Industrial Engineering
& Management Systems
Vol 12, No 3, September 2013, pp.224-233
ISSN 1598-7248 | EISSN 2234-6473 |
http://dx.doi.org/10.7232/iems.2013.12.3.224

Development of a Composite Revenue Sharing-Quantity Flexibility Contract

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(Received: January 31, 2013 / Revised: July 4, 2013 / Accepted: August 25, 2013)

ABSTRACT

In supply chain management, the supply contract can induce collaboration and coordination among the supply chain members in order to optimize supply chain performance. Numerous supply contracts have been examined; however, some difficulties related to the application of these contracts still occur. One of the solutions is to apply the composite supply contract which can assist in the supply chain coordination. This research examines the composite contract of the revenue sharing and quantity flexibility contracts in a two-stage supply chain, which comprises a retailer and a supplier. In this research, a mathematical model of the composite contract is developed; then, the applicability of the proposed composite contract is examined by investigating its capability in terms of supply chain coordination and profit allocation. In the numerical experiments, the composite revenue sharing-quantity flexibility contract showed that it is superior to both component contracts in terms of supply chain coordination and profit allocation among supply chain members.

Keywords: Supply Contract, Composite Contract, Revenue Sharing Contract, Quantity Flexibility Contract, Supply Chain Coordination

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1. INTRODUCTION

Supply chain management (SCM) is employed to express the need to integrate the key business processes, from ending customers to original suppliers in the supply network nowadays. Moreover, the management of all activities, which are related to sourcing and procurement, alternation, and logistics management activities, is also part of SCM. In SCM, coordination and collaboration among supply chain members are of critical importance for achievement of optimal supply chain performance in order to fulfill customers' satisfaction in the extremely high competition market. Optimizing supply chain performance requires the collaboration and management of actions in the supply chain. However, the supply chain members are sometimes lacking in intention to implement those actions since they are basically worried about optimizing their own benefits. For this reason, the performance of the whole supply chain is frequently insufficient. Nevertheless, optimal supply chain performance is possible to be accomplished if the supply contract can coordinate the supply chain with a set of transfer payments.

To optimize the supply chain performance, a variety of contracts have been proposed for the supply chain coordination with the main idea of aligning the supply chain members' objectives. Although many contracts have been examined, some difficulties related to the application of these contracts still occur. Practically, the contract parameters are limited to a certain range in reality. In addition, contracts are sometimes not flexible in terms of supply chain coordination or might not be practical for supply chain coordination in reality. For these reasons, many contracts are in fact no longer coordinating the supply chain or allocating the profit fairly among the supply chain members. Many researchers have tried to adjust the existing supply contracts in order to solve this problems. Another way of solution is to combine the contracts. The composite contracts can solve the problem that an original contract cannot deal with. For instance, Xiong *et al*. (2011) introduced a composite buy back and quantity flexibility contract, and this composite contract is shown effective in coordinating the supply chain and allocates the profit of the supply chain among the supply chain members. Nevertheless, using both the buyback contract and the quantity flexibility contract seems to be an inappropriate combination because both are unilaterally offered from supplier to retailer. Hence, the idea of fair trade between supplier and retailer is worth considering through the combined contract of a revenue sharing contract and a quantity flexibility contract which will be dealt with in this current research. The interesting research question is whether the composite revenue sharing and quantity flexibility contract can coordinate the supply chain and allocate the profit of the supply chain among the supply chain members or not.

2. LITERATURE REVIEW

This research bases on two supply contracts, which are revenue sharing contract and quantity flexibility contract. In order to achieve the target of this research, the literatures related to the traditional wholesale price contract, various supply chain contracts as well as supply chain coordination are reviewed in this section.

Li and Luo (2008) investigated the wholesale price contract for the newsvendor model in a two-echelon supply chain. This research confirmed that supply chain coordination cannot be conveyed by the wholesale price contract; however, it can help increase profits of the whole supply chain and each firm when combined with price discount contract. The buyback contract can solve this problem and help achieve the whole supply chain optimal profit. Zhang *et al*. (2005) considered a two-echelon sequential supply chain with a single product, which examined three types of contract, i.e., buyback contract, target rebate contract, and incremental buyback contract. They found that the buyback contract and the target rebate contract are possible for achieving supply chain coordination. Moreover, the buyback contract can also be applied for allocating the supply chain profit without restraint. In addition, the result of the incremental buyback contract resembled the result of buyback contract. Xu (2010) examined the management difficulty in a decentralized supply chain, which comprises a supplier and a manufacturer, focusing on production and procurement functions. The assumptions of this study are unsteady supplier's production yield, the immediate wholesale price and the accurate emergency production yield as well as the stochastic manufacturing demand. In this research, the supplier uses option contract for hedging against the low demand's risk and cheap instant prices. The optimal option order quantity of manufacturer and the optimal production quantity of the supplier are then derived. Gomez-Padila and Mishina (2009) analyzed the impact of an option contract in a two echelon sequential supply chain. In this research, the retailer has privilege to adjust his order after ordering a quantity with an option contract. In the two considered cases, the contract value is calculated as the performance of an option contract for supply chain coordination. However, renegotiation possibility and fluctuation rates were ignored in this study. In the first case that compares an option contract with a wholesale price contract and a buyback contract, the above research showed that the benefit of the whole supply chain increases with an option contract. In other words, with option contract, the benefits of the retailer and the supplier are highest among the three types of contract. In addition, the wholesale price contract provides the lowest benefit for a retailer, and the buyback contract grants the lowest benefit to a supplier. The second case studied a system with one retailer and multiple suppliers, which are cooperated through a wholesale price contract, a buyback contract, or an option contract. As a result, the benefit of the whole supply chain and the retailer may increase when an option contract is offered by at least one supplier. Wang *et al*. (2012) also investigated the supply contract's risk with call option in a newsvendor model by deriving two important parameters. The first parameter is a risk indicator for risk existence in using supply contract with call option, and the other is a ratio for measuring the probability of risk occurrence.

Xiong *et al*. (2011) proposed a composite contract which is the composition of buyback contract and quantity flexibility contract in a two-stage supply chain. It has been found that this composite contract has three main benefits. First, the contract can coordinate supply chain in some circumstances that the component contracts cannot. Second, the contract is more flexible in profit allocation among supply chain members when compared with a buyback contract or a quantity flexibility contract. The last but not least, the proposed composite contract is more flexible in risk allocation than a buyback contract ora quantity flexibility contract. Tsay (1999) examined the quantity flexibility contract for dealing with over order practice from retailer. In this research, the contract forces the retailer to order more than or equal to a percentage below the forecast, and the supplier has to deliver a quantity which reaches the percentage above. With the quantity flexibility contract, the costs of demand uncertainty are allocated, and the retailer and the supplier are motivated to collaborate for increasing the supply chain performance in order to reach the optimal solution for the supply chain.

Cachon and Lariviere (2001) examined the revenue sharing contract performance for coordination of a supply chain. In this study, the revenue sharing contract was demonstrated in two cases which involve one retailer–one supplier and multiple retailers–one supplier. The comparison between the revenue sharing contract and other contracts was also conducted. Even though it should be noted that a supply chain can be coordinated by the revenue sharing contract when comparing with other contracts, it is not ubiquitously applied. Hou *et al.* (2008) presented their study on dealing with coordination by use of revenue sharing contract and bargaining in a two-stage supply chain. In this research, both supplier and retailer are separate and independent organizations. It is found that the supplier' lead time, which depends on the supplier's inventory level, has a serious effect on the retailer's profit. Since the retailer usually opposes to information sharing in the centralized system and the revenue sharing contract in the decentralized system cannot help to increase the profits of both retailer and supplier, the revenue sharing contract with bargaining as a cooperated vehicle is worth to be considered. Comparing with an originally decentralized optimization, the supply chain members can reach highest profit by bargaining with this new range for achieving a mutually agreement.

How to evaluate the supply contract evaluation is also a significant issue. Cachon (2003) suggested three significant questions for evaluating contracts:1) Is the supply chain coordinated with the contract? The supply chain is considered to be coordinated when the sum of supplier's profit and retailer's profit can reach the optimal supply chain profit and no party has a profitable unilateral deviation from the set of supply chain optimal actions. 2) Is the flexibility of the contract adequate to allow the supply chain's profit sharing among supply chain members? Some contracts can coordinate supply chain only when one of the parties earns profit, and the other faces non-profit situation. This situation, if happened, will result in a situation where the contract cannot be used in reality. 3) Is the contract worth applying? An efficient contract should not only coordinate a supply chain, but also provide flexibility in profit allocation among supply chain members. All these three aspects will be taken into consideration in the research presented in this paper for the proposed hybrid contract.

3. MATHEMATICAL MODEL

In this research, supply chain coordination is examined in a classical two-stage supply chain which comprises one downstream retailer and one upstream supplier. During the selling season, the retailer is a newsvendor who faces a stochastic demand D with probability density function $\boldsymbol{F}(\boldsymbol{x})$, and cumulative density function $\boldsymbol{F}(\boldsymbol{x})$ Assume that $\boldsymbol{F}(\boldsymbol{x})$ is a differentiable and strictly increasing function on its support. Before the selling season, the retailer orders a quantity q from the supplier with a constant wholesale price w per unit. After that the retailer sells product to customer with retail price p per unit during the selling season. Under make-to-order policy, the products are produced by supplier with the cost of production c per unit. The salvage value is v per unsold unit of product. Furthermore, it is assumed that $p > c > 0$ to avoid trivial cases. Let us denote that the optimal order quantity for the coordinated supply chain as q^o and $\Pi\,(q^o)$ as the expected optimal supply chain's profit.

The proposed composite revenue sharing-quantity flexibility contract is described as follows. In the revenue sharing contract, the supplier earns a percentage of the retailer's revenue which is specified by contract parameters $\{w_r, \phi\}$, in which ϕ is the revenue's fraction that the retailer keeps, $1-\phi$ is the fraction of supplier, and w_r is the wholesale price with revenue sharing contract. Moreover, there is an additional assumption that all revenues are shared. At the same time, a quantity flexibility contract specified by contract parameters $\{w_r, \delta\}$, is also considered in which the retailer can return at most δq units of remaining inventory ($L(q)$) to the supplier with full refund, and w_a is the wholesale price with quantity flexibility. The proposed composite contract is, hence, specified by three contract parameters $\{w_r, \phi, \delta\}$, when w_r is the wholesale price with composite contract. This composite contract presents a fair trade by allowing the retailer to return δq units of leftover products to the supplier with full refund, and in return the retailer must share a percentage of his revenue to the supplier. Under the composite contract, the profit of retailer and supplier are denoted as $\pi_r(q, w_c, \phi, \delta)$ and $\pi_s(q, w_c, \phi, \delta)$, respectively. It is noted that the supply chain is considered to be coordinated when $\Pi(q, w_c, \phi, \delta) = \pi_r(q, w_c, \phi, \delta) + \pi_s(q, w_c, \phi, \delta)$. In general, this study focuses on coordination of supply chain by combination of revenue sharing and quantity flexibility contracts. In the next section, the mathematical models of all component contracts and the proposed hybrid contract will be derived.

3.1 The Newsvendor Model

The newsvendor model can be used as the basic model for development of other supply contracts. The newsvendor model comprises of two firms, a supplier and a retailer. At the beginning, the retailer orders a fixed quantity q to the supplier. Then, the supplier produces and delivers to the retailer. The following notations are used in the newsvendor model.

D = The demand during the selling season
c_s = The supplier's production cost per unit
c_r = The retailer's marginal cost per unit
c = $c_s + c_r$
p = The retail price (Assume that $p > c$)
q = The order quantity

g_r = The unit shortage cost of the retailer
g = The unit shortage cost of the supplier
g = $g_s + g_r$
ν = Salvage value
T = Transfer payment from the retailer to the supplier
$\pi_r(q)$ = The retailer's profit function
$\pi_s(q)$ = The supplier's profit function
$\prod(q)$ = The supply chain's profit function

Let $S(q)$ be the expected sales. We have

$$\begin{aligned} S(q) &= E\left[\min(q,\ D)\right] \\ &= \int_0^q xf(x)dx + \int_q^\infty qf(x)dx \end{aligned} \tag{1}$$

$$= \mathbf{q} - \int_0^q F(x)dx \tag{2}$$

Let $I(q)$ to be expected leftover inventory, we have

$$\begin{aligned} I(q) &= E[(q-D)^+] \\ &= q - S(q) \end{aligned} \tag{3}$$

Let $L(q)$ to be the lost of sales function, we have

$$\begin{aligned} L(q) &= E\left[(D-q)^+\right] \\ &= \mu - S(q) \end{aligned} \tag{4}$$

The retailer's profit function can then be derived as shown below.

$$\begin{aligned} \pi_r(q) &= pS(q) + vI(q) - g_r L(q) - c_r q - T \\ &= (p - v + g_r)S(q) - (c_r - v)q - g_r\mu - T. \end{aligned} \tag{5}$$

Also, the supplier's profit function can be determined as shown below.

$$\begin{aligned} \pi_s(q) &= T - g_s L(q) - c_s q \\ &= T - c_s q - g_s\mu + g_s S(q) \end{aligned} \tag{6}$$

The profit function of the whole supply chain is, hence, equal to the sum of the retailer profit function and the supplier profit function.

$$\begin{aligned} \Pi(q) &= \pi_r(q) + \pi_s(q) \\ &= (p - v + g)S(q) - (c - v)q - g\mu \end{aligned} \tag{7}$$

Let q^o be the supply chain's optimal order quantity. This optimal order quantity exists uniquely when the supply chain profit function is concave. In fact, the concavity of the supply chain profit function can be confirmed because of

$$\frac{d^2\Pi(q^o)}{dx^2} = -(p - v + g)f(q^o) \le 0.$$

Hence, the supply chain's optimal order quantity can be derived as:

$$q^o = \mathbf{F}^{-1}\left[\frac{p - \mathbf{c} + g}{p - v + g}\right]. \tag{8}$$

3.2 The Wholesale-Price Contract

In the wholesale-price contract, the supplier charges the retailer w per unit, in which $p > w > c$. The transfer payment in the wholesale-price contract is

$$T(q,\ w) = wq$$

The retailer's profit function can be derived as

$$\begin{aligned} \pi_r(q, w) &= pS(q) + vI(q) - g_r L(q) - c_r q - T(q, w) \\ &= (p - v + g_r)S(q) - (w - v + c_r)q - g_r\mu). \end{aligned} \tag{9}$$

The supplier's profit function can be derived as

$$\begin{aligned} \pi_s(q,\ w) &= T(q,\ w) - g_s L(q) - c_s q \\ &= g_s S(q) + (w - c_s)q - g_s\mu. \end{aligned} \tag{10}$$

The supply chain profit function can then be determined as

$$\begin{aligned} \Pi(q,\ w) &= \pi r(q,\ w) + \pi s(q,\ w) \\ &= (p - v + g)S(q) + (v - c)q - g\mu g\mu. \end{aligned} \tag{11}$$

We have

$$\frac{d^2\pi_r(q,\ w)}{dq^2} = -(p - v + g_r)f(q) \le 0.$$

Therefore, the optimal order quantity of the retailer in this contract can be determined as

$$q_r^* = F^{-1}\left[\frac{p + g_r - w - c_r}{p + g_r - v}\right]. \tag{12}$$

From (12), it should be noted that in order to reach supply chain's optimal order quantity expressed in (8) for coordinating, the supplier's profit will be negative, so the wholesale-price contract is not possible to coordinate a supply chain.

3.3 The Revenue-Sharing Contract

With this contract, the retailer is charged w_r per unit, in which $w_r < w$, and the retailer shares a percentage of his revenue to the supplier. Denote ϕ to be the retailer's portion, and $(1-\phi)$ is the portion of retailer's revenue the supplier gains, we should have $\phi \in (0, 1)$ in order to avoid trivial case.

The transfer payment with revenue-sharing contract is:

$$T_r(q,\ w_r,\ \phi)=w_r q+(1-\phi)pS(q).$$

The retailer's profit function can be derived as:

$$\begin{aligned}&\pi_r(q,\ w_r,\ \phi)\\&=pS(q)+vI(q)-g_rL(q)-c_rq-T_r(q,\ w_r,\ \phi)\\&=(\phi p-v+g_r)S(q)-(w_r+c_r-v)q-g_r\mu.\end{aligned}\tag{13}$$

The supplier's profit function can be derived as:

$$\begin{aligned}&\pi_s(q,\ w_r,\ \phi)\\&=T_r(q,\ w_r,\ \phi)-g_sL(q)-c_sq\\&=((1-\phi)p+g_s)S(q)-(c_s-w_r)q-g_s\mu.\end{aligned}\tag{14}$$

It is noted that $\pi_r(q,\ w_r,\ \phi)$ is concave in q due to

$$\frac{d^2\pi_r(q,\ w_r,\ \phi)}{dq^2}=-(\phi p-v+g_r)f(q)\le 0$$

Therefore, q_r^*, the optimal order quantity of the retailer, can be determined from

$$\frac{d\pi_r(q,\ w_r,\ \phi)}{dq}=0$$
$$\text{or}\quad q_r^*=F^{-1}\left[\frac{\phi p-w_r-c_r+g_r}{\phi p-v+g_r}\right].\tag{15}$$

The revenue-sharing contract is considered to coordinate the supply chain when $\dfrac{\phi p-w_r-c_r+g_r}{\phi p-v+g_r}=\dfrac{p-c+g}{p-v+g}$.

3.4 The Quantity Flexibility Contract

In this contract, the supplier charges w_q per unit, which normally equal to the wholesale price of the wholesale price contract, purchased by retailer and the supplier offers a credit equal to $(w_q+c_r-v)\ \min(I(q),\ \delta q)$ to the retailer for his unsold units. $\delta\in[0,1]$ is a contract parameter and is a constant.

Expected return is $E[\min(I(q),\delta q)]$, which can be determined as follows:

$$\begin{aligned}&E[\min]\ (I(q),\ \delta q)]\\&=\int_0^{(1-\delta)q}\delta q f(x)dx+\int_{1-\delta q}^{q}(q-x)f(x)dx\\&=\int_{(1-\delta)q}^{q}F(x)dx.\end{aligned}\tag{16}$$

The transfer payment with quantity-flexibility contract can be derived as

$$\begin{aligned}&T_q(q,\ w_q,\ \delta)\\&=w_qq-(w_q+c_r-v)E\left[\min(I(q),\ \delta q)\right]\\&w_qq-(w_q+c_r-v)\int_{(1-\delta)q}^{q}F(x)dx\end{aligned}\tag{17}$$

The retailer's profit function can be derived as

$$\begin{aligned}&\pi_r(q,\ w_q,\ \delta)\\&=pS(q)+vI(q)-g_rL(q)-c_r-T_q(q,\ w_q,\ \delta)\\&=(p-v+g_r)S(q)-(w_q-v+c_r)q\\&+(w_q-v+c_r)\int_{(1-\delta)q}^{q}F(x)dx-\mu g_r.\end{aligned}\tag{18}$$

The supplier's profit function can be derived as

$$\begin{aligned}&\pi_s(q,\ w_q,\ \delta)\\&=T_q(q,\ w_q,\ \delta)-g_sL(q)-c_sq\\&=g_sS(q)-(c_s-w_q)q-(w_q+c_r-v)\\&\int_{(1-\delta)q}^{q}F(x)dx-\mu g_s.\end{aligned}\tag{19}$$

It is noted that $\pi_r(q, w_r, \phi)$ concave in q, due to

$$\begin{aligned}\frac{d^2\pi_r(q,\ w_q,\ \delta)}{dq^2}&=-(p-w_q-c_r+g_r)f(q)\\&-(w_q-v+c_r)(1-\delta)^2f((1-\delta)q)<0\end{aligned}$$

So the optimal order quantity of the retailer, q_r^*, can be determined from

$$\begin{aligned}&\frac{d\pi_r(q_r^*,\ w_q,\ \delta)}{dq}=0\\&(p-w_q-c_r+g_r)(1-F(q_r^*))-(w_q-v+c_r)(1-\delta)\\&F((1-\delta)q_r^*)=0\end{aligned}\tag{20}$$

There does not exist the explicit expression for q_r^* from Eq. (20). However, the above equation has a unique solution, and q_r^* can be determined by use of bisection method.

3.5 The Proposed Composite Revenue Sharing-Quantity Flexibility Contract

In this composite contract, the supplier sells a quantity at w_c per unit, in which $w_c\le w$ (in wholesale price contract), and the supplier offer a credit equal to (w_c+c_r-v) $\min(I(q),\ \delta q)$ to the retailer for his unsold units. On the other hand, the retailer shares a percentage of his revenue to the supplier, in which ϕ is the retailer's portion, and $(1-\phi)$ is the portion of retailer's revenue the supplier receives.

The transfer payment with revenue sharing-quantity flexibility contract is

$$\begin{aligned} &T_{rq}(q,\ w_c,\ \phi,\ \delta) \\ &= w_c q + (1-\phi)pS(q) - (w_c + c_r - v)E[\min(I,\ \delta q)] \\ &= w_c q + (1-\phi)pS(q) - (w_c + c_r - v)\int_{(1-\delta)q}^{q} F(x)dx. \end{aligned} \quad (21)$$

The retailer's profit function can be derived as

$$\begin{aligned} &\pi_r(q,\ w_c,\ \phi,\ \delta) \\ &= pS(q) + vI(q) - g_r L(q) - c_r q - T_{rq}(q,\ w_c,\ \varnothing,\ \delta) \\ &= (\phi p - v + g_r)S(q) - (w_c - v + c_r)q \\ &+ (w_c - v + c_r)\int_{(1-\delta)q}^{q} F(x)dx - g_r\mu. \end{aligned} \quad (22)$$

The supplier's profit function can be derived as

$$\begin{aligned} &\pi_s(q,\ w_c,\ \phi,\ \delta) \\ &= T_{rq}(q,\ w_c,\ \phi,\ \delta) - g_s L(q) - c_s q \\ &= [(1-\phi)p + g_s]S(q) - (c_s - w_c)q \\ &- (w_c + c_r - v)\int_{(1-\delta)q}^{q} F(x)dx - g_s\mu. \end{aligned} \quad (23)$$

To determine the optimal retailer's order quantity, q_r^*, $\pi_r(q,\ w_c,\ \phi,\ \delta)$ has to be concave in q, which means

$$\frac{d^2\pi_r(q,\ w_c,\ \phi,\ \delta)}{dq^2} \le 0$$

Since the fraction of revenue the retailer keeps is ϕ, the retailer will also set his retail price such that $\phi p > w_c + c_r$ to avoid nonprofit status. Hence, $-(\phi p - w_c - c_r + g_r) f(q) - (w_c - v + c_r)(1-\delta)^2 f((1-\delta)q) < 0$

Therefore, q_r^* can be determined from

$$\frac{d\pi_r(q,\ w_c,\ \phi,\ \delta)}{dq} = 0$$

It is noted that

$$\begin{aligned} \frac{d\pi_r(q,\ w_c,\ \phi,\ \delta)}{dq} &= (\phi p - w_c - c_r + g_r)((1-F)q) \\ &-(w_c - v + c_r)(1-\delta)F((1-\delta)q) \end{aligned} \quad (24)$$

From Eq. (24), there does not exist the explicit expression for q_r^*. However, the above equation has a unique solution, and the quantity q_r^* can be determined uniquely.

The basic functions of component contracts and the proposed composite contract such as the retailer's profit function, supplier's profit function, and the retailer optimal order quantity have been derived as presented in the above sections. In the next section, each contract will be examined for its coordination ability.

4. SUPPLY CHAIN COORDINATION AND PROFIT ALLOCATION

In this section, numerical experiments are conducted in order to examine the coordination and profit allocation ability of the proposed composite revenue sharing and quantity flexibility contract.

4.1 Supply Chain Coordination

In this section, the coordination ability of the proposed composite contract will be analyzed through the comparison with the wholesale price contract as the base case, and also with the component contracts in terms of the retailer's profit, supplier's profit and the whole supply chain's profit.

Considering stochastic demand D during the selling season which follows normal distribution with mean 100 and variance 100. Assume that the supplier produces at a cost \$25/unit, and sells at a wholesale price \$65/unit. The retailer has marginal cost of \$35/unit and sells product at \$200/unit. The salvage value is \$30/unit. In addition, the penalty cost of retailer is \$100/unit, and the penalty cost of supplier is \$60/unit. With the above values of parameters, the following results are obtained:

- The optimal order quantity of the whole supply chain, which provides the maximum supply chain profit of \$13,460.1, is q^o = 113 units.
- For the wholesale price contract, the optimal order quantity of retailer is 106 units. As the result, the retailer will gain a profit of \$9,125.50, and the supplier will gain \$4,164.35. In addition, the whole supply chain profit will be \$13,289.85.
- For the revenue sharing contract, the optimal supply chain profit cannot be reached because there exists no value of $\phi \in (0,\ 1)$ that makes q_r^* to become q^o.
- For the quantity flexibility contract, the supply chain can be coordinated only when $\delta = 0.1616$. With the optimal order quantity of 113 units, the retailer and the supplier earn \$9,776.35 and \$3,683.75, respectively. It should be noted that the supplier's profit is reduced in comparison with the wholesale price contract, and hence, the supplier will not prefer to use the quantity flexibility contract.
- For the proposed composite revenue sharing and quantity flexibility contract, the interval of parameters $\delta \in$ [0.16395, 0.16331], and [0.96704, 0.97565] $\phi \in$ provides more profit for both the supplier and the retailer, and the maximum supply chain profit as well, when compared with the wholesale contract as presented in the Table 1 and Figure 1.

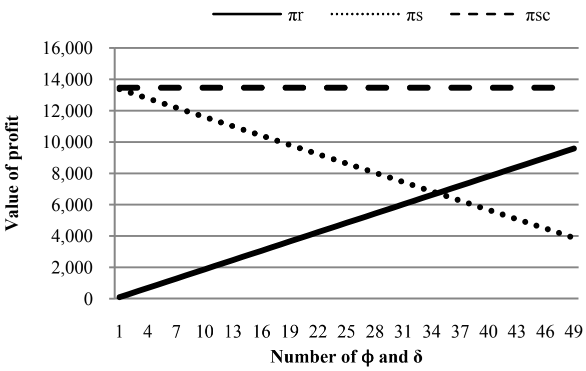

Figure 1. The profits of retailer, supplier, and the whole-supply chain in the proposed.

The comparisons between the proposed composite contract and other contracts are summarized in Table 2. It can be seen from Table 2 that the proposed revenue sharing-quantity flexibility contract can coordinate the supply chain since there always exist the values of parameters such that the supply chain profit will reach the global optimal profit in the centralized system. Interestingly, it can be seen that the supply chain can be coordinated by the proposed composite contract when it cannot be coordinated by all component contracts.

4.2 Profit Allocation

The proposed composite contract provides more flexibility in terms of profit allocation in such a way that the profit of both parties will be higher than their own profits in the wholesale price contract. In the numerical example presented above, the range of parameters at which the proposed composited contract is better than the whole sale price contract in term of profit allocation for both supplier and retailer are $\delta \in [0.16395,\ 0.16331]$, and $\phi \in [0.96704,\ 0.97565]$. The details are showed in Figure 2.

4.3 Sensitivity Analysis

In addition, sensitivity analysis was conducted in order to examine the effect of parameter changing on the proposed composite contract. Therefore, another 4 sets of numerical experiments were generated by increasing supplier's cost 5%, 10%, 15%, and 20%.

For the proposed composite revenue sharing and quantity flexibility contract, the intervals of parameters δ and ϕ that provide more profit for both the supplier and the retailer, and also help to reach the maximum supply chain profit when compared with the wholesale contract in all cases are presented in Table 3 and Figures 3 and 4.

For the revenue sharing contract, in order to reach the optimal supply chain profit the value of ϕ should be set at 3.350, 3.204, 3.042, and 2.950 when supplier's cost

Table 1. Results of the proposed composite contract

No.	ϕ	δ	π_r	π_s	π
1	0.51	0.2033	95.11	13,365.0	13,460.1
2	0.52	0.2022	292.80	13,167.3	13,460.1
3	0.53	0.2012	490.60	12,969.5	13,460.1
4	0.54	0.2001	688.24	12,771.9	13,460.1
5	0.55	0.1991	886.00	12,574.1	13,460.1
6	0.56	0.1981	1,083.72	12,376.4	13,460.1
7	0.57	0.1971	1,281.43	12,178.7	13,460.1
8	0.58	0.1961	1,479.11	11,981.0	13,460.1
9	0.59	0.1951	1,676.77	11,783.3	13,460.1
10	0.60	0.1942	1,874.55	11,585.6	13,460.1
11	0.61	0.1932	2,072.16	11,387.9	13,460.1
12	0.62	0.1923	2,269.90	11,190.2	13,460.1
13	0.63	0.1913	2,467.47	10,992.6	13,460.1
14	0.64	0.1904	2,665.17	10,794.9	13,460.1
15	0.65	0.1895	2,862.85	10,597.3	13,460.1
16	0.66	0.1886	3,060.51	10,399.6	13,460.1
17	0.67	0.1876	3,257.97	10,202.1	13,460.1
18	0.68	0.1868	3,455.76	10,004.3	13,460.1
19	0.69	0.1859	3,653.35	9,806.74	13,460.1
20	0.70	0.1850	3,850.93	9,609.17	13,460.1
21	0.71	0.1842	4,048.66	9,411.44	13,460.1
22	0.72	0.1833	4,246.19	9,213.91	13,460.1
23	0.73	0.1824	4,443.70	9,016.40	13,460.1
24	0.74	0.1816	4,641.37	8,818.72	13,460.1
25	0.75	0.1808	4,839.03	8,621.07	13,460.1
26	0.76	0.1799	5,036.48	8,423.62	13,460.1
27	0.77	0.1791	5,234.10	8,226.00	13,460.1
28	0.78	0.1783	5,431.70	8,028.40	13,460.1
29	0.79	0.1775	5,629.28	7,830.82	13,460.1
30	0.80	0.1767	5,826.84	7,633.25	13,460.1
31	0.81	0.1759	6,024.39	7,435.71	13,460.1
32	0.82	0.1751	6,221.92	7,238.18	13,460.1
33	0.83	0.1743	6,419.43	7,040.67	13,460.1
34	0.84	0.1735	6,616.92	6,843.18	13,460.1
35	0.85	0.1727	6,814.39	6,645.71	13,460.1
36	0.86	0.1719	7,011.84	6,448.26	13,460.1
37	0.87	0.1712	7,209.49	6,250.61	13,460.1
38	0.88	0.1704	7,406.90	6,053.19	13,460.1
39	0.89	0.1697	7,604.52	5,855.57	13,460.1
40	0.90	0.1689	7,801.90	5,658.19	13,460.1
41	0.91	0.1682	7,999.49	5,460.61	13,460.1
42	0.92	0.1674	8,196.83	5,263.26	13,460.1
43	0.93	0.1666	8,394.16	5,065.94	13,460.1
44	0.94	0.1659	8,591.69	4,868.40	13,460.1
45	0.95	0.1652	8,789.22	4,670.88	13,460.1
46	0.96	0.1645	8,986.72	4,473.37	13,460.1
47	0.97	0.1638	9,184.21	4,275.88	13,460.1
48	0.98	0.1630	9,381.45	4,078.65	13,460.1
49	0.99	0.1623	9,578.91	3,881.19	13,460.1

Table 2. Comparison of the profit functions with difference contracts

Supply chain with difference type of contracts	q_r^*	π_r	π_s	π_{sc}
Wholesale-price contract	106.46	9,125.50	4,164.35	13,289.85
Revenue-sharing contract	113.35	58,740.23	-45,280.13	13,460.10
Quantity-flexibility contract	113.35	9,776.35	3,683.75	13,460.10
Composite contract	113.35	Calculated in Table 1		13,460.10
Centralized system q_r^*	113.35	-	-	13,460.10

Table 3. Sensitivity analysis results of the proposed composite contract

Supplier cost	ϕ		δ		π_r		π_s		π_s	π_{sc} *
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper		
$25/unit	0.96704	0.97565	0.16331	0.16395	9,125.62	9,295.56	4,164.53	4,334.48	13,460.097	13,289.86
Increase 5%	0.96900	0.97500	0.15895	0.15938	9,144.82	9,263.17	4,041.91	4,160.26	13,305.083	13,156.79
Increase 10%	0.96890	0.97600	0.15450	0.15503	9,157.46	9,297.56	3,905.53	4,045.63	13,203.091	13,023.71
Increase 15%	0.96900	0.97600	0.14940	0.14995	9,129.45	9,267.30	3,740.83	3,878.68	13,008.126	12,864.03
Increase 20%	0.96930	0.97620	0.14604	0.14660	9,126.45	9,262.22	3,633.34	3,769.12	12,895.565	12,757.57

*Wholesale price contrast.

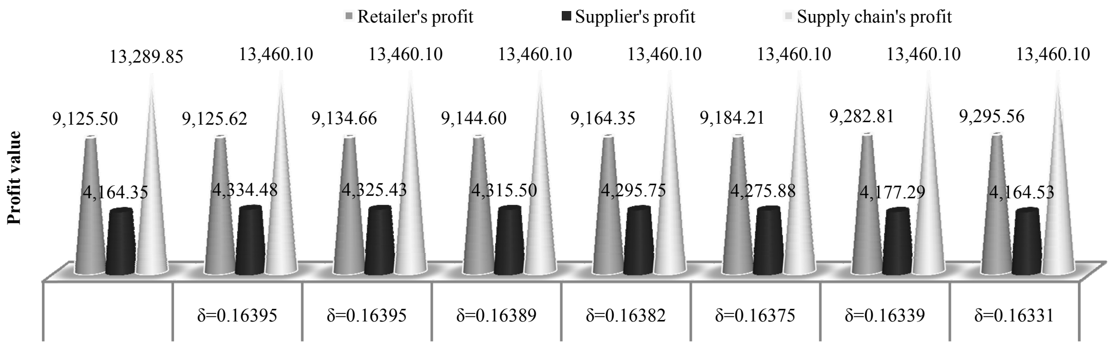

Figure 2. Comparing the profit functions of the wholesale price contract and the composite contract with different pairs of δ and ϕ.

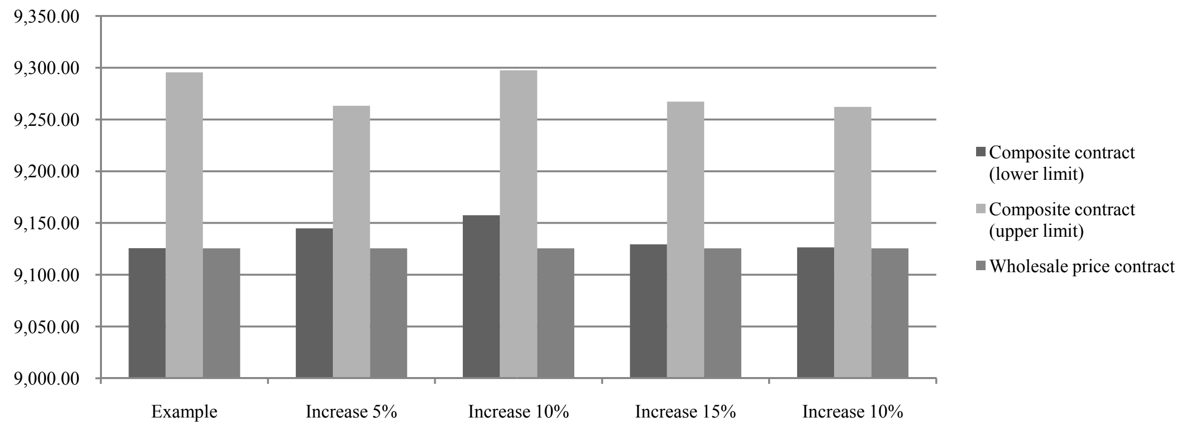

Figure 3. Comparing the retailer's profit of the composite contract, the wholesale price contract and the quantity flexibility contract with different supplier's costs.

Table 4. Comparison of the retailer's profit with different contracts

Supplier cost	Composite contract	Wholesale price contract	Quantity flexibility contract	Revenue-sharing contract
$25/unit	[9,125.62 , 9,295.56]	9,125.50	9,776.35	58,740.23
Increase 5%	[9,144.82 , 9,263.17]	9,125.50	9,756.08	55,643.39
Increase 10%	[9,157.46 , 9,297.56]	9,125.50	9,771.18	52,902.03
Increase 15%	[9,129.45 , 9,267.30]	9,125.50	9,740.04	49,721.44
Increase 20%	[9,126.45 , 9,262.22]	9,125.50	9,730.73	47,791.13

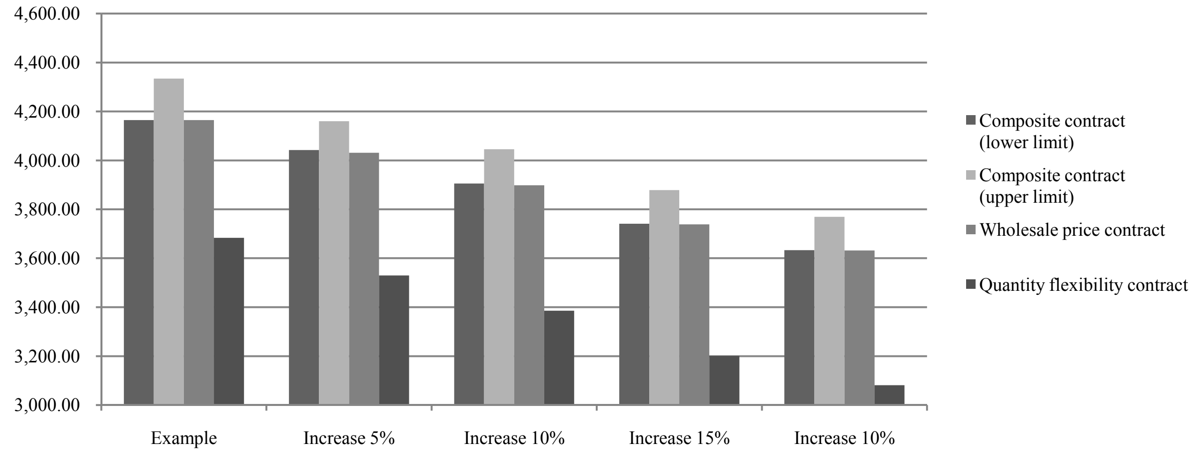

Figure 4. Comparing the supplier's profit of the composite contract, the wholesale price contract, and the quantity flexibility contract with different supplier's costs.

Table 5. Comparison of the supplier's profit with different contracts

Supplier cost	Composite contract	Wholesale price contract	Quantity flexibility contract	Revenue-sharing contract
$25/unit	[4,164.53 , 4,334.48]	4,164.35	3,683.75	-45,280.13
Increase 5%	[4,041.91 , 4,160.26]	4,031.28	3,549.00	-42,338.30
Increase 10%	[3,905.53 , 4,045.63]	3,898.21	3,431.91	-39,698.94
Increase 15%	[3,740.83 , 3,878.68]	3,738.53	3,268.08	-36,713.32
Increase 20%	[3,633.34 , 3,769.12]	3,632.07	3,164.84	-34,895.56

increases 5%, 10%, 15%, and 20%, respectively. The above values of ϕ is infeasible. In conclusion, the optimal supply chain profit cannot be reached by the revenue sharing contract because there exists no value of $\phi \in (0, 1)$ that makes q_r^* to become q^o in all cases.

For the quantity flexibility contract, when supplier's cost increases 5%, 10%, 15%, and 20%, the supply chain can be coordinated only when $\delta = 0.1571$, 0.1527, 0.1476, and 0.1442, respectively. However, it should be noted that the supplier's profit is reduced in comparison with the wholesale price contract. Hence, the supplier will not prefer to use the quantity flexibility contract in all cases as shown in Tables 4 and 5.

From the above results, we can conclude that the supply chain can be coordinated by the proposed revenue sharing-quantity flexibility contract when it cannot be coordinated by all component contracts.

5. CONCLUSIONS

In this research, a composite revenue sharing-quantity flexibility contract was developed to help achieve coordination for a two stage supply chain and to allocate profit among supply chain members better than the traditional wholesale price contract and its component contract, i.e., the revenue sharing contract and the quantity flexibility contract.

Our research results led us to conclude that the combination of revenue sharing contract and quantity flexibility contract can coordinate the supply chain better than the component contracts. Specifically, there exists the case where the proposed composite contract can coordinate the supply chain when the revenue sharing contract cannot. In addition, even though the quantity flexibility contract can coordinate the supply chain in theory, the possibility for application is weak because one of the parties might not be satisfied with her benefit when compared with the benefit from the traditional wholesale price contract. This is not a problem with the proposed composite contract because the contract parameters, i.e., δ and ϕ , can be selected such that the proposed composite contract provides better profit allocation for both parties than the traditional wholesale price contract. Also, the proposed composite contract provides more flexibility in terms of profit allocation among supply chain members.

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