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다수 고객 통합전략을 활용하는 생산 및 물류계획 수립*

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A Production-and-Scheduling for One-Vendor Multi-Buyer Model under the Consolidation Policy

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■ Abstract ■

This paper considers an integrated one-vendor multi-buyer production-inventory model where the vendor manufactures multiple products in lot at their associated finite production rates. In the model, it is allowed for each product to be shipped in lot to the buyers even before the whole product production is not completed yet. Each product lot is dispatched to the associated buyer in a number of shipments. The buyers consume their products at fixed rates. The objective is to the production and shipment schedules in the integrated system, which minimizes the total cost per unit time. The total cost consists of production setup cost, inventory holding cost and shipment cost. For the model, an iterative optimal solution procedure with shipment consolidation policy incorporated. It is then tested through numerical experiments to show how efficient and effective the shipment consolidation policy is.

Keywords : Scheduling, Production Planning, Multi-Product, Consolidation Shipment

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1. Introduction

Under the environments of supply chain management, firms realize the importance of the cooperation and coordination. This paper considers an integrated one-vendor multi-buyer production-inventory model where the vendor manufactures multiple products in lot at their associated finite production rates. In the model, it is allowed for each product to be shipped in lot to the buyers even before the whole product production is not completed yet. Each product lot is dispatched to the associated buyer in a number of shipments. The buyers consume their products at fixed rates.

There are many system issues which need to be considered in the multi-buyer case. Within the best knowledge of the authors, however, most of the work done in the literature concentrated on proposing in one-vendor one-buyer case. So, this paper considers production planning and shipment policy for an integrated one-vendor multi-buyer integrated inventory model with multiple products.

In a manufacturing setting, it is assumed that each time a machine is setup, a major setup cost is incurred independent of which product type is produced. Furthermore, when each product is produced, a minor setup cost occurs and an additional fixed setup cost is charged that depends on the product type. By coordination of production cycle and replenishments of different product types, the manufacturer can reduce his average major setup costs.

In this paper, it is assumed that each buyer purchases one product-type. This paper considers the issue associated with consolidation of products. Consolidation occurs whenever different products travel in the same vehicle [11]. Consolidation involves picking-up and dropping-off products at different origins and destinations. In this paper, consolidation means that a vehicle with an infinite capacity delivers ("drops off," or "peddles") all the required products to all the buyers through a single route, which will be called *consolidation policy* in the rest of this paper. Whenever shipment consolidation occurs, it incurs a fixed cost, such as driver wages, fuel and vehicle maintenance, etc. While shipment by use of the shipment consolidation policy becomes complex, one of the benefits associated with shipment consolidation policy is that average transportation cost can be reduced.

This paper compares three shipment policies (Policy_1, Policy_2 and Policy_SC) to manage the integrated one-vendor multi-buyer integrated inventory model with multiple products. Policy_SC is the shipment by use of the shipment consolidation policy to multi-buyers. Poicy_1 and Policy_2 are shipments by no use of the shipment consolidation policy, in other words, these policies are a kind of direct shipment policies to buyers, which will be described in later.

1.1 Literature Review

There studied many researches associated with multi-echelon inventory system. The multi-echelon inventory system was introduced by Clark and Scarf [3], and investigated extensively since 1960. In recent years, better information flow and greater cooperation between companies provides the motivation for concentrating on studying multi-echelon inventory system in the supply chain, particularly between a main manufacturer and its component buyers.

Monahan [16] researched a model about quantity discount structure so as to maximize the vendor's profit under an infinite production rate assumption. Joglekar [13] extended Monahan's model with a finite production assumption. Banerjee [2] generalized Monahan's results by adding vendor's inventory holding cost. Lee and Rosenblatt [14] considered that the vendor negotiates under a lot-for-lot by removing the basic assumption of Monahan [16] and Banerjee [2].

Goyal [4, 5] considered an economic lot size model to minimize total cost for a system of one vendor and one buyer. Banerjee [2] generalized Goyal's results by adding a finite production for the vendor. Goyal [6] also generalized Banergee's study by eliminating the lot-for-lot policy, and proved that his model can provide more economic results. Goyal and Szendrovits [10] considered a shipment policy which combines a number of increasing shipment sizes followed by a number of equal shipment sizes. A review of published work on buyer-vendor coordination models up to the year 1988 was given by Goyal and Gupta [8].

Lu [15] relaxed the lot-for-lot assumption of Goyal [6] and suggested the shipment policy with equal shipment size. Goyal [7] suggested a delivery rule with various shipment size, which involves successive shipment sizes within a lot consecutively increasing by a factor equal to the ratio between the vendor's production rate and the demand rate on the buyer. This was again based on an earlier idea from Goyal [4, 5]. Goyal [7]'s shipment policy can result in a lower joint total cost than the Lu's equal shipment size policy in one-vendor and one-buyer situation.

Hill [12] suggested a shipment policy with unequal shipment size which increases by a general fixed factor in one-vendor and one-buyer situation. He suggested that the *i*th shipment size should be determined by evaluating the product term, (First shipment size) $\cdot\ y^{i-1}$, where $1 \le y \le$ (*Production rate/Demand rate*). The resulting policy obtained by Hill [12] would provide a lower total cost policy as compared to the policy obtained by Lu [15] and Goyal [7]. It is not surprising that this more general class of policy gives rise to lower joint total cost solutions than either of the special cases, but this is at the expense of producing solutions that are less likely to be of practical interest.

Goyal and Nebebe [9] suggested a shipment policy with unequal shipment size in one-vendor and one-buyer situation that the first shipment will be of small size followed by (n-1) equal sized shipment of size; (First shipment size) · (Production rate/Demand rate). This type of policy ensures a quick delivery of the first shipment to the buyer and avoids excessive inventory levels of higher order shipments at the buyer's end.

Lu [15] suggested a delivery rule for a one-vendor multi-buyer multi-product problem under infinite production. In the problem, the lot-for-lot policy is not incorporated either. Therefore, the objective of the problem is to minimize the cost of the vendor. In other words, the problem did not consider coordination between vendor and buyers.

The remainder of the paper is presented as follows. Section 2 explains the detailed assumptions and notation, and shows the formulation of the one-vendor multi-buyer inventory problem. Section 3 analyzes the solution properties and proposes an algorithm based on the solution properties. Section 4 gives numerical examples and comparison between three shipment policies to verify the effectiveness of the proposed consolidation policy. Section 5 states some concluding remarks.

2. Assumptions and Model Formulation

The inventory system considers a single vendor (manufacturer) supplying N buyers with N different types of products. Each buyer is assumed to purchase one type of product. The vendor manufactures multi-products at finite rates. It is assumed that each time a machine is setup, a major setup cost is incurred independent of which product type is produced. Furthermore, when each product is produced, a minor setup cost occurs and an additional fixed setup cost is charged that depends on the product type. In this paper, the lot-for-lot restriction is removed, in other words, each production lot is dispatched to the buyer in a number of shipments and some of which may be made while production is still taking place.

We assume that demand rate, production rate and inventory cost are all known, and no shortage is allowed. Moreover, all replenishment lead times are assumed to be zero.

Let index i denote the i-the buyer. Buyer i purchases product type i from the vendor at a demand rate D_i, ordering cost A_i and inventory holding cost H_i. T denotes the vendor's interval between two consecutive major setups, which is the length of a production cycle, and let the inventory holding cost of the vendor be h_i ($H_i > h_i$) and the vendor's production rate of product i be P_i such that $P_i > D_i$ for $\forall i$. There are two kinds of setup costs associated with production; a major setup cost S incurred when each production cycle is started and a minor setup cost s_i incurred when product i is produced. It is further assumed that all the minor setup costs associated with each product type are negligible during each production cycle.

This paper considers shipment consolidation policy of products, which means that a vehicle with an infinite capacity delivers ("drops off," or "peddles") all the required products to all the buyers through a single route. Thus, all the shipments for each buyer occur at the same time and shipment size for each buyer is constant, respectively. This paper introduces *consolidation routing cost*, *CR*, which denotes the fixed transportation cost for shipment consolidation.

Assuming that the production of a lot is started as late as possible, the dispatching of the first shipment will return the vendor stock level to zero. Because the shipment sizes of one lot are non-decreasing, the amount of time to be spent for consuming the last shipment of one lot will be greater than the amount of time to be spent for producing the first shipment of the next lot. Therefore, the dispatching of the last shipment of one lot takes place before the production of the next lot starts.

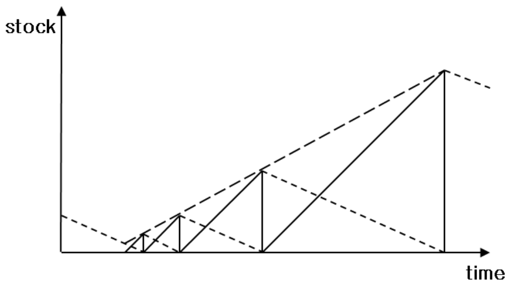
Essentially, the decision problems of the decision makers (vendor and buyer) are as follows;

a) the length of the production cycle for the vendor,
b) the economic number of shipments in which a lot will be sent to each buyer.

Thus, the objective is to determine the production cycle length and shipment schedule which minimize the total cost per unit time. Individual shipment sizes are calculated when the production cycle and the number of shipments is calculated. The cost of manufacturing setups and the cost of shipments for all policies are represented together by $\frac{1}{T}\left\{S+\sum_{i=1}^{N}(s_i+n_iA_i)\right\}$.

2.1 Policy_1

The first policy is the policy that each amount of shipment consecutively increases by the factor $\lambda_i(=P_i/D_i)$. That is, under the Policy_1, the amounts of the consecutive shipments are increased at the ratio of P_i/D_i. The stock positions associated with this policy are illustrated in <Figure 1>. The intuitive attraction of this policy is that the time for buyer to consume a shipped lot exactly balances the time for vendor to manufacture the next lot to ship within a lot.



〈Figure 1〉 Illustration of Stock Sgainst Time Under Policy_1 i.e. $\lambda_i=P_i/D_i$ (n_i = 4)

Let us consider a production cycle time T which is made up of n_i shipments. Since we are assuming $h_i>H_i$, the optimal solution must involve the vendor sending a shipment only when the buyer is just about to run out of stock.

The size of the j-th shipment within a lot is $\lambda_i^{j-1}q_i$. This shipment will allow for the buyer to last during the period of $\lambda_i^{j-1}q_i/D_i$, and during this time the average stock level of buyer i is $\frac{1}{2}\lambda_i^{j-1}q_i$. Thus, the time-weighted stockholding for buyer i during a complete production cycle is

$$\sum_{j=1}^{n_i}\frac{1}{2}\lambda_i^{j-1}q_i\times\frac{\lambda_i^{j-1}q_i}{D_i}=\sum_{j=1}^{n_i}\frac{\left(\lambda_i^{j-1}q_i\right)^2}{2D_i}=\frac{q_i^2(\lambda_i^{2n_i}-1)}{2D_i(\lambda_i^2-1)}$$

The total lot production size for product i (the sum of the n_i shipments) is

$$\sum_{j=1}^{n_i}\lambda_i^{j-1}q_i=\frac{(\lambda_i^{n_i}-1)}{(\lambda_i-1)}=D_iT$$

and the production cycle time for product i is equal to the time duration for the demand process to consume the lot quantity which is $\frac{q_i(\lambda_i^{n_i}-1)}{D_i(\lambda_i-1)}=T$. Hence, the average stock level of buyer i is the buyer's time-weighted stockholding for a cycle divided by the cycle time T;

$$\frac{q_i^2(\lambda_i^{2n_i}-1)}{2D_iT(\lambda_i^2-1)}$$

The total stock for product i in the system is at a minimum when the production of a lot is just about to start. At this point, the vendor's stock level is zero and the buyer's stock level is just enough to satisfy the demand until the first shipment of the next lot arrives which corresponds to the stock amount q_iD_i/P_i. The total stock increases at the rate of (P_i-D_i) for the time duration it takes to manufacture the lot quantity of D_iT at the rate of P_i and reaches the maximum of $\frac{D_iq_i}{P_i}+(P_i-D_i)\times\frac{D_iT}{P_i}$ at the point when the production of a lot finishes. Thus, the average total stock for product i in the system is

$$\frac{D_i q_i}{P_i} + \frac{(P_i - D_i)D_i T}{2P_i}$$

and the average stock level of vendor for the product i is the average total stock minus the average stock level of product i;

$$\frac{D_i q_i}{P_i} + \frac{(P_i - D_i)D_i}{2P_i T} - \frac{q_i^2(\lambda_i^{2n_i} - 1)}{2D_i T(\lambda_i^2 - 1)}$$

Now, the total cost per unit time, $TC(\vec{n}, T)$, can be derived as

$$\begin{aligned} & TC(\vec{n}, T) \\ & = \frac{1}{T}\left\{S + \sum_{i=1}^{N}(s_i + n_i A_i)\right\} \\ & + \sum_{i=1}^{N} H_i \times \left[\frac{D_i^2 T(\lambda_i - 1)}{P_i(\lambda_i^{n_i} - 1)} + \frac{(P_i - D_i)D_i T}{2P_i}\right] \\ & + \sum_{i=1}^{N}(h_i - H_i) \times \left[\frac{D_i T(\lambda_i - 1)(\lambda_i^{n_i} + 1)}{2(\lambda_i^{n_i} - 1)(\lambda_i + 1)}\right] \end{aligned}$$

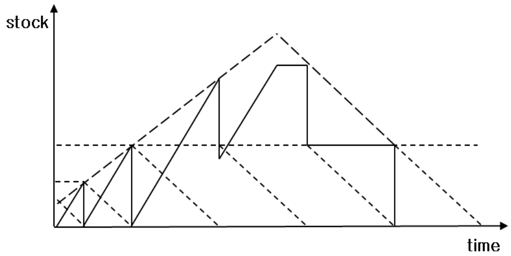
The above equation can be reduced as follows.

$$\begin{aligned} TC(\vec{n}, T) = & \frac{1}{T}\left\{S + \sum_{i=1}^{N}(s_i + n_i A_i)\right\} \\ & + \sum_{i=1}^{N}(H_i + \lambda_i h_i) \times \frac{D_i T(\lambda_i - 1)(\lambda_i^{n_i} + 1)}{2\lambda_i(\lambda_i^{n_i} - 1)(\lambda_i + 1)} \end{aligned}$$

Given all values of $\vec{n}$, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to 0) is derived as $T^* = [X(\vec{n})/Y(\vec{n})]^{1/2}$, where $X(\vec{n}) = S + \sum_{i=1}^{N}(s_i + n_i A_i)$ and $Y(\vec{n}) = \sum_{i=1}^{N}(H_i + \lambda_i h_i) \times \frac{D_i(\lambda_i - 1)(\lambda_i^{n_i} + 1)}{2\lambda_i(\lambda_i^{n_i} - 1)(\lambda_i + 1)}$.

2.2 Policy_2

The second policy is the policy that only the second shipment size increases by the factor λ_i (i.e., the size of the (n-1) equal sized shipments except for the first shipment is (First shipment size)×(Production rate/Demand rate). The stock positions associated with this policy are illustrated in <Figure 2>. It will be called "Policy_2" in the rest of the paper. The above two policies are a type of direct shipment policies.



<Figure 2> Illustration of Stock Against Time Under Policy_2 (n_i = 5)

This policy ensures a quick delivery of the first shipment to the buyer and reduces inventory levels at the buyer's side. The vendor ships the entire lot quantity, $D_i T$, in n_i shipments as given below; First shipment = q_i, followed by (n-1) shipments, $\lambda_i q_i$. Hence, the lot quantity is at $D_i T = q_i(q + (n_i - 1)\lambda_i)$. The inventory for buyer i in a cycle is given by

$$\text{Total inventory } = \frac{q_i^2}{2D_i} + \frac{(n_i - 1)(q_i \lambda_i)^2}{2D_i}$$

$$\text{Average inventory } = \frac{q_i^2(1 + (n_i - 1)\lambda_i^2)}{2D_i T}$$

The total stock for product i in the system is at the minimum when the production of a lot is just about to start. At this point, the vendor's stock level is zero and the buyer's stock level is just enough to satisfy the demand until the first shipment of the next lot arrives which corresponds to the stock amount is $q_i D_i / P_i$. The total stock increases at the rate of $(P_i - D_i)$ for the time duration it takes to manufacture the lot quantity of $D_i T$ at the rate P_i and reaches the maximum of $\frac{D_i q_i}{P_i} + (P_i - D_i) \times \frac{D_i T}{P_i}$ at the point when the production of a lot finishes. Thus, the average total stock for product i in the system is

$$\frac{D_i q_i}{P_i} + \frac{(P_i - D_i) D_i T}{2P_i}$$

and the average stock level of vendor for product i is the average total stock minus the average stock level of product i;

$$\frac{D_i q_i}{P_i} + \frac{(P_i - D_i) D_i T}{2P_i} - \frac{q_i^2 (1 + (n_i - 1)\lambda_i^2)}{2D_i T}$$

Now, the average cost per unit time, $TC(\vec{n}, T)$, can be derived as

$$TC(\vec{n}, T) = \frac{1}{T}\left\{S + \sum_{i=1}^{N} (s_i + n_i A_i)\right\}$$

$$+ \sum_{i=1}^{N} H_i \times \left[\frac{D_i T}{\lambda_i (1 + (n_i - 1)\lambda_i)} + \frac{(P_i - D_i) T}{2\lambda_i}\right]$$

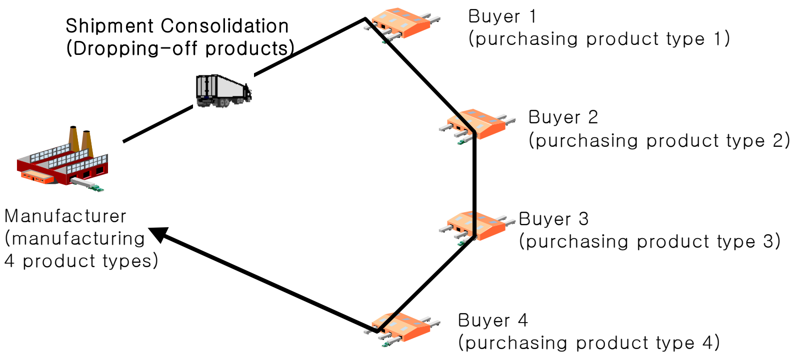
$$+ \sum_{i=1}^{N} (h_i - H_i) \times \left[\frac{D_i T (1 + (n_i - 1)\lambda_i^2}{2(1 + (n_i - 1)\lambda_i)^2}\right]$$

Given all values of $\vec{n}$, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to 0) is derived as $T^* = [X(\vec{n}) / Y(\vec{n})]^{1/2}$, where $X(\vec{n}) = S + \sum_{i=1}^{N} (s_i + n_i A_i)$, and $Y(\vec{n}) = \sum_{i=1}^{N} H_i \times \left[\frac{D_i}{\lambda_i (1 + (n_i - 1)\lambda_i)} + \frac{(P_i - D_i)}{2\lambda_i}\right] + \sum_{i=1}^{N} (h_i - H_i) \times \left[\frac{D_i (1 + (n_i - 1)\lambda_i^2)}{2(1 + (n_i - 1)\lambda_i)^2}\right]$.

3. Consolidation Policy

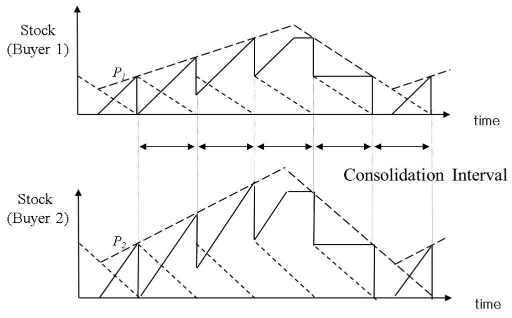
The consolidation policy in this paper can be said to be a kind of equal sized shipment policy. The number of shipments is adjusted to deliver each product to the corresponding buyer at the same time. <Figure 3> shows an example of a one-vendor 4-buyers supply chain network under the consolidation policy.

An illustration of inventory level associated a vendor and 2-buyers is depicted as in <Figure 4>



〈Figure 3〉 An Example of One-Vendor 4-Buyers Under the Consolidation Policy

In <Figure 2>, the solid line, the narrow dash line and the wide dash line, respectively, represent the vendor's stock, the buyer's stock and the overall stock.



〈Figure 4〉 An Illustration of Stock Level Associated a Vendor and 2-Buyers Under the Consolidation Policy

Now, the average cost per unit time for the consolidation policy, $TC(\vec{n}, T)$, can be derived as

$$TC(\vec{n}, T) = \frac{1}{T}\left\{S + \sum_{i=1}^{N}(s_i + n_i A_i)\right\} + \sum_{i=1}^{N} H_i \times \left[\frac{D_i^2 T(\lambda_i - 1)}{P_i(\lambda_i^{n_i} - 1)} + \frac{(P_i - D_i)D_i T}{2P_i}\right] + \sum_{i=1}^{N}(h_i - H_i) \times \left[\frac{D_i T(\lambda_i - 1)(\lambda_i^{n_i} + 1)}{2(\lambda_i^{n_i} - 1)(\lambda_i + 1)}\right]$$

These equation reduces (using L' Hôpital's Rule) as to the following.

$$TC(n, T) = \frac{1}{T}\left\{S + n \times R + \sum_{i=1}^{N} s_i\right\} + \sum_{i=1}^{N} H_i \times \left[\frac{D_i^2 T}{nP_i} + \frac{(P_i - D_i)D_i T}{2P_i}\right] + \sum_{i=1}^{N}(h_i - H_i) \times \left[\frac{D_i T}{2n}\right].$$

Given the value of n, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to 0) is derived as $T^* = [X(n)/Y(n)]^{1/2}$, where $X(n) = S + n \times R + \sum_{i=1}^{N} s_i$ and $Y(n) = \sum_{i=1}^{N} H_i \times \left(\frac{D_i^2}{nP_i} + \frac{(P_i - D_i)D_i}{2P_i}\right) + \sum_{i=1}^{N}(h_i - H_i) \times \left(\frac{D_i}{2n}\right)$.

From now an iterative optimal search procedure for the integrated multi-product inventory problem is proposed. For consolidation policy ($\lambda = 1$), $TC(n, T)$ is jointly convex in n and T, so that the optimal solution can be obtained by finding one parameter with the other parameter fixed in alternating manner. First for a particular integer value of n > 0, we find T', and then, for the fixed T', we find n' next. This fashion of alternating procedure continues, iteratively.

- Search Procedure;

Step 0 : Finding T' with particular parameter n

Step 1 : $TC(n, T)$ is a convex function of n. Given T',

T is substituted by T' in $TC(n, T)$ and then differentiated with respect to n.

$$n^2 = \frac{T}{R}\sum_{i=1}^{N}\left\{\frac{H_i D_i^2 T}{P_i} + (h_i - H_i) \times \left(\frac{D_i T}{2}\right)\right\}$$

Using Schwarz's result (1973), when *n* is found as an integer, then the optimal solution of $TC(n, T')$ is *n* itself, which is an integer such that

$$n(n-1) < \frac{T}{R}\sum_{i=1}^{N}\left\{\frac{H_i D_i^2 T}{P_i} + (h_i - H_i) \times \left(\frac{D_i T}{2}\right)\right\} \le n(n+1)$$

from which n' is found.

Step 2 : Finding $T' = [X(n)/Y(n)]^{1/2}$ with parameter n'.

Step 3 : If T' is equal to its preceding T', then stop. If else, Go to *Step 1.*

4. Computational Results

At first this paper demonstrates the total cost and the shipment frequency of the three policies. Specifically, the consolidation policy is compared with policy_1 and policy_2 in production lot size and the number of shipments. Moreover this paper solves numerical examples and depicts the trend of how the cost function moves with changing parameters. Lastly this paper describes the efficiency of consolidation policy by comparing it with two types of policies (policy_1 and policy_2).

The following basic data set will be used throughout the rest of this paper;

〈Table 1〉 Basic Data in the 1 : N Inventory System (N = 5, S = 1000)

Buyer(N)	s_i	H_i	h_i	D_i	P_i
1	400	4	6	1000	3200
2	450	5	7	1200	3500
3	500	4	6	2000	3400
4	400	6	8	1000	2800
5	350	3	5	1500	3200

〈Table 2〉 Basic Shipment Data in the 1 : N Inventory System

	Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5
A_i(50~150)	80	150	110	130	70
A_i(150~250)	170	220	250	190	240
A_i(250~350)	290	340	260	300	330

With each of the above data sets of shipment cost, test problems are solved for each of three policies and their results are compared each other. For each of these examples, we determine;

(a) The optimal solution based on all shipments being increased by the factor λ (Policy_1).
(b) The optimal solution based on only second shipment being increased by the factor λ (Policy_2).
(c) The optimal solution based on consolidation policy (Policy_SC).

In the above tables, the symbol×means the optimal number of shipments obtained from the Policy_SC and "Ratio" means the ratio of the routing cost to the total shipment cost, thus Ratio = (Routing cost/total shipment cost)×100. The consolidation policy outperforms Policy_1 and Policy_2 until the ratio is almost equal to 80% of the total shipment cost.

To illustrate the relations between parameters and total cost for each policy, the following figures are presented.

In the above figures, the value of buyer's holding cost is increased by 1 from the basic data. The total costs of Policy_1 and Policy_2 are presented with three ranges of the shipment costs and the variation of the total cost is presented with difference of holding costs.

In the cases of shipment cost ranges 50~150 and 150~250, Policy_1 gives the lower total cost until difference of holding cost is 2. In the case of shipment cost range 250~350, Policy_1 gives the lower total cost until difference of holding cost is 5. Policy_2 achieves the lower total cost

〈Table 3〉 The Total Cost and the Shipment Frequency of the Three Policies in the 1 : N Inventory System

		Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5	Total cost*	T^*
Shipment cost		80	150	110	130	70		
Policy_1		3	3	4	3	4	15395.1	0.636565
Policy_2		3	3	5	3	4	15412	0.650143
	Ratio	The number of shipments (n^*)						
Policy_SC	0.6	5					14995.6	0.629517
	0.7	4					15401.9	0.598889
	0.8	4					15758.4	0.612753

		Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5	Total cost*	T^*
Shipment cost		170	220	250	190	240		
Policy_1		2	2	3	2	2	17932.7	0.612291
Policy_2		2	2	3	2	2	18020.9	0.609291
	Ratio	The number of shipments (n^*)						
Policy_SC	0.7	3					17573.4	0.608534
	0.8	2					18093.2	0.626534
	0.9	2					18810.4	0.534386

		Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5	Total cost*	T^*
Shipment cost		290	340	260	300	330		
Policy_1		3	3	4	3	3	22023.6	0.719229
Policy_2		2	2	3	3	2	22706.2	0.590146
	Ratio	The number of shipments (n^*)						
Policy_SC	0.7	3					21870.9	0.575376
	0.8	2					23212.4	0.476641
	0.9	2					23841.7	0.489563

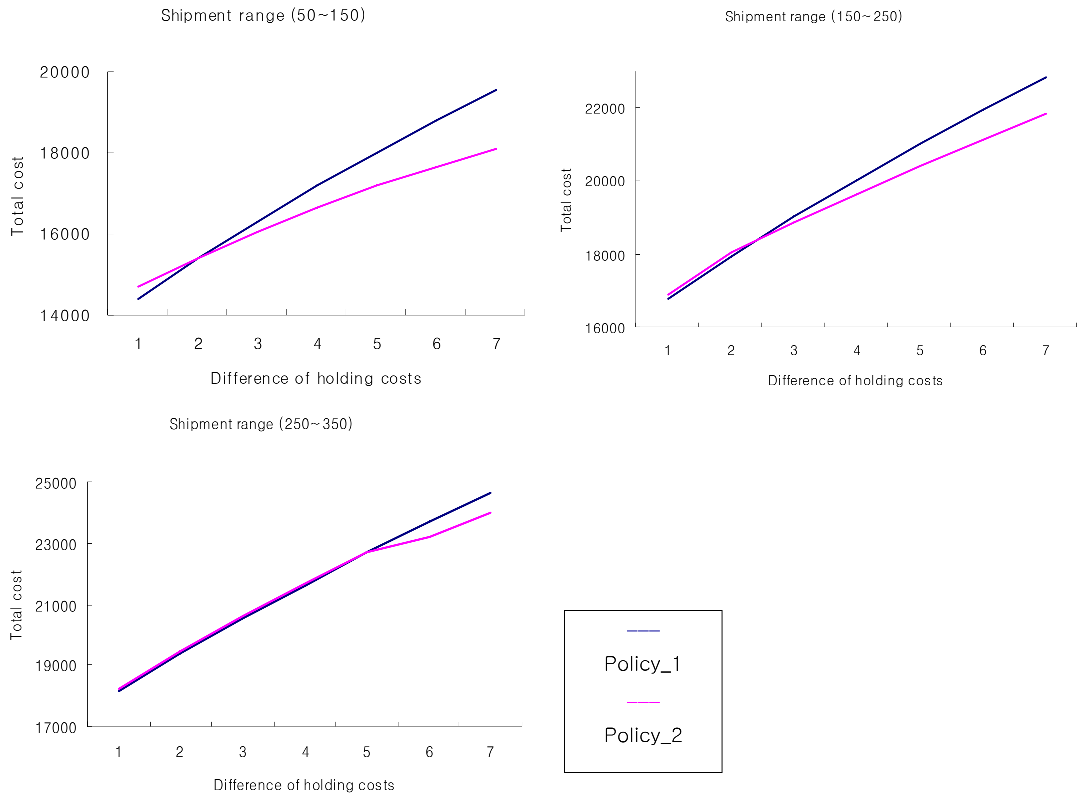
than Policy_1 as the difference of holding costs increases, although Policy_1 achieves the lower total cost in the beginning. The difference of holding cost is offset against the increasing shipment cost. That is to say, Policy_2 outperforms Policy_1 when the buyer's holding cost is relatively high.

In the above figure, vendor's holding costs are at ($h_1 = 7$, $h_2 = 8$, $h_3 = 7$, $h_4 = 9$, $h_5 = 6$) and the demand rates are increased by 100 from the basic data. The total costs of Policy_1 and Policy_2 are compared each other as demand rate increases. In this case, Policy_2 gives the lower total cost until the increment of demand rates is 300.

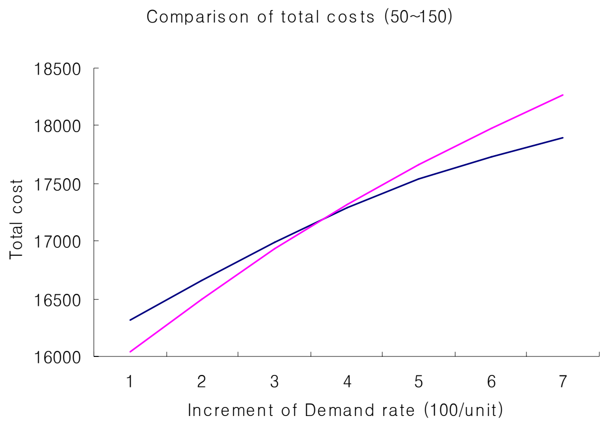
Policy_1 achieves the lower total cost than Policy_2 as demand rates increase, (i.e., as demand rates get closer to production rates), although the total cost obtained under Policy_2 is smaller in the beginning.

Therefore, we can conclude that Policy_1 outperforms Policy_2 when demand rates are relatively high.

In <Figure 7>, the shipment costs are increased by 20 from the basic data. The break

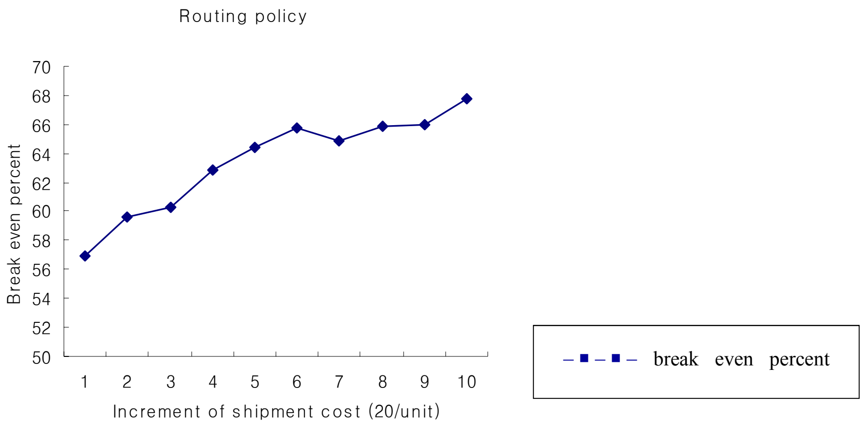


〈Figure 5〉 Relations between Total Cost of Each Policy and Difference of Holding Costs



〈Figure 6〉 Total Cost Comparison between Policy__1 and Policy__2 with Increment of Demand Rate

even percent is represented with respect to shipment cost. The break even percent means the ratio of the total shipment cost to the routing cost when the smaller value between Policy_1 and Policy_2's total costs meets consolidation policy's total cost. It is showed that the break



〈Figure 7〉 Break Even Percent with Increment of Shipment Cost

even percent generally increases as the total shipment cost increases. In other words, the consolidation policy outperforms the other policies when the total shipment cost is relatively high.

In <Table 4>~<Table 6>, a variety of different problems with variable major setup cost, minor setup costs, shipment costs and vendor's holding costs, buyer's holding costs, demand rates and production rates are considered to compare two types of inventory systems one against the other and explain the effectiveness and efficiency of the consolidation policy. For the efficiency investigation, 30 problems are generated with 5, 7, and 9 retailers. To assess the performance of the consolidation policy, the optimal solutions obtained with each of the ratios are compared with the best solutions from the other policies. Column "B.E.P" means the ratio, "Break even percent," of the total shipment

〈Table 4〉 Computational Results with N = 5, S~U(800~1300), s_i~(300, 500), A_i~(50, 350), H_i~(3, 6), h_i~(2, 5)+H_i, D_i~(1000, 2000), P_i~(2500, 3500)

	# of buyer	Policy_1	Policy_2	Consolidation Policy			
		Total cost	Total cost	60% cost	70% cost	80% cost	B.E.P
1	5	19488.7	19111.8	18013.4	18694.8	19237.7	77.7
2	5	20347.4	20290.9	18679.3	19359.5	20016.6	84.3
3	5	19246.4	19108.1	18039.1	18666.2	19431.5	75.8
4	5	19697.5	19728.3	18312.9	18945.9	19558.4	81.4
5	5	20461.7	20517.4	19163.7	19857.9	20867.2	76
6	5	19757.7	19606.3	18242.5	19010.5	19583.6	80.4
7	5	19158.2	19206	18015.6	18651.6	19351.7	77.2
8	5	17924	17878.1	17299.9	18061.4	18580.9	67.6
9	5	19132.3	19328.5	18541.1	19166.8	19925.1	69.4
10	5	20450.3	20096.5	19296.7	19949.1	20580.8	72.3

〈Table 5〉 Computational Results with N = 7, S ~ U(800 ~ 1300), s_i ~ (300, 500), A_i ~ (50, 350), H_i ~ (3, 6), h_i ~ (2, 5) + H_i, D_i ~ (1000, 2000), P_i ~ (2500, 3500)

	# of buyer	Policy_1	Policy_2	Consolidation Policy			
		Total cost	Total cost	60% cost	70% cost	80% cost	B.E.P
1	7	27429.1	27529.9	26115.5	27434.1	28337.3	70
2	7	26510.4	26231.6	24589.8	25528.3	26660.7	76.2
3	7	27167.1	27304.5	25681.5	26621.6	27529.5	76
4	7	25568.2	25465.1	24266.3	25197.7	25933.6	73.6
5	7	28866.4	29223	27804.3	28756.5	29678.3	71.2
6	7	26187	25894.6	24209.1	25112.9	26219.4	77.1
7	7	27502	27413.9	25869.3	26810.8	27898.1	75.5
8	7	26460.2	26423	25007.6	25872.9	26710.2	76.6
9	7	28191	28150.9	26481.2	27605.2	28428.6	76.6
10	7	28013.4	28067.1	26391.6	27618.7	28515.3	74.4

〈Table 6〉 Computational Results with N = 9, S ~ U(800 ~ 1300), s_i ~ (300, 500), A_i ~ (50, 350), H_i ~ (3, 6), h_i ~ (2, 5) + H_i, D_i ~ (1000, 2000), P_i ~ (2500, 3500).

	# of buyer	Policy_1	Policy_2	Consolidation Policy			
		Total cost	Total cost	60% cost	70% cost	80% cost	B.E.P
1	9	32612.3	32389.4	30370.3	31835.2	32831.1	75.6
2	9	30926.2	31115.6	30138.5	31321.7	32284.9	66.7
3	9	34003.1	34238.1	32547.8	33902.6	34917.8	71
4	9	37695.1	38070.3	35253.3	37149.6	38395.7	74.4
5	9	33365.6	33604.7	31732.5	32909.6	34046	74
6	9	33397.2	33025.5	30669.1	31831.7	33165	79
7	9	29852.6	29754.3	27965.6	28973.3	30405.1	75.5
8	9	36638.2	37071	34885.4	36150.7	37373.2	74
9	9	35792.8	35827.4	33102.7	34346.7	35547.1	81.2
10	9	33662.3	33780.7	31299.6	32775.8	33829.1	78.4

cost to the routing cost when the smaller value between Policy_1 and Policy_2's total costs meets the consolidation policy's total cost.

It cannot be claimed that one policy is always better than the other policy, between Policy_1 and Policy_2. The lower total cost is obtained by the consolidation policy when routing cost is lower than 75.3% of the total shipment cost on average.

5. Conclusion

This paper considers an integrated multi-product inventory model with shipment consolidation policy incorporated. The proposed Policy_SC is a consolidation policy that all products are consolidated together. Thus, the associated routing cost is substituted for the associated shipment cost. The objective of the model is to determine that production and shipment schedules which minimize the total cost per unit time for a vendor manufacturing products to supply to multi buyers. Three policy models are constructed to find the lower mean total annual cost for which an iterative optimal solution procedure is derived and tested for its performance by comparing the consolidation policy with policy_1 and policy_2. Based on the computational experiments, it is concluded that the proposed consolidation policy is efficient and effective when the total shipment cost is heavy.

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