

A Fast Converged Solution for Power Allocation of OFDMA System

Sungho Hwang* and Ho-Shin Cho[†]

Abstract – In this paper, we propose a fast adaptive power allocation method for an orthogonal frequency division multiple access (OFDMA) system that employs an adaptive modulation and coding (AMC) scheme. The proposed scheme aims to reduce the calculation complexity of greedy adaptive power allocation (APA), which is known as the optimal algorithm for maximizing the utility argument of power. Unlike greedy APA, which starts power allocation from “0”, the proposed algorithm initially allocates a certain level of power determined by the water-filling scheme. We theoretically demonstrate that the proposed algorithm has almost the same capability of maximizing the utility argument as the greedy APA while reducing the number of operations by $2M$, where M is the number of AMC levels.

Keywords: Adaptive power allocation, Discrete utility function, OFDMA, AMC

1. Introduction

The orthogonal frequency division multiple access (OFDMA) system is a multi-carrier system. As such, the system capacity is obtained by the sum of the individual subcarrier capacities, which highly depends on the amount of power allocated to each subcarrier. Naturally, the subcarrier capacity increases when more power is allocated. To maximize the overall system capacity, optimally controlling and assigning the power for each individual carrier within the limits of the total transmission power would be ideal. Such power allocation has been a difficult and challenging issue for many researchers. Lagrangian methods have been used as potential solutions for capacity maximization. Water-filling, in which more power is allocated to stronger subcarriers, is known as the optimal solution for capacity maximization using Lagrangian methods [1].

However, water-filling is inappropriate for obtaining the best performance when the adaptive modulation and coding (AMC) scheme is employed, in which the power allocation is performed with a number of discrete power levels. Regarding this issue, several adaptive power allocation (APA) schemes, which are sometimes called the “bit-loading” problem, have been investigated to increase the system utility in an OFDMA system with discrete AMC levels [3]-[5]. APA schemes have been developed with slightly different objectives and constraints. In [3], the maximum utility rate has been obtained under a limited power constraint, whereas in [5], the total transmission power is minimized to achieve a target data-rate of the system, which is the sum of the individual data-rates of all users. These schemes have an

iterative procedure to successfully achieve each objective by using optimal [3], heuristic [4], or sub-optimal [5] algorithms. In particular, as an optimal algorithm, greedy APA (the modified Levin-Campello algorithm) maximizes the utility argument of power at each bit assignment. However, because zero power is initially allocated to all subcarriers, the processing time for iterative allocation of power to all subcarriers until the total allocated power reaches the limit is excessively long. In this paper, we propose a fast convergent power allocation method to significantly reduce the processing time for an OFDMA system that employs AMC.

The remainder of this paper is organized as follows. In section 2, we explain the OFDMA channel architecture and some system parameter. In section 3, we provide problem formulation and proposed algorithm. Section 4 describes the computer simulation model and its numerical results. Finally, conclusions are presented in Section 5.

2. System Model

We assume that AMC has M discrete channel states and the channels are already allocated to users by means of one of the existing sub-carrier allocation methods such as maximum C/I or proportional fairness (PF). In this paper, we address only the power allocation to each sub-carrier. The channel state of sub-carrier i is denoted by ρ_i , which the corresponding user experiences. It is defined as $\rho_i = |h_i|^2/N_i$, where N_i and h_i are the noise power density and frequency response, respectively, of sub-carrier i [2]. Thus, the channel state vector is defined by

$$c = [\rho_1 \ \rho_2 \ \cdots \ \rho_S]^T, \quad (1)$$

where S denotes the number of sub-carriers.

[†] Corresponding Author: School of Electronics Engineering, Kyungpook National University, Korea. (hscho@ee.knu.ac.kr)

* LIG Nex1 R&D Center, Korea. (sungho.hwang@lignex1.com)

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3. Problem Formulation and Proposed Algorithm

In this section, we formulate an optimization problem to maximize the system utility for the allocated sub-carriers and provide a power allocation algorithm to obtain an optimal solution.

Let $U_i(\cdot)$ be the utility function for sub-carrier i and P_{total} be the total transmission power. If sub-carrier i has an allocated power P_i , then its utility is $U_i(P_i)$. Thus, we are interested in the following problem:

$$\text{Problem (P):} \quad \max_{P_i} \sum_{i=1}^S U_i(P_i), \quad (2)$$

$$\text{Subject to} \quad \sum_{i=1}^S P_i \leq P_{total}, \quad P_i \geq 0, \quad (3)$$

where $U_i(P_i) = a_i \log(1 + b_i P_i \rho_i)$ is a differentiable non-decreasing concave function, a_i is the weight vector for the allocated user at sub-carrier i to generalize the problem, and b_i is the factor for controlling the bit error rate of the allocated user at sub-carrier i . By changing the value of a_i , Problem (P) can cover throughput maximization as well as PF allocation, and b_i is determined by $b_i = -1.5/\ln(5 \cdot \text{BER}_i)$ [8], where BER_i is the bit error rate of the allocated user at sub-carrier i . Then, the optimal power allocation for problem (P) has the following solution [1]:

$$P_i^* = \left(\frac{a_i}{\lambda} - \frac{1}{b_i \rho_i} \right)^+, \quad (4)$$

where $(x)^+ = \max(x, 0)$ and λ is chosen such that the power constraint is satisfied:

$$\sum_{i=1}^S \left(\frac{a_i}{\lambda} - \frac{1}{b_i \rho_i} \right)^+ = P_{total}. \quad (5)$$

This equation represents the water-filling method of power allocation.

If the fixed sub-carrier allocation is based on the sub-carrier channel state as in the greedy or PF algorithm for system throughput, the variation of each sub-carrier channel state is sufficiently low to satisfy the following constraint:

$$P_i > 0. \quad (6)$$

Thus, the solution of problem (P), (4), is rewritten as:

$$P_i^* = \frac{a_i}{E(a_i)} \left(\frac{P_{total}}{S} + \frac{1}{S} \sum_{k=1}^S \frac{1}{b_k \rho_k} \right) - \frac{1}{b_i \rho_i}, \quad (7)$$

where $E(a_i)$ indicates the average of a_i . The term

$(1/S) \sum (1/b_k \rho_k)$ on the right-hand side indicates the average of the inverse channel states. Thus, we know that the performance improvements are marginal even though APA is employed in a continuous utility system under the following assumptions [2]:

- A1) The variation of each sub-carrier channel state is sufficiently low.
- A2) The variation of weighting factor a_i decreases as long-term utility is maximized.

By using these properties (A1 and A2), we propose a new algorithm for a discrete data rate. The proposed algorithm allocates power to maximize the system utility. This purpose is the same as that of the underlying greedy APA [3]. However, our proposed scheme initializes P_i using the P_i^* value, whereas greedy APA initializes P_i as "0." Therefore, the proposed scheme can reduce the number of iterations for power allocation.

To compare the performance of the proposed algorithm with greedy APA, we explain some variables by using Fig. 1. In this figure, P_i^* is the optimum power for the i -th sub-carrier and $P_i[n]$ is the minimum required power of level n for the i -th sub-carrier. The relationship between (δ_1) and (δ_2) as well as that between (δ_2) and (δ_3) in Fig. 1 are explained by **Theorems 1** and **2**, respectively. The inequality of the slopes for (δ_1) and (δ_4) are shown in the **Claim**.

The proposed algorithm is described in the following procedures:

- 1) For initialization,
 - if $P_i[n] \leq P_i^* < P_i[n+1]$
 - $n^* = n$;
 - end if
 - $P_i = P_i[n^*]$.
- 2) The remaining power, $P_{total} - \sum_i P_i$, is allocated using greedy APA.

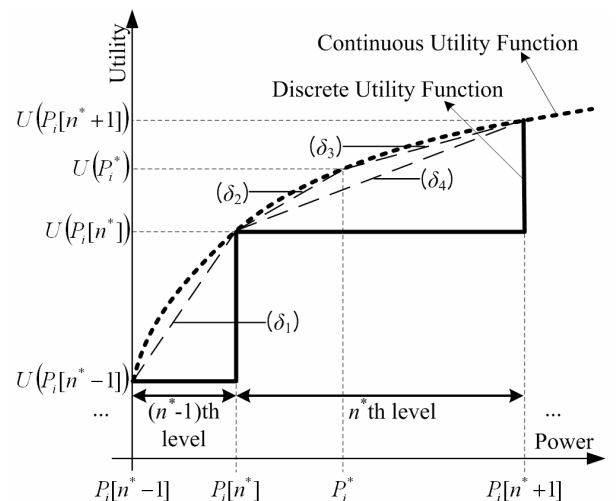


Fig. 1. Some variables for a new algorithm

Theorem 1: If $U_i(P_i)$ is a differentiable non-decreasing concave function, for every i , the following property is satisfied:

$$\frac{\Delta U_i^L}{\Delta P_i^L} > \frac{\Delta U_i^*}{\Delta P_i^*} > U_i'(P_i^*), \quad (8)$$

where $\Delta U_i^L = U_i(P_i[n^*]) - U_i(P_i[n^* - 1])$, $\Delta P_i^L = P_i[n^*] - P_i[n^* - 1]$, $\Delta U_i^* = U_i(P_i^*) - U_i(P_i[n^*])$, and $\Delta P_i^* = P_i^* - P_i[n^*]$.

Proof: Eq. (8) is achieved because $U_i(P_i)$ is a non-decreasing and concave function and $P_i^* > P_i[n^*] > P_i[n^* - 1]$ ■

Theorem 2: If $U_i(P_i)$ is a differentiable non-decreasing concave function, for every i and j , the following property is satisfied:

$$U_i'(P_i^*) = U_j'(P_j^*). \quad (9)$$

Proof: If P_i^* and P_j^* are optimal, then any change in allocation will not increase the average utility. Let $0 < \Delta P < P_i^*$, $0 < \Delta P < P_j^*$, and two arbitrary users exchange an amount of power equal to ΔP with each other. The new average utility will then be equal to or less than the optimal one. This inequality is given by

$$U_i(P_i^*) + U_j(P_j^*) \geq U_i(P_i^* - \Delta P) + U_j(P_j^* + \Delta P), \quad (10)$$

which is equivalent to

$$\frac{U_i(P_i^*) - U_i(P_i^* - \Delta P)}{\Delta P} \geq \frac{U_j(P_j^* + \Delta P) - U_j(P_j^*)}{\Delta P}. \quad (11)$$

When $\Delta P \rightarrow 0$, we have

$$\lim_{x \rightarrow (P_i^*)^-} U_i'(x) \geq \lim_{x \rightarrow (P_j^*)^+} U_j'(x) \quad (12)$$

and similarly, we obtain

$$\lim_{x \rightarrow (P_j^*)^-} U_j'(x) \geq \lim_{x \rightarrow (P_i^*)^+} U_i'(x). \quad (13)$$

Using (12) and (13), we obtain

$$U_i'(P_i^*) = U_j'(P_j^*) \quad \blacksquare$$

Claim: If $U_j(P_j)$ is a non-decreasing concave function and variables α_j and β_i are given by

$$\frac{\Delta U_j^N}{\Delta P_j^N} = \frac{U_j(P_j[n^* + 1]) - U_j(P_j[n^*])}{P_j[n^* + 1] - P_j[n^*]} = \alpha_j U_j'(P_j^*) \quad (14)$$

and

$$\frac{\Delta U_i^L}{\Delta P_i^L} = \frac{U_i(P_i[n^*]) - U_i(P_i[n^* - 1])}{P_i[n^*] - P_i[n^* - 1]} = \beta_i U_i'(P_i^*), \quad (15)$$

where $\Delta U_j^N = U_j(P_j[n^* + 1]) - U_j(P_j[n^*])$, $\Delta P_j^N = P_j[n^* + 1] - P_j[n^*]$, then, for every i and j ,

$$\beta_i > \alpha_j \approx 1. \quad (16)$$

Proof: $U_j(P_j)$ is a non-decreasing concave function and $P_j[n^* + 1] > P_j^* > P_j[n^*]$. Therefore, approximate equality and inequality are given by

$$\frac{U_j(P_j[n^* + 1]) - U_j(P_j[n^*])}{P_j[n^* + 1] - P_j[n^*]} \approx U_j'(P_j^*), \quad (17)$$

From **Theorem 1** and (17), for every i and j , variables α_j and β_i satisfy the inequalities $\alpha_j \approx 1$ and $\beta_i > 1$, respectively ■

Therefore, the following equation is satisfied by using **Theorem 2** and **Claim** for every i and j :

$$\frac{\Delta U_i^L}{\Delta P_i^L} > \frac{\Delta U_j^N}{\Delta P_j^N}. \quad (18)$$

Thus, if $P_i[n^*]$ is allocated to every i -th sub-carrier before $P_i[n^* + 1]$ is allocated to any j -th sub-carrier, the performance of greedy APA is guaranteed.

For the iteration component, the greedy APA algorithm finds the sub-carrier with the maximum value of system utility per power. Thus, the computational complexity of a single iteration is $O(S)$. We can ignore other components of the iteration and initialization processes because their computational complexity is much smaller than $O(S)$. Therefore, if A_i is the number of power allocations of sub-carrier i , the computational complexity of greedy APA is

$$\sum_{i=1}^{N_c} A_i O(S). \quad (19)$$

We can rewrite (19) as follows because $\max A_i$ is M :

$$O(M S^2). \quad (20)$$

For the proposed scheme, the computational complexity of the initialization is $O(S)$. Furthermore, the computational complexity of the proposed scheme is $O(S)$ during a single iteration because the iteration component is the same as that of greedy APA. However, because of the initialization, the remaining power is given by $\sum_i \Delta P_i^*$, which is always less than $\sum_i \Delta P_i^N$. Therefore, the power allocation to each sub-carrier occurs only once or not at all. Finally, the

maximum computational complexity of the proposed scheme is

$$O(S^2). \quad (21)$$

4. Numerical Results

This section, the performance of the proposed scheme is evaluated in terms of system utility and the number of operations by comparing it with the performances of the greedy APA [3], adaptive transmission scheme (ATS) [4], or water-filling [1] schemes under random varying radio channel conditions. As assumed in section III, the overall system capacity function is a non-decreasing concave function. Therefore, we use the overall system capacity as the system utility for a simple simulation. Although the discrete utility function is considered for the proposed

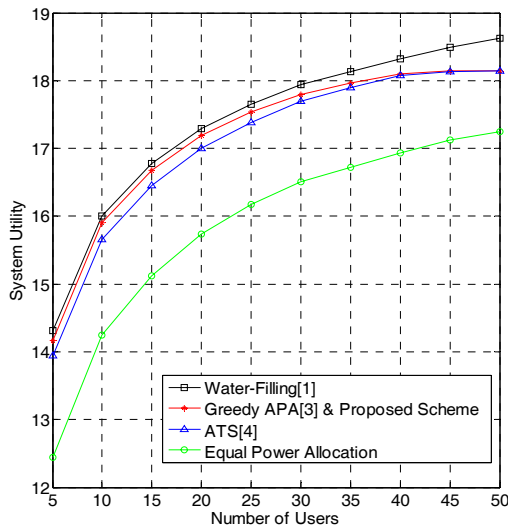


Fig. 2. Average system utility versus number of users

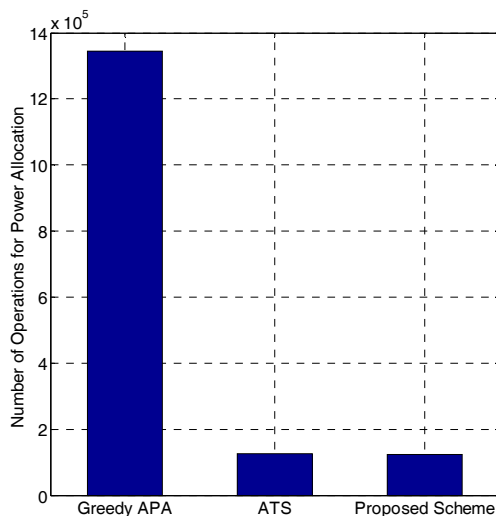


Fig. 3. Number of operations for power allocation

scheme, greedy APA, and ATS, the continuous utility function is considered for the water-filling scheme.

The OFDMA system, proposed by the IEEE 802.16 WMANS standard, is considered herein with 240 sub-carriers and 7 AMC levels. The probability density function of the signal-to-noise ratio (SNR) is assumed to be an exponentially distributed random variable with mean values uniformly distributed between [0, 16] dB [6]. We suppose that user traffic is sufficiently heavy to fully occupy a buffer.

Fig. 2 shows the average system utility for 100 symbol times. In Fig. 2, the maximum C/I method with the dynamic sub-carrier allocation algorithm is used for all power allocation schemes. The system utility of each scheme increases as the number of users increases owing to multiuser diversity. The water-filling scheme, which is the solution to the optimization problem for the continuous system utility function, shows maximum system utility. The proposed scheme outperforms ATS and equal power allocation with the discrete system utility function and shows the same performance as greedy APA.

Fig. 3 shows the number of operations for power allocation, which is the total number of iterations for 100 symbols. The proposed scheme reduces the number of operations by almost thirteen times compared with greedy APA. This is because the average AMC level for each selected user in our simulation is about 6. Therefore, the proposed scheme needs less than one operation for each sub-carrier, however, greedy APA needs more than six operations for each sub-carrier. This result corresponds to (20) and (21).

5. Conclusion

In this paper, we proposed a fast APA algorithm to maximize the discrete system utility for an OFDMA system employing AMC. The proposed scheme showed the same performance as greedy APA, which is the optimum solution for a discrete system utility function. In addition, it reduced the number of operations by almost $2M$ times that of greedy APA. Therefore, the proposed scheme can be used in an actual system that employs AMC to maximize the system utility within a short processing time.

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Sungho Hwang He received the B.S., M.S., and Ph. D. degrees in electrical engineering from Kyungpook National University in 2004, 2006, and 2010, respectively. In 2010, he joined LIG Nex1 R&D center as a Research Engineer. His current research interests

include radio resource management, MAC protocol design, and RADAR resource management.



Ho-Shin Cho He received the B.S., M.S., and Ph. D. degrees in electrical engineering from the Korea Advanced Institute of Science and Technology in 1992, 1994, and 1999, respectively.

From 1999 to 2000, he was a Senior Member of the Research Staff with the Electronics and Telecommunications

Research Institute, where he was involved in developing a base station system for IMT-2000. From 2001 to 2002, he was a faculty member of the School of Electronics, Telecommunications, and Computer Engineering, Hankuk Aviation University. In 2003, he joined faculty of the School of Electronics Engineering, Kyungpook National University. In 2010, He was a visiting professor at University of Connecticut, USA. His current research interests include radio resource management, MAC protocol design, traffic modeling, self-organized network in wireless mobile communication systems and underwater acoustic sensor networks. Prof. Cho was awarded a rising researcher fellowship of KRF in 1998, and is a member of IEEE, IEICE, IEEK, ASK, and KICS.