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Generalization of a Transformation Formula for the Exton's Triple Hypergeometric Series X_{12} and X_{17}

Junesang Choi*
Department of Mathematics, Dongguk University, Gyeongju 780-714, Republic of Korea
e-mail : junesang@mail.dongguk.ac.kr

Arjun K. Rathie
Department of Mathematics, School of Mathematical & Physical Sciences, Central University of Kerala, Riverside Transit Campus, Padennakad P.O. Nileshwar, Kasaragod-671 328, India
e-mail : akrathie@gmail.com

Abstract. In the theory of hypergeometric functions of one or several variables, a remarkable amount of mathematicians's concern has been given to develop their transformation formulas and summation identities. Here we aim at generalizing the following transformation formula for the Exton's triple hypergeometric series X_{12} and X_{17}:

$$\begin{aligned}(1+2z)^{-b}\, X_{17}\left(a,\, b,\, c_3;\, c_1,\, c_2,\, 2\,c_3;\; x,\, \frac{y}{1+2z},\, \frac{4z}{1+2z}\right) \\ = X_{12}\left(a,\, b;\, c_1,\, c_2,\, c_3+\frac{1}{2};\, x,\, y,\, z^2\right).\end{aligned}$$

The results are derived with the help of two general hypergeometric identities for the terminating ${}_2F_1(2)$ series which were very recently obtained by Kim et al. Four interesting results closely related to the Exton's transformation formula are also chosen, among ten, to be derived as special illustrative cases of our main findings. The results easily obtained in this paper are simple and (potentially) useful.

1. Introduction and Preliminaries

In 1982, Exton [6] published a very interesting and useful research paper in

* Corresponding Author.

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which he encountered a number of triple hypergeometric functions of second order whose series representations involve such products as $(a)_{2m+2n+p}$ and $(a)_{2m+n+p}$ and introduced a set of 20 distinct triple hypergeometric functions X_1 to X_{20} and also gave their integral representations of Laplacian type which include the confluent hypergeometric functions ${}_0F_1$, ${}_1F_1$, a Humbert function ψ_1 and a Humbert function ϕ_2 in their kernels. It is not out of place to mention here that the Exton's functions X_1 to X_{20} have been studied a lot until today, see, for example, the works [2], [3], [4], [5], [8], [9], [10] and [11]. Moreover, Exton [6] presented a large number of very interesting transformation formulas and reducible cases with the help of two known results which are called in the literature as Kummer's first and second transformations or theorems.

Here we are interested in the following Exton's triple hypergeometric series X_{12} defined by

$$X_{12}\,(a,\, b;\, c_1,\, c_2,\, c_3;\, x,\, y,\, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+2r}}{(c_1)_m\,(c_2)_n\,(c_3)_r} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^r}{r!} \tag{1.1}$$

and its integral representation is given by

$$\begin{aligned} X_{12}\,(a,\, b;\, c_1,\, c_2,\, c_3;\, x,\, y,\, z) &= \frac{1}{\Gamma(a)\,\Gamma(b)} \\ \cdot \int_0^\infty \int_0^\infty e^{-s-t}\, s^{a-1}\, t^{b-1}\, {}_0F_1 &\left[\begin{matrix} -\,; \\ c_1\,; \end{matrix}\; x\, s^2\right] {}_0F_1 \left[\begin{matrix} -\,; \\ c_2\,; \end{matrix}\; y\, s\, t\right] {}_0F_1 \left[\begin{matrix} -\,; \\ c_3\,; \end{matrix}\; z\, t^2\right] ds\, dt \end{aligned} \tag{1.2}$$

and Exton's triple hypergeometric series X_{17} defined by

$$X_{17}\,(a,\, b,\, c;\, d_1,\, d_2,\, d_3;\, x,\, y,\, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+r}\,(c)_r}{(d_1)_m\,(d_2)_n\,(d_3)_r} \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^r}{r!} \tag{1.3}$$

and its integral representation is given by

$$\begin{aligned} X_{17}\,(a,\, b,\, c;\, d_1,\, d_2,\, d_3;\, x,\, y,\, z) &= \frac{1}{\Gamma(a)\,\Gamma(c)} \\ \cdot \int_0^\infty \int_0^\infty e^{-s-t}\, s^{a-1}\, t^{c-1}\, {}_0F_1 &\left[\begin{matrix} -\,; \\ d_1\,; \end{matrix}\; x\, s^2\right] \psi_2\,(b;\, d_2,\, d_3;\, ys,\, zt)\; ds\, dt, \end{aligned} \tag{1.4}$$

where ψ_2 denotes a Humbert function (see, for example, [13, p. 25]).

In the above definitions and their integral representations, the *generalized hypergeometric series* ${}_pF_q$ is defined by (see [12, p. 73]):

$$\begin{aligned} {}_pF_q \left[\begin{matrix} \alpha_1,\, \ldots,\, \alpha_p\,; \\ \beta_1,\, \ldots,\, \beta_q\,; \end{matrix}\; z\right] &= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \frac{z^n}{n!} \\ &= {}_pF_q(\alpha_1,\, \ldots,\, \alpha_p;\, \beta_1,\, \ldots,\, \beta_q;\, z) \quad (p,\, q \in \mathbb{N}_0)\,, \end{aligned} \tag{1.5}$$

where $(\lambda)_\nu$ denotes the Pochhammer symbol or the *shifted factorial*, since

$$(1)_n = n! \quad (n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\};\ \mathbb{N} := \{1,\, 2,\, 3,\, \cdots\}),$$

which is defined (for $\lambda,\, \nu \in \mathbb{C}$), in terms of the familiar Gamma function Γ, by

$$= \begin{cases} 1 & (\nu = 0;\ \lambda \in \mathbb{C} \setminus \{0\}) \\ \lambda(\lambda+1)\cdots(\lambda+n-1) & (\nu = n \in \mathbb{N};\ \lambda \in \mathbb{C}), \end{cases} \tag{1.6}$$

it being *understood conventionally* that $(0)_0 := 1$ and $\mathbb{C}$ the set of complex numbers.

The precise three-dimensional region of convergence of (1.1) and (1.3) can be seen in the monograph of Srivastava and Karlsson [13, pp. 87–107].

In the theory of hypergeometric functions of one or several variables, a remarkable amount of mathematicians's concern has been given to develop their transformation formulas and summation identities. Here we aim at generalizing the following transformation formula for the Exton's triple hypergeometric series X_{12} and X_{17}:

$$\begin{aligned} &(1+2z)^{-b}\, X_{17}\left(a,\, b,\, c_3;\, c_1,\, c_2,\, 2\,c_3;\ x,\ \frac{y}{1+2z},\ \frac{4z}{1+2z}\right) \\ &\qquad = X_{12}\left(a,\, b;\, c_1,\, c_2,\, c_3 + \frac{1}{2};\, x,\, y,\, z^2\right). \end{aligned} \tag{1.7}$$

It is noted that, for convenience and simplicity, we will use

$$\sum_{m,\,n,\,r=0}^{\infty} \quad \text{for} \quad \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}.$$

In order to obtain a natural generalization of the Exton's transformation formula (1.7), we require the following results recently obtained by Kim et al. [7]:

$$\begin{aligned} &{}_2F_1\left[\begin{matrix} -2n,\, \alpha\,; \\ 2\alpha + j\,; \end{matrix}\ 2\right] \\ &\quad = \mathcal{A}_j\, \frac{\Gamma(\alpha)\,\Gamma(1-\alpha)\,\left(\frac{1}{2}\right)_n \left(\alpha + \left[\frac{j+1}{2}\right]\right)_n}{\Gamma\left(\alpha + \frac{1}{2}j + \frac{1}{2}|j|\right)\,\Gamma\left(1 - \alpha - \left[\frac{j+1}{2}\right]\right)\,\left(\alpha + \frac{1}{2}j\right)_n\,\left(\alpha + \frac{1}{2}j + \frac{1}{2}\right)_n} \end{aligned} \tag{1.8}$$

and

$$\begin{aligned} &{}_2F_1\left[\begin{matrix} -2n-1,\, \alpha\,; \\ 2\alpha + j\,; \end{matrix}\ 2\right] \\ &\quad = \frac{\mathcal{B}_j}{2\alpha + j}\, \frac{\Gamma(-\alpha)\,\Gamma(\alpha+1)\,\left(\frac{3}{2}\right)_n \left(1 + \alpha + \left[\frac{j}{2}\right]\right)_n}{\Gamma\left(\alpha + \frac{1}{2}j + \frac{1}{2}|j|\right)\,\Gamma\left(-\alpha - \left[\frac{j}{2}\right]\right)\,\left(\alpha + \frac{1}{2}j + \frac{1}{2}\right)_n\,\left(\alpha + \frac{1}{2}j + 1\right)_n}, \end{aligned} \tag{1.9}$$

where $n \in \mathbb{N}_0$, $j = 0, \pm 1, \ldots, \pm 5$, $[x]$ is the greatest integer less than or equal to x and its modulus is denoted by $|x|$, and the coefficients $\mathcal{A}_j$ and $\mathcal{B}_j$ are given in the following table.

TABLE

j	$\mathcal{A}_j$	$\mathcal{B}_j$
5	$-4(1-\alpha-2n)^2+2(1-\alpha)(1-\alpha-2n)$ $+(1-\alpha)^2+22(1-\alpha-2n)$ $-13(1-\alpha)-20$	$4(\alpha+2n)^2-2(1-\alpha)(\alpha+2n)$ $+(1-\alpha)^2+34(\alpha+2n)$ $+(1-\alpha)+62$
4	$2(\alpha+1+2n)(\alpha+3+2n)-\alpha(\alpha+3)$	$4(\alpha+2n+3)$
3	$-\alpha-4n-2$	$-3\alpha-4n-6$
2	$-\alpha-1-2n$	-2
1	-1	1
0	1	0
-1	1	1
-2	$1-\alpha-2n$	2
-3	$1-\alpha-4n$	$3-3\alpha-4n$
-4	$2(1-2\alpha-n)(3-\alpha-2n)-(1-\alpha)(4-\alpha)$	$4(1-\alpha-2n)$
-5	$4(1-\alpha-2n)^2-2(1-\alpha)(1-\alpha-2n)$ $-(1-\alpha)^2+8(1-\alpha-2n)+7\alpha-7$	$4(\alpha+2n)^2+2(1-\alpha)(\alpha+2n)$ $-(1-\alpha)^2-16(\alpha+2n)+\alpha-1$

2. Main Transformation Formulas

A natural generalization of Exton's transformation formula (1.7) to be established is as follows:

$$
\begin{aligned}
&(1+2z)^{-b}\, X_{17}\left(a,\, b,\, c_3;\, c_1,\, c_2,\, 2\,c_3+j;\; x,\, \frac{y}{1+2z},\, \frac{4z}{1+2z}\right)\\
&= \frac{\Gamma(c_3)\,\Gamma(1-c_3)}{\Gamma\left(c_3+\frac{1}{2}j+\frac{1}{2}|j|\right)\,\Gamma\left(1-c_3-\left[\frac{j+1}{2}\right]\right)}\\
&\quad\cdot \sum_{m,\,n,\,r=0}^{\infty} \mathcal{C}_j\, \frac{(a)_{2m+n}\,(b)_{n+2r}\,\left(c_3+\left[\frac{j+1}{2}\right]\right)_r}{(c_1)_m\,(c_2)_n\,\left(c_3+\frac{1}{2}j\right)_r\,\left(c_3+\frac{1}{2}j+\frac{1}{2}\right)_r}\, \frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^{2r}}{r!}\\
&\quad - \frac{2bz}{2c_3+j}\, \frac{\Gamma(-c_3)\,\Gamma(1+c_3)}{\Gamma\left(c_3+\frac{1}{2}j+\frac{1}{2}|j|\right)\,\Gamma\left(-c_3-\left[\frac{j}{2}\right]\right)}\\
&\quad\cdot \sum_{m,\,n,\,r=0}^{\infty} \mathcal{D}_j\, \frac{(a)_{2m+n}\,(b+1)_{n+2r}\,\left(1+c_3+\left[\frac{j}{2}\right]\right)_r}{(c_1)_m\,(c_2)_n\,\left(c_3+\frac{1}{2}j+\frac{1}{2}\right)_r\,\left(c_3+\frac{1}{2}j+1\right)_r}\, \frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^{2r}}{r!}\\
&\qquad (j=0,\,\pm 1,\,\ldots,\,\pm 5),
\end{aligned}
\tag{2.1}
$$

where the coefficients $\mathcal{C}_j$ and $\mathcal{D}_j$ here can be obtained by simply changing n and α into r and c_3, respectively, in the Table of $\mathcal{A}_j$ and $\mathcal{B}_j$.

Proof. For convenience and simplicity, by denoting the left-hand side of (2.1) by S and using the series definition of X_{17} as given in (1.3), after a little simplification, we have

$$S = \sum_{m,\,n,\,p=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+p}\,(c_3)_p}{(c_1)_m\,(c_2)_n\,(2\,c_3+j)_p}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^p}{p!}\,2^{2p}\,(1+2z)^{-(b+n+p)}.$$

Using the binomial theorem (see [12, p. 58]) for the last factor, we get

$$S = \sum_{m,\,n,\,p,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+p}\,(c_3)_p}{(c_1)_m\,(c_2)_n\,(2\,c_3+j)_p}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^p}{p!}\,2^{2p}\,\frac{(b+n+p)_r}{r!}\,(-1)^r\,2^r\,z^r.$$

Using the identity $(b)_{n+p}\,(b+n+p)_r = (b)_{n+p+r}$, after a little simplification, we obtain

$$S = \sum_{m,\,n,\,p,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+p+r}\,(c_3)_p}{(c_1)_m\,(c_2)_n\,(2\,c_3+j)_p}\,(-1)^r\,2^{2p+r}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^{p+r}}{p!\,r!}.$$

Now using the following well known formal manipulation of double series (see [12, p. 56]; for other manipulations, see also [1, Eq. (1.4)]):

$$\sum_{r=0}^{\infty}\sum_{p=0}^{\infty} A(p,r) = \sum_{r=0}^{\infty}\sum_{p=0}^{r} A(p,r-p),$$

after a little simplification, we have

$$S = \sum_{m,\,n,\,r=0}^{\infty}\sum_{p=0}^{r} \frac{(a)_{2m+n}\,(b)_{n+r}\,(c_3)_p}{(c_1)_m\,(c_2)_n\,(2\,c_3+j)_p}\,(-1)^{r-p}\,2^{p+r}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^r}{p!\,(r-p)!}.$$

Using the following formula

$$(r-p)! = \frac{(-1)^p\,r!}{(-r)_p} \qquad \left(0 \leqq p \leqq r;\ r,\,p \in \mathbb{N}_0\right),$$

after a little simplification, we get

$$S = \sum_{m,\,n,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+r}(-2)^r}{(c_1)_m\,(c_2)_n}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^r}{r!}\sum_{p=0}^{r}\frac{(-r)_p\,(c_3)_p}{(2\,c_3+j)_p}\,\frac{2^p}{p!}.$$

Using the definition of ${}_pF_q$ in (1.5) for the inner series, we obtain

$$S = \sum_{m,\,n,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+r}(-2)^r}{(c_1)_m\,(c_2)_n}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^r}{r!}\,{}_2F_1\left[\begin{matrix} -r,\,c_3\,; \\ 2\,c_3+j\,; \end{matrix}\ 2\right].$$

Separating r into even and odd integers and making use of the following identities:

$$(\alpha)_{2r} = 2^{2r}\,(\alpha)_r\left(\frac{\alpha}{2}\right)_r \quad \text{and} \quad (\alpha)_{r+1} = \alpha\,(\alpha+1)_r,$$

after a little simplification, we have

$$\begin{aligned} S = &\sum_{m,\,n,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b)_{n+2r}}{(c_1)_m\,(c_2)_n\,\left(\frac{1}{2}\right)_r}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^{2r}}{r!}\;{}_2F_1\left[\begin{matrix}-2r,\,c_3\,;\\ 2\,c_3+j\,;\end{matrix}\;2\right] \\ &- b\,z \sum_{m,\,n,\,r=0}^{\infty} \frac{(a)_{2m+n}\,(b+1)_{n+2r}}{(c_1)_m\,(c_2)_n\,\left(\frac{3}{2}\right)_r}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^{2r}}{r!}\;{}_2F_1\left[\begin{matrix}-2r-1,\,c_3\,;\\ 2\,c_3+j\,;\end{matrix}\;2\right]. \end{aligned}$$

Finally, using the known results (1.8) and (1.9), after a little simplification, we easily arrive at the right-hand side of (2.1). This completes the proof of (2.1). □

3. Special Cases

In our main formula (2.1), if we take $j = 0,\ \pm 1$ and ± 2, after a little simplification, and we interpret the respective resulting right-hand side with the definition of the triple hypergeometric series X_{12} given in (1.1), we get the following very interesting results:

The case $j = 0$.

$$\begin{aligned} &(1+2z)^{-b}\,X_{17}\left(a,\,b,\,c_3;\,c_1,\,c_2,\,2\,c_3;\;x,\,\frac{y}{1+2z},\,\frac{4z}{1+2z}\right) \\ &\qquad = X_{12}\left(a,\,b;\,c_1,\,c_2,\,c_3+\frac{1}{2};\,x,\,y,\,z^2\right). \end{aligned} \tag{3.1}$$

The case $j = 1$.

$$\begin{aligned} &(1+2z)^{-b}\,X_{17}\left(a,\,b,\,c_3;\,c_1,\,c_2,\,2\,c_3+1;\;x,\,\frac{y}{1+2z},\,\frac{4z}{1+2z}\right) \\ &\qquad = X_{12}\left(a,\,b;\,c_1,\,c_2,\,c_3+\frac{1}{2};\,x,\,y,\,z^2\right) \\ &\qquad\quad - \frac{2bz}{2\,c_3+1}\,X_{12}\left(a,\,b+1;\,c_1,\,c_2,\,c_3+\frac{3}{2};\,x,\,y,\,z^2\right). \end{aligned} \tag{3.2}$$

The case $j = -1$.

$$\begin{aligned} &(1+2z)^{-b}\,X_{17}\left(a,\,b,\,c_3;\,c_1,\,c_2,\,2\,c_3-1;\;x,\,\frac{y}{1+2z},\,\frac{4z}{1+2z}\right) \\ &\qquad = X_{12}\left(a,\,b;\,c_1,\,c_2,\,c_3-\frac{1}{2};\,x,\,y,\,z^2\right) \\ &\qquad\quad + \frac{2bz}{2\,c_3-1}\,X_{12}\left(a,\,b+1;\,c_1,\,c_2,\,c_3+\frac{1}{2};\,x,\,y,\,z^2\right). \end{aligned} \tag{3.3}$$

The case $j = 2$.

$$
\begin{aligned}
&(1+2z)^{-b}\, X_{17}\left(a,\, b,\, c_3;\, c_1,\, c_2,\, 2\,c_3+2;\; x,\, \frac{y}{1+2z},\, \frac{4z}{1+2z}\right)\\
&\quad = X_{12}\left(a,\, b;\, c_1,\, c_2,\, c_3+\frac{3}{2};\, x,\, y,\, z^2\right)\\
&\qquad - \frac{2bz}{c_3+1}\, X_{12}\left(a,\, b+1;\, c_1,\, c_2,\, c_3+\frac{3}{2};\, x,\, y,\, z^2\right)\\
&\qquad + \frac{4b(b+1)\, z^2}{(c_3+1)\,(2c_3+3)}\, X_{12}\left(a,\, b+2;\, c_1,\, c_2,\, c_3+\frac{5}{2};\, x,\, y,\, z^2\right).
\end{aligned} \tag{3.4}
$$

The case $j = -2$.

$$
\begin{aligned}
&(1+2z)^{-b}\, X_{17}\left(a,\, b,\, c_3;\, c_1,\, c_2,\, 2\,c_3-2;\; x,\, \frac{y}{1+2z},\, \frac{4z}{1+2z}\right)\\
&\quad = X_{12}\left(a,\, b;\, c_1,\, c_2,\, c_3-\frac{1}{2};\, x,\, y,\, z^2\right)\\
&\qquad + \frac{2bz}{c_3-1}\, X_{12}\left(a,\, b+1;\, c_1,\, c_2,\, c_3-\frac{1}{2};\, x,\, y,\, z^2\right)\\
&\qquad + \frac{4b(b+1)\, z^2}{(c_3-1)\,(2c_3-1)}\, X_{12}\left(a,\, b+2;\, c_1,\, c_2,\, c_3+\frac{1}{2};\, x,\, y,\, z^2\right).
\end{aligned} \tag{3.5}
$$

Remark. Clearly Equation (3.1) is the Exton's result given in Equation (1.7) and the Equations (3.2) to (3.5) are closely related to it. The other special cases of (2.1) can also be expressed in terms of X_{12} in the similar manner.

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