# Symmetric Paths: Their Structures and Relations 

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#### Abstract

Seungho Nam. 2013. Symmetric Paths: Their Structures and Relations. Language and Information 17.1, 1-16. The goal of this paper is two-fold: (i) the paper aims to characterize unique semantics of so called "symmetric" locatives like across the street - this will provide a guiding semantics for annotating a variety of paths; and (ii) the paper claims that we need "symmetric" paths to give a unified account of the various semantic effects of symmetric locatives. The paper illustrates several semantic effects induced by symmetric locatives: (i) symmetric underspecification, (ii) path-/event-quantification, (iii) static symmetric relations, and (iv) the symmetric inference by the adverb back. The paper defines the semantic class of symmetric locatives, and accounts for the symmetry effects in terms of properties and relations of Path Structure proposed by Nam (1995). (Seoul National University)


Key words: paths, symmetric path, symmetric locatives, orientations, quantification, underspecification

## 1. Symmetric/Asymmetric Paths

In the literature, paths are defined as a sequence of regions, i.e., a function from natural numbers from 0 through $k$ into the set of regions. Then it is natural and intuitive to claim that directional PPs in English like the ones in (1) denote a path $\pi$ which represents a trajectory of (a) participant(s) of a motion event.
(1) a. John ran from the school to the library.
$\pi=$ <the_school, the_library>

[^0]b. John ran to the library. $\pi=<\pi_{0}$, the_library $>$
c. John ran into the library. $\pi=<\pi_{0}, \overline{\text { inside(the_library) }}>$
d. John ran through the forest. $\pi=<\pi_{0}, \overline{\circledR\left(\text { (the_forest), } \pi_{2}\right.}>$

Directional PPs in English, as well noted in the literature, are also used to denote a stative path which specifies a spatial property or a relation.
(2) a. The highway goes from here to San Francisco. $\pi=$ <here, San_Francisco>
b. John spied on Mary from the control tower. $\pi=<$ control_tower, $\pi_{1}>$
c. John saw Mary through the window. $\pi=<\pi_{0}$, (R)(the-window), $\pi_{2}>$
d. John shouted at Mary over the fence.
$\pi=<\pi_{0}$, ®(the_fence), $\pi_{2}>$
Thus, for instance, from the control tower in (b) denotes a static path and the path interprets the locations of the participants: i.e., 'John is at the control tower, but Mary was not'; and through the window in (c) denotes a path which gives the following inference, i.e., 'John and Mary were on the opposite sides of the window.' In other words, John was at the region $\pi_{0}$ and Mary at $\pi_{2}$ and the region occupied by the window was in between $\pi_{0}$ and $\pi_{2}$.'

The goal of this paper is two-fold: (i) We want to characterize unique semantics of so called "symmetric" locatives like across the street - this will provide a guiding semantics for annotating a variety of paths; and (ii) we support the claim that we need "symmetric" paths to give a unified account of the unique semantics of symmetric locatives. The proposed account is based on Nam's (1995) enriched version of the traditional topology of Tarski (1927), and defines spatial paths as a sequence of regions. (See also Rescher and Garson (1968) for another view to topological logic.) Thus the paths are inherently time-free and they are used to interpret non-motional events as well as motional ones - Mary was sitting/walking across the street.

Nam (1995) extends the ontological domain of space with path, orientation and region, and identifies the following semantic types of English spatial prepositional phrases.
(3) a. Topological Invariants - PPs with at, in, on
b. Orientational Locatives - PPs with in front of/behind, above/below, on top of/under, etc.
c. Directional Locatives - PPs with to, from, into, out of, towards, etc.
d. Symmetric Locatives - PPs with across, through, over, past, around, etc.

The first two types (a) and (b) usually denote a location/region where an event takes place, and further they often denote a goal location of a motion event (e.g., walk in the room, or put it on the table). The other types (c) and (d) denote a path anchored by a reference object, which might be a goal, a source or a route, e.g., to/from/across the lake. We note that symmetric locatives denote a "symmetric (underspecified) path," whereas directional locatives denote an asymmetric (directed) path. A symmetric path does not specify the source or goal regions unless they are supplied by another phrase (e.g., from here) or by context.

This paper aims to characterize the semantics of symmetric locatives, so we will illustrate some of their symmetry effects in section 2. But to give a formal theory of their semantics, in section 3 we introduce a general logic of spatial properties and relations.

Locative expressions denoting a symmetric path are also distinctive in terms of their linguistic variations in natural language. Unlike other types of locative expressions, symmetric locatives show up as a verb phrase or a verbal affix. The former strategy is used in Korean, Japanese, Malay, Chinese, and so forth; and the latter strategy is used in German (promotional prefixes), Dutch, Kinyarwanda, Chickasaw (applicative affixes), and so forth. Thus, many languages employ verbal elements to denote symmetric paths. (4) and (5) below show the verbal locatives in Chinese and Korean, respectively. These languages, however, use pre-/postpositional phrases for other directional locatives.
(4) yuehan zou-guo-le gongyuan.

John walk-pass-Asp park
'John walked through the park.'
(5) Koni-ka kele-se kil-ul kenne-ka-ass-ta.

Koni-Nom run-Conn road-Acc cross-go-Past-Decl
'Koni walked across the street.'
The verb guo 'to pass' in (4) takes gongyuan to form a symmetric locative, and kenne 'to cross' requires a reference object to derive a symmetric path 'across the street.'

Such verbal realization as in Chinese and Korean seems to suggest that the symmetric locatives may not be interpreted simply as a path but should be interpreted as a separate event like 'passing the park' or 'crossing the street.' Thus we may claim that the symmetric PPs in English can be interpreted as denoting a separate event rather than denoting a path. But this paper will support Nam's (1995) claim that English symmetric PPs should determine a "symmetric" path with its source and goal underspecified.

## 2. Semantic Effects of Symmetric Paths

### 2.1 Symmetric Paths and Symmetric Relations

As we saw in the previous section, symmetric locatives like across the forest denote a path, but they do not specify the goal and source regions of the path. This is why they are said to be "underspecified." Most directional locatives also denote
an underspecified path, so the PP to the library denotes a path which contains a specific goal region of 'the library' but lacks a specific source region.

Further, the paths denoted by directional locatives in general contribute to the interpretation of the locations of event participants or the event itself (see Nam (1995), Kracht (2002), and Fong (1997) for detailed discussion of 'argument orientation of locative PPs'). And we note that the symmetric locatives identify a symmetric (binary) relation between the regions occupied by the argument(s) in a motional/static events. The PP in (6a) does not determine a unary property but a binary relation 'being on the opposite sides of the window,' and this relation specifies the spatial relation between the subject and the object arguments. Thus (6a) entails (6b), and the same is true of the entailment in (7a) and (7b). The sentences in (8) also show that the symmetric PPs determine a symmetric binary relation.
(6) a. John saw Mary through the window.
b. $\models$ John and Mary were on the opposite sides of the window.
a. John touched Mary across the table.
b. $\models$ John and Mary were on the opposite sides of the table.
(8) a. John shouted at Mary over the fence.
b. John heard the baby cry around the corner.

Notice that the binary relations induced by symmetric locatives are symmetric and are not reducible in terms of unary relations (properties). That is, they are not paraphrasable as a boolean compound of unary properties, so inherently binary.

### 2.2 Symmetric Paths can be Quantified

Another special property of symmetric paths is that they can be interpreted as a unit for segmenting a motion event into countable sub-events. When a frequency adverbial occurs with a symmetric locative as in (9), they derive an ambiguity: One reading of (9a) is an event-counting reading, where John's jogging happens twice everyday; and the other reading is a path-counting reading, where the total length of John's jogging for a day amounts to two turns around the park. (9b) shows that separate frequency adverbials can occur to quantify over either events or paths. Thus three times refers to the number of paths and twice the number of events. If a frequency adverbial refers to a fraction as in (10), we do not get an event-counting reading.
(9) a. John jogs around the park twice everyday.
b. John jogs around the park three times twice a day
(10) a. John jogs around the park twice and half everyday.
b. John swam across the pool twice and half everyday.
(11) a. John drove down into the city twice everyday.
b. *John drove down into the city twice and half everyday.

We note here that path-counting reading is not available for non-symmetric locatives. Thus in (11) with a non-symmetric locative, we only get event-counting reading.

### 2.3 Symmetric Paths are Bi-directional

Consider the following entailment pattern, which is derived by a symmetric locative and the adverb back.
a. John walked across the street, and came back.
b. $\models$ John walked across the street twice.
a. John ran around the corner, and came back.
b. $\models$ John ran around the corner twice

This entailment pattern is due to the symmetric nature of the paths determined by the locative PPs, across the street and around the corner. In other words, symmetric locatives do not refer to the direction of movement. So for (12a) to be true, it does not matter which side of the street John started from, but he only had to cross the street walking. Other symmetric PPs with over, past, and through follow the same entailment pattern. It is not surprising, however, non-symmetric locatives do not follow the entailment pattern.
(14) a. John walked in/into the room, and came back.
b. \# John walked in/into the room twice.
a. John ran in front of City Hall, and came back.
b. \# John ran in front of City Hall twice

The PPs in (14) are directional locatives, and the PP in front of the City Hall in (15) is an orientational one. Unlike (12-13), (a)-sentences of (14-15) do not entail (b)-sentences.

### 2.4 Symmetric Paths and Locative Perspective

Some locatives require a locative perspective (or locative point of view) to get a proper interpretation. Here by locative perspective, we refer to a spatial setting which determines a relevant spatial orientation in question. For example, across the street in (16a) does not tell us which side of the street the subject argument 'John' is located. The sentence may be given a full truth-conditions only when another perspective point is supplied to determine on which side of the street the PP locates 'John.' The locative perspective may be supplied by the utterance context, so the utterance place 'here' often serves as the perspective point, but not always. (16b) shows that the locative perspective can be overtly expressed by a from-phrase, which by no means denotes a starting/source point of a movement. When the same PP modifies a motion verb like ran in (16c), however, from-phrase denotes a source point of the movement path. Thus, (16c) does not require a locative perspective.
a. John is sitting across the street
b. John is sitting across the street from here
c. John ran across the street (from here)
a. The village is through the forest from here.
b. The post office is around the corner from here.
c. The boys were playing over the wall.
d. The library is past the book store.

The sentences in (17) above give more examples with symmetric locatives whose interpretation requires a locative perspective. Non-symmetric locatives in (18) below do not allow a locative perspective. Thus, (18b) and (19b) are ungrammatical.
a. John is sitting in front of the car.
b. *John is sitting in front of the car from here.
a. John is sitting inside the room.
b. *John is sitting inside the room from here.

## 3. The Natural Logic of Space

### 3.1 Mereological Primitives: Regions and Relations

Nam's (1995) logic of space is based on the primitive notion of region which enables us to treat locative PPs as denoting properties and relations over regions. Then English locative PPs supply such relevant properties and relations for its compositional interpretation. Based on these ontological primitives, we define other notions like path and orientation.

We start with a mereological space $\Sigma$ structured by the primitive part-to-whole relation $\subseteq$. The elements in $\Sigma$ are called regions, so $\Sigma$ is the set of regions.
(20) The space $<\Sigma, \subseteq, \phi$, Between, Nearer $>$ :
a. $\Sigma$ : The set of regions
b. $\phi$ : The special element (the empty region)

For any region $A \in \Sigma, \phi \subseteq A$.
c. $\subseteq$ : Binary part-to-whole relation in $\Sigma$

- Reflexive, transitive, and antisymmetric: i.e., partial order
d. Between: Ternary betweenness relation in $\Sigma$
- Transitive: If Between(X,Y,Z) and Between(Z,Y,U), then Between(X,Y,U)
- Symmetric on 2nd-3rd arguments: Between(X,Y,Z), then Between(X,Z,Y)
e. Nearer: Ternary relative nearness relation in $\Sigma$
- Irreflexive: $\neg \operatorname{Nearer}(\mathrm{X}, \mathrm{Y}, \mathrm{X}) \& \neg \operatorname{Nearer}(\mathrm{Y}, \mathrm{X}, \mathrm{X})$
- Transitive: If Nearer(X,Y,Z) \& Nearer(Z,Y,U), then Nearer(X,Y,U)
- Asymmetric on 1st-3rd arguments: If Nearer(X,Y,Z), then
$\neg \operatorname{Nearer}(\mathrm{Z}, \mathrm{Y}, \mathrm{X})$
(20) introduces two primitive ternary relations among regions, which impose geometric structures on the space $\Sigma$. First, the betweenness relation (Between) is given as transitive and symmetric on its second and third arguments, and BETWEEN ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) is intended to mean " X lies between Y and Z ." The other ternary relation, Nearer( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) is intended to mean " X is nearer to Y than Z is," and it is given as irreflexive, transitive, and asymmetric on its first and third arguments.

Tarski's (1959) axiom system for Elementary Geometry introduces a ternary primitive relation, betweenness, and a quaternary primitive relation, equidistance ( $\mathrm{AB}=\mathrm{CD}$ : "A is as distant from B as C is from D. "). Our formalism with the relative nearness is richer than Tarski's (1959), since the relative nearness relation can easily define the equidistance relation. Robinson (1959) shows that it is impossible for one or more binary relations to serve as the primitive notions in Euclidean geometry ${ }^{1}$ Thus the ternary nature of spatial relations reveals that the space is more complex than the temporal domain which is usually characterized by the binary relations, precedence and overlap relations (e.g., Kamp (1979); van Benthem (1983)).

### 3.2 Path Structure

Meanings of the spatial expressions in English can be categorized into two groups: (i) movement-directional, and (ii) stative-locational. Movement-directional meaning emerges when the sentence describes change of location, and stative-locational meaning is identified when an event takes place in a stative context without changing location. Our logic accommodates this contrast in terms of path and orientation.

Path is one of the basic concepts discussed in the literature on spatial language and it is claimed to be a crucial notion in perception/cognition of movement or journey, and it is one of the main cognitively motivated devices for representing changes of location (see Miller and Johnson-Laird (1976), Cresswell (1978a), Cresswell (1978b), and Jackendoff (1990) among others). We define a path as a sequence of regions. The notion of path we are interested in here is not only a physical one but also an abstract one. The same physical path then can be represented by different abstract paths: If John flew from Seoul to Hawaii and then to San Francisco, John's path can be represented either as <Seoul, Hawaii, San_Francisco> or as $<$ Seoul, San_Francisco $>$. The abstract notion of path now lacks the continuity of physical movement (or geometric lines), and renders our semantics of space sufficiently elegant as to interpret spatial expressions.

We define Path Structure as a set of paths which are partially ordered by the subpath relation. Paths are defined as "time-free" sequences of regions. A sequence is a function with a domain of natural numbers from zero through some $k$, repre-

[^1]sented as $[0, k]$ in (21).
Path Structure: $<\Pi(\Sigma), \subseteq,+>$ :
a. $\Pi(\Sigma)$ : The set of paths in a local space $\Sigma$.
b. A path $\pi$ is a sequence of regions, i.e., $\pi \in[[0, k] \rightarrow \Sigma]$ for some $k \in \aleph$, where $[0, k]=\{n \in \aleph \mid 0 \leqslant n \leqslant k\}$, and satisfies the following: $\forall \pi \in \Pi(\Sigma)$, and $\forall i \in \operatorname{Domain}(\pi), \pi(i-1) \not \ddagger \pi(i) \notin \pi(i+1)$.
c. $\subseteq$ : Binary relation (subpath) between paths defined by:

Let $\pi$ and $\pi^{\prime}$ be paths, then $\pi^{\prime}$ is a subpath of $\pi, \pi^{\prime} \subseteq \pi$, if
(i) $\operatorname{Domain}\left(\pi^{\prime}\right) \subseteq \operatorname{Domain}(\pi)$ and Range $\left(\pi^{\prime}\right) \subseteq \operatorname{Range}(\pi)$, and
(ii) there is some $i \in \operatorname{Domain}(\pi)$ such that $\pi^{\prime}(0)=\pi(i)$ and for all $j \in \operatorname{Domain}\left(\pi^{\prime}\right), \pi^{\prime}(j)=\pi(i+j)$.
d. +: Concatenation function in $[\Pi(\Sigma) \times \Pi(\Sigma) \rightarrow \Pi(\Sigma)]$ defined as:

Let $\pi$ and $\pi^{\prime}$ be arbitrary paths with Domain $(\pi)=[0, n]$ and Domain $\left(\pi^{\prime}\right)$ $=[0, m]$.
The concatenation of $\pi$ and $\pi^{\prime}, \pi+\pi^{\prime}$ is defined by:
$\pi+\pi^{\prime}(k)=\left\{\begin{array}{lll}\pi(k) & \text { if } & 0 \leq k<n \\ \pi(k)=\pi^{\prime}(0) & \text { if } & k=n \\ \pi^{\prime}(k-n) & \text { if } & n<k \leq n+m\end{array}\right.$
(21b) imposes a general condition on the path structure which is linguistically motivated by the following sentences:
a. *John drove from San Francisco to California.
b. *John drove from California to San Francisco.

The locative PPs in (22a) and (22b) make the sentences meaningless since they fail to refer to a legitimate path. Thus we want to rule out paths such that some region in the path-sequence is included in the next region of the path, or vice versa. There is no path in $\Pi(\Sigma)$ like the following: <San_Francisco, California> or $<$ California, San_Francisco>.

From the definition of subpath relation, we can prove the relation is reflexive, transitive, and antisymmetric. Also notice that the concatenation function is not a total function, i.e., for some path $\pi$ and $\pi^{\prime},\left(\pi+\pi^{\prime}\right)$ is not defined. This is due to the condition given in the definition, i.e., the last region of $\pi$ and the first region of $\pi^{\prime}$ have to coincide, i.e., $\pi(k)=\pi^{\prime}(0)$. In other words, concatenation of $\pi$ and $\pi^{\prime}$ exists iff the goal of $\pi$ and the source of $\pi^{\prime}$ are identical (goal and source are defined below).
(23) Definitions:

Let $\pi$ be a path with $\operatorname{Domain}(\pi)=[0, k]$, then
the goal of $\pi, \pi_{g}$, is $\pi(k)$; and the source of $\pi, \pi_{s}$, is $\pi(0)$.
Now we define a very special relation between paths - converse relation. For all paths $\pi$, we have a path which reverses the ordering of $\pi$. We define:

## (24) Definition:

Let $\pi$ be a path with $\operatorname{Domain}(\pi)=[0, k]$, then
$\pi^{-1}$, the converse of $\pi$ (or " $\pi$-converse") is defined by:
$\operatorname{Domain}\left(\pi^{-1}\right)=[0, k]$, and for all $i \in \operatorname{Domain}\left(\pi^{-1}\right), \pi^{-1}(i)=\pi(k-i)$.
By the definition, $\pi^{-1}(0)=\pi(k), \pi^{-1}(k)=\pi(0)$, i.e., the source of $\pi^{-1}$ is the goal of $\pi$ and the goal of $\pi^{-1}$ is the source of $\pi$. We crucially rely on the notion of pathconverse for the semantics of symmetric locatives and some special adverbs/verbs like back and return.

We note two advantages of our semantics of path: (i) our notion of path is not temporal, so paths are introduced as purely spatial entities to interpret both static and motional events; (ii) it is flexible enough to accommodate cyclic paths that allow some regions can occur more than once in a single path.
3.2.1 TRAV Relation for Motion Events. The intuitive notion of path involves a movement of an object. Now to represent the notion of movement through a path, we introduce a predicate TRAV which is a ternary relation in $\mathbf{E} \times \Pi(\Sigma) \times \mathbf{T}$, where $\mathbf{E}$ is the universe of individuals, $\Pi(\Sigma)$ the set of paths in the local space $\Sigma$, and $\mathbf{T}$ the set of time intervals. Informally, $\operatorname{TRAV}(x, \pi, T)$ means ' $x$ traverses the path $\pi$ during the interval $T$.' Here $\pi$ is a sequence of regions with its domain in natural numbers, $\aleph$, and $T$ a linearly ordered set of time points. In order to define this predicate formally, we use the function $\circledR \in[(\mathbf{E} \times \mathbf{T}) \rightarrow \Sigma]$ which assigns a unique region to each individual object at an interval. Thus, for some object $x$, and a time interval $T, \circledR(x)(T)$ denotes the region which $x$ occupies during the interval $T$. Now formally,

Definition:
$\operatorname{TRAV}(x, \pi, T)$ is true iff there is an "order-preserving" map $\mu$ from Range $(\pi)$ to $T$ such that for all $i \in \operatorname{Domain}(\pi), \operatorname{INTR}(®(x)(\mu(\pi(i)), \pi(i))$.

The last clause of the above definition uses a binary relation INTR (to be read 'intersect') between two regions, so the clause means 'the region that $x$ occupies at the time point mapped from the $i$-th region of the path $\pi$ intersects with the $i$-th region of $\pi$.' The relation INTR will be defined in more detail in (34). We take an interval $T$ as a linearly ordered set of time points, but the domain of $\mu$ is not an ordered set but a sequence (functions from $K \subseteq \aleph$ into $\Sigma$ ). Thus we use the term "order-preserving" in a special sense defined as follows:
(26) Definition:

For a path $\pi$ and an interval $T$, a function $\mu$ from Range $(\pi)=\{\pi(i) \mid i \in$ $\operatorname{Domain}(\pi)\}$ to $T$ is order-preserving iff for all $i, j \in \operatorname{Domain}(\pi)$, if $i \leq j$, then $\mu(\pi(i)) \leq_{T} \mu(\pi(j))$ and $\mu\left(\pi_{s}\right)=0_{T}$ and $\mu\left(\pi_{g}\right)=1_{T}$.

An interval $T$ is ordered by the precedence relation $\left(\leq_{T}\right)$, and the least element of $T, 0_{T}$, refers to the initial point of the interval $T$, and the greatest element $1_{T}$ refers to the terminal point of $T$.

The TRAV relation will be used to interpret sentences referring to a path and a movement. For example, John ran into the house will be interpreted to be true
iff 'John ran' and 'John traversed the path $\pi$, i.e., TRAV(john, $\pi, T$ ), such that the source of the path is outside the house and the goal is inside the house'.
3.2.2 Part-to-whole Inference. Given the Path Structure and the relation TRAV, we can account for some general entailment patterns induced by spatial expressions. (27) and (28) illustrate an entailment pattern based on the part-to-whole relation between regions.
(27) a. John came from Seoul. entails
b. $\models$ John came from Korea. (given Seoul is in Korea)
a. John drove from Paris to Amsterdam. entails
b. $\models$ John drove from Paris to the Netherlands. and
c. $\models$ John drove from France to the Netherlands.

The PPs in (27) and (28) determine a path referring to a source or a goal region, and part-to-whole relation holds between Seoul and Korea, etc. Thus (27a) and (28a) entail (27b) and (28b-c), respectively.
3.2.3 Sub-path Inference. Let us now consider (29) and (30) below where the entailment patterns involve the "subpath" relation: (29a) entails (29b-c), and (30a) entails (30b-e). The intuition is, if an object traversed a path $\pi$, then it must have traversed the subpaths of $\pi$.
(29) a. John flew from Los Angeles to San Diego, and then to Las Vegas.
b. $\models$ John flew from Los Angeles to Las Vegas.
c. $\models$ John flew from San Diego to Las Vegas.
(30) a. John drove through the forest from here to the village. entails
b. $\models$ John drove from here to the village.
c. $\models$ John drove from here (through the forest).
d. $\models$ John drove (through the forest) to the village.
e. $\models$ John drove through the forest.

### 3.3 Orientation Structure

Natural language expressions refer to spatial orientations to locate some object in the space. For example, the PP in (31) refers to the front-orientation of the car to locate the subject argument 'John.'
(31) John is sitting in front of the car.

Thus the sentence is true only if John's region is on the front-orientation of the car. Roughly, the front-orientation of a car can be characterized as a half axis moving out from the car in the direction to its front part. That is, we take orientations as spatial objects like rays which have a designated region (= Origin) and a direction. Rays, in their geometric sense, are collections of atomic regions. Orientation Structure is defined as the set of orientations with the containment relation $\left(\subseteq_{R}\right)$ between them.

Orientation Structure: $<R(\Sigma), \subseteq_{R}>$ :
a. $R(\Sigma)$ : The set of orientations in a local space $\Sigma$
b. $\subseteq_{R}$ : Binary relation (containment) between orientations.
c. An orientation $\rho \in R(\Sigma)$ is a linearly ordered set $(\rho,<)$ of atomic regions such that
i. there is a unique least element $(\operatorname{Origin}(\rho))$,
ii. there is an atomic region $\mathrm{X} \in \rho$ such that
for all atomic regions $\mathrm{Y} \in \Sigma, \mathrm{Y} \in \rho$ iff either
$\operatorname{Between}(\mathrm{Y}$, $\operatorname{Origin}(\rho)$, X ) or $\operatorname{Between(X,~} \operatorname{Origin}(\rho)$, Y), and
iii. for all atomic regions $\mathrm{X}, \mathrm{Y} \in \rho, \mathrm{X}<\mathrm{Y}$ iff $\operatorname{Nearer}(\mathrm{X}, \operatorname{Origin}(\rho)$, Y)

The linear order for an orientation is intended to be the relative distance relation from the origin of the orientation, as defined in (32c.iii). A linear order is total, irreflexive, asymmetric, and transitive. The betweenness condition given in (32c.ii) guarantees that for any orientation $\rho$, there is a line such that all the atomic regions in $\rho$ belong to the line. Orientations will be used to interpret symmetric locatives as well as orientational locatives.

Object internal properties determine their intrinsic orientations, e.g., front/back, top/bottom, left/right, and in/out orientations ${ }^{2}$ We claim here that such objects get their intrinsic orientations due to their different parts. In other words, if an object has inherent front and back parts, it can be assigned front/back orientations determined by them. For example, a car has an inherent front part, no matter how it is determined (whether it is determined by the normal direction of movement or by a formal characteristics of its front part), so we can assign front or back orientation to it. In other words, we think of intrinsic orientation as a derived concept from parts of objects.

In front of the car, for example, involves the front orientation of 'the car', which is a linearly ordered set of minimal regions with its origin at the center of 'the car' and directed to its front side. The following illustrate some objects with their intrinsic orientations:

> man, car: Top/Bottom, Front/Back, Right/Left, In/Out
> telephone: Top/Bottom, Front/Back, In/Out
> vase, boulder: Top/Bottom, In/Out
> tree: Top/Bottom
> rocket: Front/Back
> box, ball: In/Out

Finally we define a binary relation INTR ("intersection") between regions and extend it to hold between a region and an orientation.

[^2]
## (34) Definitions

a. A region A is called an intersection of all elements of a set R of regions $(\cap R)$ if A is non-empty part of each element of R and if there is no region $B$ such that $B$ is a part of each element of $R$ and $A$ is a proper part of B.
b. Regions A and B intersect each other, i.e., $\operatorname{INTR}(\mathrm{A}, \mathrm{B})$, if there is an intersection of $\{A, B\}$.
c. For X a region and $\rho$ a set of regions, $\operatorname{INTR}(\mathrm{X}, \rho)$,

X intersects $\rho$, iff there is a region $\mathrm{A} \in \rho$ such that $\mathrm{A} \subseteq \mathrm{X}$.
(34b) defines INTR relation to hold between regions, and (34c) extends the relation to hold between a region and a set of regions. Thus a region may bear the INTR relation to either an orientation or a path.

## 4. Symmetric Locatives and Symmetric Paths

English uses prepositions to form symmetric locatives. They include PPs with across, through, over, around, and past, and their semantics crucially involve betweenness relation. In section I, we noted an issue on "symmetric" path: That is, whether we should refer to symmetric path - underspecified w.r.t. their goal and source - to interpret symmetric locatives. We might propose that the symmetric PPs in English should be interpreted as an independent event. For instance, (35a) below may be interpreted as (35b) - with a symmetric path, or (35c) - consisting of two events but without referring to a symmetric path.
a. John ran across the street.
b. John's running event went through the path $<\pi_{s}$, the_street, $\left.\pi_{g}\right\rangle$
c. John_running $\left(e_{1}\right)$ and John_crossing_street $\left(e_{2}\right)$

But the symmetry effects we illustrated in section 2 suggest that we need symmetric paths to give proper interpretation to the symmetric PPs in English. That is, if we interpret them as denoting a symmetric path, we can easily account for path-/event-quantification, static symmetric relations, and the symmetric inference by the adverb back.

This section first gives a little more detailed semantics of across and through, then characterizes semantically the class of symmetric locatives. Their lexical semantics will be stated in terms of path structures determined by the locatives. Here, without losing any structural property, we will take an underspecified path as equivalent to the set of fully specified paths compatible with the underspecified one. Now (36) and (37) define the meaning of across and through as a unary preposition.
across $\alpha$ :
For $m$ a one-place motion predicate, and for $\alpha$ a noun phrase denoting an individual, interpret the one-place predicate $m+$ across $+\alpha$, as follows:
$(\operatorname{across}(\boldsymbol{\alpha}))(\boldsymbol{m})(x)=1$ iff $\boldsymbol{m}(x)$ and $\operatorname{TRAV}(x, \pi, T)$, where
$\operatorname{BETWEEN}\left(\circledR(\boldsymbol{\alpha}), \pi_{s}, \pi_{g}\right)$, and for some $t \in T, \operatorname{ON}(\circledR(x, t), \circledR(\boldsymbol{\alpha}, t))$

## (37) through $\alpha$ :

For $m$ a one-place motion predicate, and for $\alpha$ a noun phrase denoting an individual, interpret the one-place predicate $m+$ through $+\alpha$, as follows:
$(\operatorname{through}(\boldsymbol{\alpha}))(\boldsymbol{m})(x)=1$ iff $\boldsymbol{m}(x)$ and $\operatorname{TRAV}(x, \pi, T)$, where
$\operatorname{Between}\left(\circledR(\boldsymbol{\alpha}), \pi_{s}, \pi_{g}\right)$, and for some $t \in T, \operatorname{IN}(\circledR(x, t), \circledR(\boldsymbol{\alpha}, t))$
The only difference between across and through defined above is their last condition on the intermediate location of the moving object: $\mathrm{ON}(\circledR(x, t), \circledR(\boldsymbol{\alpha}, t))$ for across $\alpha$ and $\operatorname{IN}(®(x, t), \circledR(\boldsymbol{\alpha}, t))$ for through $\alpha{ }^{3}$ Thus, across requires the object $x$ to be "on" the reference object, whereas through requires $x$ to be "in" the reference object. The following examples illustrate how those PPs are interpreted in a sentence, and we see that (38) entails 'John was on the street,' and (39) 'John was in the tunnel.'
(38) John ran across the street is true iff
for some past time interval $T, \operatorname{run}(\mathbf{j o h n})(T)$ and $\operatorname{TRAV}(\mathbf{j o h n}, \pi, T)$ for a path $\pi$, where Between $\left(\circledR\right.$ (the street), $\left.\pi_{s}, \pi_{g}\right)$ and for some $t \in T$, $\mathrm{ON}(\circledR(\mathbf{j o h n}, t), \circledR($ the street, $t))$.
(39) John drove through the tunnel is true iff for some past time interval $T$, drive (john)( $T$ ) and TRAV(john, $\pi, T$ ) for a path $\pi$, such that Between $\left(\circledR(\right.$ the tunnel $\left.), \pi_{s}, \pi_{g}\right)$, and for some $t \in T$, $\mathrm{IN}(®($ john,$t), \circledR($ the tunnel,$t))$.

## 5. Symmetry Effects Explained Away

Now we account for the symmetry effects illustrated in 2.1-2.4 by identifying some unique semantics of symmetric locatives. We characterize symmetric locatives as ones determining a set of paths which is closed under "path-converse" relation. That is, if a path $\pi$ is determined by a symmetric locative $f$, then $\pi^{-1}$ is also determined by $f$.
(40) Definition: Symmetric Locatives

For $f$ a locative modifier, $f$ is a symmetric locative iff $f$ determines a set $\Pi_{f}$ of paths such that $\forall \pi \in \Pi_{f}, \pi^{-1} \in \Pi_{f}$, i.e., $\Pi_{f}$ is closed with respect to "path-converse" relation.

For example, as we see in (38), the PP across the street determines a path such that Between( $\circledR$ (the street), $\pi_{s}, \pi_{g}$ ), and since Between is symmetric on second and third arguments, BETWEEN( $®($ the street $\left.), \pi_{g}, \pi_{s}\right)$. Then by definition, $\pi_{s}=$ $\pi^{-1}{ }_{g}$, and $\pi_{g}=\pi^{-1}{ }_{s}$, and so Between(®(the street), $\pi^{-1}{ }_{s}, \pi^{-1}{ }_{g}$ ). Therefore, the same PP also determines $\pi^{-1}$, thus symmetric. This definition now identifies the class of locatives which induces the symmetric effects illustrated in 2.1-2.4.

First of all, consider (6a) below where the stative seeing event is modified by a symmetric locative through the window. Our semantics of symmetric locatives

[^3]should derive the entailment (6b) from (6a). By referring to a symmetric path like ( 6 c ), we can easily represent the stative symmetric relations between the two arguments, John (experiencer) and Mary (theme), i.e., (6d).
(6) a. John saw Mary through the window.
b. $\models$ John and Mary were on the opposite sides of the window.
c. $\pi=<\pi_{s}$, , $\left(\right.$ the_ window), $\pi_{g}>$
d. $\pi_{s}=\circledR($ john $)$, and $\pi_{g}=\circledR($ mary $)$.

Such interpretation as above is available for static events, since the proposed Path Structure defines a path as a time-free sequence of regions and the betweenness relation represented in a path derives a symmetric spatial relation between the event participants.

Now as illustrated in 2.2, (9a) repeated below shows a path-counting/eventcounting ambiguity and (10) shows clearly that a path-counting reading is available.
(9) a. John jogs around the park twice everyday.
b. John jogs around the park three times twice a day.
(10) a. John jogs around the park twice and half everyday.
b. John swam across the pool twice and half everyday.

The frequency adverbs may induce a path quantification only when the symmetric locatives like around the park and across the pool are interpreted as denoting a symmetric path rather than a separate event like 'turning around the park' or 'crossing the pool.' Thus the path quantification reading of (9a) is the following: 'Everyday John jogs and John traverses a path $\pi$ where $\pi$ contains at least two sub-paths determined by around the park.

Finally, we noted in 2.3 that the adverb back in the following gives a restitutive reversal reading, which is not just a repetitive reading.
(41) a. John ran across the street, and came back.
b. John ran across the street is true iff
for some past time interval $T, \operatorname{run}(\mathbf{j o h n})(T)$ and $\operatorname{TRAV}(\mathbf{j o h n}, \pi, T)$
for a path $\pi$, where $\operatorname{BEtWEEN}\left(\circledR(\right.$ the street $\left.), \pi_{s}, \pi_{g}\right)$ and for some $t \in T, \mathrm{ON}(®($ john,$t), \circledR($ the street, $t))$.
c. $\operatorname{TRAV}\left(\mathbf{j o h n}, \pi^{-1}, T^{\prime}\right)$, for some $T^{\prime}$ such that $T<T^{\prime}$.

The reversal reading of back cannot be derived if we interpret the symmetric locative across the street as denoting an extra event like 'crossing the street.' According to the proposed semantics of across the street in (41b), however, we can get the reversal reading of back in (41a) by just adding the reverse trajectory (41c) along the converse path $\pi^{-1}$ of the one denoted by across the street.

## 6. Implications and Further Questions

The natural logic of symmetric paths proposed here is ready to give a semantics for the annotation framework of "ISO-Space" of Pustejovsky et al. (2011). The ISO-Space would annotate a sentence like (42) below with a "relative QSLINK" which refers to a symmetric relation "ACROSS" between a motion event "running" and a related location "the_street".


Then, the sentence can be tagged with an "event-path" consisting of <event[motion], path>. The event-path will specify its moving object "john" of the source event "running" and its underspecified path, whose midpoint is the related location "the_street." But its beginning and ending locations (Source and Goal) are undetermined, so the semantics will interpret them as being at the opposite sides of the related/reference location "the_street".

Here, we leave the following questions unanswered, but we suggest that the answers, whatever they are, should be supported by the semantics proposed in this paper.
(i) How can we annotate the non-motional (stative) reading of across the street in (42) above?
(ii) Do we need "event-orientations" instead of event-path to identify the stative readings of symmetric locatives?
(iii) Does it allow more expressive power if we separate "event" from "path" rather than putting them together?

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[^1]:    ${ }^{1}$ As mentioned by Nam (1995: 70), Robinson (1959) notes Pieri's (1908) finding that, in Euclidean geometry, it is possible to define the quaternary equidistance relation $\mathrm{AB}=\mathrm{CD}$ (" A is as distant from B as C is from D ") in terms of the ternary relation $\mathrm{AB}=\mathrm{BC}$ (" A is as distant from B as C is"), that of a point being equally distant from two other points. Now the following definition shows that the relative nearness relation can define Tarski's equidistance relation. The equidistance relation $\operatorname{E-DIST}(\mathrm{X}, \mathrm{Y}, \mathrm{Z})$ which means ' Y is equally distant from X and Z is defined as follows:

[^2]:    ${ }^{2}$ Levinson (2003: 41-50) distinguishes three types of linguistic frames of spatial reference: I.e., (i) intrinsic frame of reference, (ii) relative frame of reference, and (iii) absolute frame of reference. Our term intrinsic orientation, which was formally introduced in Nam (1995), refers to spatial entities which are defined as a ray with an origin and a direction determined by the reference object. Thus, Levinson's classificatory use of the term requires the same spatial structures as ours does.

[^3]:    ${ }^{3}$ Nam (1995: 100-102) defines the binary relations IN and ON as proper-part relation ( $\subset$ ) and tangential relation between regions, respectively.

