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Instruction Using Scaffolding for Language Learner Students in Solving Mathematical Word Problems

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Communicating about mathematics is an essential component in learning mathematics and is a key standard for successful learning in a mathematics classroom using stories and storytelling as a catalyst to mathematics instruction. This, however, can make learning math for students with language deficiencies since they are working toward mastering both basic language proficiency as well as the specialized language needed for mathematics. This is a particular concern because the number of students of multicultural families is rapidly increasing. In this paper, we discuss the challenges and complexities of language-deficient students learning math in a classroom where communication is a key standard for successful learning, and suggest implications for teaching, by presenting an USA elementary teacher's scaffolding to make reading and solving word problems less intimidating for her language learner students as well as native speaking students.

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I. INTRODUCTION

Communicating about mathematics is regarded as an essential component in learning mathematics in both the USA and Korean curricula (MEST, 2011; CCSSI, 2010; NCTM, 2000). Students need multiple methods to learn to discuss, write, read, and listen to mathematical ideas in order to deepen their understanding of difficult concepts. Using stories as a catalyst to mathematics instruction can be one enjoyable and versatile method to do this. Since various contexts can be used as stories, using stories seems well situated to enhance connections as the connecting of mathematical ideas and with ideas outside the mathematics classroom.

Although storytelling (that is, reading or listening to a story and participating in a problem solving activity related to the story) can be another powerful pedagogical tool to help make learning more meaningful, this can also make learning math for students who are not fluent in the language being used in schools (hereafter, called “language learner students”) even more challenging since they are working toward mastering both basic language proficiency as well as the specialized language needed for mathematics. Furthermore, many foreign language learners lack culturally specific background knowledge that would assist them in relating to problems. This is a particular concern because the number of multicultural families has been on the rise for years in Korea: In 2012, 0.7% (46,954) of the student population was from families with multicultural background. It is estimated that students of multicultural families will be more than 1% of the student population in 2014 (MEST, 2012). Korean studies of children of multicultural families indicate that the language burden appears to be one of the influences on their unsatisfactory learning (Jo, Kang & Ko, 2013; Park, 2011). With lack of understanding that language learner students may have of mathematical problems based on their limited language proficiency, it is difficult to know whether their math abilities are being fairly assessed.

In this paper, we discuss the challenges and complexities of language-learner students learning math in a classroom where communication is a key standard for successful learning, and suggest implications for teaching, by presenting an USA elementary teacher’s scaffolding to make reading and solving word problems less intimidating for her language learner students as well as native speaking students.

II. RELATED LITERATURE

1. The complexity of the language of mathematics

Research supports that math language is semantically and syntactically specialized, which imposes particular difficulties for language learner students, who are learning this academic language alongside the social language they need for the purpose of basic communication.

Lexical items such as *denominator, divisor, quotient quadrilateral, parallelogram,* and *isosceles* are special terms only found in the context of mathematics that math students must learn in order to be successful (Rubenstein & Thompson, 2002). Also, there are multiple ways to refer to the same function in mathematics. For example, words referring to the function of addition include *sum, add, plus, and, combine,* and i*ncreased by*. Students must not simply learn one term, but all related terms to be able to fully comprehend math text. (Chamot & O'Malley, 1994). Additionally, there is the issue of polysemy, or diversity of meanings, in the lexicon of mathematics. Students may be familiar with the common uses of words like *quarter, remainder, and place*, but these words have a different meaning when used in mathematics (Chamot & O'Malley, 1994). Sometimes the same mathematical word is used in more than one way within the field of mathematics itself. The word *round*, for example, can refer to the shape of a circle or the function of rounding a number to the nearest tenth. Likewise, the word s*quare* can refer to a shape and also to a number times itself (Rubenstein & Thompson, 2002).

Math language makes use of certain syntactic structures. Some of the syntactic features include greater/less than, *n* times as much as, divided by, and if…then (Chamot & O'Malley, 1994).

2. The benefits of student-written word problems

Researchers have advocated for students writing their own math word problems as a way to increase their ability to comprehend and solve such problems, as well as to increase problem solving ability in general.

One of the benefits of having students write word problems deals with motivation. Students tend to get excited when word problems are about them or their classmates. They have a natural curiosity that can be tapped into during the mathematics class by making the content of word problems more relevant (Winograd, 1992). Students who write their own math word problems tend to be more interested in the problem solving process and write word problems that require more complicated skills to solve than they currently possess (Winograd & Higgins, 1994). Another benefit relates to personal experience. Because context is important for the comprehension and solving of math word problems (Chapman, 2006), students who have had personal experience with the situations described in the word problems they read will likely have more success understanding and solving them. When students write their own problems, they make use of contexts and situations with which they are intimately familiar. This helps to alleviate the problem of students having to solve word problems that are culturally unfamiliar to them. The use of personal experience in creating math word problems can make problems more meaningful and comprehensible to students (Barwell, 2003). Lastly, by having students write their own math word problems, students can share their math problems with the class as a whole group by writing the problems they created on the board and inviting their peers to solve them. It is an opportunity for students to hear feedback about the problems they create. It leads small groups or whole class negotiation of meaning about the problem and how best to solve it. The discussion also gives students practice talking about math and using math language. A host of the literature stresses the benefits of students sharing their original problems with their peers (Chamot, Dale, O'Malley & Spanos, 1992; Chamot & O'Malley, 1994; Hildebrand, Ludeman, Mullin, 1999; Winograd, 1990). This collaborative learning is particularly powerful for students who are not fluent in the language being used in their school.

3. Scaffolding

Research suggests that students need scaffolding to learn how to solve and write word problems (Chamot & O'Malley, 1994; Spanos, 1993; Winograd, 1990). The following step procedure (Modified WPP) can be used to help guide students through the problem solving process, which was adapted after slight modifications from Spanos' Word Problem Procedure (1993):

1. Read the problem out loud.
2. Talk about the vocabulary and circle words you don't understand.
3. Ask your partner or teacher for help with what these words mean.
4. What does the problem ask you to find? Write it below. (What question does it ask you?)
5. Draw a picture to represent the problem.
6. What should you do to solve the problem? Add? Subtract? Multiply? Divide? etc.
7. Solve the problem below.
8. Check your answer.
9. Explain how you got your answer to your partner.
10. Explain your answer to the rest of the group.
11. Write a similar problem.

At the beginning, teachers can model the process showing students their own teacher-generated word problems created from their own experience. Teachers can discuss how they chose their topics and teachers and students can discuss how these problems could be solved. As students work on solving these teacher written problems, the teacher will provide instruction of steps (such as the above eleven steps) to help students solve the problems. Teachers can then begin to provide students with word problems that contain missing information. Students first provide simple information as they fill in blanks. As time goes on, students provide more information and eventually write an entire word problem on their own. When students follow the steps of this procedure, they are able to practice reading, writing, listening and speaking. This procedure is especially beneficial for students with limited language proficiency because it is a means to breaking down and understanding word problems.

III. RESEARCH METHOD

Clearly, the language burden is great for students working toward language proficiency in the area of math. With such an instructional emphasis placed on communication that involves reading and discussing math stories and math story problems, teachers need to concentrate on making sure language learner students, as well as native speaking peers, are able to comprehend and solve such problems. To suggest its implications for math instruction in Korean schools, we present the process and findings of an action research study describing an USA elementary school teacher's methods to improve math comprehension of her language learner students as she scaffolded the teaching of students to write their own word problems.

1. Settings

This case study was completed in a small urban elementary school with student population of around 450 in a Midwest state in USA. The classroom teacher had been teaching for four years and worked in collaboration with a language-support teacher in the areas of reading and math at the time of the study. The elementary students who participated in this study were six third-grade English language learner students. The students were primarily Spanish-speaking and had been educated in the USA school system from 1.5 to 5 years. The level of English proficiency of the students was intermediate. Their academic progress scores from the previous year showed that they did not meet or partially met state standards in reading and/or math. Data was collected during a daily 30-minute small group intervention that the classroom teacher voluntarily offered for the students after school over 10 weeks. During the 10 weeks, the teacher focused on math an average of one day per week for 30 minutes each session. In all, the group received a total of 12 sessions of instruction totaling 6 hours focused on solving word problems.

2. Data collection and procedure

1) Pre/Post-test

To determine a baseline of ability to solve math word problems, the students were first given a written pre-test using five word problems. After collecting students' papers, the teacher then gave them these same five problems written in number sentence form in a different order on a separate paper. Test items were framed in both formats so that the teacher could see which format students struggled with the most. The following is an example of word problem and corresponding number sentence:

Example word problem: Last year a basketball player scored 413 points. This year he has scored 366 points. How many more points must he score to have the same score as last year?

Corresponding number sentence: 413 – 366 = _____

In order to discover if teaching students to write their own word problems helped students to solve word problems with more accuracy, a post-test, similar to the pre-test but using different problems, was administered at the end of instruction.

2) Scaffolded instruction

Following the pre-test, ten weeks of instruction was focused on learning a procedure for solving word problems. The teacher began by teaching students her modified WPP. The teacher first modeled how to follow steps one through ten with a few problems so that students knew what is expected of them. Once the students understood the process, the teacher then assigned students to partners and gave each set of partners a word problem to solve. Students used the modified WPP to solve their word problem and discuss with the group how they solved it.

Once students were using steps one through ten with more confidence, the teacher introduced the final step (Write a similar problem) that asks students to write their own word problem. The teacher scaffolded the process of writing word problems by using a word problem that students had solved already using the WPP. The problem is as follows:

Joe's truck weighs 2,143 kilograms. The truck can carry 3,402 kilograms of rocks. What is the total weight of the truck and full load?

The teacher then gave students a copy of the problem, leaving blanks for them to fill in the numbers which were missing. The problem looked like this:

Joe's truck weighs __________ kilograms. The truck can carry _________ kilograms of rocks. What is the total weigh of the truck and full load?

Together as a group, the teacher and the students discussed changing the numbers and practiced solving the problem with new sets of numbers. Finally, the students were given the following version of the same problem:

Joe's ______ weighs ______ kilograms. The ______ can carry ______ kilograms of______. What is the total weigh of the ______ and full load?

As a group, the teacher and the students discussed different vehicle choices and also options for different loads. Students re-wrote the problem choosing one of the options they generated together. Students took turns reading their new problems aloud to the group and practiced solving them, also together as a group. Instruction continued this way with a few more problems, with partners of students solving word problems using the WPP, and eventually writing their own version of the problem with more and more information being provided by students.

3) Problem-solving interview

To see if the instruction was effective in increasing their comprehension of math word problems, at the end of the ten-week instruction period, the students individually participated in a problem-solving session, where they solved a math word problem while explaining how they were solving it. Students' problem-solving sessions were recorded and then transcribed for later analysis. To analyze the data, the teacher used a checklist of strategies from her modified WPP to code what students did and said during the think-aloud.

Checklist:

- Circled words he/she didn't understand / Discussed unfamiliar vocabulary
- Discussed what the problem was asking them to find
- Drew a picture to represent the problem
- Discussed the function needed to solve the problem (addition, subtraction, multiplication, division)
- Solved the problem correctly
- Checked his/her answer

IV. RESULTS

1. Pre/Post-test comparison

All students in the case study showed improvement in their scores on the word problem portion of the tests (Table 1 for results). For all students, the scores for the word problem portion of the pre-test were lower than their scores for the corresponding number sentence portion. This indicates that students were able to compute number sentence problems with more accuracy than they were able to comprehend and solve word problems. For some students, like Student 1 and Student 5, their ability to solve both the word problems and number sentences was fairly accurate already. The other students had a more pronounced difference between their scores for the word problems and number sentences. For all but one students, the scores for the word problem portion of the post-test were the same as their scores for the corresponding number sentence portion, with the exception of Student 3, whose word problem score was 1 point lower than her number sentence score. These scores suggest that students' abilities to solve word problems were largely equivalent to their ability to compute number sentence problems. The scores for the number sentence portion of the test largely stayed the same, with the exception of Students 4 and 6 who each increased by 2 points.

Table 1. Pre-test and post-test comparison*

Student	Pre-test		Post-test	
	Word Problem Score	Number Sentence Score	Word Problem Score	Number Sentence Score
1	4	5	5	5
2	2	5	5	5
3	3	5	4	5
4	1	3	5	5
5	4	5	5	5
6	1	5	3	3

*Each score is out of 5 points

2. Problem-solving results

The following word problem was used for the individual problem-solving sessions:

Mary went to the store and bought 7 boxes of crayons. Each box had 12 crayons in it. How many crayons did Mary buy in all?

None of the students did the first item on the checklist (Circled words he/she didn't understand / Discussed unfamiliar vocabulary). There appeared to be no unfamiliar words in the word problem that presented an issue for any of the participants. On the item two on the checklist (Discussed what the problem is asking you to find), all of the students had an understanding of what the problem was asking them to find, but only two students (Students 3 and 5) explicitly stated the question that the problem required them to answer. The other students showed their understanding in other ways like drawing a picture or ultimately finding the correct answer to the problem. Student 3 was confused in attaching a label to her answer of 84, first saying "boxes" and then changing her label to "crayons." This suggests that her understanding of what the problem was asking her to find perhaps was not as clear as some of the other students. On the third item (Drew a picture to represent the problem), three students (Students 2, 4, and 6) drew a picture that consisted of seven boxes to represent the boxes of crayons with either the number "12" or twelve tally marks to represent the number of crayons in each box. One student (Student 3) wrote a column of 12s seven times to show that there were seven boxes of crayons with twelve crayons in each box, but did not draw the boxes. The last two students (Students 1 and 5) did not draw pictures, but instead directly went to writing the number sentence of 12×7. It should be noted that the two students who did not draw pictures were the same students who had high scores on the word problem portion of their pre-test. The other students who either drew pictures or wrote 12 seven times had lower scores on the word problem portion of the pre-test.

Regarding the fourth item (Discuss the function needed to solve the problem), all students ultimately decided that the function needed to solve their problem was either multiplication or repeated addition, which is essentially the same as multiplication. Some students (Students 1, 2, and 5) decided upon the function and discussed their reasons why more confidently than others. Student 3 first said she needed to multiply, and then said she should add. When asked why, she said she thought she should add because the problem used the words "in all." Usually kids are taught these key words "in all" to mean they should add. It seems that this information from past learning is what confused Student 3 in this case. After the teacher prompted her to do whatever she needed to solve the problem, she wrote seven 12s on her paper and solved the problem using repeated addition. Student 6 first indicated that she should multiply and then changed her mind, saying she needed to divide. The teacher asked the student what she could do on her paper to figure out what she needed to do. She was able to draw a picture and find the correct solution to the problem. She was, however, still unable to verbalize the mathematical function she had used at the end. On the sixth item (Solved the problem correctly), all six students were able to successfully solve the problem, although some were faster and more confident than others. On the seventh item (Checked his/her answer), none of the students really did this step. They were all satisfied with their final answer once they had arrived at it.

V. CONCLUSIONS

One major finding from this case study is that, overall, the teacher's modified WPP was a helpful tool for all students to use while solving math word problems, but different steps of the procedure seemed to be most beneficial to students based on their level of math problem solving ability. It appears that the step which required students to draw pictures to represent math word problems was a beneficial strategy for all students, but was especially useful for the students that had lower scores on the word problem portion of the pre-test. These students that struggled in their ability to solve math word problems were the more emergent students in the study based on their math ability. During the instruction phase of the case study, it was not until they drew a picture to represent the problem that they were able to go on to successfully solve it. Also, it was noticed this strategy being particularly helpful during the problem-solving sessions. The more advanced math students were able to solve the problem-solving problem without drawing a picture. The other, more emergent math students were unable to solve the problem until they drew a picture.

Another interesting finding from this case study is that the more advanced math problem solvers who scored quite high on the pre-test were also benefiting from learning the teacher's modified WPP, most specifically the step which requires students to explain their thinking about how they solved a problem. During the instruction phase of the study, it was noticed that the more advanced students usually knew how to solve math word problems with relative ease, but had difficulty explaining how they arrived at their answers. They would simply say, "I'm not sure, I just knew the answer." In fact, this was an area upon which all language learners need to improve. As they practiced using the WPP more and more, these students improved their ability to explain the steps they took to solve the problem. This oral language practice was beneficial to all the participants in the case study.

The final step that asked students to write their own math word problem was beneficial to all of the students. It required students to go beyond the mere reading and solving of word problems, which is what they normally have to do. The dissection and reconstruction of problems that students needed to do to change an existing problem led students to really comprehend those problems more deeply. This was the step, as the literature review indicated, that really engaged students and got them interested in working with word problems. Students were really excited about creating problems that were their own. They were proud to share their problems and were also very eager to solve each other's problems. The level of engagement while creating, sharing, and solving student-authored math word problems was much higher than when students were simply solving problems that were given to them to solve.

VI. IMPLICATIONS FOR TEACHING STUDENTS WORKING TOWARD LANGUAGE PROFICIENCY

Although the presented case study was limited in the number of participants and the amount of time devoted to the instruction of math word problems, results of the study suggest some implications for teaching students who are limited in language proficiency.

It is clear that teachers need to spend more time during math class teaching a procedure- not a solution of a problem but a series of processes-that students can follow in order to solve word problems. Doing so would seem to have great benefits for students working toward language proficiency, as they are improving their language skills alongside their math problem solving skills. With the rest of the math content teachers are required to teach and teachers' misconception that language learner students cannot be successful in solving word problems until they are more fluent in the language being used in school (Basurto, 1999), there seems precious little time to devote to really delving deeply into word problems. But in doing so, teachers can give their language learner students more chance for success in really comprehending and solving such problems. This can only help them perform better on math assessments that are largely made up of questions in word problem form. Especially for language learner students, this procedure through scaffolded instruction helps them to break down problems in a systematic way to aid in their comprehension. This means that teachers would be well advised to differentiate their instruction to meet the needs of language learners. Also, the scaffolding process in teaching students to write their own word problems seems an important step for teachers to do by starting small and working towards having students provide more and more information as they alter an existing math word problem.

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