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Mathematical Knowledge for Teaching and Teachers' Backgrounds: A study of U.S. Teachers

Yeon Kim*

Teachers' mathematical knowledge for teaching (MKT) strongly influence student achievement (Hill, Rowan, & Ball, 2005). However, there is generally no record that shows the levels of teachers' MKT in schools and districts. Because of the limited resources, schools and districts need some criteria to decide how to distribute their resources to help teachers improve MKT. This study explores what factors in teachers' backgrounds are related to their levels of MKT. This article examines the relationships between the MKT of 336 elementary teachers and their demographic, educational, and teaching backgrounds. To measure MKT, I used a series of multiple-choice problems. The number of years teaching mathematics is not critical to these measures. Implications are discussed with regard to policy for mathematics teacher education.

I. Introduction

Over twenty years ago, the National Commission on Teaching and America's Future (1996) strongly advocated supporting excellent teachers in order to improve U.S. schools based on the belief that what teachers know and can do is the most important influence on what students learn. Interest in how teacher resource and school affect student achievement has focused on teacher quality and qualifications. The issues related to teacher qualifications were expanded after the federal No Child Left Behind (NCLB) Act of 2001, which required highly qualified teachers. This means teachers must have a bachelor's degree, be licensed by the state to teach in their particular subject(s), and demonstrate core competencies in the subjects in which they teach. A different argument is that the paper qualifications are not enough indicators of at least one key teacher resource, content knowledge for teaching. For example, in teaching mathematics, teachers need to know and use mathematics in ways useful for helping students have a sense of mathematics as well as selecting appropriate mathematical questions and influential ways of representing the subject so that students can easily understand (Ball, Thames, & Phelps, 2008). A variety of research indicates that teachers' knowledge affects students' opportunities to learn mathematics and their achievement (Baumert et al., 2010; Hill et

* University of Michigan (kimyeon@umich.edu)

al., 2005; Rockoff, Jacob, Kane, & Staiger, 2011). However, generally, there are no school or district records that show the levels of teachers' content knowledge for teaching. However, because of the limited resources that schools and districts have, they need criteria to decide which teachers they will support to improve teachers' content knowledge for teaching, such as opportunities to participate in professional development. Districts generally have information about teachers, such as their gender, ethnicity, teaching experience, and a teaching credential. Schools might have more information such as the teachers' attitude toward teaching mathematics or number of courses that they took. By using this information, I explore what factors are really critical to decide which teachers are expected to obtain more resources than other teachers in order to improve their mathematical knowledge for teaching (MKT). In particular, this research investigates whether there are relationships between the teachers' MKT and each of the following factors: gender, ethnicity, having a credential, teachers' attitude toward teaching mathematics, the number of years teaching mathematics, teaching all subjects, the number of mathematics classes and the number of methods-of-teaching mathematics classes. If so, to what extent do these factors contribute to the teachers' MKT?

I review the literature on MKT and then turn to results from an examination of teacher knowledge and their backgrounds. On the basis of research linking credentials and student achievement and the policy logic of NCLB, it is generally expected that more experienced teachers who have a bachelor's or higher degree in mathematics and have a regular teaching credential or a mathematics credential would be more likely to use conceptual goals and strategies in the classroom than their newer colleagues without a degree in mathematics and without certification. I investigate whether this expectation works well.

II. Theoretical Backgrounds

After Shulman (1986) identified *pedagogical content knowledge* as a special domain of teacher knowledge that intertwines aspects of teaching and learning with content from subject matter knowledge, a variety of research has been conducted. For the past decades, researchers at the University of Michigan have been developing a theory that elementary school teachers need to know mathematics for teaching. As Ball, Thames, and Phelps (2008) claim, although simply knowing mathematics for oneself means being able to calculate 307 – 168, for example, teaching a simple subtraction computation involves not only carrying out this computation but also analyzing students' responses, both correct and incorrect, interpreting students' imprecise mathematical language, and bringing errors to the surface for specific pedagogical purposes. They define MKT to be the mathematical knowledge, skill, and habits of mind entailed by the work of teaching. Hill, Schilling and Ball (2004) assert that elementary teachers' MKT is multidimensional, and includes knowledge of various mathematical topics (e.g., number and operations, algebra) and domains (e.g., knowledge of content; knowledge of students and content). Moreover, Hill, Rowan and Ball (2005) claim that teachers' MKT is significantly related to student achievement gains.

It is, however, very rare to study MKT in terms of policy. The exception is the study by Hill and Lubienski (2007), who find that teachers in schools with higher proportions of low-SES and Hispanic students performed more poorly on these measures than did teachers from other schools. Their study reports the inequitable distribution of teachers in the system, but does not make suggestions about how to support teachers in terms of MKT. As a next step, this study explores the factors and the extent to which each factor contributes to MKT.

III. Methodology

1. Data and Variables

Learning Mathematics for Teaching (LMT) items are the assessment items that reflect real mathematics tasks teachers face in classrooms (e.g., assessing student work, representing numbers and operations, and explaining common mathematical rules or procedures). Finally, LMT items are the assessment tool for MKT. There are two forms in the 2007 Internal Document; Rational Number and Proportional Reasoning. Data related to Rational Number is used in this research. This form consists of three parts: Mathematics (74 items); Background and Teaching (11 items); and Views (6 items). The mathematics section is about assessment for MKT (74 items). The background and teaching section is about teaching experience (5 items), scholarship (2 items), certification (2 items), ethnicity (1 item), and gender (1 item). The views section is about attitude toward teaching mathematics (3 items) and attitude toward mathematics knowledge (3 items).

The LMT dataset is nationally representative of elementary schools samples for mail-in surveys, using stratified probability proportional to size sampling during the 2006-2007 academic year. The target population for the elementary school sample was all currently operating public schools in the fifty U.S. states and the District of Columbia that had at least ten students in 3rd and 4th grades. The frame was stratified by geographic region, according to the U.S. Census Bureau regions, and by urbanicity, according to a locale code which is assigned by the U.S. Census Bureau based on geographic location and population attributes. Initial samples of 840 elementary schools were drawn, in which the sample sizes were allocated to each stratum using a proportionate allocation. Within each stratum, schools were selected using systematic probability proportional to size sampling. The measure of size used for the elementary school sample was the total number of students in grades 3, 4 and 5. Finally, the data used for this study regarded Rational Number items for elementary school teachers. After checking the missing values, the number of participants in this study was 336.

The variables used in the models here are described in <Table III-1>, and descriptive statistics are presented in <Table III-2>. The dependent variable in this study is a discrete variable indicating whether a teacher correctly answered each item or not. Independent variables hypothesized to affect the probability that teachers

<Table III-1> Definitions of the Variables

Variable	Description
Female	1 if female, 0 if male
White	1 if Caucasian, 0 otherwise (reference group)
Mathematics credential	1 if the teacher has the credential to teach mathematics, 0 otherwise
Teaching years	Number of years teaching mathematics
Attitude toward teaching mathematics	1 if strongly disagree, 2 if disagree, 3 if neutral, 4 if agree, 5 if strongly agree about "I enjoy teaching mathematics" statement
Teaching all subjects	1 if the teacher taught all subjects, 0 otherwise
Mathematics classes	Number of undergraduate or graduate-level mathematics classes that the teacher took (0: no classes, 1: 1-2 classes, 2: 3-5 classes, 3: 6 or more classes)
Methods-of-teaching-mathematics classes	Number of undergraduate or graduate-level methods-of-teaching-mathematics classes that the teacher took (0: no classes, 1: 1-2 classes, 2: 3-5 classes, 3: 6 or more classes)

<Table III-2> Descriptive Statistics

Variable	%	Mean	SD	Min.	Max.
Female	78.87		.4088		
White	86.61		.3411		
Mathematics credential	39.88		.4904		
Teaching years		12.6786	9.4093	0	45
Teaching all subjects	37.50		.4848		
Attitude toward teaching mathematics		4.4970	.8393	1	5
Mathematics classes		1.9832	.9089	0	3
Methods-of-teaching-mathematics classes		1.2679	.7878	0	3

correctly answered each item include demographic factors such as gender and ethnicity, having a credential to teach mathematics, the number of years to teach mathematics, teaching all subjects, the number of undergraduate or graduate-level mathematics classes, and the number of undergraduate or graduate-level methods-of-teaching-mathematics classes.

2. Choosing items

The linear regression analysis could be used in the analysis of this data (see [Appendix 1].) This analysis model explains 39% of the variance in total scores. However, it is statistically more efficient and valuable to focus on the response of the most discriminated items based on the Item Discrimination Index (Johnson, 1951), which measures the extent to which item responses can discriminate between individuals who have a high score on the test and those that get a low score. This is calculated for each response.[1] Using discrimination index (see <Table III-3>), I chose the greatest five items which are different among

1) The formula for the item discrimination index is $D.I. = \frac{R_U - R_L}{f}$. R_U and R_L indicate the response frequency of the upper and lower groups who pass the item, and f is the numbers of respondents in the upper group. Generally, 27% is used to divide the upper and lower groups. In this case, f is equal to 0.27×number of total respondents.

the respondents; 20a 32, 25, 29c and 6.

<Table III-3> Discrimination Index

Item	Discrimination Index
20a	0.8377
20d	0.7496
32	0.7496
20f	0.7165
25	0.7165
29c	0.6834
20e	0.6724
6	0.6614
20c	0.6614

3. Model Specification

Due to the dichotomous nature (correctness/ incorrectness) of the outcome variables, a logistic regression model is theoretically more appropriate (Dey & Astin, 1993; Hanushek & Jackson, 1977; Long, 1997). The base logistic regression model used is specified as

$$\log\frac{P_i}{1-P_i}=\alpha+\beta_i X_i+\gamma_i Y_i+\delta_i Z_i+\zeta_i W_i+\epsilon_i \quad (1)$$

where P_i is the probability that teacher i will correctly answer the item; X_i is a vector of demographic characteristics, such as gender and ethnicity; Y_i is a variable indicating whether a teacher has a credential to teach mathematics; Z_i is a variable indicating a teacher's attitude about teaching mathematics; W_i is a vector of teaching experience such as the number of years a teacher has taught mathematics and whether a teacher teach all subjects; a, β_i, γ_i, δ_i, and ζ_i are estimated coefficients; and ε_i represents a random error term that is logistically distributed. The dependent variable is the logarithm of the odds that a particular teacher will correctly answer each item. The model is estimated using maximum likelihood estimation. In order to examine the effects of demographic characteristics, a credential to teach mathematics, attitude for teaching mathematics, and teaching experience on what factors influence on MKT, this model is used in this study.

Also, prior educational characteristics such as the number of undergraduate or graduate-level mathematics classes that the teacher took or the number of undergraduate or graduate-level methods-of-teaching-mathematics classes that the teacher took can be variables to affect the probability that a teacher correctly answers items. Since these two variables might be collinear, each factor is added into the prior model. For investigating the effect of the number of mathematics classes that a teacher took at the undergraduate or graduate-level, only this variable is added to the base model;

$$\log\frac{P_i}{1-P_i}=\alpha+\beta_i X_i+\gamma_i Y_i+\delta_i Z_i+\zeta_i W_i+\eta_i U_i+\epsilon_i \quad (2)$$

U_i is a variable indicating the number of mathematics classes that teachers took; and η_i is the estimated coefficient. In this model, only the factor of the number of mathematics classes taken is analyzed to determine how it affects whether that teachers correctly answer each item.

For investigating the effect of the number of methods-of-teaching-mathematics classes that a teacher took in undergraduate or graduate level, only this variable is combined in the base model;

$$\log\frac{P_i}{1-P_i}=\alpha+\beta_i X_i+\gamma_i Y_i+\delta_i Z_i+\zeta_i W_i+\theta_i U_i+\epsilon_i \quad (3)$$

V_i is a variable indicating the number of methods-of-teaching-mathematics classes taken; and θ_i is the estimated coefficient. In this model, only factor of the number of methods-of-teaching-mathematics classes taken is analyzed to determine how it affects that teachers correctly answer in each item.

Ⅳ. Results

[Appendix 2] presents the results of (1), (2) and (3) logistic regression analysis used in this research. The results of each factor are as follows.

1. Model of fit

<Table IV-1> indicates the area under the ROC curve in each logistic regression model. The better the model does at predicting, the greater the area will be under the ROC curve bounded by the diagonal. Since all areas are more than 0.7, all models are fair.2)

<Table IV-1> Area under the ROC curve

	model		
item	(1)	(2)	(3)
20a	0.7362	0.7436	0.7397
32	0.7468	0.7739	0.7596
25	0.7488	0.7528	0.7484
29c	0.6838	0.6855	0.7049
6	0.7392	0.7414	0.7401

2. Gender and Ethnicity

<Table IV-2> displays the results of analysis of the five chosen items by gender and ethnicity in the base logistic model. Being female is not statistically significant except for one item (29c), whereas being white is highly significant except for one item (29c), holding all other variables constant. Therefore, gender and being white are complementary to influence the probability that teachers correctly answer in each item. Specifically, compared to male teachers, the odds that female teachers correctly answer decrease by a factor of 0.55 in item 29c, holding all other variables constant. The odds of white teachers

<Table IV-2> Logistic Regression Analysis by Gender and Ethnicity

	Female			White		
Item	Odds Ratio	S.E.	Sig.	Odds Ratio	S.E.	Sig.
20a	.6310	.1859		2.2760	.8452	*
32	.8842	.2665		3.6626	1.5357	**
25	.7144	.2191		2.8582	1.0312	**
29c	.5472	.1606	*	1.1324	.3829	
6	1.0467	.3172		5.4508	2.0229	**

Note: *p<.05. **p<.01.

2) Metrics for measure of area under ROC
- .90 ~ 1.00 : Excellent (A)
- .80 ~ .90 : Good (B)
- .70 ~ .80 : Fair (C)
- .60 ~ .70 : Poor (D)
- .50 ~ .60 : Fail (F)

correctly answering compared to non-white teachers are the greatest among all factors. They are about 128% in item 20a and 445% in item 6 higher than the odds for non-white teachers, holding all other variables constant. Therefore, a teacher's ethnicity rather than gender is a critical factor when school resources are distributed for teachers' development of MKT.

3. Credential for Teaching Mathematics and Attitude toward Teaching Mathematics

<Table IV-3> presents the results of the analysis of the five chosen items by a credential for teaching mathematics and attitude toward teaching mathematics in the base logistic model. All results are statistically significant except for having a credential teaching mathematics in item 29c and attitude toward teaching mathematics in item 32, which have *p*-value of 0.1. If teachers have credentials to teach mathematics, the odds of them correctly answering increase by a factor of 2.14 in item 6 and a factor of 4.96 in item 32, compared to teachers who do not, holding all other variables constant.

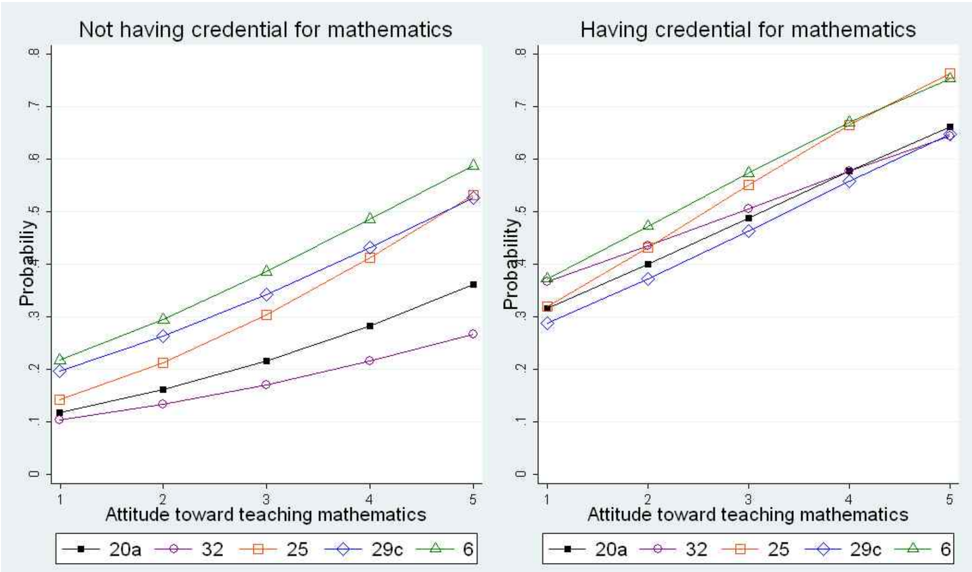
On the other hand, for a one-point increase in attitude toward teaching mathematics, the odds of teachers correctly answering increase by a factor of 1.43 in item 20a and by a factor of 1.62 in item 25, holding all other variables constant. The extent that teacher's positive attitude toward teaching mathematics is substantial in the probability that teacher correctly answer for many items.

[Figure IV-1] indicates the results of the simulation of how probabilities that teachers correctly answer vary over the range of values of attitude toward teaching mathematics between teachers who do not have credentials to teach mathematics and the other teachers who do when other variables are at their mean. Whether or not teachers have credentials to teach mathematics, the change of the probabilities is similar, that is, increasing the probabilities as the values of attitude toward teaching mathematics increase. However, the two plots have different patterns. The probabilities that teachers having credentials to teach mathematics correctly answer the five chosen items are greater than the probabilities of teachers not having credentials, in each value of attitude toward teaching mathematics. Nevertheless, the probabilities that teachers correctly answer without credentials to teach mathematics who strongly enjoy teaching mathematics are commonly greater than the probabilities that teachers correctly answer with credentials to teach mathematics who strongly

<Table IV-3> Logistic Regression Analysis by Credential for Teaching Mathematics and Attitude toward Teaching Mathematics

	Mathematics credential			Attitude toward teaching mathematics		
Item	Odds Ratio	S.E.	Sig.	Odds Ratio	S.E.	Sig.
20a	3.4507	1.0361	**	1.4329	.2386	**
32	4.9629	1.5552	**	1.3312	.2319	
25	2.8249	.8734	**	1.6169	.2544	**
29c	1.1324	.4825		1.4589	.2198	**
6	2.1430	.6778	*	1.5042	.2265	**

Note: *p<.05. **p<.01.



[Figure IV-1] Probability that Teachers Correctly Answer by Credential for Teaching Mathematics and Attitude toward Teaching Mathematics

do not enjoy teaching mathematics in four items.

The slopes of the plot for teachers' having credentials to teach mathematics are consistent over the range of values of attitude toward teaching mathematics. However, the slopes of the plot for teachers' not having credentials become greater as values of attitude toward teaching mathematics increase, and the changes of slopes are different for each item. Therefore, if teachers do not have credentials to teach mathematics, the differences among all properties in each value of attitude toward teaching mathematics becomes greater as the values of attitude toward teaching mathematics increase. When teachers do, the differences among all properties in each value of attitude toward teaching mathematics are more consistent as the values of attitude toward teaching mathematics increase than when teachers do not.

Also, even though the slopes of the left plot become greater, the gap of probabilities between when teachers do not have credentials to teach mathematics and when teachers have them becomes greater as the values of the attitude toward teaching mathematics increases because the slopes of the right plot are greater than of the left plot. For example, in item 20a, when teachers strongly disagree with "I enjoy teaching mathematics," the differences of probabilities are 0.2 between teachers who do not have credentials to teach mathematics (about 0.1) and teachers who do (about 0.3). However, when teachers strongly agree with "I enjoy teaching mathematics," the differences of probabilities are 0.3 between teachers who do not have credentials to teach mathematics (about 0.4) and teachers who do (about 0.7). In summary, having a credential to teach mathematics and a teacher's attitude to teach mathematics are critical to these measures.

<Table IV-4> Logistic Regression Analysis by Number of Years Teaching Mathematics and Teaching All Subjects

Item	Number of Years to Teach Mathematics			Teaching All Subjects		
	Odds Ratio	S.E.	Sig.	Odds Ratio	S.E.	Sig.
20a	1.0235	.0134		.7904	.2399	
32	1.0144	.0136		.9797	.3163	
25	.9901	.0130		.6040	.1760	
29c	1.0211	.0130		1.4589	.2198	**
6	1.0155	.0134		.6411	.1885	

Note: *p<.05. **p<.01.

Therefore, schools or districts need to support teachers who do not have a credential to teach mathematics and are not confident about teaching mathematics.

4. Number of Years Teaching Mathematics and Teaching All Subjects

<Table IV-4> indicates the results of the analysis of the five chosen items by the number of years teaching mathematics and whether the teacher teaches all subjects. The number of years teaching mathematics is not statistically significant in all items, and the odds ratios of all items are similar to 1. Whether a teacher taught all subjects is not statistically significant to be correct in all items except for one (29c), where the odds of teachers who teach all subjects correctly answering are about 46% higher than the odds of teachers who do not correctly answering, holding all other variables constant. Without this item, all odds of teachers correctly answering are smaller than 1, holding all other variables constant. In summary, the number of years of teaching mathematics and whether a teacher teaches all subjects are not critical to these measures; thus, they are not significant factors when school or district resources are distributed for teachers' learning opportunities.

5. Number of Mathematics Classes Taken and Number of Methods-of-Teaching-Mathematics Classes Taken

<Table IV-5> indicates the results of the analysis of the five chosen items by the number of undergraduate or graduate-level mathematics

<Table IV-5> Logistic Regression Analysis by the Number of Mathematics Classes Taken and the Number of Methods-of-Teaching-Mathematics Classes Taken

Item	Number of mathematics classes taken			Number of methods-of-teaching-mathematics classes taken		
	Odds Ratio	S.E.	Sig.	Odds Ratio	S.E.	Sig.
20a	1.4453	.2386	*	1.4018	.2297	*
32	1.8672	.3346	**	1.5607	.2626	**
25	1.4343	.2322	*	1.0078	.1678	
29c	1.1331	.1777		1.6683	.2757	**
6	1.2469	.2028		1.2172	.2065	

Note: *p<.05. **p<.01.

classes that the teacher in the (2) logistic model took and the number of undergraduate or graduate-level methods-of-teaching-mathematics classes that the teacher took in the (3) logistic model.

For a one-unit increase in the number of mathematics classes taken, the odds of teachers correctly answering increase 43% in item 25 and 87% in item 32, holding all other variables constant. For a one-unit increase in the number of methods-of-teaching-mathematics classes taken, the odds of teachers correctly answering increase 40% in item 20a and 67% in item 29c, holding all other variables constant. Even though the number of mathematics classes taken and the number of methods-of-teaching-mathematics classes taken are not statistically significant in all items, these two factors are substantially positive because all odds ratios are greater than 1.

The numbers of undergraduate or graduate-level mathematics classes and methods-of-teaching-mathematics classes that the teacher took has an affirmative effect on MKT. However, this does not mean that the more mathematics classes teachers take, the better MKT teachers have because the number of mathematics classes is not statistically significant among all five chosen items. Regarding general mathematics courses, Begle (1979) and Monk (1994) indicate that there is a weak relationship between the courses taken by teachers and their students' performances on standardized examinations. Rather, the different effects between the number of mathematics classes taken and the number of methods-of-teaching-mathematics classes taken could be one of the specific examples to indicate that MKT is multidimensional, and include knowledge of various mathematical topics such as number and operation, geometry, and algebra and domains such as knowledge of students and content and specialized content knowledge. Hence, both of these two factors are necessary to be considered together.

V. Discussion and Conclusions

Based on the research results that teachers' MKT is significantly related to student achievement gains (Hill et al., 2005), this paper explores various factors and the extent to which each factor contributes to the teachers' MKT. In summary, for a more effective in-service education for elementary school teachers, the policy initiative needs to investigate teachers' ethnicity, whether teachers have credentials to teach mathematics, and the number of undergraduate or graduate-level mathematics classes and method-of-teaching-mathematics classes. Given the limited resources to support teachers by providing additional educational opportunities, non-white teachers and teachers who do not have credentials to teach mathematics should have a priority. Also, it is necessary to motivate teachers who are not confident to teach mathematics and lack mathematics classes or method-of-teaching mathematics classes in undergraduate or graduate-level to participate in additional education related to MKT. Among the factors that this study investigated, the issues related to credential to teach mathematics and teaching experiences are discussed here.

Having a credential to teach mathematics is one of the most significant factors to influence MKT. This means that the credential to teach mathematics can provide one of the available rough indices to explain whether a teacher has MKT. It also validates that one of the requirements of NCLB, teacher certification, is critical. Therefore, it is expected that schools or districts support teachers who do not have credentials to provide more opportunities for developing their MKT than other teachers. However, it seems important to emphasize that the current research does not validate that all teachers who have a credential to teach mathematics have enough MKT to teach mathematics to teachers because there are debates about the extent to which such qualifications lead to better student achievement (Greenwald, Hedges, & Laine, 1996; Hanushek, 1996). Furthermore, this result should not be interpreted as placing more value on alternative certifications, such as Teach For America, rather than teacher education programs. In fact, there are approved teacher education institutions that offer teacher certification for their home states, such as the University of Michigan, the University Pittsburgh, etc. Both the teachers who graduate from teacher education programs that do not offer teacher certifications, and anyone who wants to obtain a teacher certification, take examinations for a teaching credential. The issue is here the improvements of both teacher education and the standard of a credential: overall improvement of the quality of educational opportunities that teachers have, and a high quality credential that people can obtain if they complete a high quality teacher education. If the quality of teacher education is not enough to help teachers get a credential, each teacher will spend other resources to achieve their credentials. If the quality of a credential or a passing grade is low, a credential would not work well. At least, the result of the analysis related to having a credential emphasizes educational opportunities for teachers, the qualities of the teacher education, and the quality of a credential.

Furthermore, contrary to the general belief that teachers who have taught mathematics for a long time have a full knowledge of teaching mathematics, the number of years teaching mathematics is not statistically significant in this study. Therefore, it would be an incorrect opinion that teachers who have a long experiences in teaching mathematics would have better MKT than teachers who have short experiences teaching mathematics. Moreover, schools or districts do not need to consider teaching experience as a factor for a distribution of the resources for developing teachers' MKT. Fundamentally, this result refutes the assumption accepted by the public in the U.S.: teaching can be learned by doing in classrooms without any education. Moreover, this analysis initiates generic questions: what can be learned in teaching experiences and where can MKT be learned? Rockoff (2004) finds that teaching experience significantly influences students' reading comprehension, but teaching experience is not statistically related to student mathematics achievement. Because the current study used limited data, it is hard to make general assertions, but it seems reliable that having a long teaching experience might not be a way that teachers can improve their MKT. Moreover, specifica educational opportunities, such as professional development or teacher education, would help teachers improve their MKT, as Smith, Desimore, and Ueno (2005) highlight preparedness to teach mathematics content and participation in professional developments.

Although this research used and analyzed data about teachers from the U.S., it provides important implication in South Korea in terms of education for teachers. There is no specific research on South Korean teachers' MKT, but it is know that they have a strong foundation in mathematics content knowledge (Leung & Park, 2002; Li, Ma, & Pang, 2008). However, it is still critical to decide which teachers need more support to improve their MKT in South Korea. Unlike the U.S., South Korea does not have issues related to teacher credentials. Generally, in South Korea, people with beginning teacher certification can be hired, and teachers after five years of teaching experience are required to have approximately one hundred professional development hours to have an advanced teacher certification. These requirements are based on the educational policy for teacher quality, but this type of systemic support for U.S. teachers is rare. However, in the U.S. as well as South Korea, the quality of the teacher education with regard to MKT has not been studied. The more critical issue is that teachers in the U.S. and South Korea with many years of teaching experience are not required to take any professional development. As previously specified, having many years of experience in teaching mathematics does not help improve a teacher's MKT. Moreover, without useful feedback, most teachers' performances plateau (The Gates Foundation, 2010), or may decrease. Schools and districts in South Korea need to encourage teachers and distribute their resources to teachers so that they make a constant effort to improve their knowledge for teaching. Continuous support for teachers to have educational opportunities to improve their MKT would improve teacher quality and, finally, the quality of school education.

VI. Limitations

The U.S. Department of Labor reports that the number of elementary school teachers, excluding special education, was about 1,538,030 in 2007. Hence, the number of participants in this study is very small even though the data used in this study has a frame stratified by geographic region and urbanicity according to the U.S. Census Bureau. To find more valid results, gathering more participants will be necessary.

Furthermore, as mentioned previously, the data used in this research is based on the frame stratified by geographic region, according to the U.S. Census Bureau regions, and by urbanicity, according to a locale code which is assigned by the U.S. Census Bureau based on geographic location and population attributes. Therefore, since the data is not randomized, it is necessary to be careful to generalize the results of this research.

Reference

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Baumert, J., Kunter, M., Werner, B., Brunner, M., Voss, T., Jordan, A., . . . Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180.

Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, DC.: Mathematical Association of America and National Council of Teachers of Mathematics.

Darling-Hammond, L. (1996). *What matters most: Teaching for America's future*. New York: National Commission on Teaching and America's Future.

Dey, E. L., & Astin, A. W. (1993). Statistical alternatives for studying college student retention: A comparative analysis of logit, probit, and linear regression. *Research in Higher Education, 34*(5), 569-581.

Greenwald, R., Hedges, L., & Laine, R. (1996). The effect of school resources on school achievement. *Review of Educational Research, 66*, 361-396.

Hanushek, E. A. (1996). A more complete picture of school resource policies. *Review of Educational Research, 66*, 397-409.

Hanushek, E. A., & Jackson, J. E. (1977). *Statistical methods for social scientists*. San Diego, CA: Academic Press.

Hill, H. C., & Lubienski, S. T. (2007). Teachers' mathematics knowledge for teaching and school context: A study of california teachers. *Educational Policy, 21*(5), 747-768.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Johnson, A. P. (1951). Notes on a suggested index of item validity: The UL Index. *Journal of Educational Psychology, 42*(8), 499-504.

Leung, F. K. S., & Park, K. M. (2002). Competent students, competent teachers? *International Journal of Educational Research, 37*(2), 113-129.

Li, Y., Ma, Y., & Pang, J. S. (2008). Mathematical preparation of prospective elementary teachers. In P. Sullivan & T. Wood (Eds.), *The international handbook of mathematics teacher education: Knowledge and beliefs in mathematics teaching and teaching development* (Vol. 1, pp. 37-62). Rotterdam, The Netherlands: Sense.

Long, J. S. (1997). *Regression models for categorical and limited dependent variables*. London: Sage Publications.

Monk, D. H. (1994). Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review, 13*(2), 125-145.

Rockoff, J. E. (2004). The impact of individual teachers on student achievement: Evidence from panel data. *The American Economic Review, 94*(2), 247-252.

Rockoff, J. E., Jacob, B. A., Kane, T. J., & Staiger, D. O. (2011). Can you recognize an effective teacher when you recruit one? *Education Finance and Policy, 6*(1), 43-74.

Shulman, L.S . (1986). Those who can understand: Knowledge growth in teaching. *Educational Research, 15*(2), 4-14.

Smith, T. M., Desimone, L. M., & Ueno, K. (2005). "Highly qualified" to do what? The relationship between NCLB teacher quality mandates and the use of reform-oriented instruction in middle school mathematics. *Educational Evaluation and Policy Analysis, 27*(1), 75-109.

The Gates Foundation, (2010). *Effective teaching: Working with teachers to develop fair and reliable measures.* Seattle, WA: Bill & Melinda Gates Foundation.

교수를 위한 수학적 지식과 교사의 배경: 미국 교사에 관한 연구

김 연 (University of Michigan)

교사가 어느 수준의 mathematical knowledge for teaching (MKT)를 가지고 있는가는 학생의 수학 성취도에 강력한 영향을 준다 (Hill, Rowan, & Ball, 2005). 그러나 학교나 지역 교육청이 교사의 MKT 수준에 대한 정보를 가지고 있지 않으므로, 어떤 교사들에게 MKT를 증진시킬 수 있는 기회와 재원을 줄 것인지 결정하기는 쉽지 않다. 본 연구는 교사들의 어떤 특징이 그들의 MKT 수준과 관련이 있는가를 탐구한다. 본 논문은 336명의 초등학교 교사들의 MKT와 그들의 인구학적 그리고 교육적 배경 및 교수 경험과의 관련성을 분석했다. 특히, 교사의 MKT를 측정하기 위해 본 연구는 객관식 평가 문항을 이용했다. 결국, 교수경력은 MKT 측정과 아무런 영향이 없음이 밝혀졌다. 본 연구의 결과와 관련하여 수학 교사 교육을 위한 정책에 대한 논의로 본 고를 마친다.

* 주제어 : 교수를 위한 수학적 지식(mathematical knowledge for teaching), 교사(teachers), 학교와 교육청의 재원 분배(distribution of school and district resources), 교사 교육 (education for teachers)

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[Appendix 1] The results of the linear regression analysis

Variable	Coefficient	Sig.
Constant	16.6134	**
Female	-2.6750	*
White	7.0843	**
Mathematics credential	7.7276	**
Teaching years	.1888	**
Teaching all subjects	-1.3720	
Attitude toward teaching mathematics	3.0025	**
Mathematics classes	1.7223	*
Methods-of-teaching-mathematics classes	1.5416	*
R^2	0.3949	**

Note: *p<.05. **p<.01.

[Appendix 2] The results of the (1), (2) and (3) logistic regression analysis

<TABLE 1> Estimating the Probability of Correctness in (1) logistic regression model: 20a

Variable	Odds Ratio	S.E.	Sig.
Female	.6310	.1859	
White	2.2760	.8452	*
Mathematics credential	3.4507	1.0361	**
Teaching years	1.0235	.0134	
Teaching all subjects	.7904	.2399	
Attitude toward teaching mathematics	1.4329	.2386	**

Note: *p<.05. **p<.01.

<TABLE 2> Estimating the Probability of Correctness in (1) logistic regression model: 32

Variable	Odds Ratio	S.E.	Sig.
Female	.8842	.2665	
White	3.6626	1.5357	**
Mathematics credential	4.9629	1.5552	**
Teaching years	1.0144	.0136	
Teaching all subjects	.9797	.3163	
Attitude toward teaching mathematics	1.3312	.2319	

Note: *p<.05. **p<.01.

<TABLE 3> Estimating the Probability of Correctness in (1) logistic regression model: 25

Variable	Odds Ratio	S.E.	Sig.
Female	.7144	.2191	
White	2.8582	1.0312	**
Mathematics credential	2.8249	.8734	**
Teaching years	.9901	.0130	
Teaching all subjects	.6040	.1760	
Attitude toward teaching mathematics	1.6169	.2544	**

Note: *p<.05. **p<.01.

<TABLE 4> Estimating the Probability of Correctness in (1) logistic regression model: 29c

Variable	Odds Ratio	S.E.	Sig.
Female	.5472	.1606	*
White	1.1324	.3829	
Mathematics credential	1.1324	.4825	
Teaching years	1.0211	.0130	
Teaching all subjects	.6746	.1933	
Attitude toward teaching mathematics	1.4589	.2198	**

Note: *p<.05. **p<.01.

<TABLE 5> Estimating the Probability of Correctness in (1) logistic regression model: 6

Variable	Odds Ratio	S.E.	Sig.
Female	1.0467	.3172	
White	5.4508	2.0229	**
Mathematics credential	2.1430	.6778	*
Teaching years	1.0155	.0134	
Teaching all subjects	.6411	.1885	
Attitude toward teaching mathematics	1.5042	.2265	**

Note: *p<.05. **p<.01.

<TABLE 6> Estimating the Probability of Correctness in (2) logistic regression model: 20a

Variable	Odds Ratio	S.E.	Sig.
Mathematics classes	1.4453	.2386	*

Note: *p<.05. **p<.01.

<TABLE 7> Estimating the Probability of Correctness in (2) logistic regression model: 32

Variable	Odds Ratio	S.E.	Sig.
Mathematics classes	1.8672	.3346	**

Note: *p<.05. **p<.01.

<TABLE 8> Estimating the Probability of Correctness in (2) logistic regression model: 25

Variable	Odds Ratio	S.E.	Sig.
Mathematics classes	1.4343	.2322	*

Note: *p<.05. **p<.01.

<TABLE 9> Estimating the Probability of Correctness in (2) logistic regression model: 29c

Variable	Odds Ratio	S.E.	Sig.
Mathematics classes	1.1331	.1777	

Note: *p<.05. **p<.01.

<TABLE 10> Estimating the Probability of Correctness in (2) logistic regression model: 6

Variable	Odds Ratio	S.E.	Sig.
Mathematics classes	1.2469	.2028	

Note: *p<.05. **p<.01.

<TABLE 11> Estimating the Probability of Correctness in (3) logistic regression model: 20a

Variable	Odds Ratio	S.E.	Sig.
Methods-of-teaching-mathematics classes	1.4018	.2297	*

Note: *p<.05. **p<.01.

<TABLE 12> Estimating the Probability of Correctness in (3) logistic regression model: 32

Variable	Odds Ratio	S.E.	Sig.
Methods-of-teaching-mathematics classes	1.5607	.2626	**

Note: *p<.05. **p<.01.

<TABLE 13> Estimating the Probability of Correctness in (3) logistic regression model: 25

Variable	Odds Ratio	S.E.	Sig.
Methods-of-teaching-mathematics classes	1.0078	.1678	

Note: *p<.05. **p<.01.

<TABLE 14> Estimating the Probability of Correctness in (3) logistic regression model: 29c

Variable	Odds Ratio	S.E.	Sig.
Methods-of-teaching-mathematics classes	1.6683	.2757	**

Note: *p<.05. **p<.01.

<TABLE 15> Estimating the Probability of Correctness in (3) logistic regression model: 6

Variable	Odds Ratio	S.E.	Sig.
Methods-of-teaching-mathematics classes	1.2172	.2065	

Note: *p<.05. **p<.01.