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Rejection Degree by Fuzzy Significance Probability

Gyu-Tag Choi*† , Il-Soo Park**, Hyun-Woo Nam***
and Jong-Choon Moon****

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Abstract: We propose some properties for fuzzy hypothesis test by fuzzy significance probability. First, we define fuzzy number data and fuzzy significance probability for repeatedly observed data with alternated error term. By the agreement index, we compare fuzzy significance probability with significance level and drawing conclusions the degree of acceptance and rejection by agreement index.

Key Words : Fuzzy number data, Degree of acceptance and rejection, Fuzzy significance probability, Agreement index

1. Introduction

We propose some properties for fuzzy hypothesis test by fuzzy significance probability by agreement index. The negation of the assertion is taken to be the fuzzy null hypothesis H_{f0} and the assertion itself is taken to be the fuzzy alternative hypothesis H_{f1} by obtained data from the fuzzy samples[1].

Kang, Choi and Lee[2] defined fuzzy hypothesis membership function, also they found the agreement index by area for fuzzy hypothesis membership function and membership function of fuzzy critical region. Thus they obtained the results by the grade for judgement to acceptance or rejection for the fuzzy hypothesis.

Thus, we introduction some properties of fuzzy hypothesis test using fuzzy significance probability by agreement index. First we define fuzzy number data for repeatedly observed data with alteration error term in random experiment. Comparing the fuzzy significance probability with significance level for the hypothesis, we draw conclusion for the degree of acceptance and rejection by agreement index.

For the fuzzy hypothesis, we consider

$$H_{f0}: \theta \simeq \psi \quad \text{or} \quad H_{f0}: \theta \prec \psi \ , \quad \theta \in \Theta \tag{1.1}$$

where "$\simeq$" is similarity and "$\prec$" is less than or similarity. We construct a set

$$\{(H_{f0}(\psi), H_{f1}(\psi)) | \psi \in \Theta\} \tag{1.2}$$

with membership function $m_H(\psi)$ where Θ is parameter space.

For satisfying fuzzy statistical data, a fuzzy number A in $\Re$ is said to be convex if for any real numbers $x, y, z \in \mathcal{R}$ with $x \le y \le z$,

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*† Gyu-Tag Choi(corresponding author) : Subdivision of Business Admniistration, KyungNam College of Information & Technology.
E-mail : gtchoi@eagle.kit.ac.kr, Tel : 051-320-1376
**Il-soo Park : Dept. of Refrigeration & Air-conditioning Engineering, TongMyong University.
***Hyun-Woo Nam : Dept. of Fire Safety Management, GyeongSan University
****Jong-Choon Moon : Dept. of Automative Engineering, GyeongSan University

$$m_A(y) \geqq m_A(x) \wedge m_A(z) \tag{1.3}$$

with $\wedge$ standing for minimum.

Also, fuzzy statistical data A is called normal if the following holds

$$\bigvee_{x} m_A(x) = 1 . \tag{1.4}$$

An $\delta-$level set of a fuzzy data number A is a set of $[A]^{\delta}$ and defined by

$$[A]^{\delta} = \{x | m_A(x) \geqq \delta, 0 < \delta \leqq 1\} \tag{1.5}$$

An $\delta-$level set of fuzzy data number A is a convex fuzzy set which is a closed and bounded interval denoted by $[A]^{\delta} = [A_l^{\delta}, A_r^{\delta}]$.

Let A and B be fuzzy numbers data in $\mathcal{R}$ and let $\odot$ be a binary operation defined in $\mathcal{R}$. Then the operation $\odot$ can be extended to the fuzzy numbers A and B by defining the relation of Zadeh's extension principle, as
given $A,B \subset \Re$, $\forall x_i, x_j, x_k \in \Re$;

$$m_{A_\delta \odot B_\delta}(z) = \bigvee_{x_k = x_i \odot x_j} (m_{A_\delta}(x_i) \wedge m_{B_\delta}(x_j)), \tag{1.6}$$

$i,j,k = 1,2,\cdots.$

A modelling the fuzziness of data were described the fuzziness of a fuzzy sample $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$. As a precise fuzzy sample x_i of n precise realizations $x_i \in \Re$ may be regarded as vector in $\Re^n$.

2. Acceptance or rejection degree

Let a test statistics T by fuzzy random sample from sample space Ω. Let $\{P_\theta, \theta \in \Omega\}$ be a family of fuzzy probability distribution, where Θ is a parameter vector of Ω.

Choose a membership function $m_T(x)$ whose value is likely to best reflect the plausibility of the fuzzy hypothesis being tested.

Let us consider membership function $m_C(x)$ of critical region C, which we will call the agreement index of $m_T(x)$ which regard to$m_C(x)$([3],[5]). Thus we have following agreement index.

Definition 2.1. Let a fuzzy membership function $m_T(x)$, $x \in R$, and consider another membership function $m_C(x)$, $x \in R$, then we have a index which call the agreement index as the ratio being defined in the following way;

$$AGI(T,C) = \frac{area(m_T(x) \cap m_C(x))}{area(m_C(x))} \in [0,1] \tag{2.1}$$

where $area(\cdot)$ is area of membership function.

Definition 2.2. We define the maximum grade membership function of rejection or acceptance degree by agreement index for real-valued function $\Re_\delta$ by $\delta-$level as

$$\Re_\delta(0) = \sup_\theta \left\{ \frac{\text{area } (\mathrm{m}_{\mathrm{T}_\delta}(\theta) \cap \mathrm{m}_{\mathrm{C}_\delta}(\theta))}{\text{area } \mathrm{m}_{\mathrm{C}_\delta}(\theta)} \right\}, \tag{2.2}$$

$$\Re_\delta(1) = 1 - \Re_\delta(0) \tag{2.3}$$

by $\delta-level$ for the fuzzy hypothesis testing.

In agreement index, we have the area by $\delta-level$ as;

$$area(m_C(x) \cap m_T(x)) = \int_{\delta_0}^{\delta_1} (C_r^{-1}(\delta) - T_l^{-1}(\delta)) d\delta$$

and

$$area\ m_{C_\delta}(\theta) = \int_{\delta_0}^{1} (C_r^{-1}(\delta) - C_l^{-1}(\delta)) d\delta \tag{2.4}$$

where C_r, C_l are right and left side line of $m_C(x)$, X_l is left side line of $m_T(x)$ and δ_0 is reliable degree and δ_1 is meeting point of $m_C(x)$ and $m_T(x)$.

3. Fuzzy significance probability

A fuzzy test of the fuzzy null hypothesis is a course of separation of a fuzzy set if values of a fuzzy random variable X for which H_{f0} is would be rejected. The fuzzy random variable whose value serves to determinable the action is called the fuzzy test statistic, and the fuzzy set of its values for which H_{f0} is to be rejected is called the fuzzy rejection region of the test. A fuzzy test of specifiable degree by a fuzzy test statistic and the fuzzy rejection region is denoted by (2.2) and (2.3).

For the fuzzy hypothesis, we consider

$$H_{f0}: \theta \simeq \psi \quad \text{or} \quad H_{f0}: \theta < \psi \ , \ \theta \in \Theta.$$

If we have D test statistics T then the significance probability is given by $P(T > x)$ or $P(T < x)$.

If the fuzzy significance probability is $P(T < x) > \alpha$ then we reject the hypothesis where α is fuzzy significance level. Considering the unknown state of nature and the possible results from applying a fuzzy test, one of the following situations will arise:

	UNKNOWN TRUE STATE OF NATURE	
TEST concludes	H_{f0} true $\theta < \psi$	H_{f0} false $\theta > \psi$
Do not reject H_{f0}	Correct degree	Wrong degree (type Ⅱ error)
Reject H_{f0}	Wrong degree (type Ⅰ error)	Correct degree

Fuzzy Type Ⅰ error : rejection degree of H_{f0} when H_0 is true.

Fuzzy Type Ⅱ error : failure degree to reject H_{f0} when H_1 is true.

The probabilities of the two types of error

$\alpha = P$[type Ⅰ error] = P[rejection degree of H_{f0} when H_{f0} is true],

$\beta = P$[type Ⅱ error] = P[not rejecting degree H_{f0} when H_{f1} is true].

The fuzzy probability α depends on the particular value of the parameter in the range covered by H_{f0}, whereas β depends on the value over the range covered by H_{f1} and $\gamma(\theta) = P$ [the fuzzy test rejects H_{f0} when the true value of the parameter is θ].

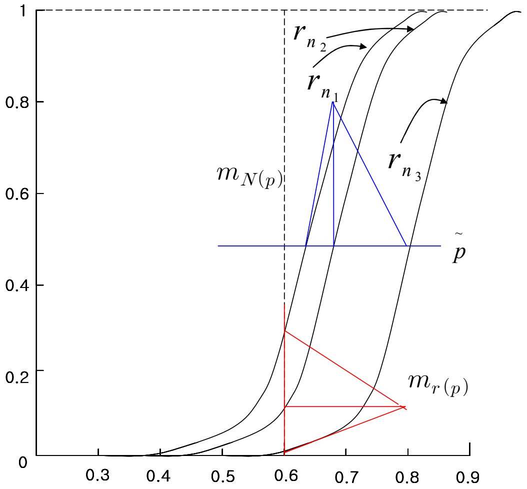

Fig. 1 A power curves

Under H_{f0}, θ is restricted to the range $\theta < \psi$, which is to the left of the middle vertical membership function in Fig. 1 In this part of the graph, the rejection probability $\gamma(\theta)$ is, by definition, the same as the type Ⅰ error fuzzy probability $\alpha(\theta)$.

Under H_1, the range of θ is $\theta > \psi$, which is to the right of the middle vertical membership function. In this range, $1 - \gamma(\theta) = P$[retain H_{f0}]=P[type Ⅱ error]=$\beta(\theta)$. Thus the graph of the fuzzy rejection probability curve $\gamma(\theta)$ of a test provides a complete picture of the performance of the test for all possible contingencies with regard to the true degree state of fuzzy nature.

4. Drawing conclusions

For drawing a conclusion, a researchers have fuzzy number samples by fictitious data for modulus of elasticity(Gpa) by Young's modulus of collected 10 amount of work for iron as;

[174, 176], [188, 192], [213, 217], [196, 200],
[183, 185], [205, 209], [207, 213], [191, 195],
[194, 198], [179, 181].

The data indicate a cause for concern that modulus of elasticity(Gpa) is about 202. Thus, the null hypothesis should be

$$H_{f0}:\ \mu \simeq \widetilde{202}\,.$$

Since the counts are spread over a wide range, an approximation by a continuous distribution is not unrealistic for inference about the fuzzy mean. Assuming that the measurements consistence a fuzzy sample from a normal population and that the level of significance $\alpha=0.1$. We employ the *t*-test

$$\tilde{t}=\frac{|\tilde{\tilde{\bar{X}}}-\widetilde{202}|}{\tilde{s}/\sqrt{10}} \tag{4.1}$$

with rejection region $t \leqq -t_{0.1/2}$ or $t \geqq t_{0.1/2}$ From the $t-$table, we determine that $t_{0.05}=1.833$ with $d.f.=9$.

Computations from the sample data yields

$$\tilde{\tilde{\bar{X}}}=[193.0+1.8\delta,\ 196.6-1.8\delta],$$

$$\tilde{s}^2=[106.311+51.511\delta+14.8\delta^2,\ 268.533-110.711\delta+14.8\delta^2].$$

If we have $\delta=1$,

$$\tilde{t}=\frac{|\tilde{\tilde{\bar{X}}}-202|}{\tilde{s}/\sqrt{10}}=[1.733, 1.733] < t\,(0.05\,;9)=1.833. \tag{4.2}$$

By (4.2), we accept the fuzzy alternative hypothesis H_{f0}. If $\delta=0.0$ then we have

$$\tilde{t}=\frac{|\tilde{\tilde{\bar{X}}}-[201+\delta,\ 203-\delta]|}{\tilde{s}/\sqrt{10}}=[0.849,\ 3.067] \tag{4.3}$$

and $t\,(0.05\,;9)=1.833$.

From (2.2) and (2.3), we have

$$\int_0^{0.927}(-1.333\delta+3.067-1.833)\,d\delta\Big/ \int_0^{1}((-1.333\delta+3.067)-(0.884\delta+0.849))\,d\delta = 0.516. \tag{4.4}$$

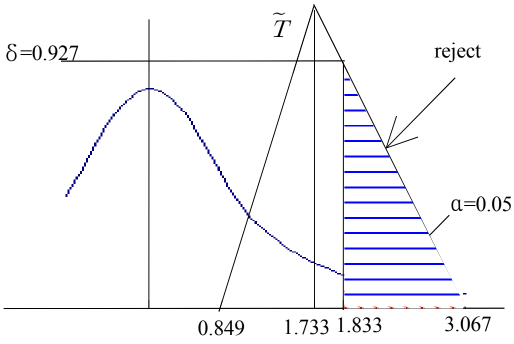

Fig. 2 Rejection degree by agreement index

Thus, we accept the fuzzy alternative hypothesis H_{f0} degree of $\Re_{\gamma}(0)=0.516$ by [Definition 3.1] as [Fig. 2], and we can show that the membership function of rejection degree by significance probability to [Fig. 3].

Another hand, we can show the rejection degree of the fuzzy hypothesis H_{f0} by significance probability as $\Re_{\gamma}(0) = 0.163$ by [Definition 3.1] and [Fig. 4].

We can compare the properties of the fuzzy test statistics with fuzzy number samples, the fuzzy test by significance probability is excellent test for statistical hypothesis rather than crisp t-test.

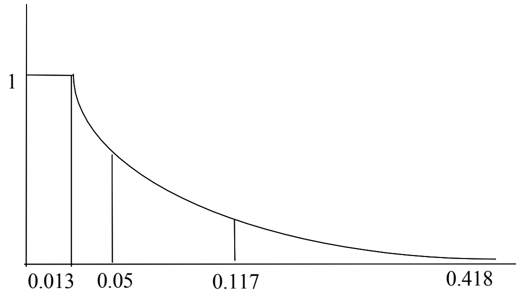

Fig. 3 Rejection degree by significance probability

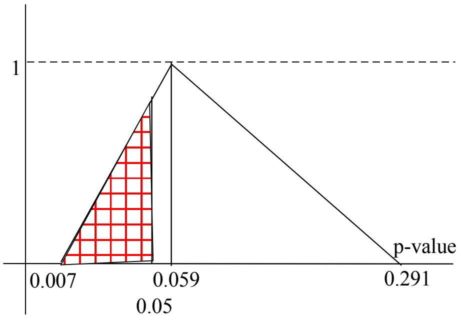

Fig. 4 Degree of significance probability

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