# Codebook-Based Interference Alignment for Uplink MIMO Interference Channels

Hyun-Ho Lee, Ki-Hong Park, Young-Chai Ko, and Mohamed-Slim Alouini

***Abstract:*** **In this paper, we propose a codebook-based interference alignment (IA) scheme in the constant multiple-input multiple-output (MIMO) interference channel especially for the uplink scenario. In our proposed scheme, we assume cooperation among base stations (BSs) through reliable backhaul links so that global channel knowledge is available for all BSs, which enables BS to compute the transmit precoder and inform its quantized index to the associated user via limited rate feedback link. We present an upper bound on the rate loss of the proposed scheme and derive the scaling law of the feedback load to maintain a constant rate loss relative to IA with perfect channel knowledge. Considering the impact of overhead due to training, cooperation, and feedback, we address the effective degrees of freedom (DOF) of the proposed scheme and derive the maximization of the effective DOF. From simulation results, we verify our analysis on the scaling law to preserve the multiplexing gain and confirm that the proposed scheme is more effective than the conventional IA scheme in terms of the effective DOF.**

***Index Terms:*** **Effective degrees of freedom (DOF), interference alignment (IA), limited feedback, multiplexing gain, uplink multiple input multiple output(MIMO) interference channel (IC).**

## I. INTRODUCTION

Interference alignment (IA) is one of the promising techniques to mitigate interference in the $K$-user interference channel (IC) that models wireless networks in which $K$ transmitters communicate with their own intended receivers, respectively [1]. Recently, intensive research efforts have been devoted to show that IA using multiple antennas can provide more degrees of freedom (DOF) compared with conventional schemes in the constant multiple-input multiple-output (MIMO) IC [2], [3]. Although most research efforts on IA have been focusing on designing the precoding matrices and receive filters for the constant MIMO IC [4], [5], such solutions are still not feasible since they require globally perfect channel state information (CSI) for the transmitter [1].

On the other hand, in practical scenarios, acquiring perfect CSI at the transmitter (CSIT) is almost impossible due to the limited-rate feedback link. Thus, IA schemes with limited feedback that quantize and feedback the channel coefficients using Grassmannian codebooks have been developed and analyzed for the $K$-user IC in [6], [7]. Specifically, the required scaling law for the number of feedback bits with respect to the signal-to-noise ratio (SNR) in order to preserve the multiplexing gain has been derived. In [8], the authors considered the effective DOF achieved by IA for the constant MIMO IC when CSI is obtained by training and fed back to the transmitter. In [9], a scheme to reduce the quantization error compared with the naive work in [7] was proposed but it entails an iterative algorithm with high computational complexity. In [10], the authors investigated the performance of IA where CSI is acquired via training and analog feedback by characterizing the effective sum rate with overhead in relation to various parameters such as SNR, Doppler spread, and feedback channel quality. The work in [11] also addressed the limited feedback design for IA under more practical network topology such as path loss or spatial correlation. In [12], an efficient feedback scheme for IA was proposed by reducing the redundant information in the channel quantization procedure. To avoid the feedback overhead issues, the author in [13] considered analog CSI feedback where the CSI quality increases with SNR on the feedback link, which implies that the multiplexing gain can be preserved as long as the SNR levels of the forward and feedback link are comparable.

In this paper, we propose a codebook-based IA scheme, which can be applicable to the $K$-user constant MIMO IC especially for the uplink scenario. Specifically, in our proposed scheme, we assume cooperation among base stations (BSs) through reliable backhaul links [14], [15] so that global CSI is available for all BSs, which implies that the precoding matrices and receive filters based on IA can be obtained at BSs. Instead of quantizing the channel coefficients as in [6], [7], it is assumed that each BS quantizes the precoding matrix using its codebook and informs the index to the associated user. We analyze the rate loss as a function of the number of feedback bits and quantify the scaling law of the feedback load to maintain the constant rate loss relative to IA with perfect CSIT. We also show that a fraction of the multiplexing gain can be still achieved if a fraction of the feedback load according to the derived scaling law is given. Similar to the results in [16], we show from our simulations that the average sum rate of the $K$-user constant MIMO IC saturates at a certain constant value if the number of feedback bits is fixed regardless of the SNR value. We also confirm that the multiplexing gain of the proposed scheme is preserved by increasing

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the number of feedback bits according to the derived scaling law. As a practical aspect, we investigate the effect of overhead such as training, cooperation, and feedback by defining the effective DOF that is similarly considered in [8]. We can see from simulations that the effective DOF of the proposed scheme outperforms that of the conventional IA scheme given the equal number of feedback bits. Moreover, the proposed scheme can significantly reduce the number of feedback bits compared with the conventional IA scheme in order to obtain the same effective DOF.

The following mathematical notations will be used throughout the paper. Upper case and lower case boldfaces are used to denote matrices and vectors, respectively. Note that $(\cdot)^{\mathsf{T}}$, $(\cdot)^{\mathsf{H}}$, $\mathrm{tr}\{\cdot\}$, $||\cdot||$, and $\mathbb{E}[\cdot]$ represent the transpose, conjugate transpose, trace, $l_2$-norm, and expectation operator, respectively. $\mathbf{I}$ denotes the identity matrix.

## II. SYSTEM MODEL

We consider a MIMO uplink cellular network with $K$ cells which forms a MIMO IC by assuming that a transmitter is chosen by a scheduler in each cell. Each transmitter (user) is equipped with $N_t$ antennas and each receiver (BS) with $N_r$ antennas. We assume a flat-fading channel, where the channel coefficients remain constant during $T$ symbols. Assuming that the $j$th transmitter attempts to send the data symbol vector $\mathbf{x}_j \in \mathbb{C}^{d_j \times 1}$ with $d_j$ independent data streams to the $j$th BS, we can represent the signal vector at the $j$th BS, $\mathbf{y}_j \in \mathbb{C}^{N_r \times 1}$, as

$$\mathbf{y}_j = \sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{V}_j\mathbf{x}_j + \sum_{i=1, i\neq j}^{K} \sqrt{\frac{P}{d_i}}\mathbf{H}_{j,i}\mathbf{V}_i\mathbf{x}_i + \mathbf{n}_j \tag{1}$$

where $\mathbf{H}_{j,i} \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix between the $i$th user and the $j$th BS, $\mathbf{V}_j \in \mathbb{C}^{N_t \times d_j}$ is the precoding matrix for the $j$th user with unit-norm column vectors $\mathbf{v}_{j,m}$ for $m = 1, 2, \cdots, d_j$, and $\mathbf{n}_j \in \mathbb{C}^{N_r \times 1}$ is the complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\mathbf{I}$. Note that in (1), $P$ is the transmit power, $\mathbb{E}[\|\mathbf{x}_j\|^2] = d_j$, and the total number of streams is denoted as $d_{\mathrm{tot}} = \sum_{j=1}^{K} d_j$. We assume that all the channel coefficients are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Denoting the receive filter for the $j$th BS as $\mathbf{U}_j \in \mathbb{C}^{N_r \times d_j}$ which consists of unit-norm column vectors $\mathbf{u}_{j,m}$ for $m = 1, 2, \cdots, d_j$, we can write the $m$th stream of the $j$th BS as the sum of the desired signal, the inter-stream interference (ISI), the inter-user interference (IUI), and the noise term, which is given by

$$\begin{aligned}\hat{x}_{j,m} =& \mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}x_{j,m} + \sum_{\substack{l=1\\ l\neq m}}^{d_j} \mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_j}}\mathbf{H}_{j,j}\mathbf{v}_{j,l}x_{j,l} \\ &+ \sum_{\substack{i=1\\ i\neq j}}^{K}\sum_{l=1}^{d_i} \mathbf{u}_{j,m}^{\mathsf{H}}\sqrt{\frac{P}{d_i}}\mathbf{H}_{j,i}\mathbf{v}_{i,l}x_{i,l} + \mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{n}_j \end{aligned}\tag{2}$$

where $x_{j,m}$ is the $m$th data stream of the $j$th user.

## III. CODEBOOK-BASED IA

In this section, we introduce a strategy of the proposed codebook-based IA by taking training, cooperation, and feedback into consideration. As stated in Section I, instead of quantizing the channel coefficients as in [6]–[8], we provide a different mechanism of IA with limited feedback for each BS in order to quantize the precoding matrix using its codebook and to inform the index to the associated user by exploiting cooperation among BSs.

### *A. Training*

To learn the channel coefficients at BSs, all users send known pilot symbols over a period $KN_t$ [17]. Then, the $j$th BS estimates the channels from all users, i.e., $\mathbf{H}_{j,i}$ for all $i$. For simplicity, we assume that all BSs can estimate the channels perfectly.

### *B. Cooperation among BSs*

We assume that all BSs can exploit cooperation and acquire global channel knowledge through reliable backhaul links [14], [15]. Depending on the manner of cooperation, we classify cooperation among BSs into two categories, namely 1) decentralized cooperation and 2) centralized cooperation as follows:

1) Decentralized cooperation: Each BS independently and simultaneously sends its estimated channels from training to other BSs. As in [17], the BSs use analog linear modulation to transmit the CSI by directly modulating the carrier with the channel matrix, $\mathbf{H}_i = [\mathbf{H}_{i,1}\mathbf{H}_{i,2}\cdots\mathbf{H}_{i,K}]$ ($N_r \times KN_t$ matrix) multiplied by a prearranged $KN_t \times T_c$ unitary spreading matrix, $\mathbf{\Phi}_i$ and receive $K$ CSI-bearing signals at the same time. It is desirable to make the $K$ unitary spreading matrix mutually orthogonal to estimate $K$ channel matrices each, i.e., $\mathbf{\Phi}_i\mathbf{\Phi}_i^{\mathsf{H}} = \mathbf{I}$ and $\mathbf{\Phi}_i\mathbf{\Phi}_j^{\mathsf{H}} = \mathbf{0}$ for $i \neq j$, and possible if $T_c - KN_t \geq KN_t(K-1)$. Here, we assume that the transmit power of the CSI-bearing signal is high enough to be reliably estimated at the other BSs. Therefore, the total time blocks required for channel information exchange is constrained on $T_c \geq K^2N_t$ for decentralized cooperation.
2) Centralized cooperation: Once an arbitrary BS is chosen as a coordinator, other BSs send their estimated channels to the coordinator. $(K-1)$ CSI bearing signals should be estimated independently at the coordinator by using the spreading matrix with the size $K(K-1)N_t$. The coordinator computes the transmit precoders and receive filters for all the networks and send the receive filters back to the other BSs. Similarly, the coordinator sends the signal bearing the receive filters, $\mathbf{U} = [\mathbf{u}_{1,1}\cdots\mathbf{u}_{1,d_1}\mathbf{u}_{2,1}\cdots\mathbf{u}_{2,d_2}\mathbf{u}_{2,1}\cdots\mathbf{u}_{K,d_K}]$ ($N_r \times \sum_{l=1,l\neq j_c}^{K} d_l$ matrix) by multiplying the spreading matrix $\mathbf{\Phi}$ ($\sum_{l=1,l\neq j_c}^{K} d_l \times \tau_c$ matrix). $j_c$ indicates the index of the BS which is selected as a coordinator. In order to separate $\sum_{l=1,l\neq j_c}^{K} d_l$ receive filters, the time blocks of the filter-bearing signal is constrained on $\tau_c \geq \sum_{l=1,l\neq j_c}^{K} d_l$. Therefore, the total time block for centralized cooperation should be conditioned on $T_c \geq K(K-1)N_t + \sum_{l=1,l\neq j_c}^{K} d_l$. The coordinator can quantize the precoding vector using its cor-

responding codebook and inform the index to the associated users at the feedback stage described in subsection II-D.

Once an arbitrary BS is chosen as a coordinator, other BSs send their estimated channels to the coordinator over a period $K(K-1)N_t$ [17] and then the coordinator can compute the transmit precoders and receive filters for all the networks. In this case, since the coordinator needs to send the receive filters for other BSs, $\sum_{l=1,l\neq j_c}^{K} d_l$ time slots are required additionally, where $j_c$ indicates the index of the BS which is selected as a coordinator.

We denote the total symbol durations required for cooperation as

$$T_c = \begin{cases} K^2 N_t, & \text{Decentralized cooperation,} \\ K(K-1)N_t + \sum_{l=1,l\neq j_c}^{K} d_l, & \text{Centralized cooperation.} \end{cases}$$

### C. IA

We assume as mentioned earlier that the BS can jointly design the precoding matrix and receive filter by using CSI through cooperation among BSs. As the specific designs for the precoding matrix and receive filter, we consider the conventional IA schemes, which can be found in [4], [5]. While IA can be realized with any receiver design, we consider a per-stream zero-forcing receiver such that the ISI and IUI can be perfectly canceled under the assumption of full CSIT, i.e., $\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,k}\mathbf{v}_{k,l}=0$ for $(j,m)\neq(k,l)$. Each BS will utilize the zero-forcing vector $\mathbf{u}_{j,m}$ for each stream even though the quantized precoder utilizes at the transmitter due to the limited feedback.

### D. Feedback

After the precoding matrix and receive filter are computed, we assume that each BS quantizes the precoding vector using its corresponding codebook and informs the index to the associated user via limited rate feedback link. For 1) analytical tractability and 2) scalability for any antenna configurations, we consider an random vector quantization (RVQ) codebook with isotropically distributed and i.i.d. vectors from the complex unit sphere [18], [20]. The codebook for the $m$th stream of the $j$th BS is denoted as $\mathcal{F}_{j,m}=\left\{\mathbf{f}_{j,m,1},\mathbf{f}_{j,m,2},\cdots,\mathbf{f}_{j,m,2^{B_{j,m}}}\right\}$, where $B_{j,m}$ is the number of feedback bits for the $m$th stream of the $j$th BS. Each BS uses an independently generated codebook per stream to ensure the number of spatial dimensions and the codebook is accessible to its associated user. By adopting the metric based on the chordal distance in [18] as $i_{j,m}=\arg\max_{1\le k\le 2^{B_{j,m}}}\left|\mathbf{f}_{j,m,k}^{\mathsf{H}}\mathbf{v}_{j,m}\right|$, the quantized transmit beamforming vector is obtained as $\hat{\mathbf{v}}_{j,m}=\mathbf{f}_{j,m,i_{j,m}}$. As a result, the quantization index is determined and fed back to the $j$th user over a period $KB_{j,m}/B_f$, where $B_f$ is the capacity of feedback links (bits per symbol duration). $B_f$ will be defined later in Section V for the purpose of illustration to analyze the effective DOF.

## IV. RATE LOSS AND SCALING LAW

In this section, we investigate the rate loss due to the limited rate of the feedback link and derive the scaling law of the number of feedback bits per stream in order to preserve a bounded rate loss.

### A. Rate Loss Analysis

The achievable rate for the $m$th stream of the $j$th BS with the infinite rate feedback is given by

$$R_{j,m}^{(\mathrm{P})}=\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right) \tag{3}$$

which indicates that all interference can be nullified under perfect CSIT.

On the other hand, in case of the limited feedback, since the quantized precoders do not fit perfectly on the aligned dimensions, both IUI and ISI cannot be perfectly eliminated in spite of using the zero-forcing vector $\mathbf{u}_{j,m}$, which incurs sum rate degradation by the resulting residual interference power. The achievable rate for the $m$th stream of the $j$th BS with the limited feedback can be written as

$$R_{j,m}^{(\mathrm{L})}=\log_2\left(1+\frac{\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2}{1+I_{j,m}}\right) \tag{4}$$

where

$$I_{j,m}=\sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,l}\right|^2+\sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\hat{\mathbf{v}}_{i,l}\right|^2 \tag{5}$$

is the residual interference power due to the limited feedback. From (3) and (4), the average rate loss for the $m$th stream of the $j$th BS is defined as $\triangle R_{j,m}=\mathbb{E}\left[R_{j,m}^{(\mathrm{P})}-R_{j,m}^{(\mathrm{L})}\right]$, where the expectation is carried out over the channel distribution and random codebooks. To characterize the performance loss of the proposed codebook-based IA due to the limited rate of the feedback link, we derive an upper bound on the rate loss as a function of the number of feedback bits, $B_{j,m}$.

Let us first derive the statistic of the residual interference power, $I_{j,m}$. We can rewrite (5) as

$$\begin{aligned} I_{j,m} &= \sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\hat{\mathbf{v}}_{j,l}\right|^2 \\ &+ \sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\hat{\mathbf{v}}_{i,l}\right|^2 \end{aligned} \tag{6}$$

where $\mathbf{e}_{j,i,m}^{\mathsf{H}}=\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}$ and $\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}=\mathbf{e}_{j,i,m}^{\mathsf{H}}/\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|$. We can decompose $\hat{\mathbf{v}}_{j,m}$ as

$$\hat{\mathbf{v}}_{j,m}=\left(\mathbf{v}_{j,m}^{\mathsf{H}}\hat{\mathbf{v}}_{j,m}\right)\mathbf{v}_{j,m}+\left(\mathbf{z}_{j,m}^{\mathsf{H}}\hat{\mathbf{v}}_{j,m}\right)\mathbf{z}_{j,m} \tag{7}$$

where $\mathbf{z}_{i,l}$ is isotropically distributed in the nullspace of $\mathbf{v}_{i,l}$. Substituting (7) into (6), we have

$$\begin{aligned} I_{j,m} &= \sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left(\sin^2\theta_{j,l}\right)\left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\mathbf{z}_{j,l}\right|^2 \\ &+ \sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left(\sin^2\theta_{i,l}\right)\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2 \end{aligned} \tag{8}$$

where $\sin^2\theta_{i,l} = \left|\mathbf{z}_{i,l}^{\mathsf{H}}\hat{\mathbf{v}}_{i,l}\right|^2$. Before deriving the upper bound of the expectation of $I_{j,m}$, we provide the following lemmas. Note that Lemmas 1, 2, and 3 give 1) the distribution of $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$, 2) the independence of $\sin^2\theta_{i,l}$, $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$, and $\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2$, and 3) the upper bound of $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]$, respectively.

**Lemma 1:** $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$ is Beta distributed with parameters $(1, N_t-2)$.

*Proof:* The proof can be found in [21, Lemma 2]. □

**Lemma 2:** The random variables, $\sin^2\theta_{i,l}$, $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$, and $\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2$, are statistically independent.

*Proof:* Since the amplitude and direction of any isotropically distributed vector are independent, $\left|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right|^2$ is independent of $\bar{\mathbf{e}}_{j,i,m}$. $\bar{\mathbf{e}}_{j,i,m}$ and $\mathbf{z}_{i,l}$ are independently and isotropically distributed within the same $(N_t-1)$-dimensional nullspace of $\mathbf{v}_{i,l}$. Similar to the Lemma 2 in [18], $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$ is a Beta$(1, N_t-2)$ random variable which is independent of quantization error $\sin^2\theta_{i,l}$. □

**Lemma 3:** The upper bound of $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]$ is given by $\bar{\lambda}_{\max}$ where is approximated as $\bar{\lambda}_{\max} \approx N_r N_t \left(\frac{N_t+N_r}{N_t N_r+1}\right)^{2/3}$.

*Proof:* From the definition, we have $\|\mathbf{e}_{j,i,m}\|^2 = \mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}\mathbf{u}_{j,m}$. Note that $\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}\mathbf{u}_{j,m} \leq \lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{H}_{j,i}\mathbf{H}_{j,i}^{\mathsf{H}}$. Therefore, we obtain the upper-bound of $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]$ as the mean of $\lambda_{\max}$ which is $\bar{\lambda}_{\max} \approx N_r N_t \left(\frac{N_t+N_r}{N_t N_r+1}\right)^{2/3}$ for $N_r N_t \leq 250$ in [19, eq. (28)]. □

Since the random variables, $\sin^2\theta_{i,l}$, $\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2$, and $\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2$, are all independent as shown in Lemma 2, the expected residual interference power can be written as

$$\begin{aligned}&\mathbb{E}\left[I_{j,m}\right]\\&= \sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\mathbb{E}\left[\left(\sin^2\theta_{j,l}\right)\right]\mathbb{E}\left[\left\|\mathbf{e}_{j,j,m}^{\mathsf{H}}\right\|^2\right]\mathbb{E}\left[\left|\bar{\mathbf{e}}_{j,j,m}^{\mathsf{H}}\mathbf{z}_{j,l}\right|^2\right]\\&+\sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\mathbb{E}\left[\left(\sin^2\theta_{i,l}\right)\right]\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right]\mathbb{E}\left[\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2\right]. \quad (9)\end{aligned}$$

From Lemmas 1 and 3, we have $\mathbb{E}\left[\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2\right] = 1/N_t - 1$ [23] and $\mathbb{E}\left[\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\right] \leq \bar{\lambda}_{\max}$, respectively. From [18, Lemma 1], the upper bound of $\mathbb{E}\left[\left(\sin^2\theta_{i,l}\right)\right]$ is given by $2^{-B_{i,l}/N_t-1}$. Therefore, we can rewrite (9) as

$$\begin{aligned}\mathbb{E}\left[I_{j,m}\right] &< \sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{j,l}}{N_t-1}}\\&+\sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{i,l}}{N_t-1}}. \quad (10)\end{aligned}$$

Consequently, from (10), we can quantify the upper bound on the rate loss due to the limited rate of the feedback link in the following theorem.

**Theorem 1:** The upper bound on the rate loss for the $m$th stream of the $j$th BS is given by

$$\triangle R_{j,m} < \log_2\left(\begin{array}{c}1+\sum_{\substack{l=1\\l\neq m}}^{d_j}\frac{P}{d_j}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{j,l}}{N_t-1}}\\+\sum_{\substack{i=1\\i\neq j}}^{K}\sum_{l=1}^{d_i}\frac{P}{d_i}\left(\frac{\bar{\lambda}_{\max}}{N_t-1}\right)2^{-\frac{B_{i,l}}{N_t-1}}\end{array}\right). \quad (11)$$

*Proof:* See Appendix. □

### *B. Scaling Law of the Number of Feedback Bits per Stream*

If the number of feedback bits per stream is fixed for all the SNR values, the residual interference power will dominate the desired signal power as SNR goes to infinity, which results in zero multiplexing gain [16]. Therefore, the upper bound on the rate loss derived in Theorem 1 can be maintained constant by increasing the number of feedback bits per stream as a function of SNR. For the simplicity of the analysis, we assume that the number of feedback bits per stream is set to $B$ for all users, i.e., $B_{j,m} = B, \forall (j,m)$ and leave the optimization for the feedback bits allocation as a future work. In the following theorem, we verify the sufficient scaling law of the feedback bits per stream to maintain the constant upper bound on the rate loss.

**Theorem 2:** The sufficient scaling law of the feedback bits per stream to maintain the rate loss no larger than $\log_2 b$ is given by

$$B \geq (N_t-1)\log_2\left(\frac{KP\bar{\lambda}_{\max}}{N_t-1}\right) - (N_t-1)\log_2(b-1). \quad (12)$$

*Proof:* Under the assumption of the equal number of feedback bits over streams, the upper bound in (11) can be simplified as

$$\begin{aligned}\triangle R_{j,m} &< \log_2\left(\begin{array}{c}1+\frac{d_j-1}{d_j}\frac{P\bar{\lambda}_{\max}}{N_t-1}2^{-\frac{B}{N_t-1}}\\+(K-1)\frac{P\bar{\lambda}_{\max}}{N_t-1}2^{-\frac{B}{N_t-1}}\end{array}\right)\\&< \log_2\left(1+\frac{KP\bar{\lambda}_{\max}}{N_t-1}2^{-\frac{B}{N_t-1}}\right). \quad (13)\end{aligned}$$

To exploit the sufficient number of feedback bits for a rate loss of no larger than $\log_2 b$, we set (13) to the maximum allowable gap of $\log_2 b$ as $\log_2\left(1+\frac{KP\bar{\lambda}_{\max}}{N_t-1}2^{-\frac{B}{N_t-1}}\right) \leq \log_2 b$ and then we can solve the number of feedback bits per stream as a function of $b$ and SNR, which is given by (12). □

Furthermore, by setting the maximum allowable rate gap per data stream as $b = 2$ in (12), we obtain the scaling law of the feedback bits per stream to preserve the rate loss less than 1 (bps/Hz) as

$$B \quad \geq \quad (N_t-1)\log_2\left(\frac{KP\bar{\lambda}_{\max}}{N_t-1}\right) \quad (14)$$

which will be later used for our simulations in Section VI.

## V. ANALYSIS ON EFFECTIVE DOF

Although our analysis in Section IV shows the performance loss due to the quantization error, it neglects the impact of overhead resulting from training, cooperation, and feedback. In this section, we consider the case that training, cooperation, feedback, and data transmission are all orthogonal in time during the coherence time, $T$, which is similarly considered in [13] and [24]. Based on the analysis in Section IV, we analyze the effective DOF discussed in [8] and [13] to characterize the overhead of the proposed codebook-based IA under practical scenarios. By taking training, cooperation, and feedback into consideration, we can compute the expected effective sum rate as

$$\bar{R}_{\text{sum}}^{(\text{L})} = \left(1 - \frac{KN_t}{T} - \frac{T_c}{T} - \frac{KB}{B_f T}\right) \sum_{j,m} \mathbb{E}\left[R_{j,m}^{(\text{L})}\right]. \quad (15)$$

Similarly, we can define the effective DOF as

$$\eta_e = \left(1 - \frac{KN_t}{T} - \frac{T_c}{T} - \frac{KB}{B_f T}\right)\eta \quad (16)$$

where $\eta = \lim_{P\to\infty} \sum_{j,m} \mathbb{E}\left[R_{j,m}^{(\text{L})}\right]/\log_2 P$. Denoting the indicator of the scaling law as $\alpha$ which shows the quantity of feedback bits according to SNR, in the following theorem, we show the lower and upper bounds of $\eta$ when the scaling law of the feedback bits per stream is $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$ for $0 \le \alpha \le 1$.

**Theorem 3:** When the scaling law of the feedback bits per stream is $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$ for $0 \le \alpha \le 1$, we have $\eta = \alpha d_{\text{tot}}$.

*Proof:* Firstly, we show the upper bound of $\eta$. By discarding the terms of the residual interference power in (5) except $\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,l}\right|^2$ [18], we have the upper bound of $\eta$ as

$$\mathbb{E}\left[R_{j,m}^{(\text{L})}\right] \le 1 + \frac{B + \log_2 e}{N_t - 1} + \log_2(N_t - 2) + \log_2 e \quad (17)$$

which can be easily derived by using Lemma 3 in Section IV and Theorem 2 in [18]. Substituting $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$ into (17), the upper bound of $\eta$ is given by $\mathbb{E}\left[R_{j,m}^{(\text{L})}\right] \le \alpha\log_2 P + O(1)$, which directly results in $\eta \le \alpha d_{\text{tot}}$.

Secondly, we show the lower bound of $\eta$. For high SNR, by ignoring the effect of the noise, we have

$$\begin{aligned}\eta \quad &\ge \quad \lim_{P\to\infty} \frac{\mathbb{E}\left[\sum_{j,m}\log_2\left(\frac{\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2}{I_{j,m}}\right)\right]}{\log_2 P} \\ &= \quad d_{\text{tot}} - \lim_{P\to\infty}\frac{\sum_{j,m}\mathbb{E}\left[\log_2 I_{j,m}\right]}{\log_2 P}. \quad (18)\end{aligned}$$

The upper bound of $\mathbb{E}\left[\log_2 I_{j,m}\right]$ is given by

$$\begin{aligned}\mathbb{E}\left[\log_2 I_{j,m}\right] \quad &\le \quad \log_2 P + \mathbb{E}\left[\log_2\left(\sin^2\theta_{i,l}\right)\right] \\ &+ \quad \mathbb{E}\left[\log_2\left(K\left\|\mathbf{e}_{j,i,m}^{\mathsf{H}}\right\|^2\left|\bar{\mathbf{e}}_{j,i,m}^{\mathsf{H}}\mathbf{z}_{i,l}\right|^2\right)\right] \\ &\le \quad (1-\alpha)\log_2 P + O(1) \quad (19)\end{aligned}$$

where the last inequality can be derived from $\mathbb{E}\left[\log_2\left(\sin^2\theta_{i,l}\right)\right] = \frac{-\log_2 e}{N_t - 1}\sum_{k=1}^{2^B}\frac{1}{k} \le \frac{-B}{N_t-1}$ in [18, Appendix III] and recalling $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$. Substituting (19) into (18), we have $\eta \ge \alpha d_{\text{tot}}$. As a result, we obtain $\eta = \alpha d_{\text{tot}}$ given for $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$ for $0 \le \alpha \le 1$. □

$B_f$ should be at least on the same order of $B$ in order to feedback the quantization index within a coherence block, $T$ [8] and, for the purpose of illustration, we set $B_f = \beta\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$, where $\beta$ is a constant. Plugging $B = \alpha(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t - 1\right)$ and $\eta = \alpha d_{\text{tot}}$ into (16), then we have

$$\eta_e = \left(1 - \frac{KN_t}{T} - \frac{T_c}{T} - \frac{\alpha(N_t-1)K}{\beta T}\right)\alpha d_{\text{tot}} \quad (20)$$

which is a quadratic function with respect to $\alpha$. By differentiating (20) with respect to $\alpha$, the maximization of $\eta_e$ is achieved when $\alpha_{\max} = \frac{\beta T}{2(N_t-1)K}\left(1 - \frac{KN_t}{T} - \frac{T_c}{T}\right)$. If $\alpha_{\max}$ is out of domain for $0 \le \alpha \le 1$, i.e., $\alpha_{\max} > 1$, $\eta_e$ is maximized when $\alpha = 1$. As a result, the effective DOF, $\eta_e$, is maximized as

$$\eta_{e,\max} = \begin{cases} \left(1 - \frac{KN_t}{T} - \frac{T_c}{T}\right)^2 \frac{\beta T}{4(N_t-1)K}\, d_{\text{tot}} \\ \quad \text{if } \xi_1 + \frac{d_{\text{tot}}}{4\xi_2} < T \le \xi_1 + \frac{d_{\text{tot}}}{2\xi_2} \\ \left(1 - \frac{KN_t}{T} - \frac{T_c}{T} - \frac{(N_t-1)K}{\beta T}\right) d_{\text{tot}} \\ \quad \text{if } T > \xi_1 + \frac{d_{\text{tot}}}{2\xi_2} \end{cases} \quad (21)$$

where $\xi_1 = KN_t + T_c$ and $\xi_2 = \beta d_{\text{tot}}/4(N_t-1)K$. Note that the lower bound of $T$ in the first condition comes from the assumption that $T$ is longer than the symbol durations corresponding to training, cooperation, and feedback, i.e., $T > KN_t + T_c + 1/\beta(N_t-1)K$.

## VI. NUMERICAL RESULTS

In this section, we present simulation results to evaluate the sum rate performance and scaling law of the proposed codebook-based IA. Subsequently, we illustrate simulation results to validate our analysis on the effective DOF. Throughout this section, we denote the $K$-user uplink MIMO IC with $N_t$ transmit antennas, $N_r$ receive antennas, and $d_j$ data streams for the $j$th user as $(N_t, N_r, K, [d_1, d_2, \cdots, d_K])$. For all the simulations, we set the maximum allowable rate gap per data stream as $b = 2$.

In Fig. 1, we illustrate the average sum rate as a function of SNR for the $(4, 5, 3, [2, 2, 2])$ system. We confirm that the proposed codebook-based IA scheme can maintain the rate loss within $d_{\text{tot}}\log_2 b = 6 \times \log_2(2) = 6$ (bps/Hz) compared with IA with perfect feedback by increasing the feedback quality according to the relationship in (14) with SNR. When the feedback quality is fixed regardless of SNR, we observe that the multiplexing gain goes to zero in high SNR region since the residual interference terms will dominate the rate in (4) as $P$ increases. In Fig. 2, we show the average sum rate as a function

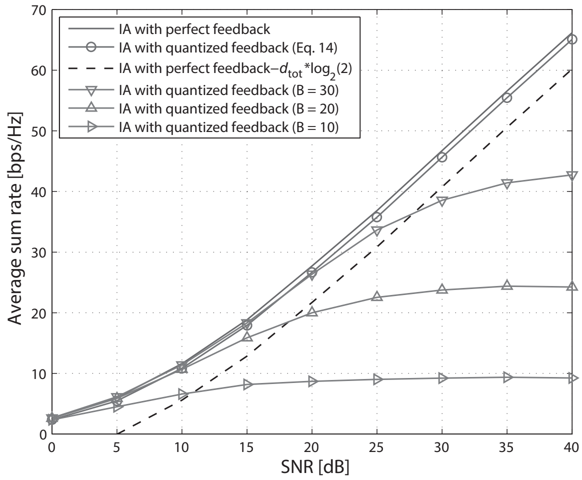


Fig. 1. Average sum rate of uplink MIMO IC for the $(4,5,3,[2,2,2])$ system with perfect feedback, scaling feedback quality, and fixed feedback quality.

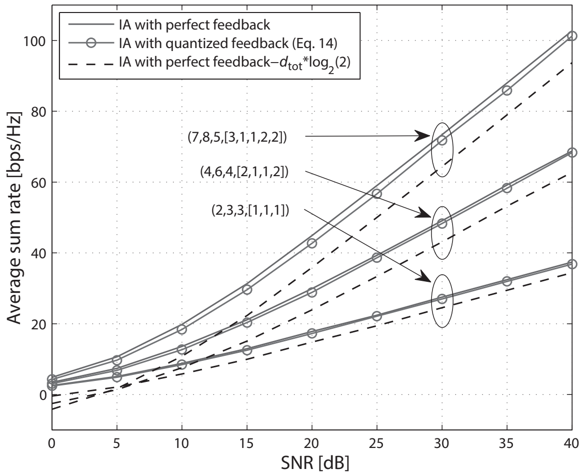


Fig. 2. Average sum rate of uplink MIMO IC with various system configurations.

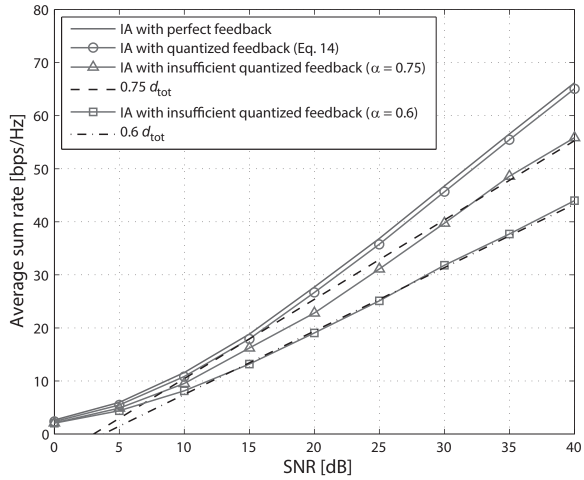


Fig. 3. Average sum rate of uplink MIMO IC for the $(4,5,3,[2,2,2])$ system with perfect feedback, scaling feedback quality, and insufficient feedback quality.

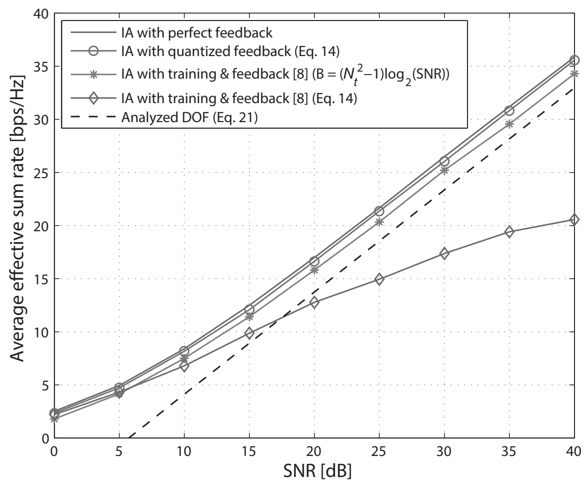


Fig. 4. Average effective sum rate of uplink MIMO IC for the $(2,2,3,[1,1,1])$ system where $T=600$.

of SNR by varying the system configurations. We can observe that the multiplexing gain of the proposed codebook-based IA scheme can be preserved by scaling $B$ according to the derived scaling law with SNR for various system configurations. Moreover, even when the data streams are allocated to transmitters asymmetrically, it is shown that the rate loss is still maintained within $d_{\rm tot}\log_2 b$ (bps/Hz). In Fig. 3, we plot the average sum rate as a function of SNR to demonstrate the validity of Theorem 3. We clearly see that an $\alpha$-fraction of the multiplexing gain can be achieved when we impose the scaling law according to $B=\alpha\,(N_t-1)\log_2\left(KP\bar{\lambda}_{\max}/N_t-1\right)$ for $0\le\alpha\le 1$.

Moreover, we verify the advantage of the proposed IA scheme over the conventional IA scheme that adopts training and channel coefficients feedback [8] by assessing the effective DOF. We consider an example for the $(2,2,3,[1,1,1])$ system with $T=600$[1] and $\beta=1$. From Fig. 4, the effective DOF of the proposed scheme from simulation is identical to that from our analysis in (21). We see that the proposed scheme can achieve higher effective DOF than the conventional scheme in [8] given the equal number of feedback bits. Moreover, to get the same effective DOF of the proposed scheme, it can be seen that much more feedback bits, $(N_t^2-1)\log_2(\mathrm{SNR})$, are required for the conventional scheme [8].

[1]We assume that the channels remain constant over a coherence block of $T=600$ channel uses which is related to two physical parameters, the coherence time $T_c$ and the coherence bandwidth (subcarrier bandwidth) $W_c$ by $T=W_cT_c$ [24]. In this paper, we assume a typical LTE scenario operating at $f_0=2.1$ GHz and the coherence bandwidth $W_c=15$ kHz with the mobility $\Delta v=5$ km/h [25]. Taking as the coherence time $T_c\simeq\sqrt{\frac{9}{16\pi}}\frac{1}{f_d}=\sqrt{\frac{9}{16\pi}}\frac{\Delta v}{c}f_0$, we approximately obtain $T=600$ channel uses.

## VII. CONCLUSION

In this paper, we considered the feedback requirements of a codebook-based IA in the constant MIMO IC especially for the uplink case. By assuming cooperation among BSs, BS can compute the transmit precoder and inform its quantized index instead of the quantized channel coefficients to the associated user. We derived an upper bound of the rate loss and the scaling law of the number of feedback bits in order to preserve the multiplexing gain of IA with perfect CSIT. It was shown that if a partial feedback load is used according to the derived scaling law, a fraction of the multiplexing gain can be only achieved. We also considered the effective DOF to see the impact of overhead arising from training, cooperation, and feedback. By comparing the proposed scheme with the conventional scheme, both analytically and numerically, the proposed scheme has been shown to be more effective one in terms of the effective DOF.

## Appendix

### A. Proof of Theorem 1

Prior to the proof of Theorem 1, we need to address the following lemma.

**Lemma 4:** The equality holds between the following expectation terms given by

$$\begin{aligned}&\mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right]\\ =\ &\mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2\right)\right]. \qquad (22)\end{aligned}$$

*Proof:* The proof can be found in [21, Lemma 1]. □

The rate loss can be written as

$$\begin{aligned}\triangle R_{j,m} &= \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right]\\ &- \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2+I_{j,m}\right)\right]\\ &+ \mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right]\\ &\overset{(a)}{\leq} \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\mathbf{v}_{j,m}\right|^2\right)\right]\\ &- \mathbb{E}\left[\log_2\left(1+\frac{P}{d_j}\left|\mathbf{u}_{j,m}^{\mathsf{H}}\mathbf{H}_{j,j}\hat{\mathbf{v}}_{j,m}\right|^2\right)\right]\\ &+ \mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right]\\ &\overset{(b)}{\leq} \mathbb{E}\left[\log_2\left(1+I_{j,m}\right)\right]\\ &\overset{(c)}{\leq} \log_2\left(1+\mathbb{E}\left[I_{j,m}\right]\right) \qquad (23)\end{aligned}$$

where (a) comes from the fact that $I_{j,m} \geq 0$ and $\log(\cdot)$ is a monotonically increasing function [18], (b) follows from Lemma 4, and (c) follows from the Jensen's inequality. Substituting (11) into (23), we derive the upper bound of the rate loss for the $m$th stream of the $j$th BS as given in (12).

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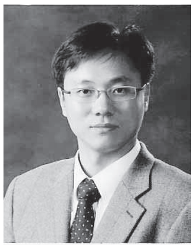

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