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Intuitionistic Fuzzy Theta-Compact Spaces

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Abstract

In this paper, we introduce certain types of continuous functions and intuitionistic fuzzy θ-compactness in intuitionistic fuzzy topological spaces. We show that intuitionistic fuzzy θ-compactness is strictly weaker than intuitionistic fuzzy compactness. Furthermore, we show that if a topological space is intuitionistic fuzzy retopologized, then intuitionistic fuzzy compactness in the new intuitionistic fuzzy topology is equivalent to intuitionistic fuzzy θ-compactness in the original intuitionistic fuzzy topology. This characterization shows that intuitionistic fuzzy θ-compactness can be related to an appropriated notion of intuitionistic fuzzy convergence.

Keywords: Intuitionistic fuzzy topology, Theta-compact

1. Introduction

The concept of an intuitionistic fuzzy set as a generalization of fuzzy sets was introduced by Atanassov [1]. Coker and his colleagues [2–4] introduced an intuitionistic fuzzy topology using intuitionistic fuzzy sets.

Many researchers studied continuity and compactness in fuzzy topological spaces and intuitionistic fuzzy topological spaces [5–8]. Recently, Hanafy et al. [9] introduced an intuitionistic fuzzy θ-closure operator and intuitionistic fuzzy θ-continuity.

In this paper, we introduce certain types of continuous functions and intuitionistic fuzzy θ-compactness in intuitionistic fuzzy topological spaces. We show that intuitionistic fuzzy θ-compactness is strictly weaker than intuitionistic fuzzy compactness. Moreover, we show that the sufficient condition in Theorem 4.5 holds for intuitionistic fuzzy θ-compact spaces; however, in general, it fails for intuitionistic fuzzy compact spaces. Furthermore, we show that if a topological space is intuitionistic fuzzy retopologized, then intuitionistic fuzzy compactness in the new intuitionistic fuzzy topology is equivalent to the intuitionistic fuzzy θ-compactness in the original intuitionistic fuzzy topology described in Theorem 4.6. This characterization shows that the intuitionistic fuzzy θ-compactness can be related to an appropriated notion of intuitionistic fuzzy convergence.

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2. Preliminaries

Let X and I denote a nonempty set and unit interval $[0, 1]$, respectively. An *intuitionistic fuzzy set* A in X is an object of the form

$$A = (\mu_A, \gamma_A),$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of non-membership, respectively, and $\mu_A + \gamma_A \leq 1$. Obviously, every fuzzy set μ_A in X is an intuitionistic fuzzy set of the form $(\mu_A, 1 - \mu_A)$.

Throughout this paper, $I(X)$ denotes the family of all intuitionistic fuzzy sets in X and intuitionistic fuzzy is abbreviated as IF.

Definition 2.1. [1] Let X denote a nonempty set and let intuitionistic fuzzy sets A and B be of the form $A = (\mu_A, \gamma_A)$, $B = (\mu_B, \gamma_B)$. Then,

(1) $A \leq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$,

(2) $A = B$ iff $A \leq B$ and $B \leq A$,

(3) $A^c = (\gamma_A, \mu_A)$,

(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$,

(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$,

(6) $\underline{0} = (\tilde{0}, \tilde{1})$ and $\underline{1} = (\tilde{1}, \tilde{0})$.

Definition 2.2. [2] An *intuitionistic fuzzy topology* on X is a family $\mathcal{T}$ of intuitionistic fuzzy sets in X that satisfy the following axioms.

(1) $\underline{0}, \underline{1} \in \mathcal{T}$,

(2) $G_1 \cap G_2 \in \mathcal{T}$ for any $G_1, G_2 \in \mathcal{T}$,

(3) $\bigcup G_i \in \mathcal{T}$ for any $\{G_i : i \in J\} \subseteq \mathcal{T}$.

In this case, the pair $(X, \mathcal{T})$ is called an *intuitionistic fuzzy topological space* and any intuitionistic fuzzy set in $\mathcal{T}$ is known as an *intuitionistic fuzzy open set* in X.

Definition 2.3. [2] Let $(X, \mathcal{T})$ and A denote an intuitionistic fuzzy topological space and intuitionistic fuzzy set in X, respectively. Then, the *intuitionistic fuzzy interior* of A and the *intuitionistic fuzzy closure* of A are defined by

$$\text{cl}(A) = \bigcap\{K \mid A \leq K, K^c \in \mathcal{T}\}$$

and

$$\text{int}(A) = \bigcup\{G \mid G \leq A, G \in \mathcal{T}\}.$$

Theorem 2.4. [2] For any IF set A in an IF topological space $(X, \mathcal{T})$, we have

$$\text{cl}(A^c) = (\text{int}(A))^c \quad \text{and} \quad \text{int}(A^c) = (\text{cl}(A))^c.$$

Definition 2.5. [3, 4] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An *intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ of X is an intuitionistic fuzzy set in X defined by

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if} \quad y = x, \\ (0, 1) & \text{if} \quad y \neq x. \end{cases}$$

In this case, x, α, and β are called the *support*, *value*, and *nonvalue* of $x_{(\alpha,\beta)}$, respectively. An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to *belong* to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.

Remark 2.6. If we consider an IF point $x_{(\alpha,\beta)}$ as an IF set, then we have the relation $x_{(\alpha,\beta)} \in A$ if and only if $x_{(\alpha,\beta)} \leq A$.

Definition 2.7. [4,10] Let $(X, \mathcal{T})$ denote an intuitionistic fuzzy topological space.

(1) An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be *quasi-coincident* with the intuitionistic fuzzy set $U = (\mu_U, \gamma_U)$, denoted by $x_{(\alpha,\beta)}qU$, if $\alpha > \gamma_U(x)$ or $\beta < \mu_U(x)$.

(2) Let $U = (\mu_U, \gamma_U)$ and $V = (\mu_V, \gamma_V)$ denote two intuitionistic fuzzy sets in X. Then, U and V are said to be *quasi-coincident*, denoted by UqV, if there exists an element $x \in X$ such that $\mu_U(x) > \gamma_V(x)$ or $\gamma_U(x) < \mu_V(x)$.

The word 'not quasi-coincident' will be abbreviated as $\widetilde{q}$ herein.

Proposition 2.8. [4] Let U, V, and $x_{(\alpha,\beta)}$ denote IF sets and an IF point in X, respectively. Then,

(1) $U\widetilde{q}V^c \iff U \leq V$,

(2) $UqV \iff U \not\leq V^c$,

(3) $x_{(\alpha,\beta)} \leq U \iff x_{(\alpha,\beta)}\widetilde{q}U^c$,

(4) $x_{(\alpha,\beta)}qU \iff x_{(\alpha,\beta)} \not\leq U^c$.

Definition 2.9. [4] Let $(X, \mathcal{T})$ denote an intuitionistic fuzzy topological space and let $x_{(\alpha,\beta)}$ denote an intuitionistic fuzzy point in X. An intuitionistic fuzzy set A is said to be an *intuitionistic fuzzy ϵ-neighborhood* (*q-neighborhood*) of $x_{(\alpha,\beta)}$ if there exists an intuitionistic fuzzy open set U in X such that $x_{(\alpha,\beta)} \in U \leq A$ ($x_{(\alpha,\beta)}qU \leq A$, respectively).

Theorem 2.10. [10] Let $x_{(\alpha,\beta)}$ and $U = (\mu_U, \gamma_U)$ denote an IF point in X and an IF set in X, respectively. Then, $x_{(\alpha,\beta)} \in \text{cl}(U)$ if and only if UqN, for any IF q-neighborhood N of $x_{(\alpha,\beta)}$.

Definition 2.11. [9] An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be an *intuitionistic fuzzy θ-cluster point* of an intuitionistic fuzzy set A if for each intuitionistic fuzzy q-neighborhood U of $x_{(\alpha,\beta)}$, $Aq\text{cl}(U)$. The set of all intuitionistic fuzzy θ-cluster points of A is called *intuitionistic fuzzy θ-closure* of A and is denoted by $\text{cl}_\theta(A)$. An intuitionistic fuzzy set A is called an *intuitionistic fuzzy θ-closed set* if $A = \text{cl}_\theta(A)$. The complement of an intuitionistic fuzzy θ-closed set is said to be an *intuitionistic fuzzy θ-open set*.

Definition 2.12. [11] Let $(X, \mathcal{T})$ and U denote an intuitionistic fuzzy topological space and an intuitionistic fuzzy set in X, respectively. The *intuitionistic fuzzy θ-interior* of U is denoted and defined by

$$\text{int}_\theta(U) = (\text{cl}_\theta(U^c))^c.$$

Definition 2.13. [2] Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ denote two intuitionistic fuzzy topological spaces and let $f : X \to Y$ denote a function. Then, f is said to be *intuitionistic fuzzy continuous* if the inverse image of an intuitionistic fuzzy open set in Y is an intuitionistic fuzzy open set in X.

Definition 2.14. [2] An intuitionistic fuzzy topological space $(X, \mathcal{T})$ is said to be *intuitionistic fuzzy compact* if every open cover of X has a finite subcover.

Definition 2.15. [9] A function $f : X \to Y$ is said to be *intuitionistic fuzzy θ-continuous* if for each intuitionistic fuzzy point $x_{(a,b)}$ in X and each intuitionistic fuzzy open q-neighborhood V of $f(x_{(a,b)})$, there exists an intuitionistic fuzzy open q-neighborhood U of $x_{(a,b)}$ such that $f(\text{cl}(U)) \leq \text{cl}(V)$.

Proposition 2.16. [12] Let $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ and $x_{(\alpha,\beta)}$ denote a function and an IF point in X, respectively.

(1) If $f(x_{(\alpha,\beta)})qV$, then $x_{(\alpha,\beta)}qf^{-1}(V)$ for any IF set V in Y.

(2) If $x_{(\alpha,\beta)}qU$, then $f(x_{(\alpha,\beta)})qf(U)$ for any IF set U in X.

Remark 2.17. Intuitionistic fuzzy sets have some different properties compared to fuzzy sets, and these properties are shown in the subsequent examples.

1. $x_{(\alpha,\beta)} \in A \cup B \not\Rightarrow x_{(\alpha,\beta)} \in A$ or $x_{(\alpha,\beta)} \in B$.
2. $x_{(\alpha,\beta)}qA$ and $x_{(\alpha,\beta)}qB \not\Rightarrow x_{(\alpha,\beta)}q(A \cap B)$.

Thus, we have considerably different results in generalizing concepts of fuzzy topological spaces to the intuitionistic fuzzy topological space.

Example 2.18. Let A, B denote IF sets on the unit interval $[0, 1]$ defined by

$$\mu_A = \frac{1}{3}\chi_{[0,\frac{1}{2}]}, \quad \gamma_A = \frac{2}{3}\chi_{[0,1]},$$

$$\mu_B = \frac{1}{3}\chi_{[\frac{1}{2},1]}, \quad \gamma_B = \frac{1}{3}\chi_{[0,1]}.$$

In addition, let $x = \frac{1}{4}, \quad (\alpha, \beta) = (\frac{1}{4}, \frac{1}{2})$. Then, $x_{(\alpha,\beta)} \in A \cup B$. However, $x_{(\alpha,\beta)} \notin A$ and $x_{(\alpha,\beta)} \notin B$.

Example 2.19. Let A, B denote IF sets on the unit interval $[0, 1]$ defined by

$$\mu_A = \frac{1}{3}\chi_{[0,\frac{1}{2}]}, \quad \gamma_A = \frac{2}{3}\chi_{[0,1]},$$

$$\mu_B = \frac{1}{3}\chi_{[\frac{1}{2},1]}, \quad \gamma_B = \frac{1}{3}\chi_{[0,1]}.$$

In addition, let $x = \frac{1}{4}, \quad (\alpha, \beta) = (\frac{1}{2}, \frac{1}{4})$. Then, $x_{(\alpha,\beta)}qA$ and $x_{(\alpha,\beta)}qB$; however, $x_{(\alpha,\beta)}\tilde{q}(A \cap B)$.

For the notions that are not mentioned in this section, refer to [11].

3. Intuitionistic Fuzzy θ-Irresolute and Weakly θ-Continuity

Definition 3.1. Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be IF topological spaces. A mapping $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is said to be *intuitionistic fuzzy θ-irresolute* if the inverse image of each IF θ-open set in Y is IF θ-open in X.

Theorem 3.2. Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be IF topological spaces. Let $\mathcal{T}_\theta$ be an IF topology on X generated using the subbase of all the IF θ-open sets in X, and let $\mathcal{U}_\theta$ be an IF topology on Y generated using the subbase of all the IF θ-open sets in Y. Then a function $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is IF θ-irresolute if and only if $f : (X, \mathcal{T}_\theta) \to (Y, \mathcal{U}_\theta)$ is IF continuous.

Proof. Trivial.

Recall that a fuzzy set A is said to be a *fuzzy θ-neighborhood* of a fuzzy point x_α if there exists a fuzzy closed q-neighborhood U of x_α, such that $U\tilde{q}A$ [13].

Definition 3.3. An intuitionistic fuzzy set A is said to be an *intuitionistic fuzzy θ-neighborhood* of intuitionistic fuzzy point $x_{(\alpha,\beta)}$ if there exists an intuitionistic fuzzy open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $\text{cl}(U) \leq A$.

Recall that a function $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is said to be a *fuzzy weakly θ-continuous* function if for each fuzzy point x_α in X and each fuzzy open q-neighborhood V of $f(x_\alpha)$, there exists a fuzzy open q-neighborhood U of x_α such that $f(U) \le \text{cl}(V)$ [13].

Definition 3.4. A function $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is said to be *intuitionistic fuzzy weakly θ-continuous* if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each intuitionistic fuzzy open q-neighborhood V of $f(x_{(\alpha,\beta)})$, there exists an intuitionistic fuzzy open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V)$.

Theorem 3.5. A function $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is IF weakly θ-continuous if and only if for each IF point $x_{(\alpha,\beta)}$ in X and each IF open θ-neighborhood N of $f(x_{(\alpha,\beta)})$ in Y, $f^{-1}(N)$ is an IF q-neighborhood of $x_{(\alpha,\beta)}$.

Proof. Let f be an IF weakly θ-continuous function, and let $x_{(\alpha,\beta)}$ be an IF point in X. Let N be an IF θ-neighborhood of $f(x_{(\alpha,\beta)})$ in Y. Then there exists an IF open q-neighborhood V of $f(x_{(\alpha,\beta)})$ such that $\text{cl}(V) \le N$. Since f is IF weakly θ-continuous, there exists an IF q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V) \le N$. Thus $U \le f^{-1}(N)$. Therefore, there exists an IF q-neighborhood U of $x_{(\alpha,\beta)}$ such that $U \le f^{-1}(N)$. Hence $f^{-1}(N)$ is an IF q-neighborhood of $x_{(\alpha,\beta)}$.

Conversely, let $x_{(\alpha,\beta)}$ be an IF point in X, and let V be an IF open q-neighborhood of $f(x_{(\alpha,\beta)})$. Then $\text{cl}(V)$ is an IF θ-neighborhood of $f(x_{(\alpha,\beta)})$. By the hypothesis, $f^{-1}(\text{cl}(V))$ is an an IF q-neighborhood of $x_{(\alpha,\beta)}$. Then there exists an IF open set U such that $x_{(\alpha,\beta)} q U \le f^{-1}(\text{cl}(V))$. Thus $f(U) \le \text{cl}(V)$. Therefore there exists an IF open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V)$. Hence f is an IF weakly θ-continuous function.

Theorem 3.6. If a function $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is IF weakly θ-continuous, then

(1) $f(\text{cl}(A)) \le \text{cl}_\theta(f(A))$ for each IF set A in X,

(2) $f(\text{cl}(\text{int}(\text{cl}(f^{-1}(B))))) \le \text{cl}_\theta(B)$ for each IF set B in Y.

Proof. (1) Let $x_{(\alpha,\beta)} \in \text{cl}(A)$, and let V be an IF open q-neighborhood of $f(x_{(\alpha,\beta)})$. Since f is IF weakly θ-continuous, there exists an IF open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V)$. Since $x_{(\alpha,\beta)} \in \text{cl}(A)$, UqA. Thus $f(U)qf(A)$. Since $f(U) \le \text{cl}(V)$, we have $\text{cl}(V)qf(A)$. Thus for each IF open q-neighborhood V of $f(x_{(\alpha,\beta)})$, $\text{cl}(V)qf(A)$. Hence $f(x_{(\alpha,\beta)}) \in \text{cl}_\theta(f(A))$.

(2) Let B be an IF set in Y and $x_{(\alpha,\beta)} \in \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$. Let V be an IF open q-neighborhood of $f(x_{(\alpha,\beta)})$. Since f is IF weakly θ-continuous, there exists an IF open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V)$. Since $\text{int}(\text{cl}(f^{-1}(B))) \le \text{cl}(f^{-1}(B))$,

$$\text{cl}(\text{int}(\text{cl}(f^{-1}(B)))) \le \text{cl}(\text{cl}(f^{-1}(B))) = \text{cl}(f^{-1}(B)).$$

Since $x_{(\alpha,\beta)} \in \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$, $x_{(\alpha,\beta)} \in \text{cl}(f^{-1}(B))$. Thus $f^{-1}(B)qU$, or $Bqf(U)$. Since $f(U) \le \text{cl}(V)$, we have $\text{cl}(V)qB$. Therefore $f(x_{(\alpha,\beta)}) \in \text{cl}_\theta(B)$. Hence we obtain $f(\text{cl}(\text{int}(\text{cl}(f^{-1}(B))))) \le \text{cl}_\theta(B)$, for each IF set B in Y.

Theorem 3.7. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ be a function. Then the following statements are equivalent:

(1) f is an IF weakly θ-continuous function.

(2) For each IF open set U with $x_{(\alpha,\beta)} q f^{-1}(U)$, $x_{(\alpha,\beta)} q$ $\text{int}(f^{-1}(\text{cl}(U)))$.

Proof. (1) $\Rightarrow$ (2). Let f be an IF weakly θ-continuous function, and let U be an IF open set with $x_{(\alpha,\beta)} q f^{-1}(U)$. Then $f(x_{(\alpha,\beta)})qU$. By the definition of IF weakly θ-continuous, there exists an IF open q-neighborhood V of $x_{(\alpha,\beta)}$ such that $f(V) \le \text{cl}(U)$. Thus $V \le f^{-1}(\text{cl}(U))$, i.e. $V \widetilde{q} (f^{-1}(\text{cl}(U)))^c$. Therefore, $x_{(\alpha,\beta)} \notin \text{cl}((f^{-1}(\text{cl}(U)))^c) = (\text{int}(f^{-1}(\text{cl}(U))))^c$. Hence we have $x_{(\alpha,\beta)} q (\text{int}(f^{-1}(\text{cl}(U))))$.

(2) $\Rightarrow$ (1). Let the condition hold, and let $x_{(\alpha,\beta)}$ be any IF point in X and V an IF open q-neighborhood of $f(x_{(\alpha,\beta)})$. Then $x_{(\alpha,\beta)} q f^{-1}(V)$. By the hypothesis,

$$x_{(\alpha,\beta)} q \text{int}(f^{-1}(\text{cl}(V))).$$

Put $U = \text{int}(f^{-1}(\text{cl}(V)))$. Then U is an IF open q-neighborhood of $x_{(\alpha,\beta)}$. Since $\text{int}(f^{-1}(\text{cl}(V))) \le f^{-1}(\text{cl}(V))$,

$$f(\text{int}(f^{-1}(\text{cl}(V)))) \le f(f^{-1}(\text{cl}(V))) \le \text{cl}(V).$$

Thus $f(U) \le \text{cl}(V)$. Therefore there exists an IF open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \le \text{cl}(V)$. Hence f is an IF weakly θ-continuous function.

4. Intuitionistic Fuzzy θ-Compactness

Definition 4.1. A collection $\{G_i \mid i \in I\}$ of intuitionistic fuzzy θ-open sets in an intuitionistic fuzzy topological space $(X, \mathcal{T})$

is said to be an *intuitionistic fuzzy θ-open cover* of a set A if $A \leq \bigvee\{G_i \mid i \in I\}$.

Definition 4.2. An intuitionistic fuzzy topological space $(X, \mathcal{T})$ is said to be *intuitionistic fuzzy θ-compact* if every intuitionistic fuzzy θ-open cover of X has a finite subcover.

Definition 4.3. A subset A of an intuitionistic fuzzy topological space $(X, \mathcal{T})$ is said to be *intuitionistic fuzzy θ-compact* if for every collection $\{G_i \mid i \in I\}$ of intuitionistic fuzzy θ-open sets of X such that $A \leq \bigvee\{G_i \mid i \in I\}$, there is a finite subset I_0 of I such that $A \leq \bigvee\{G_i \mid i \in I_0\}$.

Remark 4.4. Since every IF θ-open set is IF open, it follows that every IF compact space is IF θ-compact.

Theorem 4.5. An IF topological space $(X, \mathcal{T})$ is IF θ-compact if and only if every family of IF θ-closed subsets of X with the finite intersection property has a nonempty intersection.

Proof. Let X be IF θ-compact and let $\mathcal{F} = \{F_i \mid i \in I\}$ denote any family of IF θ-closed subsets of X with the finite intersection property. Suppose that $\bigwedge\{F_i \mid i \in I\} = \underline{0}$. Then, $\bigvee\{F_i^c \mid i \in I\} = \underline{1}$, i.e., $\{F_i^c \mid i \in I\}$ is an IF θ-open cover of X. Since X is IF θ-compact, there is a finite subset I_0 of I such that $\bigvee\{F_i^c \mid i \in I_0\} = \underline{1}$. This implies that $\bigwedge\{F_i \mid i \in I_0\} = \underline{0}$, which contradicts the assumption that $\mathcal{F}$ has a finite intersection property. Hence, $\bigwedge\{F_i \mid i \in I\} \neq \underline{0}$.

Let $\mathcal{G} = \{G_i \mid i \in I\}$ denote an IF θ-open cover of X and consider the family $\mathcal{G}' = \{G_i^c \mid i \in I\}$ of an IF θ-closed set. Since $\mathcal{G}$ is a cover of X, $\bigwedge\{G_i^c \mid i \in I_0\} = \underline{0}$. Hence, $\mathcal{G}'$ does not have the finite intersection property, i.e., there are finite numbers of IF θ-open sets $\{G_1, G_2, \cdots, G_n\}$ in $\mathcal{G}$ such that $\bigwedge\{G_i^c \mid i = 1, 2, \ldots n\} = \underline{0}$. This implies that $\{G_1, G_2, \cdots, G_n\}$ is a finite subcover of X in $\mathcal{G}$. Hence, X is IF θ-compact.

Theorem 4.6. Let $(X, \mathcal{T})$ denote an IF topological space and $\mathcal{T}_\theta$ denote the IF topology on X generated using the subbase of all IF θ-open sets in X. Then, $(X, \mathcal{T})$ is IF θ-compact if and only if $(X, \mathcal{T}_\theta)$ is IF compact.

Proof. Let $(X, \mathcal{T}_\theta)$ be IF compact and let $\mathcal{G} = \{G_i \mid i \in I\}$ denote an IF θ-open cover of X in $\mathcal{T}$. Since for each $i \in I, G_i \in \mathcal{T}_\theta, \mathcal{G}$ is an IF open cover of X in $\mathcal{T}_\theta$. Since $(X, \mathcal{T}_\theta)$ is IF compact, $\mathcal{G}$ has a finite subcover of X. Hence, $(X, \mathcal{T})$ is IF θ-compact.

Let $(X, \mathcal{T})$ be IF θ-compact and let $\mathcal{G} = \{G_i \mid G_i \in \mathcal{T}_\theta, i \in I\}$ denote an IF open cover of X in $\mathcal{T}_\theta$. Since for each $i \in I, G_i \in \mathcal{T}_\theta$, G_i is an IF θ-open set in $(X, \mathcal{T})$. Therefore, $\mathcal{G}$ is an IF θ-open cover of X in $\mathcal{T}$. Since $(X, \mathcal{T})$ is IF θ-compact, $\mathcal{G}$ has a finite subcover of X. Hence, $(X, \mathcal{T}_\theta)$ is IF compact.

Theorem 4.7. Let A be an IF θ-closed subset of an IF θ-compact space X. Then, A is also IF θ-compact.

Proof. Let A denote an IF θ-closed subset of X and let $\mathcal{G} = \{G_i \mid i \in I\}$ denote an IF θ-open cover of A. Since A^c is an IF θ-open subset of X, $\mathcal{G} = \{G_i \mid i \in I\} \cup A^c$ is an IF θ-open cover of X. Since X is IF θ-compact, there is a finite subset I_0 of I such that $\bigvee\{G_i \mid i \in I_0\} \cup A^c = \underline{1}$. Hence, A is IF θ-compact relative to X.

Theorem 4.8. An IF topological space $(X, \mathcal{T})$ is IF θ-compact if and only if every family of IF closed subsets of X in $\mathcal{T}_\theta$ with the finite intersection property has a nonempty intersection.

Proof. Trivial by Theorem 4.5.

Theorem 4.9. Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ denote IF topological spaces. Let $\mathcal{T}_\theta$ denote an IF topology on X generated by the subbase of all IF θ-open sets in X and let $\mathcal{U}_\theta$ denote an IF topology on Y generated by the subbase of all IF θ-open sets in Y. Then, a function $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is IF θ-irresolute if and only if $f : (X, \mathcal{T}_\theta) \to (Y, \mathcal{U}_\theta)$ is IF continuous.

Proof. Trivial.

Recall that a function $f : (X, \mathcal{T}) \to (Y, \mathcal{T}')$ is said to be *intuitionistic fuzzy strongly θ-continuous* if for each IF point $x_{(\alpha,\beta)}$ in X and for each IF open q-neighborhood V of $f(x_{(\alpha,\beta)})$, there exists an IF open q-neighborhood U of $x_{(\alpha,\beta)}$ such that $f(\mathrm{cl}(U)) \leq V$ ([9]).

Theorem 4.10. (1) An IF strongly θ-continuous image of an IF θ-compact set is IF compact.

(2) Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ denote IF topological spaces and let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be IF θ-irresolute. If a subset A of X is IF θ-compact, then image $f(A)$ is IF θ-compact.

Proof. (1) Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ denote an IF strongly θ-continuous mapping from an IF θ-compact space X onto an IF topological space Y. Let $\mathcal{G} = \{G_i \mid i \in I\}$ be an IF open cover of Y. Since f is an IF strongly θ-continuous function, $f : (X, \mathcal{T}_\theta) \to (Y, \mathcal{U})$ is an IF continuous function (Theorem 4.2 of [11]). Therefore, $\{f^{-1}(G_i) \mid i \in I\}$ is an IF θ-open cover of X. Since X is IF θ-compact, there is a finite subset

I_0 of I such that $\bigvee\{f^{-1}(G_i) \mid i \in I_0\} = \underline{1}$. Since f is onto, $\{G_i \mid i \in I_0\}$ is a finite subcover of Y. Hence, Y is IF compact.

(2) Let $\mathcal{G} = \{G_i \mid i \in I\}$ be an IF θ-open cover of $f(A)$ in Y. Since f is an IF θ-irresolute, for each G_i, $f^{-1}(G_i)$ is an IF θ-open set. Moreover, $\{f^{-1}(G_i) \mid i \in I\}$ is an IF θ-open cover of A. Since A is IF θ-compact relative to X, there exists a finite subset I_0 of I such that $A \leq \bigvee\{f^{-1}(G_i) \mid i \in I_0\}$. Therefore, $f(A) \leq \bigvee\{G_i \mid i \in I_0\}$. Hence, $f(A)$ is IF θ-compact relative to Y.

Theorem 4.11. Let A and B be subsets of an IF topological space $(X, \mathcal{T})$. If A is IF θ-compact and B is IF θ-closed in X, then $A \wedge B$ is IF θ-compact.

Proof. Let $\mathcal{G} = \{G_i \mid i \in I\}$ be an IF θ-open cover of $A \bigwedge B$ in X. Since B^c is IF θ-open in X, $(\bigvee\{G_i \mid i \in I\}) \vee B^c$ is an IF θ-open cover of A. Since A is IF θ-compact, there is a finite subset I_0 of I such that $A \leq (\bigvee\{G_i \mid i \in I_0\}) \vee B^c$. Therefore, $A \wedge B \leq (\bigvee\{G_i \mid i \in I_0\})$. Hence, $A \wedge B$ is IF θ-compact.

5. Conclusion

We introduced IF θ-irresolute and weakly θ-continuous functions, and intuitionistic fuzzy θ-compactness in intuitionistic fuzzy topological spaces. We showed that intuitionistic fuzzy θ-compactness is strictly weaker than intuitionistic fuzzy compactness. Moreover, we showed that if a topological space is intuitionistic fuzzy retopologized, then intuitionistic fuzzy compactness in the new intuitionistic fuzzy topology is equivalent to intuitionistic fuzzy θ-compactness in the original intuitionistic fuzzy topology.

Conflict of Interest

No potential conflict of interest relevant to this article was reported.

References

[1] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87-96, Aug. 1986. http://dx.doi.org/10.1016/S0165-0114(86)80034-3

[2] D. Çoker, "An introduction to intuitionistic fuzzy topological spaces," *Fuzzy Sets and Systems*, vol. 88, no. 1, pp. 81-89, May 1997. http://dx.doi.org/10.1016/S0165-0114(96)00076-0

[3] D. Çoker, "An introduction to fuzzy subspaces in intuitionistic fuzzy topological spaces," *The Journal of Fuzzy Mathematics*, vol. 4, no. 4, pp. 749-764, Dec. 1996.

[4] D. Çoker and M. Demirci, "On intuitionistic fuzzy points," *Notes on Intuitionistic Fuzzy Sets*, vol. 1, no. 2, pp. 79-84, 1995.

[5] W. K. Min and Y. K. Kim, "A note on fuzzy r-m precontinuity and fuzzy r-minimal compactness on fuzzy r-minimal spaces," *Journal of The Korean Institute of Intelligent Systems*, vol. 21, no. 1, pp. 128-131, 2011. http://dx.doi.org/10.5391/JKIIS.2011.21.1.128

[6] P. K. Lim, S. R. Kim, and K. Hur, "Intuitionistic smooth topological spaces," *Journal of The Korean Institute of Intelligent Systems*, vol. 20, no. 6, pp. 875-883, 2010. http://dx.doi.org/10.5391/JKIIS.2011.20.6.875

[7] J. I. Kim, W. K. Min, and Y. H. Yoo, "Fuzzy γ-compactness on fuzzy γ-minimal spaces," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 9, no. 4, pp. 281-284, Dec. 2009. http://dx.doi.org/10.5391/IJFIS.2009.9.4.281

[8] Kandil, O. Tantawy, M.Yakout, and S. Saleh, "C*-compactness in l-fuzzy topological spaces," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 9, no. 4, pp. 261-268, Dec. 2009. http://dx.doi.org/10.5391/IJFIS.2009.9.4.261

[9] I. M. Hanafy, A. M. Abd El-Aziz, and T. M. Salman, "Intuitionistic fuzzy θ-closure operator," *Boletn de la Asociacion Matematica Venezolana*, vol. 13, no. 1, pp. 27-39, 2006.

[10] I. M. Hanafy, "Intuitionistic fuzzy functions," *International Journal of Fuzzy Logic and Intelligent Systems*, vol. 3, no. 2, pp. 200-205, Dec. 2003. http://dx.doi.org/10.5391/IJFIS.2003.3.2.200

[11] S. J. Lee and Y. S. Eoum, "Intuitionistic fuzzy θ-closure and θ-interior," *Communications of the Korean Mathematical Society*, vol. 25, no. 2, pp. 273-282, 2010. http://dx.doi.org/10.4134/CKMS.2010.25.2.273

[12] I. M. Hanafy, "On fuzzy γ-open sets and fuzzy γ-continuity in intuitionistic fuzzy topological spaces," *The Journal of Fuzzy Mathematics*, vol. 10, no. 1, pp. 9-19, Mar. 2002.

[13] M. N. Mukherjee and S. P. Sinha, "On some near-fuzzy continuous functions between fuzzy topological spaces," *Fuzzy Sets and Systems*, vol. 34, no. 2, pp. 245-254, Jan. 1990. http://dx.doi.org/10.1016/0165-0114(90)90163-Z

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