Partial Relay Selection in Decode and Forward Cooperative Cognitive Radio Networks over Rayleigh Fading Channels

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Abstract

The performance of an partial relay selection on the decode-and-forward (DF) mode cognitive radio (CR) relay networks is studied, with some important factors, including the outage probability, the bit error ratio (BER), and the average channel capacity being analyzed. Different from the conventional relay selection schemes, the impact of spectrum sensing process as well as the spectrum utilization efficiency of primary users on the performance of DF-based CR relaying networks has been taken into consideration. In particular, the exact closed-form expressions for the figures of merit such as outage probability, BER, and average channel capacity over independent and identically distributed (i.i.d.) Rayleigh fading channels, have been derived in this paper. The validity of the proposed analysis is proven by simulation, which showed that the numerical results are consistent with the theoretical analysis in terms of the outage probability, the BER and the average channel capacity. It is also shown that the full spatial diversity order can always be obtained at the signal-to-noise ratio (SNR) range of [0dB, 15dB] in the presence of multiple potential relays.

Keywords: cognitive radio, cooperative networks, relays, decode and forward

1. Introduction

Cognitive radio (CR) is regarded as a promising technology to provide high bandwidth to mobile users via heterogeneous wireless network architectures and dynamic spectrum access techniques, and the significant improvement of efficiency of spectrum utilization [1]-[5].

In CR networks, however, the CR users must immediately vacate the licensed spectrum bands they are using once the primary users return to access those bands. As a result, the interruptive transmissions in unlicensed users will lead to a discontinuous data service and intolerable delay. Challenges aforementioned can be addressed by using cognitive relaying, which enables distributed cognitive users to collaborate with each other and share their distinct spectrum bands. A seamless data transmission can therefore be realized by using cooperative relays, and a mutual benefit among CR users can be brought forward by communicating with each other to minimize the miss detection probability. Besides, more benefits can be brought by using cognitive relays, such as an expanded coverage, a better immunity against signal fading, a more system-wide power saving and an increased throughput of the whole system [6]-[9].

The diversity gain of cooperative networks can thus be improved by employing multiple relays. However, in the presence of multiple relays, a sophisticated resource allocation algorithm is required to perform so as to guarantee an orthogonal channel (in carrier frequencies, time slots or codes) being allocated to each relay and avoid inter-relay-interference [10], [11]. As the number of relays increases, the cost of bandwidth penalty may even deteriorate the benefits brought by cooperative relays. In consideration of the challenges aforementioned, relay selection can be regarded as one of the most attractive methods to solve the complicated interference-mitigation issue met in the multi-relay systems. Selecting the best relay to forward data is an ideal way to effectively balance the complexity and the spectral efficiency improvement due to the full diversity order obtained [12], [13]. Only two time slots are thus required to enable two orthogonal channels (regardless of the number of relays) to be necessarily reserved in both the source and the selected relay nodes [14].

Currently, relay selection has been treated in several existed literatures (see, e.g., [15]-[17] and the references therein). Most of those works deal with the case where the primary users are absent for the relay selection. By selecting the optimal relay from multiple candidates, only one relay is active in each time slot so as to significantly improve the spectral efficiency and obtain a full spatial diversity order simultaneously. By and large, relay selection techniques in cooperative communications systems can be classified into two modes, i.e., the opportunistic relay selection [18] and the partial relay selection [19]. In the former, the channel state information (CSI) of both the source-to-relay ($S \rightarrow R$) and relay-to-destination ($R \rightarrow D$) links should be considered by the central unit. In the latter, on the other hand, the CSI of only the $S \rightarrow R$ or $R \rightarrow D$ link is necessarily considered. In [20], the closed-form expressions for the lower bound and asymptotic symbol error rate of in Rayleigh fading relay channel with opportunistic relay selection are derived. The impact of relay selection with an outdated CSI on the capacity of cooperative communication systems has been studied in [21], [22], where the closed-form expressions for channel capacity of opportunistic relay selection with four classical adaptive transmission techniques are given out by [21], and that with partial relay selection has been studied in [22]. The opportunistic relay selection is proven to have an

advantage over the partial relay selection if the SNR of links is relative high and a perfect CSI is assumed at the same time.

It is worth pointing out that the relay selection in cognitive networks got a considerable attention in recent years, with some important factors, such as mutual interference between primary users and secondary users, being taken into consideration in CR relay network [23]-[26]. The impact of interference threshold on the amplify-and-forward (AF) underlay cognitive networks has been studied in [27]. In [28], which focuses on exploiting the transmission opportunity in CR networks, cooperative communication appears to be a promising approach to improve the throughput of the secondary users by increasing the spatial and spectrum diversity orders. The author in [29] analyzes the outage probability of AF relaying cognitive networks, in which the best relay is selected based on either full CSI or partial CSI feedback. The BER of AF systems with partial relay selection in CR network is study in [30]. Furthmore, some efficient spectrum sensing schemes are also proposed in [31]-[33]. However, to the best of the our knowledge, the impact of spectrum sensing process and spectrum utilization efficiency of primary users on the overlay CR relay networks with partial relay selection has still not been studied considerably in prior works.

In this paper, partial relay selection in DF-based CR networks is studied. One of main contributions in this paper is to derive the exact closed-form expressions of some figures of merit, such as the outage probability, BER and the average channel capacity over i.i.d. Rayleigh fading channels, for the proposed relay selection mode. In particular, parameters aforementioned are proven to be dependent on spectrum sensing process as well as the spectrum utilization efficiency of primary users. Besides, a full diversity order is proven to be obtained at low SNR regime, regardless of the spectrum utilization efficiency of the primary users. At high SNR regime, on the other hand, the diversity order keeps unchanged, regardless of the number of potential relays. The validity of the proposed theoretical analysis is also proven by simulations.

The remainder of this paper is organized as follows. Section 2 introduces the system model of partial relay selection in cognitive radio relay networks. The probability density function (PDF) and cumulative distribution function (CDF) of received SNR at destination are introduced in section 3. The closed-form expressions of some critical parameters, including outage probability, BER, and average channel capacity, are derived in section 4. Section 5 gives out the numerical results. Finally, section 6 concludes this paper.

Notation: $\Re\{x\}$ and $\Im\{x\}$ are the real and imaginary part of x, respectively. γ_{ab} represents the SNR of $a \to b$ link. $f_X(.)$ and $F_X(.)$ represent the PDF and CDF of the random variable (RV) X, respectively. Pr(.) denotes the probability.

2. System Model

In this section, a CR relay network, which consists of a secondary source terminal (S), M half-duplex DF potential relays (as denoted by the set of $\Omega = \{R_i, i = 1, 2, \ldots M\}$), and a destination terminal (D) (please see **Fig. 1**), is considered. S communicates with D using a time division multiple access (TDMA) arrangement. Hence, two time slots are needed to complete one communication session. In the first time slot, S broadcasts the packets to the relays. In the second time slot, each potential relay decides whether to become a candidate for the best relay selection by examining two conditions, decoding of the received signal and availability of the SH. After the best relay selection, the selected relay forward a regenerated

replica to D. At the end of each communication session, D combines the received signals using maximal ratio combining (MRC).
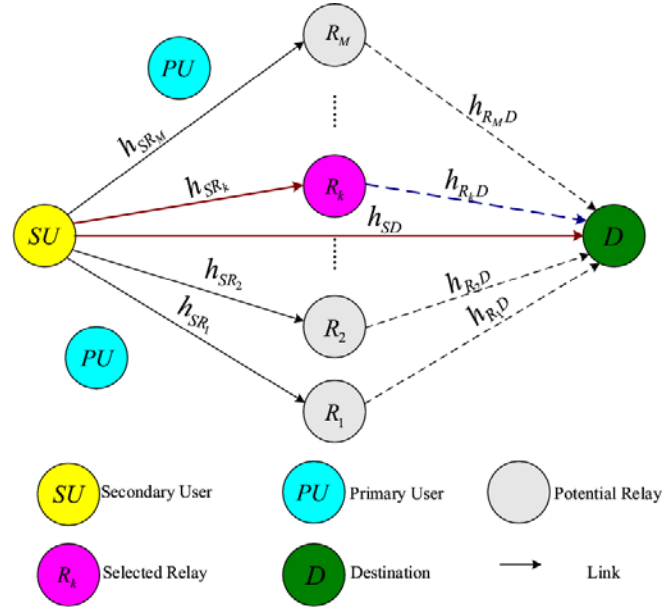

Fig. 1. A single-hop cooperating network with multi-relays in overlay cognitive radio relay networks

Without loss of generality, the wireless channels of $S \rightarrow R_i$, $R_i \rightarrow D$ and $S \rightarrow D$ links are assumed to be i.i.d. Rayleigh distributed RVs. Therefore, we have the circularly symmetric complex Gaussian channel gain h_{ab} ($a,b \in \{S, R_i, D\}$) with mean zero and variance σ^2 between node a and b, which is denoted by $h_{ab} \sim \mathcal{CN}(0, \sigma^2)$. In brief, we assume that all nodes transmit with unit power and the additive white Gaussian noise (AWGN) power is N_0, and the effective SNR is given by $\gamma_{ab} = \dfrac{|h_{ab}|^2}{N_0} = \dfrac{(\Re\{h_{ab}\})^2}{N_0} + \dfrac{(\Im\{h_{ab}\})^2}{N_0}$. From [34, Eq.(2-3-27)], γ_{ab} can be formulated as a chi-square (χ^2) random variable with 2 degrees of freedom, and its PDF is given by $f_{\gamma_{ab}}(\gamma) = \dfrac{1}{\overline{\gamma}_{ab}} e^{-\frac{\gamma}{\overline{\gamma}_{ab}}}$, where $\overline{\gamma}_{ab}$ denotes the average SNR of $a \rightarrow b$ link. Accordingly, the average SNR of each $S \rightarrow R_i$ link can be represented as $\overline{\gamma}_{SR_i}$. The average SNR of $R_i \rightarrow D$ and $S \rightarrow D$ links can be denoted by $\overline{\gamma}_{R_iD}$ and $\overline{\gamma}_{SD}$, respectively.

3. PDF and CDF of the Relay-Selection Channel

In this section, two conditions, i.e., the potential relay condition and the relay selection rule, will be analyzed, and afterwards, the PDF and CDF of the proposed relay selection channels are given out.

3.1 Potential Relay Condition

The relay may be a in the set of potential relay, Ω, only if it has successfully decoded the received packets and acquired the SH.

1) Successful Decoding: In order to decode successfully, the target rate r, measured in bits per second per Hertz, is lower than the mutual information, the successful decoding probability can be written as

$$
\begin{aligned}
P_{d_i} &= \Pr[\frac{1}{2}\log_2(1+\gamma_{SR_i}) \geq r] \\
&= \Pr[\gamma_{SR_i} \geq \gamma_{th}] \\
&= e^{-\gamma_{th}/\overline{\gamma}_{SR_i}},
\end{aligned}
\tag{1}
$$

where γ_{SR_i} and $\overline{\gamma}_{SR_i}$ denote the SNR and the average SNR of $S \to R_i$ link, respectively, and $\gamma_{th} = 2^{2r} - 1$ is the threshold SNR.

2) Successful Acquisition of SH: When the primary users is absent with probability, $\Pr[\mathsf{H}_0^{PU}]$, and present with probability, $\Pr[\mathsf{H}_1^{PU}]$, the opportunity of SH is obtained by R_i with probability P_{a_i}. The probability of R_i successful acquisition of SH can be rewritten as

$$
P_{a_i} = (1 - P_{f,i})\Pr[\mathsf{H}_0^{PU}] + (1 - P_{de,i})\Pr[\mathsf{H}_1^{PU}],
\tag{2}
$$

where the first term denotes, the probability of the secondary users obtain the SH when the primary user is absent with false alarm probability, $P_{f,i}$; the second term denotes, the probability of the secondary users obtain the SH when the primary user is present with detection probability, $P_{de,i}$.

Therefore, if the R_i have successful spectrum acquisition probability P_{a_i} and successful decoding probability P_{d_i}, the relay become successful potential relay with probability Q_i, $\Pr[R_i \in \Omega] = P_{d_i}P_{a_i}$.

Since all the $S \to R_i$ links is i.i.d. Rayleigh channels, $\overline{\gamma}_{SR_1} = \overline{\gamma}_{SR_2} = \cdots = \overline{\gamma}_{SR_M} = \overline{\gamma}_{SR}$. Therefore, $P_{d_1} = P_{d_2} = \cdots = P_{d_M} = P_d$. Likewise, $P_{a_1} = P_{a_2} = \cdots = P_{a_M} = P_a$. Hence, $\mathsf{Q}_1 = \mathsf{Q}_2 = \cdots = \mathsf{Q}_M = \mathsf{Q}$. In this case, $\Pr[|\Omega| = m]$, becomes

$$
\Pr[|\Omega| = m] = \binom{M}{m}(\mathsf{Q})^m(1-\mathsf{Q})^{M-m},
\tag{3}
$$

where $|\Omega|$ is the cardinality of Ω.

3.2 Relay Selection Rule

In this subsection, for the Rayleigh fading on the $R_i \to D$ links is i.i.d., the average SNR in each the $R_i \to D$ links is given by $\overline{\gamma}$, and is the same as the average SNR of $S \to D$ link. In other words, $\overline{\gamma}_{R,D} = \overline{\gamma}_{SD} = \overline{\gamma}$. Two scenarios, i.e., $|\Omega| = 0$ and $|\Omega| \geq 1$, will be analyzed separately as follows.

1) $|\Omega| = 0$: When the set of Ω is null, φ (with no elements content the condition of potential relay), the secondary users only have the $S \to D$ link. Hence, the effective SNR at the

destination can be given as $\gamma_d = \gamma_{SD}$. In this case, the PDF of γ_d becomes

$$f_{\gamma_d \| \Omega \|}(\gamma \mid 0) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}, \gamma \geq 0, \tag{4}$$

where $\overline{\gamma}$ is the average SNR of $S \to D$ link.

Evidently, (4) leads to

$$F_{\gamma_d \| \Omega \|}(\gamma \mid 0) = 1 - e^{-\frac{\gamma}{\overline{\gamma}}}, \gamma \geq 0. \tag{5}$$

2) $|\Omega| \geq 1$: The secondary user selects the optimal relay according to the following rule, i.e.,

$$k = \arg \max_{i: R_i \in \Omega} (\gamma_{R_i D}). \tag{6}$$

From Appendix I (18), with the partial relay selection rule (6), the conditional PDF of $\gamma_{R_k D}$ can be derived as

$$f_{\gamma_{R_k D} \| \Omega \|}(\gamma \mid m) = \frac{m}{\overline{\gamma}} \sum_{n=0}^{m-1} \binom{m-1}{n} (-1)^n e^{-\frac{(n+1)\gamma}{\overline{\gamma}}}, \gamma \geq 0. \tag{7}$$

In the presence of cooperative relays, the spatial diversity order can be improved by utilizing the relay-forward links together with the $S \to D$ link. Some combining method, e.g., MRC, can be employed to the destination to optimize the effective SNR as $\gamma_d = \gamma_{R_k D} + \gamma_{SD}$.

From Appendix I (19), in this case, the conditional PDF of γ_d for a given $|\Omega|$ can be derived as

$$f_{\gamma_d \| \Omega \|}(\gamma \mid m) = \frac{m}{\overline{\gamma}} \sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^{n+1} \frac{1}{n} \left(e^{-\frac{(n+1)\gamma}{\overline{\gamma}}} - e^{-\frac{\gamma}{\overline{\gamma}}} \right) + \frac{m\gamma}{\overline{\gamma}^2} e^{-\frac{\gamma}{\overline{\gamma}}}, \gamma \geq 0. \tag{8}$$

Evidently, (8) leads to (the detail please see Appendix II)

$$F_{\gamma_d \| \Omega \|}(\gamma \mid m) = m \sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^{n+1} \frac{1}{n(n+1)} \left((n+1) e^{-\frac{\gamma}{\overline{\gamma}}} - e^{-\frac{(n+1)\gamma}{\overline{\gamma}}} - n \right) + \frac{m}{\overline{\gamma}} \left(\overline{\gamma} - (\overline{\gamma} + \gamma) e^{-\frac{\gamma}{\overline{\gamma}}} \right), \gamma \geq 0. \tag{9}$$

Hence, the uncondition CDF of the received SNR is derived as

$$F_{\gamma_d}(\gamma) = \sum_{m=1}^{M} F_{\gamma_d \| \Omega \|}(\gamma \mid m) \Pr[|\Omega| = m] + F_{\gamma_d \| \Omega \|}(\gamma \mid 0) \Pr[|\Omega| = 0], \gamma \geq 0. \tag{10}$$

4. Performance Analysis

In this section, the closed-form expressions for the outage probability, BER and channel capacity will be derived.

4.1 Outage Probability Analysis

For a pre-set threshold γ_{th}, from (10), the outage probability of the partial relay selection scheme can be derived as

$$P_{out} = \sum_{m=1}^{M} F_{\gamma_d \parallel \Omega \mid}(\gamma_{th} \mid m) \Pr[\mid \Omega \mid = m] + F_{\gamma_d \parallel \Omega \mid}(\gamma_{th} \mid 0) \Pr[\mid \Omega \mid = 0] \tag{11}$$

4.2 Bit Error Ratio Analysis

Using [34, Eq.(2.3-10)], [35, Eq.(4)], the BER of the proposed partial relay selection can be derived as

1) $\mid \Omega \mid = 0$ *:*

$$\begin{aligned} P_e(\mid \Omega \mid = 0) &= \int_0^{\infty} F_{\gamma_d \parallel \Omega \mid}\left(\frac{x^2}{\eta} \mid 0\right) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}}{\overline{\gamma}+1}}\right), \gamma \geq 0. \end{aligned} \tag{12}$$

2) $\mid \Omega \mid \geq 1$ *:* with aid of [36, Eq.3.371], the result becomes

$$\begin{aligned} P_e(\mid \Omega \mid = m) &= \int_0^{\infty} F_{\gamma_d \parallel \Omega \mid}\left(\frac{x^2}{\eta} \mid m\right) \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= \frac{m}{2} \sum_{n=1}^{m-1} \binom{m-1}{n}(-1)^{n+1} \frac{1}{n(n+1)}\left((n+1)\sqrt{\frac{\overline{\gamma}}{\overline{\gamma}+1}} - \sqrt{\frac{\overline{\gamma}}{n+1+\overline{\gamma}}} - n\right) \\ &\quad + \frac{m}{2}\left(1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}} - \frac{1}{2}\sqrt{\frac{\overline{\gamma}}{(1+\overline{\gamma})^3}}\right), \gamma \geq 0. \end{aligned} \tag{13}$$

Hence, the unconditional BER is derived as

$$P_e = \sum_{m=1}^{M} P_e(\mid \Omega \mid = m) \Pr[\mid \Omega \mid = m] + P_e(\mid \Omega \mid = 0) \Pr[\mid \Omega \mid = 0], \tag{14}$$

and η are specified modulation constants determined by modulation format (e.g., for phase shift keying (PSK) modulation, we have $\eta = 2$ [37]).

4.3 Channel Capacity Analysis

From Appendix III, the closed-form expression for the channel capacity of the proposed partial relay selection can be given as

$$C = \sum_{m=1}^{M} \Pr[|\Omega|=m] \frac{B}{2\ln 2} \frac{m}{\overline{\gamma}} [\sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^{n+1} \frac{1}{n} \left(\frac{\overline{\gamma}}{n+1} e^{\frac{n+1}{\overline{\gamma}}} E_1 \left(\frac{n+1}{\overline{\gamma}} \right) - \overline{\gamma} e^{\frac{1}{\overline{\gamma}}} E_1 \left(\frac{1}{\overline{\gamma}} \right) \right)$$

$$+ \overline{\gamma} + (\overline{\gamma}-1) e^{\frac{1}{\overline{\gamma}}} E_1 \left(\frac{1}{\overline{\gamma}} \right)] + \Pr[|\Omega|=0] \frac{B}{2\ln 2} e^{\frac{1}{\overline{\gamma}}} E_1 \left(\frac{1}{\overline{\gamma}} \right), \quad (15)$$

where B stands for the signal bandwidth.

5. Numerical Results

In this section, we study the performance of some of the derived closed-form expressions through numerical evaluation as well as using Monte Carlo simulation. Binary phase shift keying (BPSK) modulation is considered in this paper, and this implies $\eta = 2$. Let us start with some figure of merit, including the outage probability, BER and the average channel capacity over i.i.d. Rayleigh fading channels which implies $\overline{\gamma}_{SD} = \overline{\gamma}_{RD} = \overline{\gamma}_{SR} = \overline{\gamma}$. A normalized system bandwidth is assumed.

The outage probability as a function of γ for different number of potential relays, M=0, 1, 2, 3, 4, 5, 6, is depicted in Fig.2. At low average SNR, since with larger number of relays, the destination has more opportunity to correct decode the received data. Hence, the spatial diversity order is increasing with the number of potential relays. At high average SNR, since the relay can decoded the received data packets correctly regardless of the number of potential relays, two copies are transmitted to the destination. Hence, the diversity order keeps two with opportunity, and the slope of curves keeping same as in the high average SNR values are shown in **Fig. 2**. However, the number of candidate relays which acquire SH and decode the received packets successfully is increased by increasing the number of relays. The more optimum relays can be chose from the candidate relays set with larger number of elements. Hence, the outage probability performance is improved by the larger number of relays.
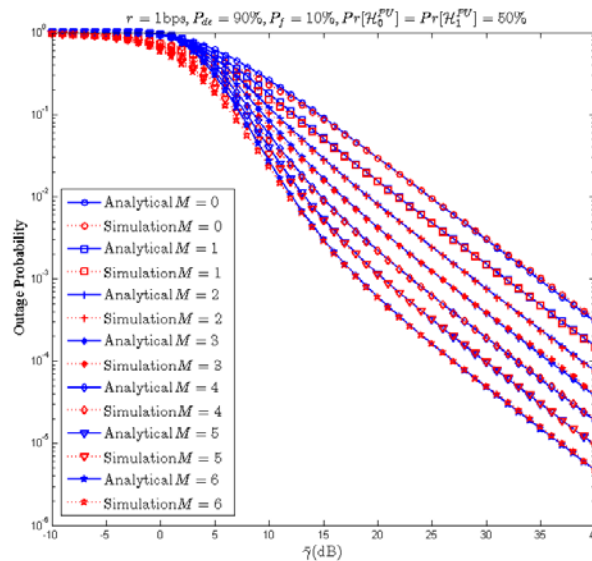

Fig. 2. Outage probability versus the average SNR of the links for different values of M using simulations and analytical results
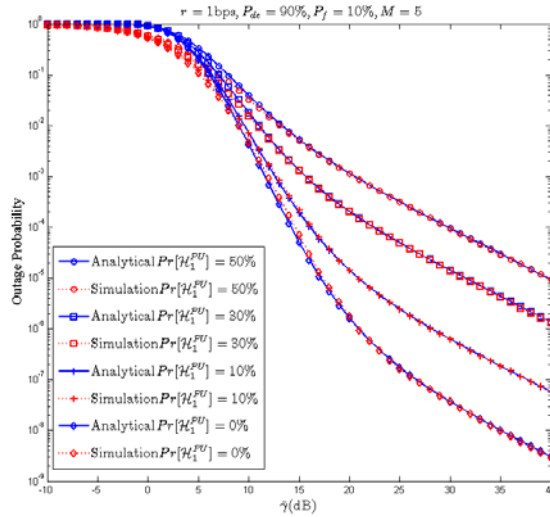
Fig. 3. Outage probability versus the average SNR of the links for different values of $Pr[\mathsf{H}_1^{PU}]$ using simulations and analytical results

By keeping $M = 5$ unchanged, for different $Pr[\mathsf{H}_1^{PU}]$, a smaller $Pr[\mathsf{H}_1^{PU}]$ implies a larger probability of acquisition SH, and the outage probability is therefore a monotonically increasing function of $Pr[\mathsf{H}_1^{PU}]$, as shown in **Fig. 3**. Note that $Pr[\mathsf{H}_1^{PU}] = 0$ implies the primary user is absent all the time. If the relays can decode successfully, it can be the candidate relays. For all $Pr[\mathsf{H}_1^{PU}] > 0$ scenarios, the primary user is presented occasionally.

BER as a function of average SNR is illustrated in **Fig. 4**. For a specific average SNR of links, spectrum sensing process and spectrum utilization efficiency of primary users, more potential relays implies a better BER performance due to an improved relay link. However, similar to the properties exhibited in outage probability performance, the slope of curves is the same when average SNR goes beyond a certain threshold.
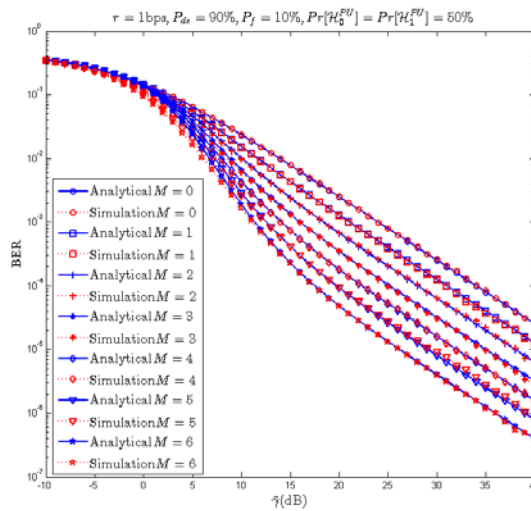

Fig. 4. BER versus the average SNR of the links for different values of M using simulations and analytical results

The effect of the probability of the primary users is present $Pr[\mathsf{H}_1^{PU}]$ on BER performance of the partial relay selection is also illustrated in **Fig. 5**. Like in the outage probability performance, BER is also a monotonically increasing function of $Pr[\mathsf{H}_1^{PU}]$. At low SNR-regime, the diversity order is impacted by the number of relays, because the cooperative networks with more relays have more opportunities to successfully delivery the data. However, at high SNR-regime, the data can be successfully decoded and forwarded, with the diversity order being uncorrelated to the number of relays. Hence, the slopes for the BER-curves in terms of SNR exhibit some changes in **Fig. 5.**

The average channel capacity as a function of average SNR for the proposed relay selection scheme is described in **Fig. 6**.The channel capacity is a monotonically increasing function of the number of available relays. Note that $M = 0$ implies the secondary users only have the $S \to D$ links. For all $M > 0$ scenarios, if the relay can decode successfully, it could be the candidate relays, and the secondary users will have the opportunity to use the relay. Hence, the capacity can be improved by the diversity order. At high average SNR, since the diversity order remain unchanged. Hence, with large number of M, further increasing M may cause the average channel capacity increase slightly. When $\overline{\gamma} <5$dB, the noise significantly impacts the performance. Therefore, it appears some gap between analytic and simulation results for capacities at low SNR-regime, as shown in **Fig. 6**. In fact, if linear coordinate instead of logarithmic coordinate was utilized, the gap would appear to be very little.
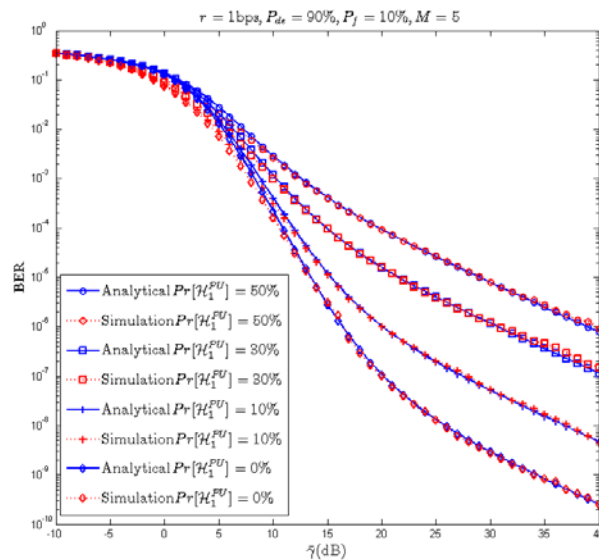

Fig. 5. BER versus the average SNR of the links for different values of $Pr[\mathsf{H}_1^{PU}]$ using simulations and analytical results
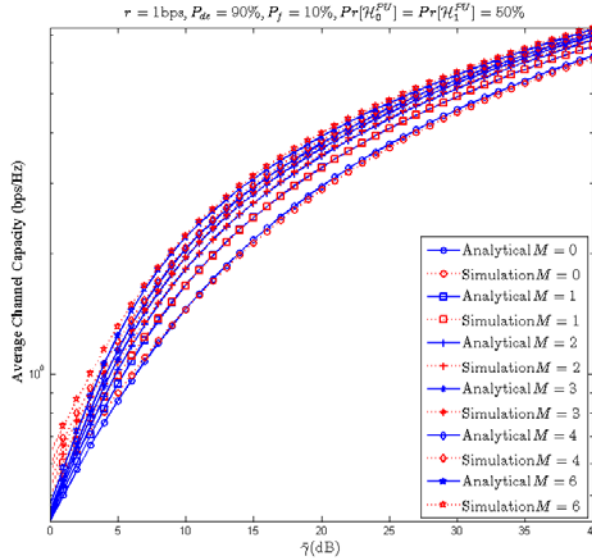
Fig. 6. Average channel capacity versus the average SNR of the links for different values of *M* using simulations and analytical results

6. Conclusion

The performance of DF cooperative cognitive networks was studied, with the exact closed-form expressions, including the outage probability, the BER and the channel capacity, being derived. The validity of the proposed theoretical close-form expression on the critical figures of merit, including the outage probability, the BER and the average channel capacity, was proven via simulations, and the theoretical analysis matches the corresponding numerical results well. It was also shown in the numerical results that some other parameters, including the number of relays, spectrum sensing process, and the spectrum utilization efficiency of primary users impact the system performance greatly in the presence of multiple potential relays. Simulation results proved that the full spatial diversity order can be achieved at low SNR by partial relay selection.

Appendix I

The conditional probability density function of γ_d

For the Rayleigh fading on the $R_i \rightarrow D$ links is independent and identically distributed (i.i.d.), the average SNR in all the $R_i \rightarrow D$ links is given by $\overline{\gamma}$, and is the same as the average SNR of $S \rightarrow D$ links. $f_{\gamma_{R_1 D}}(\gamma) = f_{\gamma_{R_2 D}}(\gamma) = \cdots = f_{\gamma_{R_M D}}(\gamma) = f_{\gamma_{RD}}(\gamma)$, we obtain the PDF of γ_{RD} as follow

$$f_{\gamma_{RD}}(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}, \gamma \geq 0. \tag{16}$$

The CDF of γ_{RD} can be derived directly from (16) as

$$F_{\gamma_{RD}}(\gamma) = \int_0^{\gamma} f_{\gamma_{RD}}(x)dx = 1 - e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \geq 0. \tag{17}$$

With the partial relay selection rule (6), the conditional PDF of γ_{R_kD} can be derived as

$$\begin{aligned}
f_{\gamma_{R_kD}|\Omega|}(\gamma \mid m) &= m(F_{\gamma_{RD}}(\gamma))^{m-1} f(\gamma) \\
&= m(1 - e^{-\frac{\gamma}{\bar{\gamma}}})^{m-1} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \\
&= m \sum_{n=0}^{m-1} \binom{m-1}{n} (-1)^n e^{-n\frac{\gamma}{\bar{\gamma}}} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \\
&= \frac{m}{\bar{\gamma}} \sum_{n=0}^{m-1} \binom{m-1}{n} (-1)^n e^{-\frac{(n+1)\gamma}{\bar{\gamma}}}, \gamma \geq 0,
\end{aligned} \tag{18}$$

where $\binom{m-1}{n} = \frac{(m-1)!}{(m-1-n)!n!}$.

After MRC, the conditional PDF of γ_d will be

$$\begin{aligned}
f_{\gamma_d|\Omega|}(\gamma \mid m) &= f_{\gamma_{SD}}(\gamma) \otimes f_{\gamma_{R_kD}|\Omega|}(\gamma \mid m) \\
&= \int_0^{\gamma} f_{\gamma_{SD}}(\gamma - x) f_{\gamma_{R_kD}|\Omega|}(x \mid m) dx \\
&= \int_0^{\gamma} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma-x}{\bar{\gamma}}} \frac{m}{\bar{\gamma}} \sum_{n=0}^{m-1} \binom{m-1}{n} (-1)^n e^{-\frac{(n+1)x}{\bar{\gamma}}} dx \\
&= \frac{m}{\bar{\gamma}^2} \sum_{n=0}^{m-1} \binom{m-1}{n} (-1)^n e^{-\frac{\gamma}{\bar{\gamma}}} \int_0^{\gamma} e^{\frac{x}{\bar{\gamma}}} e^{-\frac{(n+1)x}{\bar{\gamma}}} dx \\
&= \frac{m}{\bar{\gamma}^2} \sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^n e^{-\frac{\gamma}{\bar{\gamma}}} \frac{1}{\frac{1}{\bar{\gamma}} - \frac{n+1}{\bar{\gamma}}} \left(e^{\left(\frac{1}{\bar{\gamma}} - \frac{n+1}{\bar{\gamma}}\right)\gamma} - 1 \right) + \frac{m\gamma}{\bar{\gamma}^2} e^{-\frac{\gamma}{\bar{\gamma}}} \\
&= \frac{m}{\bar{\gamma}^2} \sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^n \frac{1}{\frac{1}{\bar{\gamma}} - \frac{n+1}{\bar{\gamma}}} \left(e^{-\frac{(n+1)\gamma}{\bar{\gamma}}} - e^{-\frac{\gamma}{\bar{\gamma}}} \right) + \frac{m\gamma}{\bar{\gamma}^2} e^{-\frac{\gamma}{\bar{\gamma}}} \\
&= \frac{m}{\bar{\gamma}} \sum_{n=1}^{m-1} \binom{m-1}{n} (-1)^{n+1} \frac{1}{n} \left(e^{-\frac{(n+1)\gamma}{\bar{\gamma}}} - e^{-\frac{\gamma}{\bar{\gamma}}} \right) + \frac{m\gamma}{\bar{\gamma}^2} e^{-\frac{\gamma}{\bar{\gamma}}}, \gamma \geq 0,
\end{aligned} \tag{19}$$

with $f_{\gamma_{SD}}(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$.

Appendix II

The conditional cumulative distribution function of γ_d

With the aid of [36, Eq.3.351], the conditional CDF of γ_d can be obtained by integrating in (19) yielding

$$
\begin{aligned}
F_{\gamma_d\|\Omega|}(\gamma \mid m) &= \int_0^\gamma f_{\gamma_d\|\Omega|}(x \mid m)dx \\
&= \frac{m}{\overline{\gamma}}\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n}\left(\frac{\overline{\gamma}}{n+1}\left(1-e^{-\frac{(n+1)\gamma}{\overline{\gamma}}}\right)-\overline{\gamma}\left(1-e^{-\frac{\gamma}{\overline{\gamma}}}\right)\right)+\frac{m}{\overline{\gamma}}\left(\overline{\gamma}-(\overline{\gamma}+\gamma)e^{-\frac{\gamma}{\overline{\gamma}}}\right) \\
&= m\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n}\left(\frac{1}{n+1}\left(1-e^{-\frac{(n+1)\gamma}{\overline{\gamma}}}\right)-\left(1-e^{-\frac{\gamma}{\overline{\gamma}}}\right)\right)+\frac{m}{\overline{\gamma}}\left(\overline{\gamma}-(\overline{\gamma}+\gamma)e^{-\frac{\gamma}{\overline{\gamma}}}\right) \\
&= m\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n(n+1)}\left((n+1)e^{-\frac{\gamma}{\overline{\gamma}}}-e^{-\frac{(n+1)\gamma}{\overline{\gamma}}}-n\right)+\frac{m}{\overline{\gamma}}\left(\overline{\gamma}-(\overline{\gamma}+\gamma)e^{-\frac{\gamma}{\overline{\gamma}}}\right),\gamma\geq 0.
\end{aligned}
\tag{20}
$$

Appendix III

The average channel capacity

With the aid of $\int_0^\infty \ln(1+t)e^{-xt}dt=\frac{1}{x}e^x E_1(x)$ and [36, Eq.3.462.15], the average channel capacity can be obtained as

$$
\begin{aligned}
C &= \mathsf{E}\{\frac{B}{2}\log_2\left(1+\gamma_d\right)\} \\
&= \sum_{m=1}^M \Pr[|\Omega|=m]\int_0^\infty \frac{B}{2}\log_2\left(1+x\right)f_{\gamma_d\|\Omega|}(x\mid m)dx \\
&\quad + \Pr[|\Omega|=0]\int_0^\infty \frac{B}{2}\log_2\left(1+x\right)f_{\gamma_d\|\Omega|}(x\mid 0)dx \\
&= \sum_{m=1}^M \Pr[|\Omega|=m]\int_0^\infty \frac{B}{2}\log_2\left(1+x\right)[\frac{m}{\overline{\gamma}}\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n}\left(e^{-\frac{(n+1)x}{\overline{\gamma}}}-e^{-\frac{x}{\overline{\gamma}}}\right) \\
&\quad + \frac{mx}{\overline{\gamma}^2}e^{-\frac{x}{\overline{\gamma}}}]dx + \Pr[|\Omega|=0]\int_0^\infty \frac{B}{2}\log_2\left(1+x\right)\frac{1}{\overline{\gamma}}e^{-\frac{x}{\overline{\gamma}}}dx \\
&= \sum_{m=1}^M \Pr[|\Omega|=m]\frac{B}{2\ln 2}\frac{m}{\overline{\gamma}}[\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n}\int_0^\infty \ln\left(1+x\right)\left(e^{-\frac{(n+1)x}{\overline{\gamma}}}-e^{-\frac{x}{\overline{\gamma}}}\right)dx \\
&\quad + \int_0^\infty \frac{x}{1+x}e^{-\frac{x}{\overline{\gamma}}}dx + \int_0^\infty \ln(1+x)e^{-\frac{x}{\overline{\gamma}}}dx] + \Pr[|\Omega|=0]\frac{B}{2\ln 2}\frac{1}{\overline{\gamma}}\int_0^\infty \ln\left(1+x\right)e^{-\frac{x}{\overline{\gamma}}}dx \\
&= \sum_{m=1}^M \Pr[|\Omega|=m]\frac{B}{2\ln 2}\frac{m}{\overline{\gamma}}[\sum_{n=1}^{m-1}\binom{m-1}{n}(-1)^{n+1}\frac{1}{n}\left(\frac{\overline{\gamma}}{n+1}e^{\frac{n+1}{\overline{\gamma}}}E_1\left(\frac{n+1}{\overline{\gamma}}\right)-\overline{\gamma}e^{\frac{1}{\overline{\gamma}}}E_1\left(\frac{1}{\overline{\gamma}}\right)\right)\overline{\gamma} \\
&\quad + (\overline{\gamma}-1)e^{\frac{1}{\overline{\gamma}}}E_1\left(\frac{1}{\overline{\gamma}}\right)] + \Pr[|\Omega|=0]\frac{B}{2\ln 2}e^{\frac{1}{\overline{\gamma}}}E_1\left(\frac{1}{\overline{\gamma}}\right),
\end{aligned}
\tag{21}
$$

where $E_1(x)$ is the exponential integral function, $E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt$.

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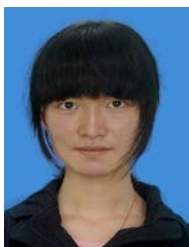
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