한국복합신소재구조학회 논문집 제5권, 제4호, 2014년 12월
J. Korean Soc. Adv. Comp. Struc. Vol. 5, No. 4, pp. 18-23 December 2014
DOI http://dx.doi.org/10.11004/kosacs.2014.5.4.018

ISSN 2093-5145(Print)
ISSN 2288-0232(Online)

복합신소재 구조물의 형상비에 따른 고유진동수의 영향

김윤영[1] · 한봉구[2]

삼성전자 반도체연구소 CAE 팀, 책임연구원[1], 서울과학기술대학교 건설시스템디자인공학과[2],

The Effect of the Aspect Ratio on the Natural Frequency of the Advanced Composite Structures

Kim, Yun Young[1] · Han, Bong Koo[2]

1 Senior Engineer, Semiconductor R&D Center, Samsung Electronics, Gyeonggi-Do, Korea
[2] Professor, Dept. of Civil Engineering, Seoul National University of Science, Seoul, Korea

Abstract: In this paper. the effects of the aspect ratio on the natural frequency of the advanced composite road structures is studied. The advanced composite structures are too difficult for such design engineers for construction and some simple but accurate enough methods are necessary. Some laminate orientations have decreasing values of D_{16}, B_{16}, D_{26} and B_{26} stiffnesses as the ply number increases. The plate aspect ratio considered is from 1 to 5. Most of the road structures have large aspect ratios, for such cases further simplification is possible by neglecting the effect of the longitudinal moment terms.

Key Words: advanced composite structures, effect of aspect ratio, natural frequency, finite difference method

1. INTRODUCTION

The problem of deteriorating infrastructures is very serious all over the world. The U.S. Civil Engineering Research Foundation (CERF) report, “High-Performance Construction Material and System : An Essential Program for America and its Infrastructure", published, in collaboration with several organizations, U.S. Department of Transportation figures as follows :

(1) The road bridge condition in U.S.A at the year 2009, 149,654 of the Americans 603,259 bridges are structurally deficient or obsolete. (structurally deficient 71,177, functionally obsolete 78,477)
(2) 199,584 of these bridges are more than 50 years old and unsuitable for current or projected traffic.
(3) Traffic delays alone will cost Americans $115 billion per year in lost work time and fuel by the year 2009.

Steel girders become rusty. The reinforcing bars embedded in concrete beams or slabs are subject to corrosion caused by electro-chemical action. Underground fuel thanks are under similar condition. In 1979, the U.S. Bureau of Standards (NIST) study showed that yearly loss caused by corrosion related damages mounted to 82 billion dollars, about 4.9% of GNP. About 32 billion dollars could be saved if existing technologies were used to prevent such losses.

These figures are in the United States of America, where various federal, state, and other agencies are doing their best in maintaining such structures in good

주요어: 복합신소재구조물, 형상비의 영향, 고유진동수, 유한차분법

Corresponding author: Han, Bong Koo
Department of Civil Engineering, Seoul National University of Science, 232 Gongneung-ro, Nowon-gu, Seoul 139-743, Korea
Tel: +82-2-970-6577, Fax: +82-2-948-0043, E-mail: bkhan@seoultech.ac.kr

Received December 12, 2014 / Revised December 22, 2014 / Accepted December 26, 2014

condition. The issue of deteriorating and damaged infrastructures and lifelines has become a critically important subject in the United States as well as Japan and Europe. The problem in developing nations, where degree of construction quality control and maintenance are in question, must be much more profound [Kim, 1995, 1999].

The advanced composite materials can be effectively used for repairing such structures. Because of the advantages of these materials, such repair job can fulfill two purposes :

(1) Repair of existing damage caused by corrosion, impact, earthquake, and others.

(2) Reinforcing the structure against anticipated future situation which will require increasing the load beyond the design parameters used for this structure.

Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in-situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The reinforced concrete slab can be assumed as a $[90^0,0^0,90^0]_r$ type specially orthotropic plate as a close approximation, assuming that the effect of B_{16}, B_{26}, D_{16} and D_{26} stiffness are negligible. Many of the bridge and building floor systems, including the girders and cross-beams, also behave as similar specially orthotropic plates. Such plates are subject to the concentrated mass/masses in the form of traffic loads, or the test equipments such as the accelerator in addition to their own masses. Analysis of such problems is usually very difficult.

The most of the design engineers for construction has academic background of bachelors degree. Theories for advanced composite structures are too difficult for such engineers and some simple but accurate enough methods are necessary.

The author has reported that some laminate orientations such as $[\alpha,\beta]_{r,}$ $[\alpha,\beta,\gamma]_{r,}$ $[\alpha,\beta,\beta,\alpha,\alpha,\beta]_{r,}$ and $[\alpha,\beta,\beta,\gamma,\alpha,\alpha,\beta]_{r,}$ with $\alpha = -\beta$, and $\gamma = 0^0$ or 90^0 and with increasing r, have decreasing values of B_{16}, B_{26}, D_{16} and D_{26} stiffness. Most of the civil and architectural structures are large in sizes and the numbers of laminae are large, even though the thickness to length ratios are small enough to allow to neglect the transverse shear deformation effects in stress analysis. For such plates, the fiber orientations given above behave as specially orthotropic plates and simple formulas developed by the reference [Kim, 1995, 1996] can be used. Most of the bridge and building slabs on girders have large aspect ratios. For such cases further simplification is possible by neglecting the effect of the longitudinal moment terms (Mx) on the relevant partial differential equations of equilibrium. In this paper, the result of the study on the subject problem is presented. Even with such assumption, the specially orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution [Kim 1995, Han & Kim 2004, 2010, 2011].

The method of vibration analysis used is the one developed by the author. He developed and reported, a simple but exact method of calculating the natural frequency of beam and tower structures with irregular cross-sections and attached mass/masses. Since 1989, this method has been extended to two-dimensional problems with several types of given conditions and has been reported at several international conferences. This method uses the deflection influence surfaces. The finite difference method is used for this purpose, in this paper.

2. METHOD OF ANALYSIS

The equilibrium equation for the specially orthotropic plate is :

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}=q(x,y) \qquad (1)$$

where $D_1 = D_{11}, D_2 = D_{22}, D_3 = D_{12}+2D_{66}$

The assumptions needed for this equation are :

(1) The transverse shear deformation is neglected.

(2) Specially orthotropic layers are arranged so that no coupling terms exist, i.e, $B_{ij}=0, (\ )_{16}=(\ )_{26}=0$.

(3) No temperature or hydrothermal terms exist.

The purpose of this paper is to demonstrate, to the practicing engineers, how to apply this equation to the slab systems made of plate girders and cross-beams.

In case of an orthotropic plate with boundary conditions other than Navier or Levy solution type, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical methods for eigenvalue problems are also very much involved in seeking such a solution [Goldberg et al. 1967, Stephen et al. 1989, Ashton 1970, Whitney 1970]. Finite difference method (F.D.M) is used in this paper. The resulting linear algebraic equations can be used for any cases with minor modifications at the boundaries, and so on.

The problem of deteriorating infrastructures is very serious all over the world. Before making any decision on repair work, reliable non-destructive evaluation is necessary. One of the dependable methods is to evaluate the in-situ stiffness of the structure by means of obtaining the natural frequency. By comparing the in- situ stiffness with the one obtained at the design stage, the degree of damage can be estimated rather accurately.

The basic concept of the Rayleigh method, the most popular analytical method for vibration analysis of a single degree of freedom system, is the principle of conservation of energy ; the energy in a free vibrating system must remain constant if no damping forces act to absorb it. In case of a beam, which has an infinite number of degree of freedom, it is necessary to assume a shape function in order to reduce the beam to a single degree of freedom system. The frequency of vibration can be found by equating the maximum strain energy developed during the motion to the maximum kinetic energy. This method, however, yields the solution either equal to or larger than the real one. Recall that Rayleigh's quotient ≥ 1. For a complex beam, assuming a correct shape function is not possible. In such cases, the solution obtained is larger than the real one.

Design engineers need to calculate the natural frequencies of such element but obtaining exact solution to such problems is very much difficult. Pretlove reported a method of analysis of beams with attached masses using the concept of effective mass. This method, however, is useful only for certain simple types of beams. Such problems can be easily solved by presented method.

A simple but exact method of calculating the natural frequency corresponding to the first mode of vibration of beam and tower structures with irregular cross sections and attached mass/masses was developed and reported. This method consists of determining the deflected mode shape of the member due to the inertia force under resonance condition. Beginning with initially “guessed” mode shape, “exact” mode shape is obtained by the process similar to iteration. Recently, this method was extended to two dimensional problems including composite laminates, and has been applied to composite plates with various boundary conditions with/without shear deformation effects and reported at several international conferences including the Eighth Structures Congress(1990) and Fourth Materials Congress(1996) of American Society of Civil Engineers.

This method is used for vibration analysis in this paper. A natural frequency of a structure is the frequency under which the deflected mode shape corresponding to this frequency begins to diverge under the resonance condition. From the deflection caused by the free vibration, the force required to make this deflection can be found, and from this force, resulting deflection can be obtained. If the mode shape as determined by the series of this process is sufficiently accurate, then the relative deflections (maximum) of both the converged and the previous one should remain unchanged under the inertia force related with this natural frequency. Vibration of a structure is a harmonic motion and the amplitude may contain a part expressed by a trigonometric function.

2.1 Simple method of vibration analysis

In this paper, simple method of vibration analysis given in as follows.

The magnitudes of the maximum deflection at a certain number of points are arbitrarily given as

$$w\ (i,j)^{(1)} = W\,(i,j)^{(1)} \tag{2}$$

where (i,j) denotes the point under consideration. This is absolutely arbitrary but educated guessing is good for accelerating convergence. The dynamic force corresponding to this (maximum) amplitude is

$$F(i,j)^{(1)} = m(i,j)[\omega(i,j)^{(1)}]^2 \mathrm{w(i,j)}^{(1)} \tag{3}$$

The “new” deflection caused by this force is a function of f and can be expressed as

$$\mathrm{w(i,j)}^{(2)} = \mathrm{f}\ \{\mathrm{m(i,j)}[\omega\mathrm{(i,j)}^{(1)}]^2\mathrm{w(i,j)}^{(1)}\}$$
$$= \sum^{k,l} \triangle(i,j,k,l)\{m(i,j)[\omega(i,j)^{(1)}]^2\mathrm{w(k,l)}^{(1)}\} \tag{4}$$

where $\triangle$ is the deflection influence surface. The relative (maximum) deflections at each point under consideration of a structural member under resonance condition, $\mathrm{w(i,j)}^{(1)}$ and $\mathrm{w(i,j)}^{(2)}$, have to remain unchanged and the following condition has to be held :

$$\mathrm{w(i,j)}^{(1)} / \ \mathrm{w(i,j)}^{(2)} = 1 \tag{5}$$

From this equation, $w(i,j)^{(1)}$ at each point of (i,j) can be obtained. But they are not equal in most cases. Since the natural frequency of a structural member has to be equal at all points of the member, i.e. $w(i,j)$ should be equal for all (i,j), this step is repeated until sufficient equal magnitude of $w(i,j)$ is obtained at all (i,j) points.

However, in most cases, the difference between the maximum and the minimum values of $w(i,j)$ obtained by the first cycle of calculation is sufficiently negligible for engineering purposes. The accuracy can be improved by simply taking the average of the maximum and the minimum, or by taking the value of $\mathrm{w(i,j)}$ where the deflection is the maximum. For the second cycle, $\mathrm{w(i,j)}^{(3)}$ is

$$\mathrm{w(i,j)}^{(3)} = \mathrm{f}\{\mathrm{m(i,j)}[\omega\mathrm{(i,j)}^{(2)}]^2\mathrm{w(i,j)}^{(2)}\} \tag{6}$$

the absolute numerics of $\mathrm{w(i,j)}^{(2)}$ can be used for convenience.

2.2 Finite Difference Method

The method used in this paper requires the deflection influence surfaces. F.D.M is applied to the governing equation of the specially orthotropic plates,

$$D_1\frac{\partial^4 w}{\partial x^4} + 2D_3\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_2\frac{\partial^4 w}{\partial y^4}$$
$$= q(x,y) - kw(x,y) + Nx\frac{\partial^2 w}{\partial x^2}$$
$$+ Ny\frac{\partial^2 w}{\partial y^2} + 2Nxy\frac{\partial^2 w}{\partial x \partial y} \tag{7}$$

where, $D_1 = D_{11},\ D_2 = D_{22}, D_3 = (D_{12} + 2D_{66})$.

The number of the pivotal points required in the case of the order of error $\triangle_2$, where $\triangle$ is the mesh size, is five for the central differences,

This makes the procedure at the boundaries complicated. In order to solve such problem, the three simultaneous partial differential equations of equilibrium with three dependent variables, w, M_x and M_y, are used instead of Eq.(7) with $N_x = N_y = N_{xy} = 0$.

$$\frac{\partial^2 M_x}{\partial x^2} - 4D_{66}\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^2 M_y}{\partial y^2}$$
$$= -q(x,y) + kw(x,y) \tag{8}$$

$$M_x = -D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2} \tag{9}$$

$$M_y = -D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2} \tag{10}$$

If F.D.M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme is very efficient to solve such equations.

In order to confirm the accuracy of the F.D.M., [A/B/A]r type laminate with aspect ratio of a/b=1m/1m=1 is considered.

For simplicity, it is assumed that $A = 0^o, B = 90^o$ and r=1. Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M. is used to solve this problem and the result is compared with the Navier solution.

Calculation is carried out with different mesh sizes. The error is less than 1%. This is smaller than the predicted theoretical errors ;

If F.D.M is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim & Han is very efficient to solve such equations [Kim 1995, Han & Kim 2004, 2010, 2011].

Since one of the few efficient analytical solutions of the specially orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M is used to solve this problem. The result is satisfactory as expected.

By neglecting the Mx terms, the sizes of the matrices needed to solve the resulting linear equations are reduced to two thirds of the “non-modified” equations.

3. NUMERICAL EXAMINATION

3.1 Structure under Consideration

The structure under consideration is as shown in Fig.1.

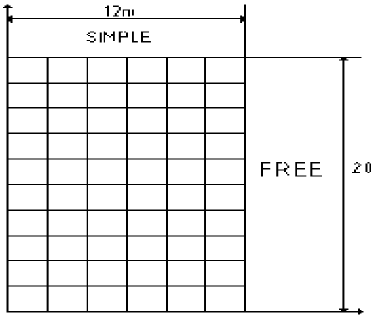

Fig. 1 Structure under Consideration

The material properties are :
$E_1 = 67.36$ GPa, $E_2 = 8.12$ GPa,
$G_{12} = 3.0217$ GPa
$\nu_{12} = 0.272$, $\nu_{21} = 0.0328$, r $=1$
Ply thickness $= 0.005$ m
Orientation : $[90^o, 0^o, 90^o]$r

The stiffnesses are :
$D_{11} = 2929$, $D_{22} = 18492$, $D_{12} = 627$
a $=$nb, n $=$an integer 1 ~ 5
and $D_{66} = 849$, b $=3$ m
Loading : q $= 286.65$ N/m^2

Analysis is carried out and the result is given by Tables from 1 to 2. Increase of the cross-beam sizes does not produce profound change of deflection, Table 1. Similar conclusion can be obtained from the frequency, Table 2.

Table 1. Deflection at the center (m)
Loading : 100 kN at the center

Case	1	2	3	4	5
$\delta(m)$	0.6765 E-01	0.6262 E-01	0.6061 E-01	0.5951 E-01	0.5881 E-01
Case i/ Case 1	1.0000	1.0803	1.1162	1.1368	1.1503

Table 2. Natural Frequency (rad/sec)
Loading : 100 kN at the center

Case	1	2	3	4	5
$w(rad/\sec)$	0.7313 E+01	0.7471 E+01	0.7539 E+01	0.7577 E+01	0.7603 E+01
Case i / Case 1	1.0000	1.0216	1.0309	1.0361	1.0397

3.2 Effects of the Aspect Ratio

The plate are considered as [a/b/a]r composite laminated plate.

In order to study the effect of Mx on the equilibrium equations, two cases are considered :

Case A : w, M_x and M_y are considered.

Case B : w and M_y are considered, M_x is neglected.

F.D.M. is used to obtain w, M_x, M_y and obtain the natural frequency.

Plates with all edges simple supported (SS), the aspect ratio and the natural frequencies at the center of the uniformly loaded plate are as shown in Table 3.

Table 3 Effects of aspect ratio (SS case)

Aspect ratio (a/b)	1	2	3	4	5
ω_A/ω_B	1.0960	1.0303	1.0189	1.0137	1.0107

Plates with one side simple and the other side free supported (SF), the aspect ratio and the natural frequencies at the center of the uniformly loaded plate are as shown in Table 4.

Table 4 Effects of aspect ratio (SF case)

Aspect ratio (a/b)	1	2	3	4	5
ω_A/ω_B	0.9957	0.9977	0.9985	0.9988	0.9991

It is concluded that, for all boundary conditions, neglecting Mx terms is acceptable if the aspect ratio (a/b) is equal to or larger than 2.

4. CONCLUSION

The result of numerical examination is quite promising. When plates with all edges are simple supported (SS), the ratios of the natural frequencies

(ω_A/ω_B) at the center of the uniformly loaded plate range from 1.0960 to 1.0107 according to its aspect ratio (a/b) from 1 to 5. In case Plates with one side simple and the other side free supported (SF), the ratios of the natural frequencies (ω_A/ω_B) at the center of the uniformly loaded plate range from 0.9957 to 0.9991 according to its aspect ratio (a/b) from 1 to 5.

It is concluded that, for all boundary conditions, neglecting the longitudinal moment (M_x) terms is acceptable if the aspect ratio (a/b) is equal to or larger than 2.

References

Ashton, J.E. (1970), “Anisotropic Plate Analysis- Boundary Condition,” J. of Composite Materials. pp. 162-171, April.

Goldberg, John E. and Kim, D.H. (1967), “The Effect of Neglecting the Radial Moment Term in Analyzing a Sectorial Plate by Means of Finite Differences,” Proc, of the Seventh International Symposium on Space Technology and Sciences, Tokyo, Japan

Han, B. K., Kim, D. H. (2004), “Simple Method of Vibration Analysis of Three Span Continuous Reinforced Concrete Bridge with Elastic Intermediate Supportm,” Journal of Korean Society for Composite Materials, Vol. 17, No. 3, pp. 23-28.

Han, B. K. and Lee, C. S. (2010), “Analysis of Rolled Beam Bridge by means of Specially Orthotropic Laminates Theory,” Journal of the Korean Society for Advanced Composite Structures, Vol 1, No 3, pp 35-40.

Han, B. K. and Suk, J. W, (2011), “Influence of Loading Sizes on Natural Frequency of Composite Laminates,” Journal of the Korean Society for Advanced Composite Structures, Vol 2, No 3, pp 12-17.

Kim, D. H. (1995), “Composite Structures for Civil and Architectural Engineering,” E&FN SPON, Chapman & Hall, London.

Kim, D. H., Han, B. K., Lee, J. H., and Hong, C.W.,(1999) “Simple Methods of Vibration Analysis of Three Span Continuous Reinforced Concrete Bridge with Elastic Intermediate Supports,” Proceeding of the Advances in Structural Engineering and Mechanics, Seoul, Vol. 2, pp. 1279 - 1284

Pagano, N .J. (1970), “Exact Solution for Rectangular Bidirectional Composites and Sandwich Plates,” Journal of Composites Materials, Vol. 4, No. 1, pp 20-34.

Stephen P. Timoshenko, and S. Woinowsky-krieger (1989), “Theory of Plates and Shells, Second Edition,” Mcgraw Hill Book Co.

Whitney, J. M. and Leissa, A. W. (1970), “Analysis of a Simply Supported Laminated Anisotropic Rectangular Plate,” J. of AIAA, Vol. 8, No. 1 pp. 28-33.