mathbf{x},\mathbf{G},\mathbf{F}\big)\Big]P_{SR}(\mathbf{x}_R=\mathbf{x}|\mathbf{x})$$
$$\overset{P}{\leq} E_{\mathbf{G}',\mathbf{F}'}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2>\|\sqrt{P_S\omega_{\min}}\mathbf{G}'+\mathbf{N}_{SD}\|^2$$
$$+\min\Big[\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\mathbf{x},\tilde{\mathbf{x}}),\|\sqrt{P_R\mu_{\min}}\mathbf{F}'+\mathbf{N}_{RD}\|^2\Big]\Big)\Big] \quad (21)$$

and

$$E_{\mathbf{G},\mathbf{F}}\Big[P\big(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R\neq\mathbf{x},\mathbf{G},\mathbf{F}\big)\Big]P_{SR}(\mathbf{x}_R\neq\mathbf{x}|\mathbf{x})$$
$$\overset{P}{\leq} E_{\mathbf{G}',\mathbf{F}'}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\mathbf{x},\mathbf{x}_R)$$
$$>\|\sqrt{P_S\omega_{\min}}\mathbf{G}'+\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2\Big)\Big]P_{SR}(\mathbf{x}\to\mathbf{x}_R). \quad (22)$$

Since multiplying the unitary matrix does not change the statistical distribution of the matrix with circularly symmetric complex Gaussian entries, the entries of $\mathbf{G}'$ and $\mathbf{F}'$ have the same distribution as the entries of $\mathbf{G}$ and $\mathbf{F}$, respectively. Therefore, we assume an $M_D\times r_S$ matrix $\mathbf{G}''$ and an $M_D\times r_R$ matrix $\mathbf{F}''$ as

$$[\mathbf{G}'']_i=[\mathbf{G}]_i \text{ for } i=1,\cdots,r_S$$

and

$$[\mathbf{F}'']_i=[\mathbf{F}]_i \text{ for } i=1,\cdots,r_R$$

and then the inequalities in (21) and (22) can be rewritten as

$$E_{\mathbf{G},\mathbf{F}}\Big[P\big(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R=\mathbf{x},\mathbf{G},\mathbf{F}\big)\Big]P_{SR}(\mathbf{x}_R=\mathbf{x}|\mathbf{x})$$
$$\overset{P}{\leq} E_{\mathbf{G}'',\mathbf{F}''}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2>\|\sqrt{P_S\omega_{\min}}\mathbf{G}''+\mathbf{N}_{SD}\|^2$$
$$+\min\Big[\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\mathbf{x},\tilde{\mathbf{x}}),\|\sqrt{P_R\mu_{\min}}\mathbf{F}''+\mathbf{N}_{RD}\|^2\Big]\Big)\Big] \quad (23)$$

and

$$E_{\mathbf{G},\mathbf{F}}\Big[P\big(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R\neq\mathbf{x},\mathbf{G},\mathbf{F}\big)\Big]P_{SR}(\mathbf{x}_R\neq\mathbf{x}|\mathbf{x})$$
$$\overset{P}{\leq} E_{\mathbf{G}'',\mathbf{F}''}\Big[P\Big(\|\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2-\sigma^2\ln 2f(\mathbf{x},\mathbf{x}_R)$$
$$>\|\sqrt{P_S\omega_{\min}}\mathbf{G}''+\mathbf{N}_{SD}\|^2+\|\mathbf{N}_{RD}\|^2\Big)\Big]P_{SR}(\mathbf{x}\to\mathbf{x}_R). \quad (24)$$

Combining (23), (24), and (13), the upper bound on the PEP in (12) can be derived. In the next subsection, we will derive the achievable diversity by finally calculating the upper bounds in (23) and (24).

C. Derivation of Diversity

Let $s=2\sqrt{P_S\omega_{\min}}\mathrm{Re}\{\mathrm{tr}(\mathbf{G}''\mathbf{N}_{SD}^\dagger)\}$, $t=2\sqrt{P_R\mu_{\min}}\mathrm{Re}\{\mathrm{tr}(\mathbf{F}''\mathbf{N}_{RD}^\dagger)\}$, $q=-\sigma^2\ln 2f(\mathbf{x},\tilde{\mathbf{x}})$, $q'=-\sigma^2\ln 2f(\mathbf{x},\mathbf{x}_R)$, $w=P_S\omega_{\min}\|\mathbf{G}''\|^2$, and $h=P_R\mu_{\min}\|\mathbf{F}''\|^2$. Then, $s\sim\mathcal{N}(0,2w\sigma^2)$ and $t\sim\mathcal{N}(0,2h\sigma^2)$.

The right-hand side (RHS) of (23) can be rewritten as $E_{w,h}\Big[P\big(t>q-h,s<-w-q\big)+P\big(t<q-h,t+s<-w-h\big)\Big]=A+B$, where $A=E_w\Big[Q\Big(\frac{w+q}{\sqrt{2w\sigma^2}}\Big)\Big]E_h\Big[Q\Big(\frac{q-h}{\sqrt{2h\sigma^2}}\Big)\Big]$ and $B=E_{w,h}\Big[\int_{-\infty}^{q-h}Q\Big(\frac{w+h+t}{\sqrt{2w\sigma^2}}\Big)\frac{\exp(-\frac{t^2}{4h\sigma^2})}{\sqrt{4\pi h\sigma^2}}dt\Big]$. Also, the RHS of (24) can be rewritten as

$$E_{q',w}\big[P\big(s<q'-w\big)\big]P_{SR}(\mathbf{x}\to\mathbf{x}_R)$$
$$=E_w\Big[Q\Big(\frac{w-q'}{\sqrt{2w\sigma^2}}\Big)\Big]P_{SR}(\mathbf{x}\to\mathbf{x}_R)$$
$$\leq\Big(\int_0^{q'}p_w(x)dx+E_w\Big[\exp\Big(-\frac{(w-q')^2}{4w\sigma^2}\Big)\Big]\Big)P_{SR}(\mathbf{x}\to\mathbf{x}_R)$$
$$=C+D \quad (25)$$

where $C=P_{SR}(\mathbf{x}\to\mathbf{x}_R)\int_0^{q'}p_w(x)dx$ and $D=E_w\Big[\exp\Big(-\frac{w}{4\sigma^2}-\frac{\frac{q'^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(\frac{q'}{2\sigma^2}\Big)P_{SR}(\mathbf{x}\to\mathbf{x}_R)$. Then, the PEP in (12) is bounded above by

$$P(\mathbf{x}\to\tilde{\mathbf{x}})=\sum_{\mathbf{x}_R\in\mathcal{A}^L}E_{\mathbf{G},\mathbf{F}}\Big[P(\mathbf{x}\to\tilde{\mathbf{x}}|\mathbf{x}_R,\mathbf{G},\mathbf{F})\Big]P_{SR}(\mathbf{x}_R|\mathbf{x})$$
$$\overset{P}{\leq}A+B+\sum_{\mathbf{x}_R\neq\mathbf{x}}(C+D). \quad (26)$$

Let $d_A=\lim_{\sigma^2\to 0}\frac{\ln A}{\ln\sigma^2}$, $d_B=\lim_{\sigma^2\to 0}\frac{\ln B}{\ln\sigma^2}$, $d_C=\lim_{\sigma^2\to 0}\frac{\ln C}{\ln\sigma^2}$, and $d_D=\lim_{\sigma^2\to 0}\frac{\ln D}{\ln\sigma^2}$. Summarizing the results of d_A,d_B,d_C and d_D, we can derive the diversity as follows.

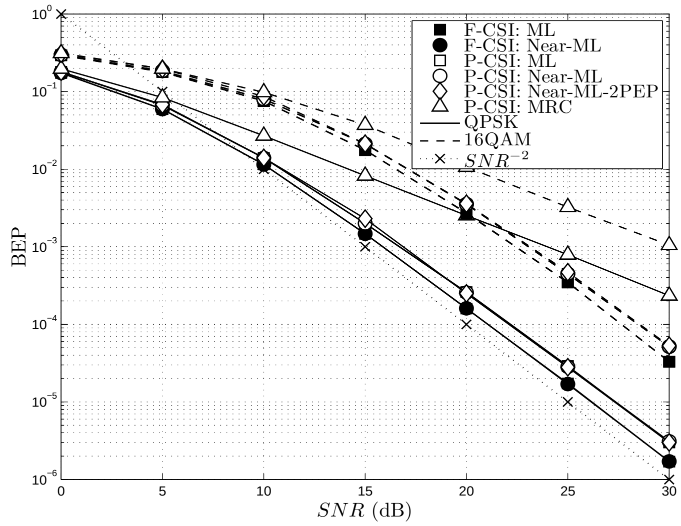

Fig. 2. Comparison of BEPs of various decoders in the uncoded single-antenna DF relay system.

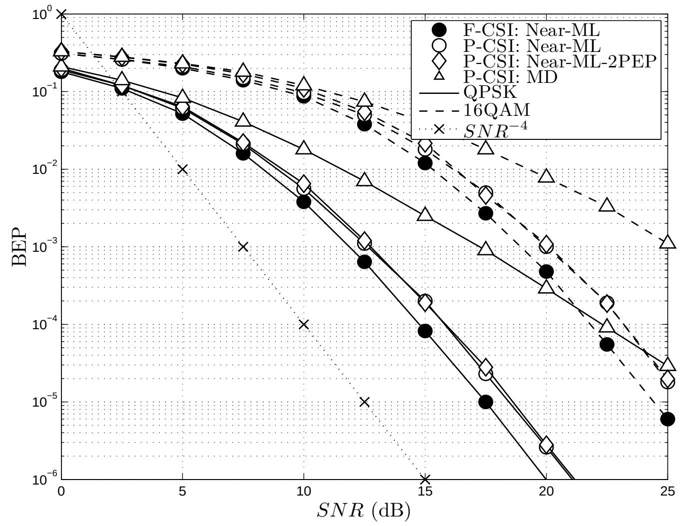

Fig. 4. Comparison of BEPs of various decoders in the multiplexing DF relay system.

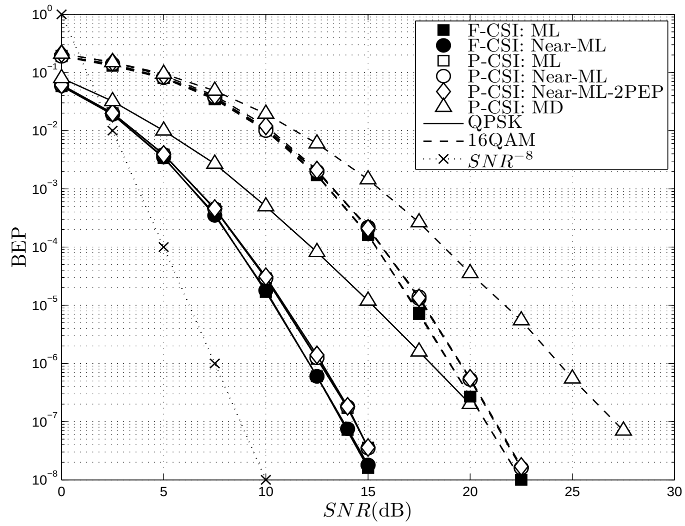

Fig. 3. Comparison of BEPs of various decoders in the Alamouti-coded DF relay system.

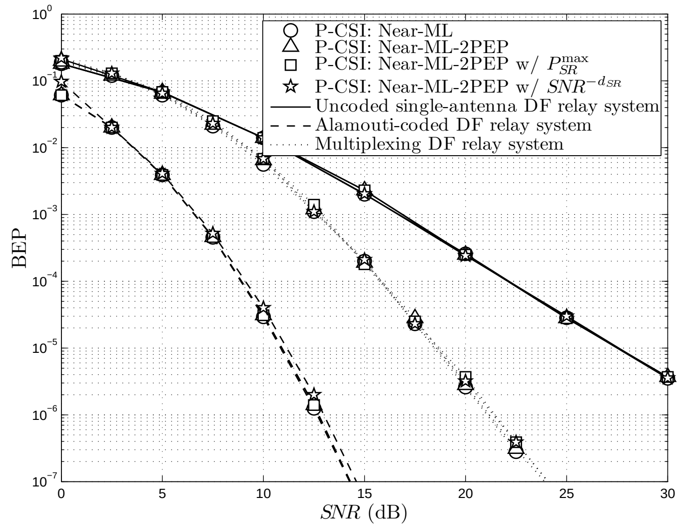

Fig. 5. Comparison of BEPs of various maximum diversity achieving decoders in the DF relay systems with partial CSI when QPSK is used.

Theorem 2: For the MIMO DF relay systems, both the near-ML and near-ML-2PEP decoders with partial CSI achieve the diversity $r_S M_D + \min[r_S M_R, r_R M_D]$ where r_S and r_R are the minimum ranks of $(\mathbf{X}_S(\mathbf{x}) - \mathbf{X}_S(\mathbf{z}))(\mathbf{X}_S(\mathbf{x}) - \mathbf{X}_S(\mathbf{z}))^\dagger$ and $(\mathbf{X}_R(\mathbf{x}) - \mathbf{X}_R(\mathbf{z}))(\mathbf{X}_R(\mathbf{x}) - \mathbf{X}_R(\mathbf{z}))^\dagger$ for all $\mathbf{z} \neq \mathbf{x}$, respectively. The full diversity $M_S M_D + M_R \min[M_S, M_D]$ is achieved when $r_S = M_S$ and $r_R = M_R$.

Proof : See Appendix-C. □

V. DISCUSSION AND MONTE CARLO SIMULATIONS

A. Characteristic of Maximum Diversity Achieving Decoders

In the previous section, we proved that the near-ML and near-ML-2PEP decoders under partial CSI have the diversity $r_S M_D + \min[r_S M_R, r_R M_D]$ which is the same as the diversity of the near-ML decoder under full CSI derived in [4]. Also, from the derivation of the diversity, especially, (29), (30), and (33), we can easily find that not only the near-ML and near-ML-2PEP decoders but also decoders whose $f(\check{\mathbf{x}}, \check{\mathbf{x}}_R \neq \check{\mathbf{x}})$ in (11) are linearly proportional to $SNR^{-d_{SR}}$ in the high SNR range achieve the maximum diversity. In this paper, we focus on the achievable diversity and the idea of two-level-value applied in the near-ML-2PEP decoder in general MIMO DF relay systems and do not mathematically compare their error performances.

On the other hand, to obtain the maximum diversity order, the maximum diversity achieving decoders need to consider the transmit signal from the relay, which increases the decoding complexity compared with the well-known MD decoder [4] that assumes the relay always decodes correctly. While the MD decoder has the complexity order $\mathcal{O}(|\mathcal{A}|^L)$, the maximum diversity achieving decoders have the complexity order $\mathcal{O}(|\mathcal{A}|^{2L})$. However, the decoding complexity also can be reduced. Once the computations of $\left\|\mathbf{Y}_{RD} - \sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\right\|^2$ are completed, we save them to $|\mathcal{A}|^L$ memories, then the decoders only need to do $2|\mathcal{A}|^L$ times computations of $\left\|\mathbf{Y}-\mathbf{H}\mathbf{X}\right\|^2$. Specially, the near-ML-2PEP decoder in (10) needs only one additional memory for $\min_{\check{\mathbf{x}}_R}\left\|\mathbf{Y}_{RD}-\sqrt{P_R}\mathbf{F}\mathbf{X}_R(\check{\mathbf{x}}_R)\right\|^2$ and $3|\mathcal{A}|^L$ times computations of $\left\|\mathbf{Y}-\mathbf{H}\mathbf{X}\right\|^2$, which means the complexity order is $\mathcal{O}(|\mathcal{A}|^L)$. For example, for the Alamouti-coded DF relay system, the complexity order is $\mathcal{O}(|\mathcal{A}|)$ which is comparable with the PL decoder [16]. Therefore, the proposed near-ML-2PEP decoder can be applied in practice without much complexity.

B. Monte Carlo Simulations

To confirm the analytical results, we give some Monte Carlo simulations in this subsection.

First, we consider the uncoded single-antenna DF relay system as a special case of MIMO DF relay systems. As an example where the ML decoder can be used at the destination, we handle the Alamouti-coded DF relay system where the source and relay transmit signals by using Alamouti code. Finally, we consider the 2×2 multiplexing DF relay system to show the case where the ML decoder is difficult to apply at the destination, where 2×2 multiplexing MIMO construction are used at both the source and relay.

Figs. 2–4 present the BEP curves of various decoders under full and partial CSI on the channel condition of $\sigma_{SR}^2 = \sigma_{SD}^2 = \sigma_{RD}^2 = 1$ for the uncoded single-antenna, Alamouti-coded, and 2×2 multiplexing DF relay systems, respectively, where 'F-CSI' means the full CSI assumption and 'P-CSI' means the partial CSI assumption at the destination. The curves with quadrature phase shift keying (QPSK) and 16QAM show that the ML, near-ML, near-ML-2PEP decoders have similar BEP performances under partial CSI and their curves are almost parallel with that of the ML and near-ML decoders under full CSI even though there are performance gaps between BEPs of the above decoders under full CSI and partial CSI. This means that they have the same diversity as proved in Theorem 2. We can also observe that the near-ML and near-ML-2PEP decoders have much better performance than the conventional maximum ratio combining (MRC) [27] and MD decoder [4] under partial CSI.

Next, we compare various maximum diversity achieving decoders by Monte Carlo simulations. As discussed in Subsection V.A, the decoders with $\lim_{SNR\to\infty} f(\check{\mathbf{x}}, \check{\mathbf{x}}_R \neq \check{\mathbf{x}}) \sim SNR^{-d_{SR}}$ will achieve the maximum diversity. As examples, we present two more decoders: 1) $f(\check{\mathbf{x}}, \check{\mathbf{x}}_R \neq \check{\mathbf{x}}) = P_{SR}^{\max} = \max_{\mathbf{x},\mathbf{z}\neq\mathbf{x}} P_{SR}(\mathbf{x} \to \mathbf{z})$ and 2) $f(\check{\mathbf{x}}, \check{\mathbf{x}}_R \neq \check{\mathbf{x}}) = SNR^{-d_{SR}}$ in (11). We call them near-ML-2PEP with $P_{SR}^{\max}$ and near-ML-2PEP with $SNR^{-d_{SR}}$, respectively. The simulation results in Fig. 5 show that all of the decoders of the near-ML, near-ML-2PEP, near-ML-2PEP with $P_{SR}^{\max}$, and near-ML-2PEP with $SNR^{-d_{SR}}$ achieve the maximum diversity in the uncoded single-antenna, Alamouti-coded, and multiplexing DF relay systems.

VI. CONCLUSION

In this paper, we have presented the ML and near-ML decoders in the MIMO DF relay systems under partial CSI and proposed the near-ML-2PEP decoder to save memory. We have also proved that the near-ML and near-ML-2PEP decoders can achieve the maximum diversity in the MIMO DF relay systems. From the simulation results, it was found that the proposed near-ML-2PEP decoder obtains similar BEP performance to the ML and near-ML decoders and much better performance than MRC and MD decoders under partial CSI. Furthermore, not only the average PEP but also other values which satisfy $\lim_{SNR\to\infty} f(\check{\mathbf{x}}, \check{\mathbf{x}}_R \neq \check{\mathbf{x}}) \sim SNR^{-d_{SR}}$ can be applied to the near-ML-2PEP decoder with the maximum diversity.

APPENDICES

A. Proof of Lemma 1

For a pair of signals, $\mathbf{x}$ and $\mathbf{z}$, the PEP can be derived by taking an expectation on the coefficient matrix of the SR channel, K, to the conditional PEP in (5), i.e.,

$$P_{SR}(\mathbf{x} \to \mathbf{z}) = E_{\mathbf{K}}\left[Q\left(\sqrt{\frac{P_S}{2\sigma^2}\left\|\mathbf{K}\left(\mathbf{X}_S(\mathbf{x}) - \mathbf{X}_S(\mathbf{z})\right)\right\|^2}\right)\right].$$

By using $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\theta})d\theta$, the PEP can be derived as

$$P_{SR}(\mathbf{x}\to\mathbf{z}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} E_{\mathbf{K}}\left[\exp\left(-\frac{P_S\left\|\mathbf{K}\left(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z})\right)\right\|^2}{4\sigma^2\sin^2\theta}\right)\right]d\theta. \tag{27}$$

Let $\mathbf{U}$ be a unitary matrix whose columns are the eigenvectors of $(\mathbf{X}_S(\mathbf{x}) - \mathbf{X}_S(\mathbf{z}))(\mathbf{X}_S(\mathbf{x}) - \mathbf{X}_S(\mathbf{z}))^\dagger$ corresponding to its eigenvalues λ_i's, where $\lambda_i \neq 0$ for $i = 1, \cdots, r$ and $\lambda_i = 0$ for $i = r+1, \cdots, M_S$. Then, we have

$$\begin{aligned}
&\left\|\mathbf{K}\left(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z})\right)\right\|^2\\
&=\mathrm{tr}\left(\mathbf{K}\left(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z})\right)\left(\mathbf{X}_S(\mathbf{x})-\mathbf{X}_S(\mathbf{z})\right)^\dagger\mathbf{K}^\dagger\right)\\
&=\mathrm{tr}\left(\mathbf{K}\mathbf{U}\mathrm{diag}(\lambda_1, \cdots, \lambda_r, 0, \cdots, 0)\mathbf{U}^\dagger\mathbf{K}^\dagger\right)\\
&=\sum_{i=0}^{r}\lambda_i\|[\mathbf{K}\mathbf{U}]_i\|^2.
\end{aligned}$$

Therefore, the expectation in (27) can be rewritten as

$$E_{\mathbf{K}}\left[\exp\left(-\frac{P_S\sum_{i=0}^{r}\lambda_i\|[\mathbf{K}\mathbf{U}]_i\|^2}{4\sigma^2\sin^2\theta}\right)\right] = \prod_{i=1}^{r}\left(\frac{\sin^2\theta}{\sin^2\theta + \frac{P_S\sigma_{SR}^2\lambda_i}{4\sigma^2}}\right)^{M_R} \tag{28}$$

where the equation is due to the fact that $\|[\mathbf{K}\mathbf{U}]_i\|^2$ is the summation of M_R i.i.d. exponential random variables with rate parameter $1/\sigma_{SR}^2$. By plugging (28) into (27) and using the result in [24], the PEP can be derived as

$$\begin{aligned}
P_{SR}(\mathbf{x} \to \mathbf{z}) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=1}^{r}\left(\frac{\sin^2\theta}{\sin^2\theta + \frac{P_S\sigma_{SR}^2\lambda_i}{4\sigma^2}}\right)^{M_R}d\theta\\
&= \frac{1}{2}\sum_{i=1}^{r}\sum_{k=1}^{M_R}A_{kl}\left[1-\sqrt{\frac{c_i}{1+c_i}}\sum_{j=0}^{k-1}\binom{2j}{j}[4(1+c_i)]^{-j}\right]
\end{aligned}$$

where $A_{kl} = \frac{\{\frac{d^{M_R-k}}{dz^{M_R-k}}\prod_{n=1,n\neq i}^{r}(\frac{1}{1+c_n z})^{M_R}\}|_{z=-c_i^{-1}}}{(M_R-k)!c_i^{M_R-k}}$ and $c_i = \frac{P_S\sigma_{SR}^2\lambda_i}{4\sigma^2}$. In addition, when $\sigma^2 \to 0$, the PEP can be simplified to

$$\begin{aligned}\lim_{\sigma^2\to 0} P_{SR}(\mathbf{x}\to\mathbf{z}) &= \lim_{\sigma^2\to 0}\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=1}^{r}\left(\frac{\sin^2\theta}{\sin^2\theta+\frac{P_S\sigma_{SR}^2\lambda_i}{4\sigma^2}}\right)^{M_R}d\theta\\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}(\sin^2\theta)^{rM_R}d\theta\cdot\lim_{\sigma^2\to 0}\prod_{i=1}^{r}\left(\frac{P_S\sigma_{SR}^2\lambda_i}{4\sigma^2}\right)^{-M_R}\\ &= \frac{1}{2}\binom{2rM_R}{rM_R}\left(\prod_{i=1}^{r}P_S\sigma_{SR}^2\lambda_i\right)^{-M_R}\lim_{\sigma^2\to 0}(\sigma^2)^{rM_R}.\end{aligned}$$

Then, we have

$$\lim_{\sigma^2\to 0}\frac{\ln P_{SR}(\mathbf{x}\to\mathbf{z})}{\ln\sigma^2} = rM_R.$$

B. Fact 1

Fact 1: [4] For an $n\times m$ matrix $\mathbf{A}$, there exist a unitary matrix $\mathbf{U}$ and a real diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1,\cdots,\lambda_n)$ such that $\mathbf{A}\mathbf{A}^\dagger = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^\dagger$. We assume $\lambda_i \neq 0, i = 1,\cdots,m$, $\lambda_j = 0, j = m+1,\cdots,n$, and $\lambda_{\min} = \min_{i=1,\cdots,m}\lambda_i$. Then, the following inequality holds for any $l\times n$ matrix $\mathbf{B}$ as

$$\|\mathbf{BA}\|^2 = \sum_{i=1}^{m}\lambda_i\big\|[\mathbf{BU}]_i\big\|^2 \geq \lambda_{\min}\|\mathbf{B}'\|^2$$

where $\mathbf{B}'$ is an $l\times m$ matrix constructed by $[\mathbf{B}']_i = [\mathbf{BU}]_i$, $i = 1,\cdots,m$.

C. Proof of Theorem 2

For A, by using $Q(x)\leq\exp(-x^2/2), x\geq 0$, we have

$$\begin{aligned}A &\leq E_w\Big[\exp\Big(-\frac{(w+q)^2}{4w\sigma^2}\Big)\Big]\\ &= E_w\Big[\exp\Big(-\frac{w}{4\sigma^2}-\frac{\frac{q^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(-\frac{q}{2\sigma^2}\Big).\end{aligned}\tag{29}$$

Since w is a r_SM_D-Erlang random variable with pdf of $p_w(x) = \frac{\lambda^n x^{n-1}\exp(-\lambda x)}{(n-1)!}$, $\lambda = P_S\omega_{\min}\sigma_{SD}^2$, $n = r_SM_D$, the expectation in (29) can be rewritten as

$$\begin{aligned}&E_w\Big[\exp\Big(-\frac{w}{4\sigma^2}-\frac{\frac{q^2}{4\sigma^2}}{w}\Big)\Big]\\ &= \int_0^\infty\frac{\lambda^n x^{n-1}\exp(-\lambda x)}{(n-1)!}\exp\Big(-\frac{x}{4\sigma^2}-\frac{\frac{q^2}{4\sigma^2}}{x}\Big)dx\\ &= \int_0^\infty\frac{\lambda^n x^{n-1}}{(n-1)!}\exp\Big(-\Big(\frac{1}{4\sigma^2}+\lambda\Big)x-\frac{\frac{q^2}{4\sigma^2}}{x}\Big)dx\\ &\overset{(a)}{=} \frac{2\lambda^n}{(n-1)!}\Big(\frac{\frac{q^2}{4\sigma^2}}{\frac{1}{4\sigma^2}+\lambda}\Big)^{\frac{n}{2}}K_n\left(2\sqrt{\frac{q^2}{4\sigma^2}\Big(\frac{1}{4\sigma^2}+\lambda\Big)}\right)\end{aligned}$$

where (a) is derived from the equation $\int_0^\infty x^{v-1}\exp(-\beta/x-\gamma x) = 2(\beta/\gamma)^{v/2}K_v(2\sqrt{\beta\gamma}), \mathrm{Re}\{\beta\} > 0, \mathrm{Re}\{\gamma\} > 0$ and $K_v(\cdot)$ is a modified Bessel function [25]. Since

$$\lim_{\sigma^2\to 0} f(\mathbf{x},\tilde{\mathbf{x}}) = \begin{cases}\lim_{\sigma^2\to 0} P_{SR}(\mathbf{x}\to\tilde{\mathbf{x}}) = (\sigma^2)^{rM_R} & \text{for the near-ML decoder}\\ \lim_{\sigma^2\to 0}\overline{P_{SR}} = (\sigma^2)^{r_SM_R} & \text{for the near-ML-2PEP decoder}\end{cases}$$

$q/\sigma^2 = -\ln 2f(\mathbf{x},\tilde{\mathbf{x}})$ is linearly proportional to $-\ln\sigma^2$ in high SNR range for both decoders, then, we have $2\sqrt{\frac{q^2}{4\sigma^2}\big(\frac{1}{4\sigma^2}+\lambda\big)} \gg 0$. Moreover, by using the asymptotic expansion $K_v(z)\sim\sqrt{\frac{\pi}{2z}}e^{-z}\big(1+\frac{4v^2-1}{8z}+\frac{(4v^2-1)(4v^2-9)}{2!(8z)^2}+\cdots\big)$ in [26], we have

$$\begin{aligned}d_A &\geq \lim_{\sigma^2\to 0}\frac{\ln\Big[E_w\Big[\exp\Big(-\frac{w}{4\sigma^2}-\frac{\frac{q^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(-\frac{q}{2\sigma^2}\Big)\Big]}{\ln\sigma^2}\\ &= \lim_{\sigma^2\to 0}\ln\Bigg[\frac{2\lambda^n}{(n-1)!}\Big(\frac{\frac{q^2}{4\sigma^2}}{\frac{1}{4\sigma^2}+\lambda}\Big)^{\frac{n}{2}}\sqrt{\frac{\pi}{4\sqrt{\frac{q^2}{4\sigma^2}\big(\frac{1}{4\sigma^2}+\lambda\big)}}}\\ &\qquad\exp\left(-2\sqrt{\frac{q^2}{4\sigma^2}\Big(\frac{1}{4\sigma^2}+\lambda\Big)}\right)\exp\Big(-\frac{q}{2\sigma^2}\Big)\Bigg]\Big/\ln\sigma^2\\ &= \lim_{\sigma^2\to 0}\frac{\ln\Big[\frac{2\lambda^n q^n}{(n-1)!}\sqrt{\frac{\pi\sigma^2}{q}}\exp\big(-\frac{q}{2\sigma^2}\big)\exp\big(-\frac{q}{2\sigma^2}\big)\Big]}{\ln\sigma^2}\\ &= \lim_{\sigma^2\to 0}\frac{\ln\Big[\frac{2\sqrt{\pi}\lambda^n\sigma^{2n}}{(n-1)!}\big(\frac{q}{\sigma^2}\big)^{n-\frac{1}{2}}\exp\big(-\frac{q}{\sigma^2}\big)\Big]}{\ln\sigma^2}\\ &= \lim_{\sigma^2\to 0}\frac{\ln\frac{2\sqrt{\pi}\lambda^n}{(n-1)!}}{\ln\sigma^2}+\frac{n\ln\sigma^2}{\ln\sigma^2}+\frac{(n-\frac{1}{2})\ln(\frac{q}{\sigma^2})}{\ln\sigma^2}+\frac{-\frac{q}{\sigma^2}}{\ln\sigma^2}.\end{aligned}\tag{30}$$

From Lemma 1, we finally achieve that $d_A \geq n + d_{SR} = r_SM_D + r_SM_R$ for both near-ML and near-ML-2PEP decoders.

For B, we have

$$\begin{aligned}B &\leq E_{w,h}\Bigg[\int_{-\infty}^{-w-h}\frac{\exp(-\frac{t^2}{4h\sigma^2})}{\sqrt{4\pi h\sigma^2}}dt\\ &\qquad+\int_{-w-h}^{q-h}\exp\Big(-\frac{(w+h+t)^2}{4w\sigma^2}\Big)\frac{\exp(-\frac{t^2}{4h\sigma^2})}{\sqrt{4\pi h\sigma^2}}dt\Bigg]\\ &= E_{w,h}\Bigg[\exp\Big(-\frac{(w+h)^2}{4h\sigma^2}\Big)\\ &\qquad+\exp\Big(-\frac{w+h}{4\sigma^2}\Big)\sqrt{\frac{w}{w+h}}\int_{-w-h}^{q-h}\frac{\exp(-\frac{(t+h)^2}{4\frac{wh}{w+h}\sigma^2})}{\sqrt{4\pi\frac{wh}{w+h}\sigma^2}}dt\Bigg]\\ &\leq 2E_{w,h}\Big[\exp\Big(-\frac{w+h}{4\sigma^2}\Big)\Big].\end{aligned}$$

Since w and h are r_SM_D and r_RM_D-Erlang random variables with parameters $P_S\omega_{\min}\sigma_{SD}^2$ and $P_R\mu_{\min}\sigma_{RD}^2$, respectively and d_B can be lower bounded as

$$d_B \geq r_SM_D + r_RM_D.\tag{31}$$

For C, we have

$$C = P_{SR}(\mathbf{x} \to \mathbf{x}_R) \int_0^{q'} \frac{\lambda^n x^{n-1} \exp(-\lambda x)}{(n-1)!} dx$$
$$= P_{SR}(\mathbf{x} \to \mathbf{x}_R) \frac{\gamma(n, \lambda q')}{(n-1)!}$$

where $\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt$. By using the series representation of $\gamma(\alpha, x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{\alpha+i}}{i!(\alpha+i)}$ [25], the above integral can be rewritten as

$$C = \frac{P_{SR}(\mathbf{x} \to \mathbf{x}_R)}{(n-1)!} \sum_{i=0}^{\infty} \frac{(-1)^i (\lambda q')^{n+i}}{i!(n+i)}.$$

From Lemma 1 and $q' = -\sigma^2 \ln 2f(\mathbf{x}, \mathbf{x}_R)$, we have

$$d_C \geq r_S M_R + r_S M_D. \quad (32)$$

Next, we consider D. For the near-ML decoder, $q' = -\sigma^2 \ln 2P_{SR}(\mathbf{x} \to \mathbf{x}_R)$, and then D can be written as $E_w\Big[\exp\Big(-\frac{w}{4\sigma^2} - \frac{\frac{q'^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(-\frac{q'}{2\sigma^2}\Big)$ which has the same form as the RHS of (29). Thus, we can use the result of d_A, i.e.,

$$d_D \geq r_S M_D + r_S M_R.$$

On the other hand, $q' = -\sigma^2 \ln 2\overline{P_{SR}}$ for the near-ML-2PEP decoder and then, we have

$$D \leq E_w\Big[\exp\Big(-\frac{w}{4\sigma^2} - \frac{\frac{q'^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(\frac{q'}{2\sigma^2}\Big) \sum_{\mathbf{x},\mathbf{z}\neq\mathbf{x}} P_{SR}(\mathbf{x} \to \mathbf{z})$$
$$= \frac{1}{2} M^L (M^L - 1) E_w\Big[\exp\Big(-\frac{w}{4\sigma^2} - \frac{\frac{q'^2}{4\sigma^2}}{w}\Big)\Big]\exp\Big(-\frac{q'}{2\sigma^2}\Big). \quad (33)$$

By using the result of d_A, we also achieve $d_D \geq r_S M_D + r_S M_R$.

From (26) and the derivations of d_A, d_B, d_C, d_D, we have the diversity

$$d \geq \min_{\mathbf{x},\tilde{\mathbf{x}}\neq\mathbf{x}} \min[d_A, d_B, d_C, d_D]$$
$$\geq r_S M_D + \min[r_S M_R, r_R M_D].$$

Since the decoders under partial CSI cannot obtain larger diversity than the ML decoder under full CSI whose diversity is $r_S M_D + \min[r_S M_R, r_R M_D]$, we have finally proved that both the near-ML and near-ML-2PEP decoders under partial CSI achieve the diversity of $r_S M_D + \min[r_S M_R, r_R M_D]$ in the MIMO DF relay systems. When $r_S = M_S$ and $r_R = M_R$, both achieve the full diversity $M_S M_D + M_R \min[M_S, M_D]$.

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