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복합재료 거동특성의 파괴해석Ⅰ- 이방성 소성 적합모델

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A Progressive Failure Analysis Procedure for Composite LaminatesⅠ - Anisotropic Plastic Constitutive Model

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Abstract: A progressive failure analysis procedure for composite laminates is developed in here and in the companion paper. An anisotropic plastic constitutive model for fiber-reinforced composite material, is developed, which is simple and efficient to be implemented into computer program for a predictive analysis procedure of composites. In current development of the constitutive model, an incremental elastic-plastic constitutive model is adopted to represent progressively the nonlinear material behavior of composite materials until a material failure is predicted. An anisotropic initial yield criterion is established that includes the effects of different yield strengths in each material direction, and between tension and compression. Anisotropic work-hardening model and subsequent yield surface are developed to describe material behavior beyond the initial yield under the general loading condition. The current model is implemented into a computer code, which is Predictive Analysis for Composite Structures (PACS), and is presented in the companion paper. The accuracy and efficiency of the anisotropic plastic constitutive model are verified by solving a number of various fiber-reinforced composite laminates with and without geometric discontinuity. The comparisons of the numerical results to the experimental and other numerical results available in the literature indicate the validity and efficiency of the developed model.

Key Words: Progressive Failure, Composite Laminate, Constitutive Relationships, Anisotropic Plasticity.

1. INTRODUCTION

The high strength- and stiffness -to-weight ratios of advanced composite materials makes these materials attractive for certain critical applications (Yener & Wolcott, 1988). Composites have been increasingly used as structural materials in the space and aerospace industry, aircraft industry, automobile industry, and in various engineering fields.

Composite materials partly behave in a nonlinear fashion, although composite materials generally have been modeled as linear elastic material. The nonlinearity of composite materials can be attributed to inherent material nonlinearity of individual constituents and to micromechanical failures such as fiber or matrix microcracking and interfacial debonding.

That the plasticity theory is capable of mathematically modeling the inelastic material behavior at macroscopic level (Yener & Yi, 1989) is well known. The available plasticity constitutive models for isotropic materials are difficult to use for composite materials. Several theoretical anisotropic plasticity constitutive models have been developed, that are

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generally for metal or are too complex for a numerical computer code.

Several theoretical anisotropic plasticity constitutive models were proposed for composite laminates (Petit & Waddoups, 1969; Hahn & Tsai, 1973; Sandhu, 1974). Aforementioned constitutive models are often used in the practical design field for their simplicity and in spite of their limitations. Their unavoidable shortcoming for a predictive analysis procedure, however, is that they cannot predict the material behavior of permanent strain accumulation by large deformation. Several anisotropic plasticity theories have been developed to describe the plastic behavior of anisotropic materials. Hill's theory (Hill, 1948, 1950) was the first anisotropic yield criterion which is a generalization of von Mises' yield criterion for isotropic material. Recently, Valliappan (1972), as well as Owen and his colleagues (Owen & Figueiras 1983; Owen & Li, 1988), used Whang's approach in the finite element analysis of anisotropic elastic-plastic materials. However, Whang's yield function is not valid for highly anisotropic material such as unidirectional composite materials, and their model does not account for differential between the tensile and compressive yield strengths (i.e., Bauschinger effect).

Aforementioned anisotropic plasticity theories can be considered as generalizations of plasticity theory for isotropic materials, with more material parameters to account for symmetries of material. There have been appeared many general theoretical anisotropic plasticity theories (Edelman & Drucker 1951; Williams & Svensson, 1971; Baltov & Sawczuck, 1965; Eisenberg & Phillips, 1968). Although these theories are very general theoretically, they are too complex to be used in a numerical method. Our work (Yener & Yi, 1990 & 1992) indicates that most constitutive models for composite materials represent slight modifications of the conventional elasticity and plasticity theories.

As mentioned earlier, general response prediction of composite structures becomes possible by developing a realistic and comprehensive analysis procedure for general loading conditions. Such an undertaking, among other considerations, requires very efficient constitutive model which can predict realistically nonlinear material behavior. Hence, the objective of current research is to develop an anisotropic plasticity constitutive model for fiber-reinforced composite laminates that is simple and efficient to be implemented into a computer program for a predictive analysis procedure of composites.

2. Constitutive Modeling of Composite Materials

2.1 Elastic Constitutive Relationship

The most general stress-strain relationship within the linear elasticity can be written in tensor notation as

$$\sigma_{ij} = D_{ijkl}\epsilon_{kl} \tag{1}$$

where σ_{ij} is the stress vector, D_{ijkl} the stress-strain matrix, and ϵ_{kl} the strain vector. In the case of fiber-reinforced composite laminate, which possesses three mutually orthogonal planes of symmetry at each layer, the constants in the stress-strain matrix are reduced to 9 (Tsai & Hahn, 1980). For the convenience of implementation into finite element computer program, the contracted notation is used. As shown in Fig. 1, the planes of symmetry are aligned with material principal axes (1,2,3) in an lamina. With the contracted notation, the stress-strain relation for a lamina becomes

$$\sigma_i = D_{ij}\epsilon_j \qquad (i,j = 1,2,...6) \tag{2}$$

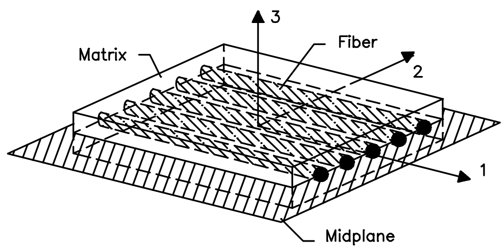

Fig. 1 Unidirectional Fiber-reinforced Composite Lamina

If the principal material directions 1,2 do not coincide with the reference axes x,y, a relation is needed between the stresses and strains. The relationship can be represented in symbolic matrix form as

$$\sigma_{1-2} = [T_\sigma]\sigma_{x-y} \tag{3a}$$

$$\epsilon_{1-2} = [T_\epsilon]\epsilon_{x-y} \tag{3b}$$

where the subscripts 1-2 and x-y refer to the material (local) and global coordinates systems and $[T_\sigma]$ and $[T_\epsilon]$ are the stress transformation and the strain transformation matrix, respectively.

2.2 Anisotropic Plasticity Theory

As pointed out, development of an anisotropic plasticity constitutive model is required for a predictive analysis of fiber-reinforced composite laminates. Incremental theory of plasticity is used as the base of development for an anisotropic plasticity theory, since it more adequately reflects the progressive behavior of material. To be a general constitutive model, work-hardening behavior is assumed. The following three fundamental elements of plasticity are developed for the anisotropic plasticity theory for fiber-reinforced composite laminates:

(a) An initial anisotropic yield surface that defines the elastic limit of material behavior in the stress space.

(b) An anisotropic hardening rule that specifies the evolution of subsequent yield surface under plastic deformation.

(c) The flow rule, clarifying the direction of the incremental plastic strain vector in strain space.

On the other hand, anisotropic materials under multi-axial loading conditions exhibit some important characteristics, including (1) different yield stresses with orientation and (2) differences between tensile and compressive yield stresses. An anisotropic yield criterion under general loading, which includes effects of different yield strengths in material principal directions and between tension and compression (Bauschinger effect), is proposed for composite materials, making in especially useful for both unidirectional and bidirectional composite lamina.

2.3. Anisotropic Yield Criterion

Considering the material characteristics of the fiber-reinforced composites given in the preceding section, a general form of the anisotropic yield criterion for composite material can be the quadratic function given as

$$f = A_{ij}(\sigma_i - \alpha_i)(\sigma_j - \alpha_j) - \kappa^2 \quad (i,j = 1,2...6) \tag{4}$$

where σ_i is the current state of stress and where three material variables A_{ij}, α_i, and κ represent the current state of plastic deformation. Anisotropic parameters describe the current state of plastic anisotropy as represented by different yield stresses with respect to material directions. The translation vector describes the current strength differentials between tensile and compressive yield stresses. The scalar parameter represents the effective size of yield surface that corresponds to the reference yield stress of material at a given point in history.

Composite material considered in this study is assumed to have three orthogonal symmetric planes (Fig. 1), and the principal axes of anisotropy are assumed as reference axes. If we neglect transverse normal stress and differences between the positive and negative shear stresses, A_{ij} and α_i can be written in the following matrix form

$$A_{ij} = [A] = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 \\ A_{12} & A_{22} & 0 & 0 & 0 \\ 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \tag{5}$$

$$\alpha_i = [\alpha] = [\alpha_1\, \alpha_2\, 0\, 0\, 0]^T \tag{6}$$

If the principal axes of anisotropy do not coincide with the reference axes but rotate by a certain angle θ, the anisotropic parameter matrix is transformed according to the stress transformation matrix as

$$[\underline{A}] = [T_\sigma]^T [A][T_\sigma] \tag{7}$$

where $[T_\sigma]$ is the stress transformation matrix as already defined by Eq. 3.

The yield function can also be expressed in explicit form as

$$f = A_{ij}\sigma_i\sigma_j - A_i\sigma_i - K \tag{8}$$

The yield functions given by Eqs. 4 and 8 are equivalent if

$$A_i = 2A_{ij}\alpha_j \tag{9}$$

and $$K = - A_{ij}\alpha_i\alpha_j + \kappa^2 \tag{10}$$

The yield function given by Eq. 8 is attractive because the strength differentials are described directly by α_i; however, a direct evaluation of α_i (due to strength differential) in Eq. 4 from uniaxial tests is quite cumbersome (Shih & Lee, 1978). The yield function given by Eq. 8 is more convenient for finding A_{ij} and A_i from experimental results, at which point α_i can be determined by using Eq. 9. Hence, Eq. 8 is used in the following developments in terms of A_{ij}, A_i, and K parameters.

For the yield function of Eq. 8, a physical interpretation can be made by uniaxial tension and compression tests in each direction, and by pure shear tests, respectively. We show uniaxial tensile yield stresses in 1- and 2-directions as X_T and Y_T on the yield surface and absolute values of compressive yield stresses as X_C and Y_C, respectively. From simple uniaxial tension and compression test in material 1-and 2-direction, we can obtain the anisotropic material parameters as

$$A_{11} = \frac{K}{X_T X_C} \qquad A_1 = A_{11}(X_T - X_C) \tag{11}$$

$$A_{22} = \frac{K}{Y_T Y_C} \qquad A_2 = A_{22}(Y_T - Y_C) \tag{12}$$

In addition, the anisotropic parameters are related to the shear yield stresses for the pure shear tests, as

$$A_{44} = \frac{K}{S_a^2} \qquad A_{55} = \frac{K}{S_t^2} \qquad A_{66} = \frac{K}{S_a^2} \tag{13}$$

where the axial shear yield stress S_a is associated with 1-2 and 1-3 planes, and the transverse shear yield stress S_t is associated with 2-3 planes. The anisotropic material parameters, which describe plastic anisotropy in material principal directions, are obtained except off-diagonal term in Eq. 5.

2.4 Evaluation of Interaction Term

In order to determine the remaining off-diagonal interaction term in the anisotropic parameter matrix (Eq. 5), either of any biaxial tests like tension-tension, tension-compression, or zero volumetric plastic strain assumption (i.e.,the incompressibility) has been used. However, the values of the interaction term from the biaxial test will apparently not be unique (Shih & Lee, 1978). An infinite number of biaxial tests indicates that it may not practical to choose the optimum test result. On the other hand, most composite materials are brittle; thus the incompressibility condition is inadequate for use (Chen & Han, 1988).

In the current plane stress formulation, the bound restriction (stability condition) for an elliptic equation is used to determine interaction term A_{12}. The yield surface of Eq. 8 is an ellipsoid in a three-dimensional stress space with σ_1, σ_2, and one of the shear stresses. Hence, the yield surface will be an ellipse in a plane of constant shear stress (Fig. 2) if the following condition in terms of anisotropic parameters is satisfied:

$$A_{11}A_{22} - A_{12}^2 > 0 \tag{14}$$

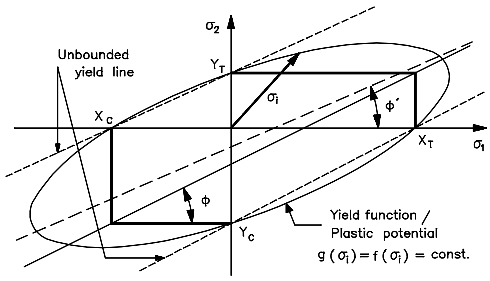

Fig. 2 Assumed Rotation of Yield Surface in a Constant Shear Stress Plane.

If the stability condition, Eq. 14 is not satisfied, the surface described by Eq. 8 will not be closed and will, therefore, be unbounded. To assure a closed yield surface to ensure that the yield surface becomes valid physically, the value of A_{12} must satisfy the stability condition Eq. 14. When an elliptic equation satisfies the stability condition, the rotation angle of the major axis of ellipse shows the following relation with the coefficients of elliptic function, which in this case are anisotropic material parameters of Eq. 8:

$$\tan 2\phi' = \frac{2A_{12}}{A_{11} - A_{22}} \tag{15}$$

where ϕ' is the rotation angle between the major axis of yield ellipse and the 1-direction of the principal axes of anisotropy, shown in Fig. 2.

In the current model, the rotation of the major axis to the strength axis in 1-direction is assumed as

$$\tan\phi = \frac{Y_T + Y_C}{X_T + X_C} \tag{16}$$

where ϕ is the assumed rotation angle, and X_T, X_C, Y_T, and Y_C are the tensile and compressive yield stresses in 1- and 2- directions, respectively. Eq. 16 is based on an assumption that the rotation of the major axis of yield ellipse is lined on the intersection line between tensile and compressive yield stresses of each direction (Fig. 2). With this assumption, therefore, the interactive effects between the normal stresses can be included in the yield function. As shown in Fig. 2, the rotation angle of yield ellipse (dashed line) does not exactly coincide with the assumed solid line, but the differences are negligible. By substitution of Eq. 16 into Eq. 15, A_{12} can be written as

$$A_{12} = \frac{(X_T+X_C)(Y_T+Y_C)}{(X_T+X_C)^2-(Y_T+Y_C)^2}(A_{11}-A_{22}) \tag{17a}$$

or,

$$A_{12} = \frac{(X_T+X_C)(Y_T+Y_C)}{(X_T+X_C)^2-(Y_T+Y_C)^2}\left(\frac{K}{X_TX_C}-\frac{K}{Y_TY_C}\right) \tag{17b}$$

The new interaction terms given by Eq. 17 automatically satisfy the stability condition, Eq. 14. The anisotropic material parameters in Eqs. 11, 12, 13, and 17 define the anisotropic initial yield criterion with the current state of stresses through Eq. 8; with further loading, however, we must determine how the material behaves beyond the initial yield criterion.

2.5 Anisotropic Work-Hardening Rule

As mentioned earlier, several noteworthy contributions for anisotropic hardening rules have been made, but most are impractical and too complex to be implemented into a numerical computer program for predictive analysis of fiber-reinforced composites.

An attempt must be made to construct an anisotropic work-hardening model to explain material behavior of fiber-reinforced composites beyond initial yield criterion. The current model allows for a nonproportional change of yield values so that the subsequent yield surface can be distorted. The basic concept of the current model is to determine the anisotropic material parameters A_{ij} in the principal material directions with varying yield strengths. The total plastic work due to the change of yield stress in each direction should be the same as the equivalent change in effective stress (Schreyer, Kulak & Kramer, 1979).

The assumption, originally developed by Baltov & Sawczuck (1965), is adopted, showing that the plastic work produced during plastic loading in any principal material direction is the same amount produced by effective stress in the irrespective direction:

$$W^p = \int \sigma_i d\epsilon_i^p = \int \overline{\sigma} d\overline{\epsilon^p} \quad (i=1,2,..,5) \tag{18}$$

where $\overline{\sigma}$ and $\overline{\epsilon}$ are the effective stress and effective plastic strain, respectively. Note that one of the stress-strain diagrams is arbitrarily chosen as the effective stress-strain diagram. In the principal material directions, the plastic work W_p of Eq. 18 can be expressed approximately as

$$W^p = \frac{1}{2E_i^p}(X_i^2 - X_{0i}^2) \quad (i=1,2,..,5) \tag{19}$$

where X_{0i}^p and X_i^p are the initial and subsequent yield strengths, and E_i^p is the plastic modulus (Fig. 3) in the principal material directions, respectively. By analogy to Eq. 19, the plastic work done by the effective stress can be expressed as

$$W^p = \frac{1}{2H'}(\overline{\sigma^2} - \overline{\sigma_0^2}) \tag{20}$$

where H' is the hardening modulus (Fig. 3) of the effective stress-effective plastic strain diagram, and $\overline{\sigma}_0^p$ is the initial effective yield stress. By equating the plastic works of Eqs. 19 and 20, we obtain

$$\frac{1}{2H'}(\overline{\sigma^2}-\overline{\sigma_0^2})=\frac{1}{2E_i^p}(X_i^2-X_{0i}^2) \tag{21}$$

From Eq. 21, the subsequent yield stress becomes

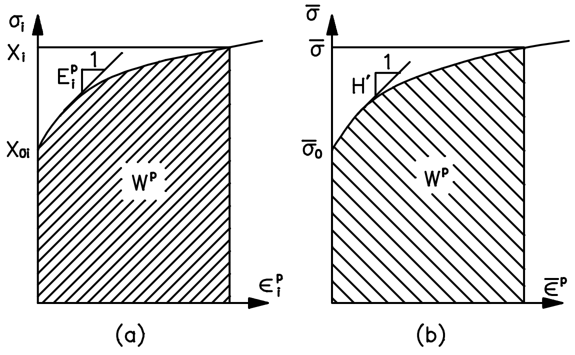

Fig. 3 Plastic work produce (a) in any Principal Material Direction and (b) by the Effective Stress.

$$\begin{aligned}
X^2(\overline{\sigma}) &= \frac{E_1^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+X_0^2\\
Y^2(\overline{\sigma}) &= \frac{E_2^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+Y_0^2\\
S_{12}^2(\overline{\sigma}) &= \frac{G_{12}^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+S_{012}^2\\
S_{23}^2(\overline{\sigma}) &= \frac{G_{23}^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+S_{023}^2\\
S_{13}^2(\overline{\sigma}) &= \frac{G_{13}^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+S_{013}^2
\end{aligned} \tag{22}$$

The evolution of the subsequent yield surface can be expressed with the individual subsequent yield strengths in Eq. 22.

The plastic modulus E_i^p can be determined in terms of the elastic modulus E_i and the tangent modulus E_i^t. The strain increment $d\epsilon$ is assumed to consist of two parts: the elastic strain increment, $d\epsilon^e$, and the plastic strain increment, $d\epsilon^p$ (Fig. 4(a)).

$$d\epsilon_i = d\epsilon_i^e + d\epsilon_i^p \tag{23}$$

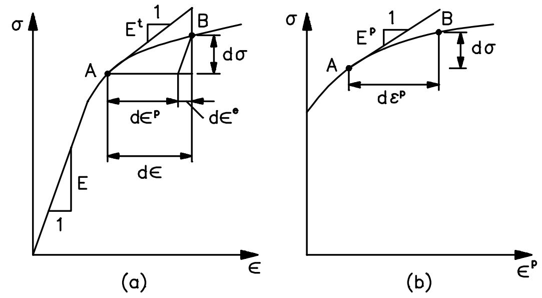

Fig. 4 (a) Tangent Modulus and (b) Plastic Modulus.

The stress increment $d\sigma$ is related to the strain increment $d\epsilon$ by

$$d\sigma_i = E_i^t d\epsilon_i \tag{24}$$

where E_i^t is the tangent modulus that changes during the plastic modulus (Fig. 4(a)). The plastic strain increment $d\epsilon^p$ and the stress increment $d\sigma$ are related by

$$d\sigma_i = E_i^p d\epsilon_i^p \tag{25}$$

where E_i^p is referred to the plastic modulus (Fig. 4(b)). The elastic strain increment shows the usual relation with the stress increment as

$$d\sigma_i = E_i d\epsilon_i^e \tag{26}$$

where E_i is the elastic modulus. The relationship between moduli E_i^t, E_i^p, and E_i:

$$\frac{1}{E_i^p}=\frac{1}{E_i^t}-\frac{1}{E_i} \tag{27}$$

By substituting the subsequent yield strengths into the initial anisotropic parameters, the anisotropic parameters for the subsequent yield criterion at any state of plastic deformation can be obtained as

$$A_{ii}(\overline{\sigma})=\frac{\overline{\sigma^2}}{X_i^2}=\frac{\overline{\sigma^2}}{\left[\frac{E_i^p}{H'}(\overline{\sigma^2}-\overline{\sigma_0^2})+X_{0i}^2\right]} \tag{28}$$

We should note that the repeated indices in Eq. 28 do not imply summation here, and that K in anisotropic parameters becomes $\overline{\sigma^2}$ because of α_i

= 0 (neglecting the Bauschinger effect). With these varying anisotropic parameters, the subsequent yield surface is defined so the shape of surface may be distorted depending on the amount of plastic deformation of each material direction.

2.6 Elastic-Plastic Constitutive Relationship

The incremental constitutive relationship for an elastic-plastic material is formulated by using the consistency condition. The derivation of the incremental elastic-plastic stress-strain relationship is well documented in many textbooks, so it is only noted that the derivation of the current model employed the associated flow rule and the strain-hardening hypothesis of the effective plastic strain.

Simply here, the incremental elastic-plastic constitutive relationship is presented as

$$d\sigma_i = D_{ij}^{ep} d\epsilon_j \tag{29}$$

where $d\sigma_i$ and $d\epsilon_j$ are the incremental stress and strain vectors, respectively, D_{ij}^{ep} is the elastic-plastic material stiffness tensor defined as

$$D_{ij}^{ep} = D_{ij}^{e} - \frac{D_i^e \dfrac{\partial f}{\partial \sigma_k} \dfrac{\partial \overline{\sigma}}{\partial \sigma_l} D_{ij}^e}{\dfrac{\partial \overline{\sigma}}{\partial \overline{\epsilon^p}} + \dfrac{\partial f}{\partial \sigma_m} D_{mn}^e \dfrac{\partial \overline{\sigma}}{\partial \sigma_n}} \tag{30}$$

where the flow vector $\partial f / \partial \sigma_{ij}$ can be written in the explicit form as

$$\frac{\partial f}{\partial \sigma_i} = \begin{Bmatrix} \dfrac{\partial f}{\partial \sigma_1} \\ \dfrac{\partial f}{\partial \sigma_2} \\ \dfrac{\partial f}{\partial \sigma_4} \\ \dfrac{\partial f}{\partial \sigma_5} \\ \dfrac{\partial f}{\partial \sigma_6} \end{Bmatrix} = \begin{Bmatrix} 2A_{11}\sigma_1 + 2A_{12}\sigma_2 + A_1 \\ 2A_{22}\sigma_2 + 2A_{12}\sigma_1 + A_2 \\ A_{44} \\ A_{55} \\ A_{66} \end{Bmatrix} \tag{31}$$

where f is the subsequent yield function, and A_{ij} is the anisotropic material parameters in the subsequent yield function. To be consistent with the strain-hardening hypothesis, the hardening modulus $\partial \overline{\sigma} / \partial \overline{\epsilon^p}$ in Eq. 30 is given as

$$H' = \frac{d\overline{\sigma}}{d\overline{\epsilon^p}} = \frac{E^T}{1 - \dfrac{E^T}{E}} \tag{32}$$

where E and E^T are the elastic and tangent moduli of the material from uniaxial test, respectively, and the tangent modulus is changed along with the plastic deformation. Eq. 32 is evaluated from a generalized effective stress-effective strain curve derived from the stress-strain curves obtained along the respective principal directions.

3. NUMERICAL ILLUSTRATIVE EXAMPLES

Anisotropic plasticity constitutive model has been presented for analysis of fiber-reinforced composite structures. Also, their models are successfully implemented into the computer program PACS (Yener & Yi, 1994). In order to verify the validity of the problem formulation and the efficiency of numerical implementation, several example problems are analyzed by using the computer program PACS. Numerical results are compared with various experimental and other numerical results.

3.1 Elastic-Plastic Analysis of Composite Laminates

In this section, the anisotropic elastic-plastic analysis capability of the computer program is verified with several composite laminates on different boundary and loading conditions. The elastic-plastic material behavior is observed and compared. Also, the behaviors of isotropic and anisotropic materials are compared.

This section aims to predict nonlinear anisotropic material behavior of the fiber-reinforced composite laminates. Petit and Waddops (1969) performed experiments on a variety of Boron/Epoxy composite laminates. The material constants are given in Table 1, and used as input data in the computer program PACS.

Table 1. The Material Constants for Boron/Epoxy (B/Ep)

Elastic constants (ksi)	Plastic constants (ksi)	Failure constants (ksi)
$E_1 = 30{,}000$ $E_2 = 3{,}080$ $\nu_{12} = 0.3$ $G_{12} = 1{,}000$	$E_{T1} = 26{,}100$ $E_{T2} = 2{,}200$ $G_{T12} = 180$ $X_0 = 132.5$ $Y_0 = 9$ $S_0 = 10$	$X_U = 200$ $Y_U = 12.5$ $S_U = 18.6$

The shear moduli in 2-3 and 3-1 plane are not given in experimental data, hence these values are assumed same as one of 1-2 plane. Hashin modeled nonlinear behavior of composite material based on the deformation theory of plasticity (Hashin, Bagchi & Rosen, 1973). Present analysis results are included in the same graph with those experimental and other numerical results.

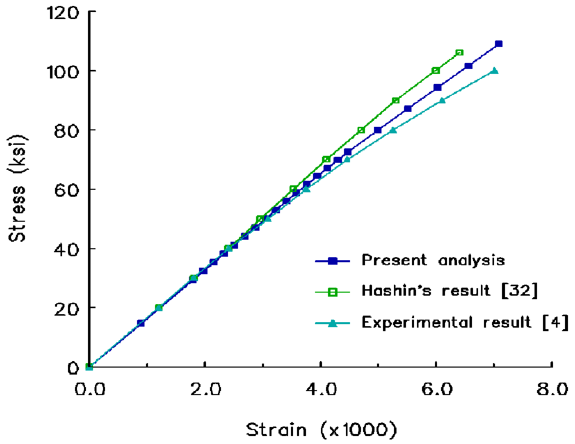

Fig. 5 Tensile stress-strain curve for $[0^0/90^0]_s$ cross-ply Boron/Epoxy laminate

Several laminate schemes are tested. To explain the notation for the laminate scheme used in this section, layer angles are distinguished by a slash and written in order of from the top, with the whole stacking sequence enclosed within square brackets. If there is more than one layer at any angles, the number of layers is denoted by subscripts within the brackets. The 0^0 layer has fibers along the x-direction in the reference coordinates system. If the laminate has a symmetric scheme referred to mid-surface, the symmetric is denoted by subscript s outside the brackets.

The predicted stress-strain curve by PACS for $[0^0/90^0]s$ cross-ply B/Ep laminate is compared with the experimental results in Fig. 5. Although the program PACS predicts high ultimate strength of laminate, material behavior shows better agreement with the experimental results than with Hashin's model. Note that curves are located the end of section.

Fig. 6 examines a $[45^0/45^0]s$ angle-ply laminate, showing quite different outcomes between test and numerical results. As mentioned previously, the shear muduli in 1-3 and 2-3 planes are assumed the same as the 1-2 plane because these were not obtained in experiments by Waddoup and Petit (1969). As a highly nonlinear nature of response is depicted in Fig. 6, a significant amount of shear strains appears in the $[45^0/45^0]s$ angle-ply laminate. The ultimate strength of the present analysis is predicted quite close to that of the experiment.

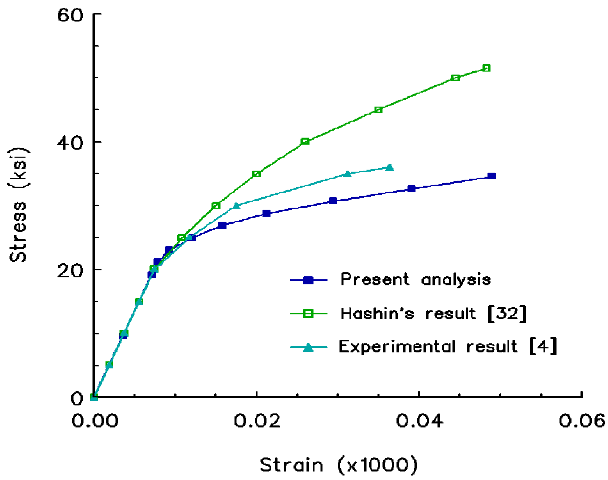

Fig. 6 Tensile stress-strain curve for $[45^0/\text{-}45^0]_s$ angle-ply B/Ep laminate.

Stress-strain curves for the $[30^0/30^0]s$ angle-ply laminate is shown in Fig. 7, showing that predicted material behaviors are in good agreement with experimental and other numerical results. The predicted ultimate strength is quite close to that of the experiment. Another angle-ply laminate $[60^0/60^0]s$, presented in Fig. 8, compares to the curves of experiment and numerical results. Close shapes of the curves are shown, but the experimental result shows more nonlinear behavior.

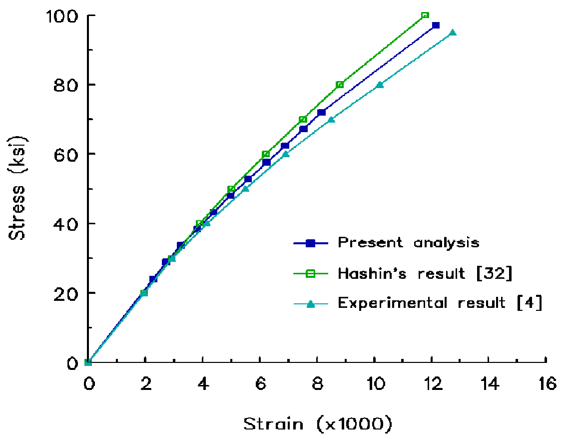

Fig. 7 Tensile stress-strain curve for $[30^0/-30^0]_s$ angle-ply B/Ep laminate.

Noting observations about the three angle-ply laminates, $[45^0/45^0]s$, $[30^0/30^0]s$, and $[60^0/60^0]s$ is interesting. Ultimate strengths of laminates become lower when fiber angles are wider from the loading direction. The $[45^0/45^0]s$ angle-ply laminate shows more nonlinear behavior than the other two angle-ply laminates because of a more significant amount of shear strain than found in the other two angle-ply laminates. Although the $[60^0/60^0]s$ and $[30^0/30^0]s$ angle-ply laminates show similar trends of material behavior, the $[60^0/60^0]s$ shows more scatter and nonlinear material behavior than the $[30^0/30^0]s$ angle-ply laminate due to more shear forces.

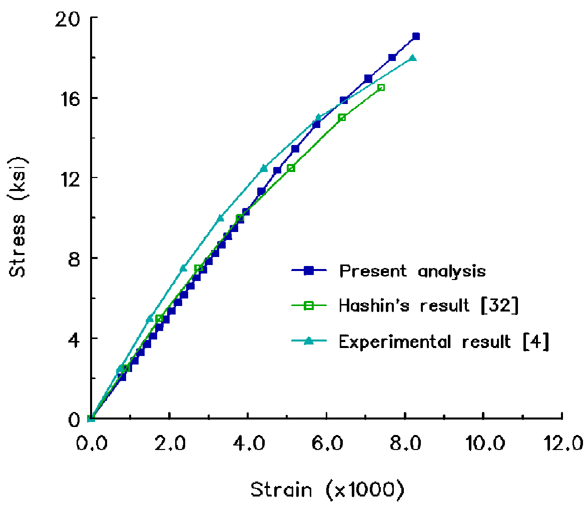

Fig. 8 Tensile stress-strain curve for $[60^0/-60^0]_s$ angle-ply B/Ep laminate.

6. CONCLUSIONS

The development and use of an analytical procedure which has capability of predicting the progressive material behavior of structures, named as a predictive analysis procedure in here, is developed for more accurate assessment of structural safety and efficiency of composite structures.

A quadratic anisotropic yield criterion in stress space is developed for general use with unidirectional and bidirectional composite lamina. The developed anisotropic work-hardening model allows for a nonproportional change of the yield values so that the subsequent yield surface can be a distorted shape.

The accuracy and efficiency of the proposed anisotropic plastic constitutive model are verified with various benchmark problems. Numerical predictions of the computer program PACS compare very well with available experimental, analytical and other numerical results. Comparisons illustrate the capability of the constitutive model. The developed constitutive model predicts progressive nonlinear behavior from the beginning of loading, in plane and through thickness direction of composite laminates.

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