Underlay Cooperative Cognitive Networks with Imperfect Nakagami$-m$ Fading Channel Information and Strict Transmit Power Constraint: Interference Statistics and Outage Probability Analysis

Khuong Ho-Van, Paschalis C. Sofotasios, and Steven Freear

Abstract: **This work investigates two important performance metrics of underlay cooperative cognitive radio (CR) networks: Interference cumulative distribution function of licensed users and outage probability of unlicensed users. These metrics are thoroughly analyzed in realistic operating conditions such as imperfect fading channel information and strict transmit power constraint, which satisfies interference power constraint and maximum transmit power constraint, over Nakagami$-m$ fading channels. Novel closed-form expressions are derived and subsequently validated extensively through comparisons with respective results from computer simulations. The proposed expressions are rather long but straightforward to handle both analytically and numerically since they are expressed in terms of well known built-in functions. In addition, the offered results provide the following technical insights: $i)$ Channel information imperfection degrades considerably the performance of both unlicensed network in terms of OP and licensed network in terms of interference levels; $ii)$ underlay cooperative CR networks experience the outage saturation phenomenon; $iii)$ the probability that the interference power constraint is satisfied is relatively low and depends significantly on the corresponding fading severity conditions as well as the channel estimation quality; $iv)$ there exists a critical performance trade-off between unlicensed and licensed networks.**

Index Terms: **Cooperative relaying, imperfect channel information, interference statistics, multipath fading, underlay CR.**

I. INTRODUCTION

The rapid growth of emerging wireless technologies and services has led to a critical dilemma regarding efficient exploitation of available spectrum resources. On the one hand, it has been shown that licensed users under-utilize significantly their traditionally allocated spectrum [1]; on the other hand, most current wireless applications compete for rather limited spectrum resources. Cognitive radio (CR) technology has the potential to improve substantially spectrum utilization efficiency by allowing unlicensed users to access opportunistically frequency bands that are allotted to licensed users [2]. A notable technology of CR systems is the *underlay mode* where users intelligently adjust their transmit power to ensure that interference at licensed users remains below an acceptable controllable level [3].

In general, the transmit power of unlicensed users is constrained by two critical power measures: $i)$ The maximum interference power that licensed users can tolerate; $ii)$ the maximum transmit power that unlicensed users are capable to operate [3]. Constraints on the transmit power of unlicensed users limit their transmission range. However, this shortage can be effectively remedied with the aid of cooperative relaying techniques which explore short-distance point-to-point communication for lower path-loss and combine identical signals from several independent channels for space diversity and thus, extending the corresponding coverage [4]. Cooperative relaying systems consist of relay nodes operating in either the amplify-and-forward (AF) or the decode-and-forward (DF) protocols.

It is also known that outage probability (OP) constitutes an important performance metric in the study of the information-theoretic performance limit [5]. Likewise, channel information (CI) plays a crucial role in aspects of system design such as spectrum allocation optimization. Nevertheless, due to the limitation of channel estimation algorithms the availability of perfect CI is practically impossible. Furthermore, multipath fading can be adequately characterized by Nakagami$-m$ distribution, which is a flexible model that also includes as a special case, for $m = 1$, Rayleigh distribution [6]. As a result, analytic evaluation of OP under imperfect Nakagami$-m$ fading channel information is realistic, general, and essential. In this context, the OP of underlay cooperative/two-hop CR networks was analyzed in [3], [7]$-$[13]. Specifically, the work in [3] studies the outage performance of underlay *DF two-hop* CR networks over *Nakagami-m* fading channels while [7]$-$[10] consider underlay *DF cooperative* CR networks over *Rayleigh* fading channels. The common ground between [3], [7]$-$[10] is the assumption of *perfect* CI and two power constraints. On the contrary, only few works investigate the effect of *channel estimation error* on the outage performance of underlay relay CR networks. In more details, [11] assumes imperfect CI between the licensed network and the unlicensed network but perfect CI among the unlicensed network while only the interference power constraint is considered. The authors in [12] study the CI imperfection on all channels but also only with the interference power constraint. Moreover, [13] presents the asymptotic analysis of cooperative CR networks considering both interference power con-

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K. Ho-Van is with the Department of Telecommunications Engineering, HoChiMinh City University of Technology, Vietnam, email: khuong.hovan@yahoo.ca.

P. C. Sofotasios and S. Freear are with the School of Electronic and Electrical Engineering, University of Leeds, England, UK, email: {p.sofotasios, s.freear}@leeds.ac.uk.

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straint and maximum transmit power constraint but it does not investigate CI imperfection on all channels simultaneously. In other words, the imperfect CI between the licensed network and the unlicensed network but the perfect CI among the unlicensed network; or the imperfect CI among the unlicensed network but the perfect CI between the licensed network and the unlicensed network. Furthermore, [11]−[13] consider only Rayleigh fading channels.

To the best of our knowledge, the analytic performance evaluation of OP in underlay DF cooperative CR networks under realistic conditions such as *imperfect Nakagami*$-m$ *fading channel information* and two power constraints, has not been addressed in the open literature. Motivated by this, the present work investigates this topic by deriving an analytic expression for the OP in such networks. The proposed analysis also includes [3], [7]−[13] as special cases while the easily computed proposed expression facilitates the performance evaluation without necessarily requiring exhaustive simulations and possibly system design optimization. It is also shown that imperfect CI degrades the system performance considerably while underlay cooperative CR networks experience the outage saturation phenomenon.

Due to channel estimation errors, the interference at licensed users can not be always maintained below an acceptable level. Therefore, thorough investigation of its statistics is undoubtedly necessary. To this end, the interference cumulative distribution function (CDF) of licensed users was derived for Rayleigh fading channels in [11], [12], [14]. Capitalizing on this, the present work investigates the interference CDF for Nakagami$-m$ fading channels. Various results illustrate that the value of the interference CDF at the maximum interference power is relatively low and significantly dependant upon the severity of fading and the channel estimator quality. Furthermore, the interference level at licensed users is inversely proportional to the OP of unlicensed users which results to the performance trade-off between licensed and unlicensed networks.

The rest of the paper is organized as follows: Section II revisits the considered system model and addresses the interference statistics of licensed users. The novel OP expression and its relation to previous works are provided in Section III. The validity and behaviour of the offered results are analyzed in Section IV and closing remarks are given in Section V.

II. SYSTEM MODEL AND INTERFERENCE STATISTICS OF LICENSED USERS

We consider the underlay cooperative CR network in Fig. 1 as in [3], [8]. The unlicensed network performs two-stage cooperative relaying where the source U_S communicates with the destination U_D with the aid of the relay U_R. This communication causes interference to licensed user(s), namely L_x. During stage 1, U_S transmits its information while in stage 2 it remains idle and U_R, operating in the DF protocol, is activated only after successful decoding of U_S's information. Then, U_D restores U_S's information either with maximum ratio combining of both signals from U_S in stage 1 and U_R in stage 2 if U_R is active, or based on the signal from U_S in the stage 1 if U_R is idle.

We assume independent frequency-flat Nakagami$-m$ fading channels with the parameters $\{m, \lambda\}$, which can assist in ob-

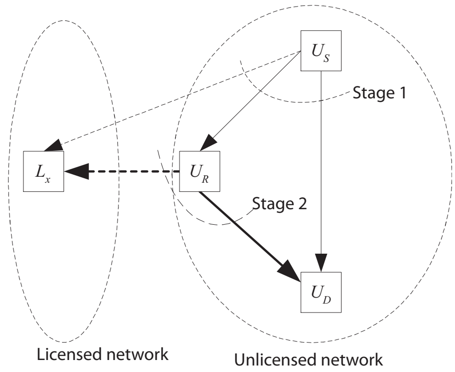

Fig. 1. System model.

taining useful insights on the behaviour and performance of underlay cooperative CR networks in different fading conditions. The probability density function (pdf) of the magnitude of the channel coefficient, $|b_{u,q}|$, between user u and user q can be expressed as, $f_{|b_{u,q}|}(x) = 2\Lambda^m x^{2m-1} e^{-\Lambda x^2}/\Gamma(m)$, where $u \in \{U_S, U_R\}$, $q \in \{U_R, U_D, L_x\}$, $x > 0$, $\Lambda = m/\lambda$ with $\lambda = \mathcal{E}\left\{|\mathrm{b_{u,q}}|^2\right\}$. Furthermore, $\Gamma(.)$ is the Gamma function [15, eq. (8.310.1)] and $\mathcal{E}\{.\}$ denotes statistical expectation.

The received signal, $a_{u,q}$, at the user q is modeled as $a_{u,q} = b_{u,q}c_u + d_{u,q}$, where[1] $d_{u,q} \sim \mathcal{CN}(0, \mathrm{N_0})$ is the noise at user q whereas c_u is the transmitted information symbol. The symbol energy of c_u is denoted as $\alpha_u = \mathcal{E}\{|\mathrm{c_u}|^2\}$ and is strictly set as $\alpha_u = \min\left(I_T/\left|b_{u,L_x}\right|^2, P_m\right)$ in order to satisfy both interference power constraint, $\alpha_u \leq I_T/\left|b_{u,L_x}\right|^2$, and maximum transmit power constraint, $\alpha_u \leq P_m$, [3]. Here, I_T is the maximum interference power that licensed users can tolerate while P_m is the maximum transmit power of unlicensed users.

Based on the linear minimum mean-square error channel estimation, the corresponding estimation error model can be expressed as $b_{u,q} = \widehat{b}_{u,q} + \xi_{u,q}$, where $\xi_{u,q} \sim \mathcal{CN}(0, \tau)$ is the channel estimation error, $\widehat{b}_{u,q}$ is the estimate of the $u-q$ channel which follows the Nakagami$-m$ distribution with parameters $\{m, \zeta\}$ and $\xi_{u,q}$ is statistically independent of $\widehat{b}_{u,q}$, [16]. Furthermore, the corresponding variances of $\xi_{u,q}$, and $\widehat{b}_{u,q}$ are expressed as $\tau = \lambda/(1+\rho\theta\lambda)$ and $\zeta = \rho\theta\lambda^2/(1+\rho\theta\lambda)$, respectively, with $\rho > 0$ accounting for the quality of the estimator and $\theta = I_T/(N_0\lambda)$ denoting the transmit signal-to-noise ratio[2] (SNR). To this effect, by letting $h_{u,q} = |\hat{b}_{u,q}|^2$ it follows that the corresponding pdf is given by [6, eq. (2.21)], namely,

[1] $d \sim \mathcal{CN}(\mathrm{q}, \mathrm{p})$ denotes a circular symmetric complex Gaussian random variable with mean q and variance p.

[2] According to the channel estimation error model in [16], the transmit SNR is the ratio of the transmit power to noise variance, α_u/N_0. When allocated to pilot symbols, the transmit power must be assumed to be constant, e.g., α_p. Under the interference power constraint, α_p must guarantee the interference power at the licensed user below I_T, i.e. $\alpha_p|b_{u,L_x}|^2 \leq I_T$. As such, on average $\alpha_p\lambda \leq I_T$, by selecting $\alpha_p = I_T/\lambda$, and so, $\theta = \alpha_p/N_0 = I_T/(N_0\lambda)$. In general, the transmit power of pilot symbols should be $\alpha_p = \min(I_T/\lambda, P_m)$. However, either $\alpha_p = I_T/\lambda$ or $\alpha_p = \min(I_T/\lambda, P_m)$ eventually reflects the channel estimation error variance τ. Thus, by changing τ indirectly through changing ρ, we still capture the effect of channel estimation error on the system performance. Since the channel estimation is outside the scope of our paper, the setting of $\theta = I_T/(N_0\lambda)$ is just an example to illustrate this effect.

$f_{h_{u,q}}(x) = \beta^m x^{m-1} e^{-\beta x}/\Gamma(m)$, where $x > 0$ and $\beta = m/\zeta$.

For imperfect CI, the user u modifies its transmit power according to [14, eq. (2)], $\alpha'_u = \min\left(I_T/|\hat{b}_{u,L_x}|^2, P_m\right)$, which yields the interference power at the licensed user L_x as $Z = \alpha'_u |b_{u,L_x}|^2 = \min(I_T/|\hat{b}_{u,L_x}|^2, P_m)|b_{u,L_x}|^2$. Since $|\hat{b}_{u,L_x}|^2 \neq |b_{u,L_x}|^2$, the corresponding interference power can not be always guaranteed below I_T, which can be proven detrimental to the operation of licensed users. Therefore, the question that naturally arises is: *What is the percentage of the interference power constraint met under imperfect channel information?* To this end [11], [12], [14] provided the interference CDF, which is defined as the percentage that the interference at licensed users is below a certain level, for Rayleigh fading channels. Our present work generalizes this concept by considering Nakagami$-m$ fading, which is more realistic and includes [11], [12], [14] as a special case for $m = 1$.

Subsequently, we set $W_1 = \sqrt{X} = |b_{u,L_x}|$ and $W_2 = \sqrt{Y} = |\hat{b}_{u,L_x}|$. However, before deriving the CDF of Z, it is essential to determine the joint pdf of X and Y.

Lemma 1: Given the Nakagami$-m$ distributions of $W_1 = |b_{u,L_x}|$ and $W_2 = |\hat{b}_{u,L_x}|$ with parameters (m, λ) and (m, ζ), respectively, the joint pdf of X and Y is expressed as,

$$f_{X,Y}(x,y) = \frac{(xy)^{m/2} e^{-\frac{\eta_2 x + \eta_1 y}{\eta_1\eta_2(1-\varphi)}} I_{m-1}\left(\frac{2\sqrt{\varphi x y}}{\sqrt{\eta_1\eta_2}(1-\varphi)}\right)}{\sqrt{xy}\,\Gamma(m)(1-\varphi)\varphi^{(m-1)/2}(\eta_1\eta_2)^{(m+1)/2}} \quad (1)$$

where $I_m(.)$ is the modified Bessel function of the first kind and mth order; $\eta_1 = \mathcal{E}\left\{W_1^2\right\}/m = \lambda/m$; $\eta_2 = \mathcal{E}\left\{W_2^2\right\}/m = \zeta/m$; $\varphi = \zeta\Gamma(m+2)/(m\Gamma(m)\lambda) + m\tau/\lambda - m$.

Proof: The joint pdf of W_1 and W_2 is [17, eq. (1)],

$$f_{W_1,W_2}(x,y) = \frac{4(xy)^m e^{-\frac{\eta_2 x^2 + \eta_1 y^2}{\eta_1\eta_2(1-\varphi)}} I_{m-1}\left(\frac{2\sqrt{\varphi} x y}{\sqrt{\eta_1\eta_2}(1-\varphi)}\right)}{\Gamma(m)\eta_1\eta_2(1-\varphi)(\eta_1\eta_2\varphi)^{(m-1)/2}} \quad (2)$$

where φ denotes the correlation coefficient which emerges by its definition, i.e., $\varphi = \mathrm{cov}\left(W_1^2, W_2^2\right)/\sqrt{\mathrm{var}\left\{W_1^2\right\}\mathrm{var}\left\{W_2^2\right\}}$, [17]. By utilizing [18, eq. (2-1-149)] it immediately follows that, $\mathrm{var}\left\{W_1^2\right\} = \lambda^2/m$ and $\mathrm{var}\left\{W_2^2\right\} = \zeta^2/m$. To this effect, the covariance of W_1^2 and W_2^2 is straightforwardly computed with the aid of the standard identity, $\mathrm{cov}(W_1^2, W_2^2) = \mathcal{E}\{W_1^2 W_2^2\} - \mathcal{E}\{W_1^2\}\mathcal{E}\{W_2^2\}$, which after long but basic algebraic manipulations yields, $\mathrm{cov}(W_1^2, W_2^2) = \Gamma(m+2)/\Gamma(m)\zeta^2/m^2 + \tau\zeta - \lambda\zeta$. By substituting $\mathrm{var}\left\{W_1^2\right\}$, $\mathrm{var}\left\{W_2^2\right\}$ and $\mathrm{cov}(W_1^2, W_2^2)$ into the definition of φ yields $\varphi = \zeta\Gamma(m+2)/(m\Gamma(m)\lambda) + m\tau/\lambda - m$. Subsequently, by setting $W_1 = \sqrt{X}$ and $W_2 = \sqrt{Y}$, the joint pdf of X and Y is readily deduced from that of W_1 and W_2 in [19, eq. (6-115)], namely, $f_{X,Y}(x,y) = |J(x,y)|\, f_{W_1,W_2}\left(\sqrt{x}, \sqrt{y}\right)$, where the Jacobian $J(x,y)$ is $1/\left(4\sqrt{xy}\right)$, [19, eq. (6-114)]. Thus, substituting (2) yields (1), which completes the proof. □

The CDF of Z is $\Pr\{Z < z\} = 1 - \Pr\ \{\min(I_T/Y, P_m)\ X > z\} = 1 - \Pr\{X > z_1, X/Y > z_2\} = 1 - \int_{z_1}^{\infty}\int_0^{\frac{x}{z_2}} f_{X,Y}(x,y)\,dy\,dx$ where $\Pr\{X\}$ denotes the probability of the event X while $z_1 = z/P_m$ and $z_2 = z/I_T$. To this effect and with the aid of (1) and setting $t = \sqrt{y}$, it follows that,

$$\Pr\{Z < z\} = 1 - A_1 \int_{z_1}^{\infty}\int_0^{\sqrt{x/z_2}} \frac{t^m I_{m-1}\left(\frac{2\sqrt{\varphi x} t}{\sqrt{\eta_1\eta_2}(1-\varphi)}\right)}{x^{\frac{1-m}{2}} e^{\frac{x}{\eta_1(1-\varphi)}} e^{\frac{t^2}{\eta_2(1-\varphi)}}} dt\,dx \quad (3)$$

where $A_1 = 2(\eta_1\eta_2\varphi)^{(1-m)/2}/[\Gamma(m)(1-\varphi)\eta_1\eta_2]$. Notably, the inner integral can be expressed in terms of the Marcum $Q-$function, $Q_m(a,b)$ in [6, eq. (4.60)]. Therefore, by setting $y = t\sqrt{2/[(1-\varphi)\eta_2]}$ one obtains,

$$\Pr\{Z < z\} = 1 - \underbrace{\int_{z_1}^{\infty} \frac{x^{m-1}}{\tau_1 e^{\frac{x}{\eta_1}}} dx}_{\mathcal{T}_1} + \underbrace{\int_{z_1}^{\infty} \frac{Q_m(\sqrt{\tau_2 x}, \sqrt{\tau_3 x})}{x^{1-m}\tau_1 e^{\frac{x}{\eta_1}}} dx}_{\mathcal{T}_2} \quad (4)$$

where $\tau_1 = \eta_1^m\Gamma(m)$, $\tau_2 = 2\varphi/[\eta_1(1-\varphi)]$, $\tau_3 = 2/[\eta_2 z_2(1-\varphi)]$. Evidently, deriving an exact closed-form expression for $\Pr\{Z < z\}$ is subject to analytical solution of the integrals $\mathcal{T}_1$ and $\mathcal{T}_2$. Firstly, it is observed that $\mathcal{T}_1$ has the same algebraic representation as the upper incomplete gamma function, $\Gamma(a; x)$, [15, eq. (8.350.2)]. Thus, by performing the necessary change of variables one obtains $\mathcal{T}_1 = \Gamma(m; z_1/\eta_1)/\Gamma(m)$.

In what follows, we derive an analytic expression for $\mathcal{T}_2$.

Theorem 1: The following representations is valid for $\mathcal{T}_2$,

$$\mathcal{T}_2 \simeq \sum_{l=0}^{g}\sum_{i=0}^{m+l-1} \frac{\Gamma(g+l)\, g^{1-2l} a^{2l} b^{2i} \Gamma(m+l+i; A_2 z_1)}{\Gamma(m) c^{-m} l! i! 2^{l+i}(g-l)! A_2^{m+l+i}} \quad (5)$$

where $A_2 = c + \left(a^2 + b^2\right)/2$, $a = \sqrt{2\varphi/[(1-\varphi)\eta_1]}$, $b = \sqrt{2/[(1-\varphi)\eta_2 z_2]}$ and $c = 1/\eta_1$.

Proof: The $\mathcal{T}_2$ term can be expressed w.r.t a, b, c yielding, $\mathcal{T}_2 = \frac{c^m}{\Gamma(m)}\int_{z_1}^{\infty} x^{m-1} e^{-cx} Q_m\left(a\sqrt{x}, b\sqrt{x}\right) dx$. A simple representation for $Q_m(a,b)$ was reported in [20, eq. (6)]. Performing the necessary change of variables and substituting in (4) yields,

$$\mathcal{T}_2 \simeq \sum_{l=0}^{g}\int_{z_1}^{\infty} \frac{c^m\Gamma(g+l) a^{2l} x^{m+l-1}\Gamma\left(m+l; \frac{b^2 x}{2}\right) dx}{l! 2^l \Gamma(m)(g-l)! g^{2l-1}\Gamma(m+l) e^{\left(c+\frac{a^2}{2}\right)x}}. \quad (6)$$

By subsequently applying [15, eq. (8.352.2)] and performing the necessary change of variabless, it follows that,

$$\mathcal{T}_2 \simeq \sum_{l=0}^{g}\sum_{i=0}^{m+l-1} \frac{c^m\Gamma(g+l) g^{1-2l} a^{2l} b^{2i}}{\Gamma(m) l! i! 2^{l+i}(g-l)!}\int_{z_1}^{\infty} \frac{x^{m+l+i}}{x e^{A_2 x}} dx. \quad (7)$$

The above integral can be expressed in terms of the $\Gamma(a; x)$ function. Therefore (5) is deduced, which completes the proof. □

It is evident that the probability that the interference power constraint is satisfied is expressed as $\Pr\{Z < I_T\}$.

III. OUTAGE PROBABILITY ANALYSIS

By recalling that $a_{u,q} = b_{u,q}c_u + d_{u,q}$ and $b_{u,q} = \hat{b}_{u,q} + \xi_{u,q}$, it follows straightforwardly that $a_{u,q} = \hat{b}_{u,q}c_u + (\xi_{u,q}c_u + d_{u,q})$. To this effect, the received SNR can be expressed as $\psi_{u,q} = |\hat{b}_{u,q}|^2\mathcal{E}\left\{|\mathrm{c_u}|^2\right\}/\mathcal{E}\{|\mathrm{\xi_{u,q}c_u + d_{u,q}}|^2\} =$

$\alpha'_{\rm u}|\hat{\rm b}_{\rm u,q}|^2/(\alpha'_{\rm u}\tau+N_0)$. By performing the necessary variable change yields $\psi_{u,q}=\min\left(I_T/h_{u,L_x},P_m\right)h_{u,q}/[\min(I_T/h_{u,L_x},P_m)\tau+N_0]$. Given also the transmission rate of the unlicensed network D and according to the foundations of communication theory, the receiver is in outage if the inequalities $D\geq\frac{1}{2}\log_2\left(1+\psi\right)$ or $\psi\leq k$ hold, where $k=2^{2D}-1$ whereas ψ is the received SNR and the $1/2$ factor exhibits the two-stage nature in cooperative relaying. In the present relaying protocol, U_D is in outage either when both U_R and U_D are in outage i.e., $\psi_{U_S,U_R}<k,\ \ \psi_{U_S,U_D}<k$, or when only U_D is in outage i.e., $\psi_{U_S,U_R}\geq k$ and $\psi_{U_S,U_D}+\psi_{U_R,U_D}<k$. Thus, the OP of underlay cooperative CR networks can be expressed as,

$$P_{out}=\underbrace{\Pr\left\{\psi_{U_S,U_D}<k,\psi_{U_S,U_R}<k\right\}}_{\mathcal{R}_1}+\underbrace{\Pr\left\{\psi_{U_S,U_D}+\psi_{U_R,U_D}<k,\psi_{U_S,U_R}\geq k\right\}}_{\mathcal{R}_2}. \quad (8)$$

By recalling the general form for ψ_{U_S,U_D} and ψ_{U_S,U_R}, the $\mathcal{R}_1$ term in (8) can be expressed as follows,

$$\begin{aligned}\mathcal{R}_1&=\Pr\left\{\frac{A_3h_{U_S,U_D}}{A_3\tau+N_0}<k,\frac{A_3h_{U_S,U_R}}{A_3\tau+N_0}<k\right\}\\&=\int_0^\infty\Pr\left\{h_{U_S,U_D}<\frac{k\left(A_4\tau+N_0\right)}{A_4}\right\}\\&\quad\times\Pr\left\{h_{U_S,U_R}<\frac{k\left(A_4\tau+N_0\right)}{A_4}\right\}f_{h_{U_S,L_x}}\left(x\right)dx\end{aligned} \quad (9)$$

where $A_3=\min\left(I_T/h_{U_S,L_x},P_m\right)$ and $A_4=\min\left(I_T/x,P_m\right)$. Since $h_{u,q}$ is Nakagami$-m$ distributed, it follows that:

- Preliminary $\mathcal{A}$: $\Pr\left\{h_{u,q}<x\right\}=\gamma\left(m;\beta x\right)/\Gamma\left(m\right)$.
- Preliminary $\mathcal{B}$: $\Pr\left\{h_{u,q}>x\right\}=\Gamma\left(m;\beta x\right)/\Gamma\left(m\right)$.

where $\gamma\left(a;x\right)$ is the lower incomplete gamma function [15, eq. (8.350.1)]. With the aid of preliminary $\mathcal{A}$ one obtains,

$$\mathcal{R}_1=\underbrace{\int_\mu^\infty\frac{f_{h_{U_S,L_x}}\left(x\right)dx}{\Gamma^2(m)\gamma^{-2}(m;kA_5)}}_{\mathcal{R}_{11}}+\underbrace{\int_0^\mu\frac{f_{h_{U_S,L_x}}\left(x\right)dx}{\Gamma^2(m)\gamma^{-2}(m;kA_6)}}_{\mathcal{R}_{12}} \quad (10)$$

where $A_5=\beta(\tau+vx)$, $A_6=\beta(\tau+\chi)$, $\chi=N_0/P_m$, $v=N_0/I_T$ and $\mu=\chi/v$. Evidently, deriving a closed-form expression for $\mathcal{R}_1$ is subject to evaluation of $\mathcal{R}_{11}$ and $\mathcal{R}_{12}$.

Theorem 2: The following expression is valid for $\mathcal{R}_{11}$,

$$\begin{aligned}\mathcal{R}_{11}&=\frac{\Gamma\left(m;\beta\mu\right)}{\Gamma\left(m\right)}-\sum_{i=0}^{m-1}\sum_{p=0}^{i}\binom{i}{p}\frac{2k^iv^p\Gamma(p+m;\beta\mu A_7)}{\Gamma(m)e^{\beta\tau k}i!(\tau\beta)^{p-i}A_7^{p+m}}\\&+\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{p=0}^{i}\sum_{s=0}^{j}\binom{i}{p}\binom{j}{s}\frac{(\tau\beta)^{i+j-p-s}k^{i+j}\Gamma(A_9;\beta\mu A_8)}{i!j!\Gamma(m)A_8^{A_9}e^{2k\beta\tau}v^{-p-s}}\end{aligned} \quad (11)$$

where $\binom{n}{k}=\frac{n!}{k!(n-k)!}$, $A_7=1+kv$, $A_8=1+2kv$, and $A_9=p+s+m$.

Proof: By applying [15, eq. (8.352.1)] one obtains,

$$\mathcal{R}_{11}=\frac{\beta^m}{\Gamma\left(m\right)}\int_\mu^\infty\left(1-\sum_{i=0}^{m-1}\frac{\left[\beta k\left(\tau+vx\right)\right]^i}{i!e^{\beta k(\tau+vx)}}\right)^2\frac{x^{m-1}}{e^{\beta x}}dx. \quad (12)$$

To this effect and with the aid of the following two preliminaries:

- Preliminary $\mathcal{C}$ [15, eq. (1.111)]: The binomial expansion $(a+b)^n\triangleq\sum_{i=0}^n\binom{n}{i}a^ib^{n-i}$ which holds for $a,b\in\mathbb{R}$, $n\in\mathbb{N}$.
- Preliminary $\mathcal{D}$: $\int_x^\infty\frac{t^a}{e^{bt}}dt=\int_{bx}^\infty\frac{y^ae^{-y}}{b^{a+1}}dy=\frac{\Gamma(a+1;bx)}{b^{a+1}}$.

the proof of Theorem 2 can be completed. □

Likewise, the $\mathcal{R}_{12}$ integral is solved in the next Theorem.

Theorem 3: The following expression holds for $\mathcal{R}_{12}$,

$$\mathcal{R}_{12}=\gamma^2\left(m;\beta k\left(\tau+\chi\right)\right)\gamma\left(m;\beta\mu\right)/\Gamma^3\left(m\right). \quad (13)$$

Proof: The proof follows by applying preliminary $\mathcal{A}$. □

A. Derivation of $\mathcal{R}_2$

With the aid of ψ_{U_S,U_R}, ψ_{U_S,U_D}, and ψ_{U_R,U_D} one obtains,

$$\begin{aligned}\mathcal{R}_2&=\Pr\left\{\frac{A_3h_{U_S,U_D}}{A_3\tau+N_0}+\psi_{U_R,U_D}<k,\frac{A_3h_{U_S,U_R}}{A_3\tau+N_0}\geq k\right\}\\&=\int_0^k\int_0^\infty\frac{\Pr\left\{h_{U_S,U_D}<\frac{(k-y)A_{10}}{A_4}\right\}f_{h_{U_S,L_x}}\left(x\right)}{\left[\Pr\left\{h_{U_S,U_R}\geq\frac{kA_{10}}{A_4}\right\}f_{\psi_{U_R,U_D}}\left(y\right)\right]^{-1}}dxdy\end{aligned} \quad (14)$$

where $f_{\psi_{U_R,U_D}}\left(y\right)$ is the pdf of ψ_{U_R,U_D} and $A_{10}=A_4\tau+N_0$.

Theorem 4: The pdf of ψ_{U_R,U_D} can be expressed as,

$$\begin{aligned}f_{\psi_{U_R,U_D}}\left(y\right)&=\frac{\gamma\left(m;\beta\mu\right)A_6^my^{m-1}e^{-A_6y}}{\Gamma^2\left(m\right)}\\&+\sum_{a=0}^{m-1}\sum_{b=0}^{a}\binom{a}{b}\frac{v^{b+1}y^a\Gamma(b+m;\beta\mu A_{11})}{\Gamma(m)a!(\tau\beta)^{b-a}e^{y\beta\tau}A_{11}^{b+m}}\\&\times\left(\frac{\tau\beta}{v}-\frac{a}{vy}+\frac{(\mu\beta A_{11})^{b+m}e^{-\beta(\mu+\chi y)}}{A_{11}\Gamma(b+m;\beta\mu A_{11})}+\frac{b+m}{A_{11}}\right)\end{aligned} \quad (15)$$

where $A_{11}=1+vy$.

Proof: The proof is provided in Appendix 1. □

Capitalizing on the solution of Theorem 4 and utilizing preliminaries $\mathcal{A}$ and $\mathcal{B}$, the $\mathcal{R}_2$ integral can be expressed as,

$$\begin{aligned}\mathcal{R}_2&=\int_0^k\int_\mu^\infty\frac{\gamma\left(m;\left(k-y\right)A_5\right)f_{h_{U_S,L_x}}\left(x\right)}{\Gamma^2\left(m\right)\left[\Gamma(m;kA_5)f_{\psi_{U_R,U_D}}\left(y\right)\right]^{-1}}dxdy\\&+\int_0^k\int_\mu^\infty\frac{\gamma\left(m;\left(k-y\right)A_6\right)f_{h_{U_S,L_x}}\left(x\right)}{\Gamma^2\left(m\right)\left[\Gamma(m;kA_6)f_{\psi_{U_R,U_D}}\left(y\right)\right]^{-1}}dxdy\end{aligned} \quad (16)$$

and equivalently,

$$\begin{aligned}\mathcal{R}_2&=\underbrace{\int_0^k\underbrace{\int_\mu^\infty\frac{\gamma\left(m;\left(k-y\right)A_5\right)f_{h_{U_S,L_x}}(x)}{\Gamma^2\left(m\right)\Gamma^{-1}\left(m;kA_5\right)}dx}_{\mathcal{R}_{21\rm i}}f_{\psi_{U_R,U_D}}\left(y\right)dy}_{\mathcal{R}_{21}}\\&+\underbrace{\int_0^k\frac{\Gamma\left(m;kA_6\right)\gamma\left(m;\left(k-y\right)A_6\right)}{\Gamma^3\left(m\right)\gamma^{-1}\left(m;\beta\mu\right)}f_{\psi_{U_R,U_D}}\left(y\right)dy}_{\mathcal{R}_{22}}.\end{aligned} \quad (17)$$

Deriving (17) is subject to solving $\mathcal{R}_{21}$ and $\mathcal{R}_{22}$. However, prior derivation of some essential analytic results is necessary.

Lemma 2: The following closed-form solutions are valid,

$$\begin{aligned}
&f_{212a}^{a,i,o,l,d;k,\beta,v,\mu}\\
&=\int_0^k \frac{y^a(k-y)^i A_{12}^{-o}\Gamma\left(o;\beta\mu A_{12}\right)}{(1+vy)^d\Gamma^{-1}\left(l;\beta\left(1+vy\right)\mu\right)}dy\\
&=\frac{\Gamma\left(o\right)\Gamma\left(l\right)}{e^{2\beta\mu A_7}}\sum_{w=0}^{o-1}\sum_{u=0}^{l-1}\frac{(\beta\mu)^{w+u}B\left(i+1,a+1\right)}{w!u!A_8^{o-w}k^{-i-a-1}}\\
&\times F_1\left(a+1,o-w,d-u,i+a+2;kv/A_8,-kv\right)
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
f_{212b}^{c,i,o,j;k,\beta,v,\mu,\chi}&=\int_0^k \frac{y^c(k-y)^i\Gamma\left(o;\beta A_{12}\mu\right)}{A_{12}^o(1+vy)^j e^{\beta\chi y}}dy\\
&=\sum_{l=0}^{o-1}A_{13}\frac{F_1\Big(c+1,o-l,j,i+c+2;\frac{kv}{A_8},-kv\Big)}{l!(\beta\mu)^{-l}k^{-(i+c+1)}A_8^{o-l}\Gamma^{-1}(o)e^{\beta\mu A_8}}
\end{aligned} \tag{19}$$

where $A_{12} = 1 + (2k - y)v$, $A_{13} = B\left(i+1,c+1\right)$ while $B(\cdot,\cdot)$ and $F_1(\cdot,\cdot,\cdot,\cdot;\cdot,\cdot)$ denote the Beta function [15, eq. (8.380.1)] and Appell hypergeometric function [15, eq. (9.180.1)], respectively.

Proof: The proof is completed by expanding (18) and (19) according to [15, eq. (8.352.2)] and using [15, eq. (3.211)]. □

Lemma 3: The following closed-form expressions are valid,

$$\begin{aligned}
f_{221a}^{k,a,v}&=\int_0^k \frac{y^a}{1+vy}dy\\
&=\frac{(-1)^a\ln(A_7)}{v^{a+1}}+\sum_{q=1}^{a}\binom{a}{q}\frac{(-1)^{q+a}(A_7^q-1)}{v^{a+1}q}
\end{aligned} \tag{20}$$

and

$$f_{221b}^{a,b,c;k,v,\mu,\beta,\chi}=\int_0^k \frac{y^a\Gamma(b;\beta\mu(1+vy))}{(1+vy)^c e^{-\beta\chi y}}dy \tag{21}$$

$$=\begin{cases}\sum_{q=0}^{b-1}\sum_{l=0}^{a}\binom{a}{l}\frac{\Gamma(b)(\beta\mu)^q\ln(A_7)}{(-1)^{a+l}q!v^{a+1}e^{\beta\mu}}, & A_{14}=-1\\ \sum_{q=0}^{b-1}\sum_{l=0}^{a}\binom{a}{l}\frac{(-1)^{a-l}\Gamma(b)(A_7^{q+l-c+1}-1)}{q!v^{a+1}(q+l-c+1)(\beta\mu)^{-q}e^{\beta\mu}}, & A_{14}\neq-1\end{cases}$$

where $A_{14} = q + l - c$.

Proof: Setting $t = 1 + vy$ yields $f_{221a}^{k,a,v} = v^{-a-1}\int_1^{A_7}(t-1)^a/t dt$ while using preliminary $\mathcal{C}$ and basic integration yields (20). Likewise, by applying [15, eq. (8.352.2)] one obtains,

$$f_{221b}^{a,b,c;k,v,\mu,\beta,\chi}=\frac{\Gamma\left(b\right)}{e^{\beta\mu}}\sum_{q=0}^{b-1}\frac{(\beta\mu)^q}{q!v^{a+1}}\int_1^{1+kv}\frac{(t-1)^a}{t^{c-q}}dt \tag{22}$$

which straightforwardly yields (21) and completes the proof. □

As already mentioned, the derived results are important in the derivation of analytic expressions for $\mathcal{R}_{21}$ and $\mathcal{R}_{22}$.

Theorem 5: Given the derived functions $f_{212a}^{a,i,o,l,d;k,\beta,v,\mu}$, $f_{212b}^{c,i,o,j;k,\beta,v,\mu,\chi}$ in Lemma 2 and $F_{\psi_{U_R,U_D}}\left(y\right)$ in (28) of Appendix 1, the following closed-form expression is valid,

$$\begin{aligned}
\mathcal{R}_{21}&=\sum_{j=0}^{m-1}\sum_{p=0}^{j}\binom{j}{p}\frac{k^jv^p\Gamma(p+m;\beta\mu A_7)\mathcal{R}_{211}}{j!(\tau\beta)^{p-j}\Gamma(m)e^{\beta\tau k}A_7^{p+m}}\\
&+\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{p=0}^{i}\sum_{s=0}^{j}\binom{i}{p}\binom{j}{s}\frac{k^j(\tau\beta)^{i+j-p-s}\mathcal{R}_{212}}{i!j!v^{-p-s}\Gamma(m)e^{2\beta\tau k}}
\end{aligned} \tag{23}$$

where $\mathcal{R}_{211} = F_{\psi_{U_R,U_D}}\left(k\right)$ and

$$\begin{aligned}
\mathcal{R}_{212}&=\frac{\gamma\left(m;\beta\mu\right)f_{212b}^{m-1,i,p+s+m,0;k,\beta,v,\mu,\chi}}{\left(\tau+\chi\right)^{-m}\Gamma^2\left(m\right)\beta^{p+s}}\\
&+\sum_{a=0}^{m-1}\sum_{b=0}^{a}\binom{a}{b}\left\{\frac{\mu^{b+m}\tau^a f_{212b}^{a,i,p+s+m,1;k,\beta,v,\mu,\chi}}{a!v^{-b-1}\tau^b\Gamma(m)\beta^{p+s-a}e^{\beta\mu}}\right.\\
&+\frac{v^b f_{212a}^{a,i,p+s+m,b+m,b+m;k,\beta,v,\mu}}{a!\Gamma(m)\tau^{b-a-1}\beta^{p+s+m+b-a-1}}\\
&-\frac{v^b f_{212a}^{a-1,i,p+s+m,b+m,b+m;k,\beta,v,\mu}}{\Gamma(m)\Gamma(a)\tau^{b-a}\beta^{p+s+m+b-a}}\\
&\left.+\frac{(b+m)f_{212a}^{a,i,p+s+m,b+m,b+m+1;k,\beta,v,\mu}}{a!\Gamma(m)\beta^{p+s+m+b-a}v^{-b-1}\tau^{b-a}}\right\}.
\end{aligned} \tag{24}$$

Proof: The proof is provided in Appendix 2. □

Likewise, a novel analytic expression for $\mathcal{R}_{22}$ is derived below.

Theorem 6: Given $f_{221a}^{k,a,v}$ and $f_{221b}^{a,b,c;k,v,\mu,\beta,\chi}$ in Lemma 3, the following exact closed form representation is valid for $\mathcal{R}_{22}$,

$$\mathcal{R}_{22}=F_{\psi_{U_R,U_D}}\left(k\right)-\sum_{i=0}^{m-1}\sum_{p=0}^{i}\frac{(-1)^p\beta^i(\tau+\chi)^i\mathcal{R}_{221}}{p!(i-p)!k^{p-i}e^{\beta k(\tau+\chi)}} \tag{25}$$

where

$$\begin{aligned}
\mathcal{R}_{221}&=\sum_{a=0}^{m-1}\sum_{b=0}^{a}\binom{a}{b}\frac{v^{b+1}(\tau\beta)^{a+1}}{a!\Gamma(m)\tau^b\beta^b}\left\{\frac{f_{221b}^{a+p,b+m,b+m;k,v,\mu,\beta,\chi}}{v}\right.\\
&-\frac{f_{221b}^{a+p-1,b+m,b+m;k,v,\mu,\beta,\chi}}{a^{-1}v\tau\beta}+\frac{\mu^{b+m}f_{221a}^{k,a+p,v}}{\tau\beta^{1-b-m}e^{\beta\mu}}\\
&\left.+\frac{f_{221b}^{a+p,b+m,b+m+1;k,v,\mu,\beta,\chi}}{\tau\beta(b+m)^{-1}}\right\}+\frac{\gamma\left(m;\beta\mu\right)A_6^m k^{m+p}}{(m+p)\Gamma^2\left(m\right)}.
\end{aligned} \tag{26}$$

Proof: The proof is provided in Appendix 3. □

B. Special Case

The validity of the derived expressions is demonstrated through comparisons with results from computer simulations and with expressions [3], [7]−[13]. Particularly for the case of perfect Rayleigh channel information, it is shown that (10) and (17) coincide with [8, eq. (6)] and [8, eq. (15)], respectively.

Corollary 1: For the case of perfect Rayleigh fading channel information, (10) coincides with [8, eq. (6)].

Proof: By setting $m = 1$ and $\tau = 0$, the $\mathcal{R}_{11}$ and $\mathcal{R}_{12}$ terms are simplified as, $\mathcal{R}_{11} = \Gamma\left(1;\beta\mu\right) - 2\Gamma\left(1;\mathrm{A}_7\beta\mu\right)/\mathrm{A}_7 + \Gamma\left(1;\mathrm{A}_8\beta\mu\right)/\mathrm{A}_8 = \mathrm{e}^{-\beta\mu} - 2\mathrm{e}^{-\beta\mu\mathrm{A}_7}/\mathrm{A}_7 + \mathrm{e}^{-\beta\mu\mathrm{A}_8}/\mathrm{A}_8$ and $\mathcal{R}_{12} = \gamma^2\left(1;\beta\chi k\right)\gamma\left(1;\beta\mu\right) = \left(1-e^{-\beta\chi k}\right)^2\left(1-e^{-\beta\mu}\right)$. Thus, by substituting accordingly in (10) one obtains, $\mathcal{R}_1 = e^{-\beta\mu A_8}/A_8+\left(1-e^{-\beta\chi k}\right)^2\left(1-e^{-\beta\mu}\right)-2e^{-\beta\mu A_7}/A_7+e^{-\beta\mu}$.

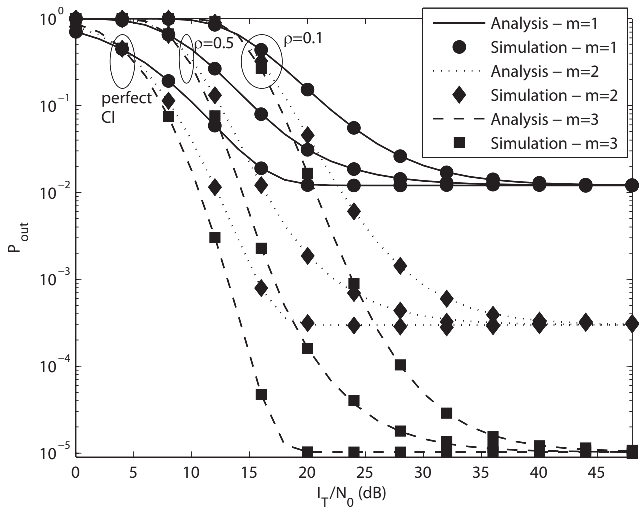

Fig. 2. Outage probability versus I_T/N_0.

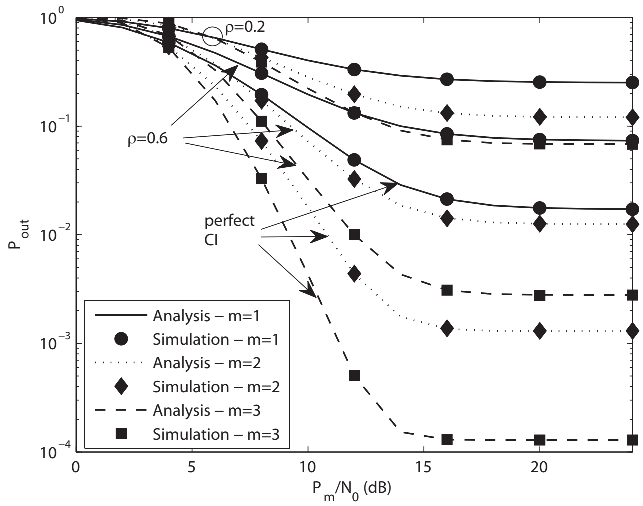

Fig. 3. Outage probability versus P_m/N_0.

To this effect, performing the necessary change of variables $\mathcal{R}_1$ coincides [8, eq. (6)], completing the proof. □

Corollary 2: Given perfect Rayleigh fading channel information, (17) coincides with [8, eq. (15)].

Proof: The proof follows immediately with the aid of the contributions in Corollary 1, Lemma 2, and Lemma 3. □

IV. NUMERICAL RESULTS

This section presents various results that validate the derived expressions and demonstrate the performance behavior of underlay cooperative CR networks over imperfect Nakagami$-m$ fading. Specifically, Fig. 2 illustrates simulated and numerical results for various imperfect channel information scenarios[3], $\rho = \{0.1, 0.5, \infty\}$, required transmission rate of $D = 1$ bps/Hz, fading power $\lambda = 1$, severity of fading $m = \{1, 2, 3\}$, $P_m/N_0 = 15$ dB, $I_T/N_0 \in [0, 48]$ dB. It is observed that analysis and simulation are in excellent agreement which validates with clarity the accuracy of the offered expression. It is also shown that the channel estimation error affects considerably the corresponding OP. Furthermore, the outage performance is, as expected, enhanced with respect to decrease of fading severity.

In the same context, Fig. 2 demonstrates some rather interesting results: 1) For low-to-moderate values of I_T, the corresponding OP is inversely proportional to I_T. This is based on the fact that I_T controls the transmit power of unlicensed users, and hence the higher the I_T, the higher the transmit power which ultimately reduces the OP; 2) an outage saturation occurs in the large I_T regime; 3) the error floor level appears to be independent of the channel estimation quality i.e., ρ. The performance saturation is due to the fact that the transmit power of unlicensed users is $\min\left(I_T/|\hat{b}_{u,L_x}|^2, P_m\right)$. Consequently, as I_T exceeds a certain threshold, $\min\left(I_T/|\hat{b}_{u,L_x}|^2, P_m\right) \simeq P_m$ determines fully the corresponding transmit power, independent of the channel estimation quality, $|\widehat{b}_{u,L_x}|^2$ or ρ, resulting in identical OP for any subsequent increase of I_T and ρ.

[3]The larger the ρ, the better the quality of the channel estimator. The perfect channel information corresponds to $\rho = \infty$, and therefore, $\tau = 0$ and $\zeta = \lambda$.

Fig. 3 depicts the OP versus P_m/N_0 for $I_T/N_0 = 15$ dB, $D = 1$ bps/Hz, $\lambda = 1$, and $\rho = \{0.2, 0.6, \infty\}$, $m = \{1, 2, 3\}$. It is seen that analytical results are in excellent agreement with the simulated results, which verifies the validity of (8). In addition, the performance trend for varying P_m/N_0 is similar to that for changing I_T/N_0 (Fig. 2). In other words, the OP decreases with the increase in P_m/N_0 and quickly reaches the error floor at large values of P_m/N_0. This behavior can be explained in the similar manner as Fig. 2. Nevertheless, the error floor level in Fig. 3 differs from that in Fig. 2 in that the former depends on ρ while the latter is independent of it. This phenomenon is explained as follows: For large values of P_m, $\min\left(I_T/|\hat{b}_{u,L_x}|^2, P_m\right) \simeq I_T/|\hat{b}_{u,L_x}|^2$ completely controls the transmit power of unlicensed users. Consequently, the saturation level is dependent upon $|\widehat{b}_{u,L_x}|^2$ and hence, the better quality of the channel estimator, i.e., larger ρ, reduces this level. Moreover, the results in Fig. 3 are reasonable since the outage performance is dramatically enhanced as ρ and m increase.

Fig. 4 depicts the interference statistics of licensed users versus the the channel estimation quality for $\lambda = 1$, $I_T/N_0 = 10$ dB, $P_m/N_0 = 15$ dB and $m = \{1, 2, 3\}$. It is clearly observed that the analytical results are in good agreement with the corresponding simulated results which justifies the accuracy of (5). Also, the probability that the interference power constraint is satisfied i.e., $\Pr\{Z < I_T\}$ is relatively small e.g., $\Pr\{Z < I_T\} < 0.6$. This indicates that channel estimation errors can detrimentally affect the performance of licensed networks. Moreover, the results are reasonable as under imperfect channel estimation condition, $\Pr\{Z < I_T\}$ increases with respect to the better quality of the channel estimator i.e., larger ρ. Also, this probability is significantly reduced in moderate fading conditions which indicates explicitly that the less severe the fading conditions, the more damage unlicensed networks cause to licensed networks. In contrast, as seen in Figs. 2 and 3, the performance of unlicensed networks is considerably improved

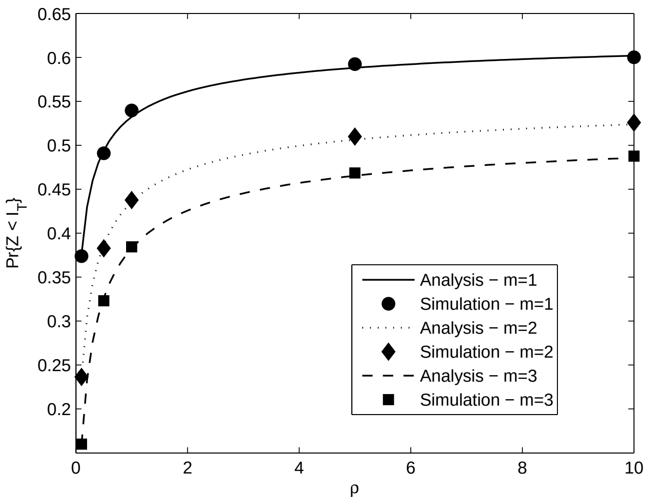

Fig. 4. $P_r\left\{Z < I_T\right\}$ versus ρ.

with respect to better fading conditions. Hence, for system design under practical conditions, such as imperfect channel information, it is important to account for the performance trade-off between licensed and unlicensed networks.

V. CONCLUSION

The present work was devoted to the analytic evaluation of the OP of unlicensed users in underlay cooperative CR networks with imperfect Nakagami$-m$ fading channel information and both interference power constraint and maximum transmit power constraint. A novel analytic expression was derived and validated by computer simulations. This expression was subsequently employed in generating several results which revealed that channel estimation error dramatically deteriorates the system performance while the outage saturation phenomenon appears under certain conditions. Furthermore, it was shown that the channel estimation quality determines the saturation level as I_T is a constant but not for fixed P_m. It was also shown that due to channel estimation error the interference at licensed users can not be guaranteed below an acceptable level at all times and based on this the corresponding statistics was investigated both analytically and through simulation. Various results illustrated that the probability that the interference is below the maximum interference power is relatively low and significantly dependent upon the severity of fading conditions as well as the channel estimation quality. Finally, it was shown that the interference at licensed users is inversely proportional to the OP of unlicensed users, which constitutes the performance trade-off between licensed and unlicensed networks that should be considered in future system designs.

Appendix 1

The CDF of ψ_{U_R,U_D} is $F_{\psi_{U_R,U_D}}(y) = \Pr\left\{\psi_{U_R,U_D} < y\right\} = \Pr\{h_{U_R,U_D} < y(A_{15}\tau + N_0)/A_{15}\} = \int_0^\infty \Pr\{h_{U_R,U_D} < y(A_4\tau + N_0)/A_4\} f_{h_{U_R,L_x}}(x)dx$ with $A_{15} = \min(I_T/h_{U_R,L_x}, P_m)$, which can be rewritten as, $F_{\psi_{U_R,U_D}}(y) = \int_\mu^\infty \gamma\left(m; yA_5\right) f_{h_{U_R,L_x}}(x)/\Gamma(m)dx + \int_0^\mu \gamma\left(m; yA_6\right) f_{h_{U_R,L_x}}(x)/\Gamma(m)dx$. By substituting $f_{h_{u,q}}(x)$ and utilizing [15, eq. (8.352.1)] yields

$$F_{\psi_{U_R,U_D}}(y) = \int_\mu^\infty \left(1 - \sum_{i=0}^{m-1} \frac{(yA_5)^i}{i! e^{yA_5}}\right) \frac{\beta^m x^{m-1}}{\Gamma(m)\, e^{\beta x}} dx + \gamma\left(m; yA_6\right)\gamma\left(m; \beta\mu\right)/\Gamma^2(m). \quad (27)$$

With the aid of preliminaries $\mathcal{C}$ and $\mathcal{D}$ it follows that,

$$F_{\psi_{U_R,U_D}}(y) = \frac{\Gamma\left(m;\beta\mu\right)}{\Gamma(m)} + \frac{\gamma\left(m; yA_6\right)\gamma\left(m;\beta\mu\right)}{\Gamma^2(m)} - \sum_{i=0}^{m-1}\sum_{p=0}^{i}\binom{i}{p}\frac{v^p y^i \tau^i \beta^i \Gamma\left(p+m;\beta A_{11}\mu\right)}{i!\Gamma(m)\tau^p\beta^p A_{11}^{p+m} e^{\beta\tau y}} \quad (28)$$

Taking the derivative of (28) with respect to y, (15) is deduced thus, completing the proof of Theorem 4.

Appendix 2

Utilizing once more $f_{h_{u,q}}(x)$ and [15, eq. (8.352.1)] and [15, eq. (8.352.2)], the inner integral of the first term in (17) can be expressed as follows,

$$\begin{aligned}\mathcal{R}_{21\mathrm{i}} &= \int_\mu^\infty \left[1 - \sum_{i=0}^{m-1}\frac{(k-y)^i A_5^i}{i! e^{(k-y)A_5}}\right]\sum_{j=0}^{m-1}\frac{k^j\beta^m A_5^j x^{m-1}}{j!\Gamma(m) e^{kA_5+\beta x}}dx \\ &= \sum_{j=0}^{m-1}\sum_{p=0}^{j}\binom{j}{p}\frac{k^j v^p \tau^j \beta^j \Gamma\left(p+m;\beta A_7\mu\right)}{j!\Gamma(m)A_7^{p+m}\tau^p\beta^p e^{\beta\tau k}} \\ &\quad - \sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{p=0}^{i}\sum_{s=0}^{j}\binom{i}{p}\binom{j}{s}\frac{e^{\beta\tau(y-2k)}(\tau\beta)^{i+j}(k-y)^i A_{16}}{i!j!\Gamma(m)(\tau\beta)^{p+s}k^{-j}v^{-p-s}}\end{aligned} \quad (29)$$

where $A_{16} = \Gamma(A_9; \beta\mu A_{12})/A_{12}^{A_9}$. By substituting (29) into the first term of (17) one obtains,

$$\begin{aligned}\mathcal{R}_{21} &= \sum_{j=0}^{m-1}\sum_{p=0}^{j}\frac{k^j v^p \tau^j \beta^j A_{17}}{j!\Gamma(m)\tau^p\beta^p e^{\beta\tau k}}\underbrace{\int_0^k f_{\psi_{U_R,U_D}}(y)\,dy}_{\mathcal{R}_{211}} \\ &\quad - \sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{p=0}^{i}\sum_{s=0}^{j}\binom{i}{p}\binom{j}{s}\underbrace{\int_0^k \frac{A_{18} f_{\psi_{U_R,U_D}}(y)}{i!j!(k-y)^{-i}}dy}_{\mathcal{R}_{212}}\end{aligned} \quad (30)$$

where $A_{17} = \Gamma(p+m;\beta\mu A_7)/A_7^{p+m}$ and $A_{18} = A_{16}k^j v^{p+s}(\tau\beta)^{i+j-p-s}e^{\beta\tau(y-2k)}/\Gamma(m)$. By recalling that $\mathcal{R}_{211} = F_{\psi_{U_R,U_D}}(k)$, substituting (15) in $\mathcal{R}_{212}$, performing some basic algebraic manipulations and utilizing $f_{212a}^{a,i,o,l,d;k,\beta,v,\mu}$ and $f_{212b}^{c,i,o,j;k,\beta,v,\mu,\chi}$ in Lemma 2, (24) is deduced and therefore, completing the proof of Theorem 5.

Appendix 3

By applying [15, eq. (8.352.1)] in (17), one obtains,

$$\begin{aligned}\mathcal{R}_{22} &= \int_0^k \left(1 - \sum_{i=0}^{m-1} \frac{(k-y)^i A_6^i}{i! e^{(k-y)A_6}}\right) f_{\psi_{U_R,U_D}}(y)\, dy \\ &= A_{19} - \sum_{i=0}^{m-1} \sum_{p=0}^{i} \binom{i}{p} \frac{(-1)^p A_6^i k^i}{i! e^{kA_6} k^p} \underbrace{\int_0^k \frac{f_{\psi_{U_R,U_D}}(y)}{y^{-p} e^{-yA_6}} dy}_{\mathcal{R}_{221}}\end{aligned} \tag{31}$$

where $A_{19} = F_{\psi_{U_R,U_D}}(y)$. By subsequently inserting (15) in $\mathcal{R}_{221}$, it immediately follows that,

$$\begin{aligned}\mathcal{R}_{221} &= \sum_{a=0}^{m-1} \sum_{b=0}^{a} \binom{a}{b} \frac{v^b (\tau\beta)^{a+1}}{\tau^b \beta^b a! \Gamma(m)} \underbrace{\int_0^k \frac{\Gamma(b+m; \beta\mu A_{11})}{y^{-a-p} e^{-\beta\chi y} A_{11}^{b+m}} dy}_{f_{221b}^{a+p, b+m, b+m; k, v, \mu, \beta, \chi}} \\ &- \sum_{a=1}^{m-1} \sum_{b=0}^{a} \binom{a}{b} \frac{v^b (\tau\beta)^{a-b}}{\Gamma(a)\Gamma(m)} \underbrace{\int_0^k \frac{\Gamma(b+m; \beta\mu A_{11}) e^{\beta\chi y}}{y^{1-a-p} A_{11}^{b+m}} dy}_{f_{221b}^{a+p-1, b+m, b+m; k, v, \mu, \beta, \chi}} \\ &+ \sum_{a=0}^{m-1} \sum_{b=0}^{a} \binom{a}{b} \frac{v^b \tau^a \beta^{a+m} \mu^{b+m}}{v^{-1} a! \Gamma(m) \tau^b e^{\beta\mu}} \underbrace{\int_0^k \frac{y^{a+p}}{A_{11}} dy}_{f_{221a}^{k, a+p, v}} + \int_0^k \frac{A_{20} A_6^m dy}{y^{1-m-p}} \\ &+ \sum_{a=0}^{m-1} \sum_{b=0}^{a} \binom{a}{b} \frac{v^b (b+m) \tau^a \beta^a}{a! \Gamma(m) v^{-1} \tau^b \beta^b} \underbrace{\int_0^k \frac{\Gamma(b+m; \beta\mu A_{11})\, dy}{y^{-a-p} A_{11}^{b+m+1} e^{-\beta\chi y}}}_{f_{221b}^{a+p, b+m, b+m+1; k, v, \mu, \beta, \chi}}\end{aligned} \tag{32}$$

where $A_{20} = \gamma(m; \beta\mu)/\Gamma^2(m)$. To this effect and since $\int_0^k y^{m+p-1} dy = k^{m+p}/(m+p)$ while $f_{221a}^{k,a,v}$ and $f_{221b}^{a,b,c;k,v,\mu,\beta,\chi}$ are given in closed form in Lemma 3, substituting (32) in (31) completes the proof of Theorem 6.

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Khuong Ho-Van received the B.E. (with the first-rank honor) and the M.S. degrees in Electronics and Telecommunications Engineering from HoChiMinh City University of Technology, Vietnam, in 2001 and 2003, respectively, and the Ph.D. degree in Electrical Engineering from University of Ulsan, Korea in 2006. During 2007-2011, he joined McGill University, Canada as a postdoctoral fellow. Currently, he is an assistant professor at HoChiMinh City University of Technology. His major research interests are modulation and coding techniques, diversity technique, digital signal processing, and cognitive radio.

Paschalis C. Sofotasios was born in Volos, Greece in 1978. He received the M.Eng. degree in Electronic and Communications Engineering from the University of Newcastle upon Tyne, UK, the M.Sc. degree in Satellite Communications Engineering from the University of Surrey, UK and the Ph.D. degree in Electronic and Electrical Engineering from the University of Leeds, UK. Since October 2010, he has been a Research Fellow at the University of Leeds. During Fall 2011 he was a Visiting Researcher at the CORES Lab of the University of California, Los Angeles (UCLA). His research interests are in communication theory and systems with emphasis on fading channel characterization and modelling, cognitive radio, cooperative systems and free-space-optical communications.

Steven Freear gained his doctorate in 1997 and subsequently worked in the electronics industry for 7 years as a VLSI system designer. He was appointed Lecturer (Assistant Professor) and then Senior Lecturer (Associate Professor) at the School of Electronic and Electrical Engineering at the University of Leeds in 2006 and 2008, respectively. His main research interest is concerned with advanced analogue and digital signal processing for ultrasonic instrumentation and wireless communication systems. He teaches digital signal processing, microcontrollers/microprocessors, VLSI and embedded systems design, hardware description languages at both undergraduate and postgraduate level. Dr Freear is Editor-in-Chief of the IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control (UFFC) and an Associate Editor of the International Journal of Electronics.