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# An energy-efficiency approach for bidirectional amplified-and-forward relaying with asymmetric traffic in OFDM systems

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#### Abstract

Two-way relaying is an effective way of improving system spectral efficiency by making use of physical layer network coding. However, energy efficiency in OFDM-based bidirectional relaying with asymmetric traffic requirement has not been investigated. In this study, we focused on subcarrier transmission mode selection, bit loading, and power allocation in a multicarrier single amplified-and-forward relay system. In this scheme, each subcarrier can operate in two transmission modes: one-way relaying and two-way relaying. The problem is formulated as a mixed integer programming problem. We adopt a structural approximation optimization method that first decouples the original problem into two suboptimal problems with fixed subcarrier subsets and then finds the optimal subcarrier assignment subsets. Although the suboptimal problems are nonconvex, the results obtained for a single-tone system are used to transform them to convex problems. To find the optimal subcarrier assignment subsets, an iterative algorithm based on subcarrier ranking and matching is developed. Simulation results show that the proposed method can improve system performance compared with conventional methods. Some interesting insights are also obtained via simulation.

*Keywords:* Energy efficiency; bidirectional relaying; two-way relay; amplify-and-forward (AF); orthogonal frequency-division multiplexing (OFDM)

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## 1. Introduction

Conventional one-way relay transmission is less spectrally efficient with respect to two-way relay transmission owing to half-duplex constraints. To overcome this disadvantage, bidirectional relaying has been proposed, which utilizes physical layer network coding (PNC) technology, enabling two users to communicate with each other with the help of an intermediate relay. In previous studies, two main protocols were used: a broadcast three-phase protocol and a two-phase protocol, which form the foundation for future studies on bidirectional communication [1]. In [1–4], the problem of capacity, achievable rate regions, and outer bounds of the bidirectional relaying system were addressed. The techniques and methodology developed in the literature for one-way relaying have been found to also work for bidirectional relaying: randomized space-time block coding in [5–6], relay buffering in delay-tolerant networks [7], and diversity-multiplexing trade-off [8].

In practice, there exist some systems that have distinct data rate requirement for uplink and downlink paths. The methods previously developed for these systems assumed symmetric traffic conditions. However, when these methods are applied to asymmetric traffic conditions, lower energy efficiency is experienced. Several studies on bidirectional relay systems with asymmetric traffic have attempted to solve this issue [9–13]. An asymmetric modulation scheme that is capable of making full use of the stronger link to improve the overall transmission rate and ensures the reliability of the weaker link was proposed in [9]. In [10], a hierarchical modulation (HM)-based scheme and a hybrid-constellation-based relay scheme are considered to enhance the performance of asymmetric bidirectional relaying. In [11], a closed-form asymptotic expression is given for the system outage probability for an amplify-and-forward (AF) bidirectional protocol with imperfect CSI estimation and asymmetric traffic requirement. A new method that allocates the transmission time and the rates in both directions for asymmetric traffic conditions was introduced in [13].

Multicarrier modulation such as orthogonal frequency-division multiplexing (OFDM), which divides the data into many substreams, has a natural advantage for finding a way to satisfy asymmetric traffic requirements from the standpoint of the frequency domain, which is an approach that has not been properly investigated. Resource allocation in bidirectional OFDM relay systems had been previously investigated [12, 15, 20]. In [12], the authors studied power allocation, mode selection, and subcarrier assignments with quality-of-service (QoS) considerations for OFDM bidirectional decode-and-forward (DF) relay systems. By using a dual method, the problem was efficiently solved. A two-step approach to power allocation for OFDM signals over a two-way AF relay was proposed in [15], in which the authors show that the received SNRs of the two users are equal under fixed total power constraints at each subcarrier for OFDM bidirectional AF relaying. In [20], power allocation for bidirectional AF multiple-relay multiple-user networks was formulated as a geometric programming (GP) problem by maximizing the instantaneous sum rate or minimizing the symbol error rate.

In this study, we focus on a two-hop system with an AF relay node, in which the received signal is linearly scaled by a scaling coefficient and then forward to its destination, which can be another user or a base station. The objective of our proposed optimization is to minimize the total power consumption under the asymmetric traffic requirement constraint. In a situation where two users with different data requirements exist, more subcarriers will naturally be assigned to the user with the higher data rate requirement. The extra subcarriers will have to

work in one-way transmission mode. This ultimately means that some subcarriers will operate in one-way transmission mode, whereas others will operate in two-way transmission mode. A combinational optimal problem with transmission mode selection, power allocation, and bit loading is formulated. Unfortunately, the problem is a mixed integer program and NP hard. Owing to this difficulty, we extend the proof for directional relaying in [14] and the structure property for symmetric two-way relaying in [15]. We develop a suboptimal algorithm to solve the proposed NP-hard problem. In the first step, we decouple the original problem into two suboptimal problems with fixed subcarrier subsets. In the second step, we find the optimal subcarrier assignment subsets that minimize the total power consumption.

The main contribution of this work can be broadly classified into two parts:

1) We formulate a joint optimization problem of transmission mode selection, power allocation, and bit loading for OFDM bidirectional AF relaying. Our objective is to minimize the energy consumption per bit. The problem is a mixed integer programming problem and NP hard. Because of nonconvexity, the method developed for DF relaying in [12] is not pragmatic. Therefore, we develop a two-step algorithm to find the asymptotically optimal solution.

2) We investigate the influence of the asymmetric ratio on system energy efficiency; the simulation reveals that there is an asymmetric traffic ratio that minimizes the energy consumption per bit. This is an interesting result for the transmission design.

# 2. System model

Consider a two-hop relay network that consists of two user nodes ( $S_a$  and  $S_b$ ) and one AF relay node (R) without line of sight between  $S_a$  and  $S_b$ . All the nodes are equipped with one antenna, and the number of subcarriers is N. The channels operate under the assumption of reciprocity, which means that the channels from user  $S_i$  ( $S_i = S_a, S_b$ ) to the relay and from the relay to user  $S_i$  are the same. The block diagram of the network is shown in Fig. 1.



Fig. 1. The OFDM bidirectional relay channel, where terminals  $S_a$  and  $S_b$  exchange messages via a relay node R.

The channel fading coefficient between  $S_a$  and relay R and that between user  $S_b$  and relay R at subcarrier n are denoted by  $h_n$  and  $g_n$ , respectively, and they are modeled as independent Rayleigh distribution variables that remain constant over the entire block transmission.

To model the asymmetric traffic, we define an asymmetric traffic ratio  $\alpha$  as the ratio of the transmission data rate requirements of users  $S_b$  to  $S_a$ . We assume that user  $S_a$  has a higher data rate requirement in this paper; hence,  $0 < \alpha \le 1$ , and  $R_b = \alpha R_a$ .

#### 3. Problem formulation

We use subscripts 1, 2 to denote one-way relaying and two-way relaying, respectively. We first introduce the following sets of variables:

 $P_{k,n,l}$  indicates the power of subcarrier *n* at node *k* for one-way relay transmission,  $k \in \{a, r\}$ .

 $P_{k,n,2}$  indicates the power of subcarrier *n* at node *k* for two-way relay transmission,  $k \in \{a,b,r\}$ .

 $r_{a,n,1}$  indicates the data rate of subcarrier *n* at  $S_a$  for one-way relay transmission.

 $r_{k,n,2}$  indicates the data rate of subcarrier *n* at node *k* for two-way relay transmission,  $k \in \{a,r\}$ .

## 3.1 One-Way Relay Network

For one-way relaying, in the first slot, user  $S_a$  transmits  $x_{a,n}$  to R, and the received signal at relay node is denoted as

$$y_{r,n} = \sqrt{P_{a,n,1}} h_n X_{a,n} + \eta_r \tag{1}$$

In the second time slot, relay *R* amplifies  $y_{r,n}$  with a scaling factor  $\omega_{n,l}$  and retransmits it to the destination.

$$y_{b,n} = \omega_{n,1} g_n (\sqrt{P_{a,n,1}} h_n x_{a,n} + \eta_r) + \eta_b$$
<sup>(2)</sup>

where  $\omega_{n,1} = \sqrt{P_{r,n,1} / (h_n^2 P_{a,n,1} + \sigma^2)}$ . Thus, we have  $P_{r,n,1} = \omega_{n,1}^2 (h_n^2 P_{a,n,1} + \sigma^2)$ , and  $\eta_r$  and  $\eta_b$  are the additive Gaussian noises at relay node *R* and the destination  $S_b$  with zero mean and variance  $\sigma^2$ , respectively. Hence, the received SNR at  $S_b$  is

$$\gamma_{n} = \frac{P_{a,n,l}\omega_{n,l}^{2}g_{n}^{2}h_{n}^{2}}{(\omega_{n,l}^{2}g_{n}^{2}+1)\sigma^{2}}$$
(3)

The instantaneous rate per subcarrier of user  $S_a$  for one-way relaying is given as

$$\boldsymbol{r}_{a,n,1} = \boldsymbol{C}(\boldsymbol{\gamma}_n) \tag{4}$$

where C(x) = (1/2)log (1+x).

## 3.2 Two-Way Relay Network

A two-phase protocol for two-way relay transmission includes the MAC phase and the BC phase. In the first slot, users  $S_a$  and  $S_b$  simultaneously transmit to relay R, and the received signal at subcarrier n is

$$y_{r,n} = \sqrt{P_{a,n,2}} h_n x_{a,n} + \sqrt{P_{b,n,2}} g_n x_{b,n} + \eta_r$$
(5)

In the second slot, relay *R* scales the received signal with the scaling factor  $\omega_{n,2}$  and broadcasts it to  $S_a$  and  $S_b$ . The received signal at  $S_a$  and  $S_b$  can be expressed as

$$y_{a,n} = \sqrt{P_{a,n,2}} \omega_{n,2} h_h^2 x_{a,n} + \sqrt{P_{b,n,2}} \omega_{n,2} h_n g_n x_{b,n} + \omega_{n,2} h_n \eta_r + \eta_a$$
(6a)

$$y_{b,n} = \sqrt{P_{a,n,2}} \omega_{n,2} h_n g_n x_{a,n} + \sqrt{P_{b,n,2}} \omega_{n,2} g_n^2 x_{b,n} + \omega_{n,2} g_n \eta_r + \eta_b$$
(6b)

where 
$$\omega_{n,2} = \sqrt{\frac{P_{r,n,2}}{P_{a,n,2}h_n^2 + P_{b,n,2}g_n^2 + \sigma^2}}$$
; thus, we have  $P_{r,n,2} = \omega_{n,2}^2(P_{a,n,2}h_n^2 + P_{b,n,2}g_n^2 + \sigma^2)$ .

where  $\eta_a$ ,  $\eta_b$ , and  $\eta_r$  are the additive Gaussian noises at user nodes  $S_a$  and  $S_b$ , and relay *R* with zero mean and variance  $\sigma^2$ , respectively. Because the user nodes know perfectly well what they have sent, self-interference cancellation could be used to remove interference. The residual signals  $\tilde{y}_{a,n}$  and  $\tilde{y}_{b,n}$  are

$$\tilde{y}_{a,n} = \sqrt{P_{b,n,2}} \omega_{n,2} h_n g_n x_{b,n} + \omega_{n,2} h_n \eta_r + \eta_a$$
(7a)

$$\tilde{\boldsymbol{y}}_{b,n} = \sqrt{\boldsymbol{P}_{a,n,2}}\,\boldsymbol{\omega}_{n,2}\boldsymbol{h}_{n}\boldsymbol{g}_{n}\boldsymbol{x}_{a,n} + \boldsymbol{\omega}_{n,2}\boldsymbol{g}_{n}\boldsymbol{\eta}_{r} + \boldsymbol{\eta}_{b} \tag{7b}$$

The received SNRs at  $S_a$  and  $S_b$  can be written as

$$\gamma_{a,n} = \frac{P_{b,n,2}\omega_{n,2}^2 h_n^2 g_n^2}{(\omega_{n,2}^2 h_n^2 + 1)\sigma^2}$$
(8a)

$$\gamma_{b,n} = \frac{P_{a,n,2}\omega_{n,2}^2 h_n^2 g_n^2}{(\omega_{n,2}^2 g_n^2 + 1)\sigma^2}$$
(8b)

For the two-way relaying transmission mode, the achievable rates of users  $S_a$  and  $S_b$  over subcarrier *n* can be respectively written as

$$\mathbf{r}_{a,n,2} = \mathbf{C}(\boldsymbol{\gamma}_{b,n}) \tag{9a}$$

$$r_{b,n,2} = C(\gamma_{a,n}) \tag{9b}$$

#### 3.3 Energy-Efficient OFDM Bidirectional AF Relaying with Asymmetric Traffic

In this study, we seek an optimal solution for the asymmetric traffic requirement for two users with multi-subcarrier modulation, such as OFDM. Based on the valid assumption that a high-date-rate user may require more subcarriers for transmission, we investigate the transmission model selection and rate allocation for each subcarrier such that the total system power consumption is minimum. Mathematically, the joint optimization problem can be formulated as (P1):

P1: Min 
$$\frac{\sum_{n=1}^{N} \rho_n^* (P_{a,n,1} + P_{r,n,1}) + \rho_n (P_{a,n,2} + P_{b,n,2} + P_{r,n,2})}{R_a + R_b}$$
 (10a)

subject to 
$$\sum_{n=1}^{N} \rho_n^* r_{a,n,1} + \rho_n r_{a,n,2} \ge R_a$$
(10b)

$$\sum_{n=1}^{N} \rho_n \mathbf{r}_{b,n,2} \ge \mathbf{R}_b \tag{10c}$$

$$r_{a,n,1} \le R_{\max}, 0 \le r_{a,n,2} \le r_{\max}, 0 \le r_{b,n,2} \le r_{\max}$$
 (10d)

$$\rho_n^* = 1 - \rho_n \in \{0, 1\} \tag{10e}$$

where  $R_a$  and  $R_b$  are the minimal data rate requirements for  $S_a$  and  $S_b$ , respectively;  $r_{max}$  is the maximal data rate at each subcarrier;  $\rho = \{\rho_n \rho_n^*\}$  is the set of binary assignment variables that indicates whether a one-way relaying or a two-way relaying transmission mode is adopted at subcarrier n;  $P = \{P_{a,n,l}, P_{r,n,l}, P_{a,n,2}, P_{b,n,2}, P_{r,n,2}\}$  is the set of power variables; and  $R = \{r_{a,n,l}, r_{a,n,2}, r_{b,n,2}\}$  is the set of rate variables.

It noteworthy that the constraints in (10b) and (10c) are nonconcave functions with respect to the power allocation vectors and the scaling factor, and hence cannot be solved by techniques relying on the Karush–Kuhn–Tucker (KKT) conditions. Moreover, the existence of a binary assignment variable makes an optimal solution intractable.

### 4. Proposed schemes for optimal transmission

The problem in P1 is a combinational optimal problem. The search for the optimal solution can be decoupled into two separate subproblems: (1) optimal power allocation for both

one-way relaying and two-way relaying under fixed subcarrier assigned subsets and (2) discrete optimization of the subcarrier assigned subsets. At first glance, we have to assign 0 or 1 to  $\rho_n$ , which requires an exhaustive search over all possible permutations of (n = 1, 2..., N), with exponential complexity, where each subcarrier has five possibilities of bit loading for the case of {0,2,4,6,8} that used in the simulation, whose computational complexity is  $10^N$ . Fortunately, by using the optimal linear processing structure of channel pairing [14, 16, 17], an exhaustive search can be avoided, thus yielding a simple algorithm for searching the optimal subcarrier number for two-way relay transmission.

## 4.1 Optimal Bit Loading and Power Allocation for Fixed Subcarrier Assignment Subset

Assuming that the two subcarrier subsets are given and the same data rate per subcarrier (see remark 1) applies for each user in the two-way relay transmission mode, the optimal problem P1 is decoupled into two suboptimal problems, i.e.,

$$\operatorname{Min} \frac{\sum_{n=1}^{N_{1}} (P_{a,n,2} + P_{b,n,2} + P_{r,n,2})}{R + \sum_{n=1}^{N_{1}} r_{b,n,2}}$$
S.t:
$$\sum_{n=1}^{N_{1}} r_{b,n,2} \ge R_{b}, 0 \le r_{a,n,2} \le r_{\max}, 0 \le r_{b,n2} \le r_{\max}$$
(11)

and

$$\operatorname{Min} \frac{\sum_{n=1}^{N-N_{1}} (P_{a,n,1} + P_{r,n,1})}{\sum_{n=1}^{N-N_{1}} r_{a,n,1}}$$
(12)  
S.t: 
$$\sum_{n=1}^{N-N_{1}} r_{a,n,1} \ge R_{a} - R, 0 \le r_{a,n,1} \le r_{\max}$$

where  $R = \sum_{n=1}^{N_1} r_{a,n,2}$ ,  $N_I$  is the number of subcarrier subsets allocated for two-way relay

transmission.

The assumption of the same data rate per subcarrier for two-way relay transmission makes problem P1 solvable. Although we can not guarantee the solution is optimal, form (17), the energy consumption will sharply increase as the number of bits increases. Only when the subcarrier channel for one-way relay transmission is poor does an asymmetric data rate in two-way relay transmission mode lead to lower energy consumption, e.g. for the case of two subcarrier pairs, the power of using the best subcarrier pair to transmit in asymmetric mode is less than the method we proposed.

Remark 1: For two-way relaying, energy-efficient bit loading at each user is given by

$$\mathbf{r}_{a,n,2} = \mathbf{r}_{b,n,2} \tag{13}$$

*Proof*: With some mathematical manipulations, we can decouple (11) into an N parallel minimal optimal problem as follows:

$$\operatorname{Min} \sum_{n=1}^{N_{1}} (P_{a,n,2} + P_{b,n,2} + P_{r,n,2}) / (r_{a,n,2} + r_{b,n,2})$$
(14)

To investigate the optimal data rate  $r_{a,n,2}$  and  $r_{b,n,2}$  at each subcarrier, we assume a fixed total power  $P_T(n) = P_{a,n,2} + P_{b,n,2} + P_{r,n,2}$ . Therefore, the per-subcarrier energy-efficiency optimal problem becomes a series of sum rate maximization problems that are identical to the optimal problem in [15]. The optimal result indicates that the received SNRs for all users are the same, which therefore validates (13).

It can be verified that the data rate constraints are nonconvex, and thus, the suboptimal problems (11) and (12) are nonconvex functions, which have been the subject of significant research by several authors [15–21]. For the case of individual power constraints at the source node and the relay, they either follow an iterative procedure with the assumption that for the optimization of the power allocation at one node the power allocation at the other node is given [17–18], using a dual-decomposition method to decouple the problem [18], or they utilize the symmetric attributes of the power allocation at each node per subcarrier [19]. For the cases of total system power constraints, the following three approaches are noteworthy: [17] adopted a tight approximation for the SNR setting. [20] utilized geometric programming (GP) to optimize the power allocation by maximizing the instantaneous sum rate and minimizing the SER for one-way relay transmission. [15] proposed a two-step method for two-way relaying that first allocates power optimally across the subcarriers and then determines the optimal power allocation at each subcarrier.

As is illustrated in [15] and [21], by means of dual decomposition, the per-subcarrier problems are nonconvex, and it is difficult to find a closed-form solution. We propose a method based on the fact that closed-form solutions for the per-subcarrier energy-efficiency problem can be derived if an equality data rate is imposed on each subcarrier. The per-subcarrier optimization problem for (11) is written as

$$\operatorname{Min} \frac{P_{a,n,2} + P_{b,n,2} + P_{r,n,2}}{r_{a,n,2} + r_{b,n,2}}$$
S.t:  $r_{a,n,2} = r_{b,n,2} = r(n)$ 
(15)

By using the Lagrange multiplier method, we can easily derive the optimal solution as [22]

$$\begin{cases} |\omega_{n,2}^{*}|^{2} = \frac{1}{g_{n}h_{n}}\sqrt{\frac{2\gamma_{n}}{2\gamma_{n}+1}} \\ P_{a,n,2}^{*} = \frac{\gamma_{n}}{h_{n}^{2}}\sigma^{2}(1+\frac{1}{|\omega_{n,2}^{*}g_{n}|^{2}}) \\ P_{b,n,2}^{*} = \frac{\gamma_{n}}{g_{n}^{2}}\sigma^{2}(1+\frac{1}{|\omega_{n,2}^{*}h_{n}|^{2}}) \end{cases}$$
(16)

where  $\gamma_n = 2^{2r_{a,n,2}} - 1 = 2^{2r_{b,n,2}} - 1$ . Then, the total power consumption per subcarrier is

$$P_{T,n,2} = 2\sigma^2 \gamma_n (\frac{1}{h_n^2} + \frac{1}{g_n^2}) + \frac{2\sigma^2}{g_n h_n} \sqrt{2\gamma_n (1 + 2\gamma_n)} .$$
(17)

By taking  $E_{n,2} = P_{T,n,2} / 2C(\gamma_n)$ , we can rewrite the optimal problem (11) as

$$\operatorname{Min} \sum_{n=1}^{N_{1}} E_{n,2}$$
(18)  
.t:  $\sum_{n=1}^{N_{1}} r_{b,n,2} \ge R_{b}, 0 \le r_{a,n,2} = r_{b,n2} \le r_{\max}$ 

**Remark 2**: The problem in (18) is a convex optimization problem.

*Proof:* Because  $E_{n,2}$  is convex with respect to  $\gamma_n$ , and  $\gamma_n$  is an increasing function of  $r_{a,n,2}$  or  $\sum_{n=1}^{N_1} -$ 

 $r_{b,n,2}$ ,  $\sum_{n=1}^{N_1} E_{n,2}$  is also convex with respect to  $r_{a,n,2}$  or  $r_{b,n,2}$ .

S

As in (15), we get the per-subcarrier optimization problem for (12) as

Min 
$$\frac{P_{a,n,1} + P_{r,n,1}}{2r_{a,n,1}}$$
 S.t.  $r_{a,n,1} = r(n)$  (19)

By using the Lagrange multiplier method, we can easily derive the optimal solution as

$$\begin{cases} \mathcal{P}^{*}_{a,n,1} = \frac{\gamma_{n}}{h_{n}^{2}} \sigma^{2} (1 + \frac{1}{|\omega^{*}_{n,1}g_{n}|^{2}}) \\ |\omega^{*}_{n,1}|^{2} = \frac{1}{g_{n}h_{n}} \sqrt{\frac{\gamma_{n}}{\gamma_{n}+1}} \end{cases}$$
(20)

Then, the total power consumption per subcarrier is

$$P_{T,n,1} = \gamma_n \sigma^2 (1/h_n^2 + 1/g_n^2) + (2\sigma^2/g_n h_n) \sqrt{\gamma_n (1+\gamma_n)}$$
(21)

where  $\gamma_n = 2^{2r_{a,n,1}} - 1$ ,  $E_{n,1} = P_{T,n,1} / C(\gamma_n)$ , Thus, we can rewrite the optimal problem (12) as

$$\operatorname{Min} \sum_{n=1}^{N-N_{1}} E_{n,1}$$
S.t.  $\sum_{n=1}^{N-N_{1}} r_{a,n,1} \ge R_{a}(1-\alpha), 0 \le r_{a,n,1} \le r_{\max}$ 
(22)

Remark 3: The problem in (22) is a convex optimization problem.

*Proof:* Because  $E_{n,1}$  is convex with respect to  $\gamma_n$ , and  $\gamma_n$  is an increasing function of  $r_{a,n,1}$ ,

 $\sum_{n=1}^{n} E_{n,1}$  is also convex with respect to  $r_{a,n,1}$ .

Because problems (18) and (22) are convex optimization problems, we can find the optimal solution relying on the KKT conditions. With the KKT conditions, the data rate at each subcarrier can be expressed as a function of the Lagrange multiplier  $\lambda$ , which enables an effective searching method such as the bisection method for the optimal solution.

# 4.2 Finding the Suboptimal Subcarrier Assignment Subset for One-Way and Two-Way Relaying Transmissions

In previous sections, we analyzed the optimal problem with a fixed subcarrier assignment subset. Here, we develop a suboptimal solution to find a subcarrier assignment subset for both

one-way and two-way relaying transmissions. To facilitate the solution, we define a ranking and match operator.

**Definition 1**: For two-hop multicarrier modulation transmission, the ranking operator at each hop with respect to the subcarriers' channel gain is defined as  $Rank\{h_n\}_1^N$  and  $Rank\{g_n\}_1^N$  in descending order, respectively.

For a length-*N* vector  $s = \{s[1], ..., s[n], ..., s[N]\}$ , the ranking operator  $Rank\{s[n]\}_1^N$  is another length-*N* vector, whose elements are obtained by permuting the elements of *s*, with  $Rank[n1] \ge Rank[n2]$  for arbitrary n1 < n2.

By indexing and matching the subcarriers, the maximum transmission rate is achieved [14] [16]. Then, we can find a simple way to assign subcarriers to different transmission modes (one-way relay transmission mode or two-way relay transmission mode). Throughout the rest of this paper, we shall replace n with (n) to denote indexed subcarriers.

**Lemma 1**: (see [14], lemma 3)

For both one-way relaying and two-way relaying modes, priority is given to subcarriers with better channel conditions based on the ranking index

*Proof:* This lemma is the direct result of the ranking and match operator. For equal transmission rates, the power consumption per bit for one-way relaying  $E_{(n),1}$  and two-way relaying  $E_{(n),2}$  are monotonically decreasing functions with respect to h(n) and g(n), and because h(n) and g(n) are decreasing with respect to (n), we conclude the result that  $E_{(n),1}$  and  $E_{(n),2}$  are increasing functions with respect to (n).

**Lemma 2**: Two-way relay transmission has a higher priority of channel occupancy than one-way relay transmission.

Proof:

$$E_{(n),1} - E_{(n),2} = \frac{\gamma_{(n)}\sigma^{2}(\frac{1}{h_{(n)}^{2}} + \frac{1}{g_{(n)}^{2}}) + \frac{2\sigma^{2}}{g_{(n)}h_{(n)}}\sqrt{\gamma_{(n)}(1+\gamma_{(n)})}}{C(\gamma_{(n)})} - \frac{2\gamma_{(n)}\sigma^{2}(\frac{1}{h_{(n)}^{2}} + \frac{1}{g_{(n)}^{2}}) + \frac{2\sigma^{2}}{g_{(n)}h_{(n)}}\sqrt{2\gamma_{(n)}(1+2\gamma_{(n)})}}{2C(\gamma_{(n)})}$$
$$= \frac{(2-\sqrt{2})\sigma^{2}\sqrt{\gamma_{(n)}(1+\gamma_{(n)})}}{C(\gamma_{(n)})g_{(n)}h_{(n)}} \geq 0$$
(23)

Because  $E_{(n),1} - E_{(n),2} > 0$ , two-way relay transmission consumes less power per bit than one-way relay transmission. The two-way relay transmission mode therefore has a higher privilege to use subcarriers with good channel conditions.

**Lemma 3**: For the cases of two subcarrier transmission systems, the rate requirement for one-way relaying is  $r_1$  and that for two-way relaying is  $r_2$ , and the matched subcarrier with good channel conditions should be assigned to one-way relay transmission when  $\gamma_1 > 2\gamma_2$ .

*Proof:* For fixed transmissions  $r_1$  and  $r_2$ , we investigate the total system power consumption. Two schemes are possible: scheme 1 allocates h(1) and g(1) to one-way relay transmission and h(2) and g(2) to two-way relay transmission; scheme 2 exchanges the matched subcarriers for one-way and two-way relay transmissions. If the total power consumption of scheme 1 is less than that of scheme 2, scheme 1 is suggested.

The total power consumption of scheme 1 is

$$P_{\tau}(1) = P_{\tau,1}(1) + P_{\tau,2}(1)$$

$$= \gamma_1 \sigma^2 \left(\frac{1}{h_{(1)}^2} + \frac{1}{g_{(1)}^2}\right) + \frac{2\sigma^2}{h_{(1)}g_{(1)}} \sqrt{\gamma_1(1+\gamma_1)} + 2\gamma_2 \sigma^2 \left(\frac{1}{h_{(2)}^2} + \frac{1}{g_{(2)}^2}\right) + \frac{2\sigma^2}{h_{(2)}g_{(2)}} \sqrt{2\gamma_2(1+2\gamma_2)}$$
(24)

The total power consumption of scheme 2 is

$$P_{T}(2) = P_{T,1}(2) + P_{T,2}(2)$$

$$= \gamma_{1}\sigma^{2}(\frac{1}{h_{(2)}^{2}} + \frac{1}{g_{(2)}^{2}}) + \frac{2\sigma^{2}}{h_{(2)}g_{(2)}}\sqrt{\gamma_{1}(1+\gamma_{1})} + 2\gamma_{2}\sigma^{2}(\frac{1}{h_{(1)}^{2}} + \frac{1}{g_{(1)}^{2}}) + \frac{2\sigma^{2}}{h_{(1)}g_{(1)}}\sqrt{2\gamma_{2}(1+2\gamma_{2})}$$
(25)

The difference of power consumptions between scheme 1 and scheme 2 is

$$P_{T}(1) - P_{T}(2) = \gamma_{1}\sigma^{2}\left(\frac{1}{h_{(1)}^{2}} + \frac{1}{g_{(1)}^{2}} - \frac{1}{h_{(2)}^{2}} - \frac{1}{g_{(2)}^{2}}\right) + \sqrt{\gamma_{1}(1+\gamma_{1})}\sigma^{2}\left(\frac{2}{h_{(1)}g_{(1)}} - \frac{2}{h_{(2)}g_{(2)}}\right) + 2\gamma_{2}\sigma^{2}\left(\frac{1}{h_{(2)}^{2}} + \frac{1}{g_{(2)}^{2}} - \frac{1}{h_{(1)}^{2}} - \frac{1}{g_{(1)}^{2}}\right) + \sqrt{2\gamma_{2}(1+2\gamma_{2})}\sigma^{2}\left(\frac{2}{h_{(2)}g_{(2)}} - \frac{2}{h_{(1)}g_{(1)}}\right) = (2\gamma_{2} - \gamma_{1})A + \left(\sqrt{2\gamma_{2}(1+2\gamma_{2})} - \sqrt{\gamma_{1}(1+\gamma_{1})}\right)B$$
(26)

When  $P_{\tau}(1) - P_{\tau}(2) < 0$ , scheme 1 is preferred, and we have

$$2\gamma_2 \mathbf{A} + \sqrt{2\gamma_2(1+2\gamma_2)}\mathbf{B} < \gamma_1 \mathbf{A} + \sqrt{\gamma_1(1+\gamma_1)}\mathbf{B}$$
(27)

where  $A = \sigma^2 (1 / h_{(2)}^2 + 1 / g_{(2)}^2 - 1 / h_{(1)}^2 - 1 / g_{(1)}^2)$ ,  $B = \sigma^2 (2 / h_{(2)} g_{(2)} - 2 / h_{(1)} g_{(1)})$ , and because  $h_{(1)} > h_{(2)}$  and  $g_{(1)} > g_{(2)}$ , we can conclude A > 0 and B > 0. Because  $f(x) = Ax + B\sqrt{x(1+x)}$  is a monotonically increasing function, for the inequality to hold, we must have  $\gamma_1 > 2\gamma_2$ .

Lemmas 1–3 imply that we can adopt a linear search procedure to seek optimal subcarrier assignment subsets. Initially,  $N_I$  ( $R_b / r_{max} \le N_1 \le N - (R_a - R_b) / r_{max}$ ) pairs of matched subcarriers (( $L(1), L(2), ..., L(n), ..., L(N_I)$ ), where  $L(n) = \{h_{(n)}, g_{(n)}\}$ , are scheduled for two-way relay transmission and the rest for one-way relay transmission, and we execute the link exchange iteration process (based on Lemma 3) and reassign the power until the overall power cannot be further reduced. Then we find the optimal  $N_I^*$  that results in the minimal power consumption; the detailed algorithm follows.

The energy-efficiency multicarrier AF relay asymmetric transmission algorithm Initialize:  $R_a$ ,  $R_b$ ,  $r_{max}$ ,

**Iteration:** for  $R_b / r_{max} \le N_1 \le N - (R_a - R_b) / r_{max}$ 

(a) Execute the power allocation and bit loading for two-way relay transmission according to (18) and one-way relay transmission according to (22).

Compute 
$$P_T(N_1) = \sum_{(n)=1}^{N_1} P_{T,(n),1} + \sum_{(n)=N_1+1}^{N} P_{T,(n),2}$$
.

(b) Exchange the link between Subset2 =  $(L(1), L(2), ..., L(N_1))$  and Subset1 =  $(L(N_1+1), L(N_1+2), ..., L(N))$  according to Lemma 3.

(c) Redo the power allocation according to (18) and (22) under the new subcarrier subsets obtained in (b) to decrease the total power  $P_T(N_I)$ .

(d) Update  $P_T(N_I)$ . Execute (b) and (c) until total power does not decrease.  $P_T^*(N_I)$  is the minimal power consumption solution for  $N_1$ .

**Termination:** 

Find 
$$N_1^* (R_b / r_{max} \le N_1^* \le N - (R_a - R_b) / r_{max}$$
, with  $P_T^* (N_1^*) = \underset{r_{max} = N_1 \le N_1 \le N_1 - \frac{R_a - R_b}{r_m}}{Min_{r_m}} P_T^* (N_1)$ 

*Discussion:* Executing the link exchange iteration at each  $N_1$  will result in a global optimal solution, the time complexity is  $O(N^3 \log(N))$ . Because of the computational complexity is relatively high, especially for a large number of subcarriers, we develop a suboptimal solution that seeks the optimal subset number  $N_1^*$  first and performs the subcarrier pair exchange iteration. Furthermore, if we execute the process stated in step (b) for all subcarrier pairs, the convergence speed will be accelerated, and the time complexity is reduced to  $O(N^2)$ .

## 5. Simulation results

We perform the computer simulation for OFDM bidirectional AF relay transmission with asymmetric traffic to evaluate the system performance and the influence of energy efficiency on different asymmetric levels. The proposed scheme is compared with the traditional one-way relay method, which needs four time slots for two users to transmit the information (two time slots for each user). Note that the Rayleigh distribution is assumed in this study; the subcarrier number used is N = 64; multiple quadrature amplitude modulation (MQAM) is employed for each subcarrier; and the possible bit loading for each subcarrier is  $\{0, 2, 4, 6, 8\}$ , which means that  $r_{max} = 8$ . The corresponding modulations are no modulation, 4QAM, 16QAM, 64QAM, and 256QAM. The noise variance is set to 1.2e-10. We assume the relay is located at the center of the line between two users. We first compare the total power consumption of bidirectional relaying with the traditional one-way relaying method under different asymmetric traffic ratios, as depicted in Fig. 2. The data rate for  $S_a$  is  $R_a=40$  bit/s in Fig. 2(a) and  $R_a = 60$  bit/s in Fig. 2(b); for a different asymmetric traffic ratio  $\alpha$ , the data rate of  $S_b$  is  $R_b = \alpha \cdot R_a$  bit/s. The simulation results show that for both the one-way relay transmission system and the bidirectional relay transmission system, the power consumption is a monotonically increasing function in response to the asymmetric traffic ratio. This is because as the asymmetric traffic ratio increases, for fixed data rate  $R_a$ , more data needs to be sent. From Fig. 2, it is clear that the bidirectional OFDM relaying method needs less energy for transmission compared with the classical one-way OFDM relaying method. This is an interesting result, indicating that the bidirectional OFDM relaying method has an advantage over the classical one-way OFDM relaying method, which was depicted in Lemma 2.



Fig. 2. Total energy consumption versus the data rate requirements of the two users.

We present the subcarrier number allocated for two-way relay transmission in **Fig. 3**. It is observed that the data rate requirement has a significant impact on the subcarrier allocation. What's more, the figure shows that more subcarriers need to be assigned to the two-way relay transmission mode as the asymmetric traffic ratio increases.



Fig. 3. Subcarrier number allocated for two-way relay transmission,  $R_a=40$ , and 60bit/s.

To investigate the energy efficiency of the proposed scheme, we simulate the power consumption per bit with different asymmetric traffic ratios. Fig. 4 indicates that the energy consumption per bit of the bidirectional OFDM relaying method is less than that of the one-way OFDM relaying method. For the one-way OFDM relaying method, the curve decreases at first and then rises, because when the traffic ratio is small, the best subcarrier pairs can be used to transmit for user  $S_b$ , and the energy consumed per bit decreases, so the curve decreases. However, as the traffic ratio increases, some subcarriers with bad channel conditions are used for transmission, which decreases the energy efficiency, so the curve rises. For the bidirectional OFDM relaying method, the energy consumption per bit transmission almost decreases as the traffic ratio increases. This is because two-way relay transmission has higher energy efficiency than one-way relay transmission under the same conditions. Nevertheless, when the traffic ratio is high, some subcarrier pairs with bad link quality take part in the transmission. Although two-way relay transmission has higher energy efficiency, the average energy consumption for the extra bit in bad subcarrier pairs may exceed the average system energy consumption. This will counteract the effect of the two-way relay transmission, thereby decreasing the system energy efficiency. This is represented as an increase in the traffic ratio ( $\alpha = 0.7-0.9$ ). These results indicate that even when the uplink and downlink paths have the same data rate, asymmetric traffic transmission strategies can still improve the energy efficiency of the system.



Fig. 4. Energy consumption per bit with different asymmetric traffic ratios,  $R_a$ =40bit/s.

#### 6. Conclusion

In this paper, we described the joint optimization problem of power allocation, bit loading, and subcarrier transmission mode selection with asymmetric traffic in an OFDM-based bidirectional AF relay transmission system. By using the attributes for two-way relay transmission and subcarrier pairing, we decouple the primary problem into two subproblems. We first optimize the power allocation for both one-way relay transmission and two-way relay transmission under fixed subcarrier subsets, and find the optimal subsets under subcarrier pairing. The simulation results showed that our proposed scheme outperforms the traditional one-way OFDM relaying method. Furthermore, the best energy efficiency for OFDM bidirectional AF relaying is obtained at a fixed symmetric traffic ratio, which makes this a useful technique for bidirectional relaying design.

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