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Mathematical Model for Revenue Management with Overbooking and Costly Price Adjustment for Hotel Industries

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ABSTRACT

Revenue management (RM) has been widely used to model products characterized as perishable. Classical RM model assumed that price is the sole factor in the model. Thus price adjustment becomes a crucial and costly factor in business. In this paper, an optimal pricing model is developed based on minimization of soft customer cost, one kind of price adjustment cost and is solved by Lagrange multiplier method. It is formed by expected discounted revenue/bid price integrating quantity-based RM and pricing-based RM. Quantity-based RM consists of two capacity models, namely, booking limit and overbooking. Booking limit, built by assuming uncertain customer arrival, decides the optimal capacity allocation for two market segments. Overbooking determines the level of accepted order exceeding capacity to anticipate probability of cancellation. Furthermore, pricing-based RM models occupancy/demand rate influenced by internal and competitor price changes. In this paper, a mathematical model based on game theoretic approach is developed for two conditions of deterministic and stochastic demand. Based on the equilibrium point, the best strategy for both hotels can be determined.

Keywords: Revenue Management, Costly Price Adjustment, Overbooking, Game Theory, Cancellation, Capacity Allocation

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1. INTRODUCTION

Hotel has products characterized as perishable and categorized as fixed inventory. Consequently, there are many uncertainties in the hotel industries. Firstly, there is possibility of cancellation and no shows when customers order by call and then disappear in the D-day or cancel the order near D-day. Secondly, because of its fixed capacity, the hotel should optimize the rooms' utilities towards the uncertainty of the number of customers. Thirdly, assuming that price is the sole factor considered by customers, pricing decision affects the expected revenue so price adjustment becomes a costly decision (Chen *et al*., 2010). To avoid such losses, it needs proper management of capacity allocation and pricing. One of the methods that can be used is revenue management (RM) (Bitran and Gilbert, 1996; Netessine *et al*., 2002). This method allows companies to sell the same product with multilevel price (Bertsimas and De Boer, 2005) and provides a method to allocate the fixed capacity for all price levels to optimize both the resources' utility and revenue. Talurri and Van Ryzin (2004) describe the definition of RM as stated below:

"RM is concerned with such demand-management decisions referred to as either sales decisions (we

are making decisions on where and when to sell and to whom and at what price) and the methodology and systems required to make them."

The first RM research had been conducted by Littlewood in 1972 as the development of overbooking. RM for the case of the hotel industry has been carried out (Britan and Gilbert, 1996; Netessine and Shumsky, 2004).

Basically, RM is a method to anticipate the behavior of consumers and competitors to maximize revenue (Dolgui and Porth, 2010). There are several strategies to increase revenue (Dolgui and Porth, 2010): lowering production costs, increasing market share, and setting price. Of the three strategies, setting price is the fastest and easiest way to optimize revenue. Therefore, in this case, price becomes the sole factor considered in the RM model.

Because of the existing competition in the hospitality business, the game theory approach is also used to demonstrate the price competition among the players. Netessine and Shumsky (2004) conducted a study which introduces a method of capturing duopoly competition. The assumption used in that study is that the price offered to consumers consists of only 2 types: full price and discount price. This study complements the weakness of Bitran and Gilbert (1996) which assumes there is no competition.

Furthermore, Dai *et al*. (2004) continued the study for oligopoly competition, but it takes a different assumption that the competition of few firms (oligopoly) is partially developed by doing duopoly competition repetitively. To describe that condition, Dai *et al*. (2004) apply the game theory. It describes a mathematical model to analyze the system when firms face stochastic and deterministic demand. The purpose of this study is to develop a pricing strategy in the context of RM and also to determine the Nash equilibrium.

The previous studies use the assumption that no costs incurred due to wrong pricing decisions. Chen *et al*. (2010) improve the situation by adding price adjustment cost in the model of price adjustment cost, especially for managerial and physical cost.

This paper proposes a model based on the model which has been developed by Dai *et al*. (2004). However, some improvements are proposed. In this paper, duopoly game is considered and the models are developed under either the stochastic or deterministic demand condition. This paper considers not only pricing strategy but also capacity-allocation strategy including overbooking and booking limit. Moreover, pricing strategy, in this study, includes price adjustment cost, particularly soft customer cost. It is different from other forms of price adjustment cost.

Wolman (2000) classified three types of price adjustment cost which are managerial cost, physical cost, and customer cost. Managerial cost is all costs related to the process of price decision while physical cost is all costs to inform the price changes, such as printing price label. Both managerial and physical costs are successfully developed by Chen *et al*. (2010). There are two types of customer cost: hard and soft customer cost. Hard customer cost is similar to managerial cost but it is particularly to communicate directly with customer about the price changes. Nonetheless, soft customer cost contains intangible cost representing business risk when the company changes the price dynamically. Small changes of price may affect the customers' decision in which the business competition exists. This theory is clearly published by Sweezy, Hall, and Hitch in 1990s called "kinked demand curve" theory of oligopoly. When the company lowers the price, it is not guaranteed that company will gain more customers because competitors may follow its strategy. Thus, optimal pricing model with soft customer cost can evaluate the price changes that still maintain the customers while optimizing the revenue under competition.

The price term used in this study refers to Donalghy (as cited in Göthesson and Riman, 2004) in which the price is divided into two categories namely business and leisure travelers. Leisure travelers are consumers who can afford to pay the discount price, while business travelers are the customers who order directly (walk-ins) with full price. Meanwhile, because of the model simplification and demand aggregation, demand changes for each class (more than two classes) cannot be performed. This condition becomes the model limitation in this study.

2. DEVELOPMENT OF QUANTITY-BASED RM MODEL

Quantity-based RM model, also called capacity model, can be seen as monopoly decision because it is influenced by customer arrival instead of the competitors. The purpose of this model is to allocate the capacity and to optimize the revenue. Thus, the model is applied for any demand uncertainty and any conditions, both stochastic and deterministic. In the case of no fixed capacity management, the model includes capacity and backlog. However, for the hotel industry with a fixed capacity, it only considers the proportion of capacity allocation and cancellation called by booking limit and overbooking models.

2.1 Booking Limit Model

The purpose of the model is to allocate capacity for leisure travelers and business travelers. The assumption that leisure travelers come first followed by the business traveller is used. The number of rooms which is allocated for leisure travellers is called booking limit (b). When the limit is reached, then the remaining rooms (protection level) will be offered to the next customer called business travelers who have willing to pay full price. If the hotel sets more booking limit, then the revenue could not reach optimal point because there is possibility that the number of business travelers is more than the prediction. In contrast, if booking limit is too low, there is possibility of unsold room that may reduce capacity utility. Therefore, it is necessary to develop booking limit and protection level model. Booking limit and protection level are influenced by the ratio of discount (p_{Di}) and full fare (p_{Fi}) namely price ratio (r).

Assumed the arrival of demand follows Poisson distribution. Thus, cumulative distribution function (CDF) of Poisson distribution as formulated in Eq. (1) can be used to determine the optimal booking limits and protection levels. Based on Little wood's rule (Talluri and Van Ryzin, 2004), the optimal protection level (y^*) is obtained for each discount ratio by Eq. (2) and the booking limit is obtained by Eq. (3). Then, the results generate capacity models using regression analysis as shown in Eq. (4). It shows that booking limit relates positively with price ratio. When price ratio becomes lower, hotel should protect more rooms and vice versa.

$$F = \sum_{k=0}^{y} \frac{e^{-\lambda}\lambda^k}{k!} \quad (1)$$

$$y^* = F^{-1}\left(1 - \frac{p_{Di}}{p_{Fi}}\right) \quad (2)$$

$$b = C - y^* \quad (3)$$

$$b\left(p_{Di},\ p_{Fi}\right) = m_i + n_i \frac{p_{Di}}{p_{Fi}} \quad (4)$$

Where

- F : cumulative exponential distribution function with λ as parameter
- C : capacity
- m_i, n_i : intercept for the regression model of booking limit of hotel i

2.2 Overbooking Model

Unlike the booking limit that is developed to optimize the room utilization by allocating the capacity, overbooking (O) is a concept to anticipate cancellation and no shows by receiving orders over the capacity. In this paper, overbooking is modeled as deterministic or non-dynamic model, so the cancellation and no show are same.

There are three concepts for modeling overbooking: probability, economics, and quality levels. Because the determination of cost and quality levels needs further research, probability approach is used. The calculation of the optimal overbooking shown in Eq. (5), where q is the mean of customer show rate according to the probability distribution and C is the hotel capacity.

$$O(q) = \frac{C}{q} \quad (5)$$

3. DEVELOPMENT OF PRICING-BASED RM MODEL

Pricing-based RM model is the basic demand model which models demand as a function of price (Talluri and Van Ryzin, 2004) as clearly explained that price is the sole factor considered in the RM model. According to Talluri and Van Ryzin (2004), base model for pricing-based RM depicted in Eq. (6). Assumed that there are two firms called firm 1 and firm 2 which sell identical product, offer price at p_1 and p_2 . It is also assumed that all customers buy only from the firms offering the lowest price. It means that perfect competition is suggested in this model.

$$d_1(p_1,\ p_2) = \begin{cases} d(p_1) & \text{if } p_1 < p_2 \\ d(p_1)/2 & \text{if } p_1 = p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases} \quad (6)$$

Meanwhile, in this paper, perfect competition is not suggested as the form of competition but two-person constant-sum games is preferred. This form explains that the interests of the players are not diametrically opposed. Such games show how the player can win or lose more than other player. The more one player wins, the less the other can win or the more he must lose (Rapoport, 1971).

Furthermore, due to capacity differences of the two players/hotels, comparison of the demand room becomes irrelevant. Therefore, the occupancy rate is used as parameter of demand model (D). Total demand is obtained from multiplying the occupancy rate model (d) and its capacity (C) as shown in Eq. (7).

$$D\left(p_{Di},\ p_{Dj}\right) = C_i d\left(p_{Di},\ p_{Dj}\right) \quad (7)$$

In this section, mathematical model based on game theoretic approach is developed for two conditions, duopoly deterministic and duopoly stochastic.

3.1 Duopoly Deterministic Occupancy Rate Model

Based on theory of duopoly competition and the assumption that customer behaves myopically, hotel occupancy rates are influenced by competitor and discount price itself. Talluri and Van Ryzin (2004) stated the assumption used for static Bertrand model of price competition used as in the perfect competition, consumer only buy product from the company offering the lowest price. Based on that statement, we may conclude that in the pricing model, there is a rule that the higher the discount hotel rates A, the lower the demand of hotel B and vice versa. In conclusion, the demand of hotel A correlates with discount price of hotel A negatively and discount price of hotel B positively. This relationship is mathematically explained by Eq. (8).

$$d\left(p_{Di},\ p_{Dj}\right)=\alpha_i-\beta_i p_{Di}+\gamma_{ij} p_{Dj} \tag{8}$$

For the data which has seasonality factor, it has to add binary factor (x_i) representing peak season or nonpeak season as shown in Eq. (9). It is valued by 1 if peak season occurs and 0 if peak season does not occur.

$$d\left(p_{Di}, p_{Dj}\right)=\alpha_i-\beta_i p_{Di}+\gamma_{ij} p_{Dj}+\theta x_i \tag{9}$$

While

- d : occupancy rate (ratio of demand and capacity)
- $\alpha_i, \beta_i, \gamma_{ij}$: intercept/coefficient of occupancy rate model of hotel i towards hotel competitor (hotel j)
- θ : coefficient of peak season factor

If the hotels have peak season which can be detected by plotting graphic or using statistical tool, such as autocorrelation, then Eq. (9) is applied to develop occupancy rate model. In contrast, if the hotels do not have peak season pattern, Eq. (8) is then applied.

3.2 Duopoly Stochastic Occupancy Rate Model

Since the condition that price affects demand, stochastic demand model becomes a function of price. Nevertheless, stochastic model is different from deterministic model because of its random property. Thus, cumulative density function of price (CDF) is used to model occupancy rate or demand. Talluri and Van Ryzin (2004) developed demand model as function of price stated in Eq. (10).

$$d_{i,STO}=N\left(1-F\left(p_{Di}\right)\right) \tag{10}$$

Where N is market size and $F(p_{Di})$ is CDF of discount price.

However, the model (Eq. (10)) can be applied only for monopoly condition. Based on the same principle as Talluri and Van Ryzin (2004), in this study, duopoly stochastic model is developed as a function of CDF of discount price from both competitor and hotel itself.

All residual demand of hotel A goes to hotel B and vice versa (Talluri and Van Ryzin, 2004). Also, it is assumed that there is no customer out of the system. Thus, a constant-sum game assumption is applied. The demand function has described as the inverse of CDF of price depicted in Eq. (11).

$$d_{i,STO}=\begin{cases} N\left(\dfrac{1-F^{+}\left(p_{Di}\right)+F^{-}\left(p_{Dj}\right)}{2}\right) & if p_{Di}<p_{Dj} \\ N\left(\dfrac{1-F^{-}\left(p_{Di}\right)+F^{+}\left(p_{Dj}\right)}{2}\right) & if p_{Di}\geq p_{Dj} \end{cases} \tag{11}$$

Based on Figure 1, for example, if hotel B decides Rp125,000 as its discount fare, the customers who have the purchasing power over Rp125,000 interpreted as $1-F\left(p_{DB}\right)$ become the expected demand of hotel B. Assumed that the duopoly competition follows the principle of constant-sum games in which customer are not allowed to leave the system, then $F\left(p_{DB}\right)$ is considered as potential market for hotel A. This situation is described by Eq. (12). $F^{+}\left(p_{Di}\right)$ is CDF of price of hotel i if $p_{Di}<p_{Dj}$ and $F^{-}\left(p_{Di}\right)$ is CDF of price of hotel I if $p_{Di}\geq p_{Dj}$, We set N as the sum of occupancy rate for both hotels.

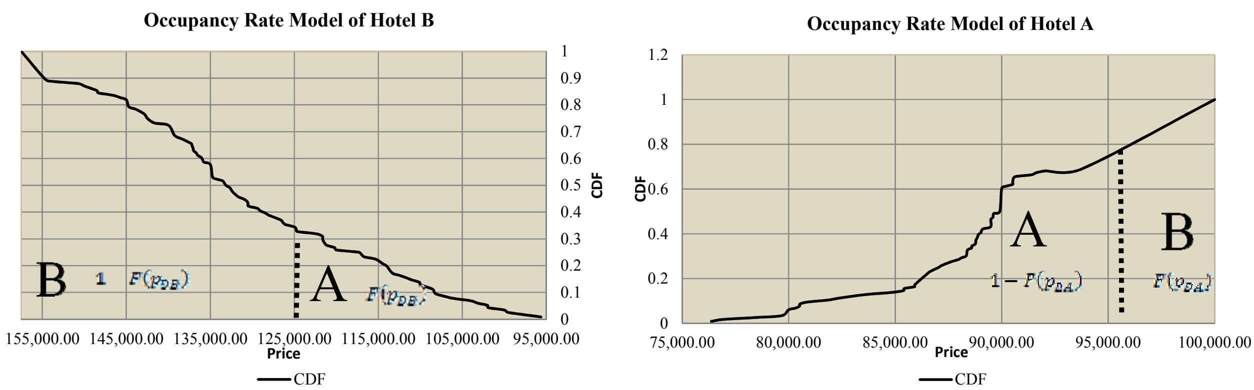

Figure 1. The concepts of duopoly stochastic model. CDF: cumulative distribution function.

4. EXPECTED DISCOUNTED REVENUE MODEL

The expected discounted revenue model becomes the basis for optimum pricing model and other analysis. It is developed based on quantity-based RM and pricing-based RM model. Eq. (12) shows the general model based on three conditions. If overbooking is applied and $D_i \geq C_i$ occurs, the excessive demand should be distributed to another hotel with cost notated as $cost_{ij}$.

$$\max \pi(p_D) = \begin{cases} p_{Di}b_i + p_{Fi}(O_i - b_i) - cost_{ij}(O_i - C_i)\, p_{Di}b_i \\ + p_{Fi}(O_i - b_i) - cost_{ij}(O_i - C_i) & if\ D_i \geq C_i \\ p_{Di}b_i + p_{Fi}(D_i - b_i)\, p_{Di}b_i + p_{Fi}(D_i - b_i) & if\ D_i \leq C_i \\ p_{Di}D_i\, p_{Di}D_i if D_i \geq b_i \end{cases}$$

$$\text{subject to } D \geq 0 \tag{12}$$

By substituting Eq. (12) with booking limit model (4) and occupancy rate model for deterministic (8) or (9) and stochastic (11), the problem can be stated as follows.

$$\max \pi(p_D) = \begin{cases} p_{Di}(m_i + n_i r_i) + p_{Fi}(O_i - m_i - n_i r_i) \\ -\text{cost}_{ij}(O_i - C_i) & if\ D_i \geq C_i \\ p_{Di}(m_i + n_i r_i) + p_{Fi}(C_i d_i - m_i - n_i r_i) \\ & if\ D_i \leq C_i \\ p_{Di}C_i d_i & if\ D_i \leq b_i \end{cases}$$

$$\text{subject to } D_i \geq 0 \tag{13}$$

Furthermore, there are two steps of validation. Firstly, validation for pricing-based RM model to check whether it has no significant gap with actual demand. Secondly, validation for quantity-based RM model uses revenue parameter to check if that model represents actual quantity-system. If both demand and revenue prediction has no significant differences from the actual one, it may be concluded that quantity-based RM can illustrate how hotel allocates its capacity in the actual system. Ultimately, revenue model can be used for further analysis.

5. OPTIMAL PRICING MODEL

Three models have been developed:quantity-based RM model, pricing-based RM model, and revenue model. Once validated, further analysis including optimal pricing and Nash equilibrium can be explored. Case study can be used to validate the proposed models. If there is significant difference between model and actual, the model should be revised accordingly.

Optimal pricing model considers price adjustment cost, particularly soft customer cost. It indicates the cost of losing customer that can reduce demand rate because of price fluctuation. Based on the theory of kinked demand curve, the lower price does not always gain more customers. On the other hand, raising prices always causes loss of customers. Therefore, the analysis is necessary to determine the optimal price while maintaining the consumer. Eq. (14) is the minimization function of soft customer cost for deterministic condition.

$$\text{Min } f_{\text{det}} = \beta_i (P_i^t)^2 - \begin{pmatrix} \beta_i P_i^{t-1} + \gamma_i (P_j^t - P_j^{t-1}) + \\ \theta_i (x_i^t - x_i^{t-1}) \end{pmatrix} P_i^t \tag{14}$$

$$\text{subject to } \quad \alpha_{\text{i}} - \beta_i P_i^t + \gamma_i \gamma P_j^t + \theta_i x_i^t \leq 1$$

For the stochastic models the minimization model can be seen in Eq. (15).

$$\text{Min } f_{sto} = f_{sto} = \frac{NC}{2} \begin{bmatrix} \beta_{i,sto} P_{i,sto}^t - \beta_{i,sto} P_i^{t-1} \\ + \beta_{j,sto} \left(P_j^{t-1} - P_j^t \right) \end{bmatrix} \left(P_i^t \right)$$

subject to:

$$1 - \alpha_{i,sto} + \beta_{i,sto} P_i^t + \alpha_{j,sto} + \beta_{j,sto} P_j^t \leq 1 \tag{15}$$

By using Lagrange multiplier, the best response function can be obtained as shown in Eq. (16) for deterministic model and Eq. (17) for stochastic model.

$$P_i^* = \frac{P_i^{t-1}}{2} + \frac{\gamma_i}{2\beta_i}\left(2P_j^t - P_j^{t-1}\right) + \frac{\theta_i}{2\beta_i}\left(2x_i^t - x_i^{t-1}\right) + \frac{\alpha_i - 1}{2\beta_i} \tag{16}$$

Proof. The proof can be seen in Appendix 1.

$$P_{i,sto}^* = \frac{P_i^{t-1}}{2} + \frac{\beta_{j,sto}}{2\beta_{i,sto}}\left(2P_j^t - P_j^{t-1}\right) + \frac{-\alpha_{i,sto} + \alpha_{j,sto}}{2\beta_{i,sto}} \tag{17}$$

Proof. The proof can be found in Appendix 2.

6. NASH EQUILIBRIUM

In finite games, there are some equilibrium points (Turocy and Von Stengel, 2011). Those points are the best solution for both players related with competitors' response. Thus, equilibrium price occurs when both hotels do their best responses. So, equilibrium price can be determined when we set $P_i^* = P_j^* = P^*$. We analyze Nash equilibrium for deterministic and stochastic models. Eq. (18) explains deterministic equilibrium point and Eq. (19) explains stochastic equilibrium point.

$$P_{i,\det}^* = \frac{P_i^{t-1}}{2} + \frac{\gamma_i\left(\alpha_j - 1\right) + \beta_j(\alpha_i - 1)}{2(\beta_i\beta_j - \gamma_i\gamma_j)} + \frac{\left(\gamma_i\theta_j + \beta_j\theta_i\right)\left(2x_i^t - x_i^{t-1}\right)}{2(\beta_i\beta_j - \gamma_i\gamma_j)} \tag{18}$$

Proof. The proof can be found in Appendix 3.

$$P^*_{i,sto} = \frac{2P_i^{t-1}}{3} + \frac{\beta_{j,sto}}{3\beta_{i,sto}} P_j^{t-1} + \frac{-\alpha_{i,sto} + \alpha_{j,sto}}{3\beta_{i,sto}} \quad (19)$$

Proof. The proof can be found in Appendix 4.

7. CASE STUDY

The proposed models are applied in the case of competition between two hotels located in the same region. Both hotels have similar customers segmentation. Actually, hotel A has two pricing classes but hotel B has more than two classes so the price of hotel B used in this analysis use average price of hotel B. That condition influences how to decide optimal booking limit so it needs to be justified as shown in Table 5 in Section 7.5.

7.1 Booking Limit

Booking limit model provides how to allocate capacity for two types of customers based on customer arrival rate indicated by the demand in 2011. Also, in this case, optimal booking limit should be generated into linier regression because the model will be combined with other models to describe revenue model which consists of pricing and quantity-based RM models including overbooking and booking limit.

There are several steps to develop this model. Firstly, using assumption that customer arrival follows Poisson distribution, where λ as parameter of Poisson distribution is calculated and then F-value stated in Eq. (1), optimal protection level (y^*) stated in Eq. (2), and optimal booking limit (b) stated in Eq. (3) are determined for each price ratio. The ratio used is based in the data that lies in the range of 45%–99% for hotel A and 65%–99% for hotel B. Secondly, regression method is applied to determine the intercept and coefficient which are shown in Eq. (4). Price ratio and optimal booking limit (b) which has been calculated are used as input for regression analysis. Eqs. (20) and (21) shows booking limit model for both hotels.

$$b(P_{DA},\ P_{FA}) = 33.416 + 16.948\frac{P_{DA}}{P_{FA}},\ R^2 = 0.946 \quad (20)$$

$$b(P_{DB},\ P_{FB}) = 11.290 + 11.972\frac{P_{DB}}{P_{FB}},\ R^2 = 0.939 \quad (21)$$

Nevertheless, in the real system, there are some hotels which have more than two classes. This condition does not fit with the assumption used in this model in which the model is used only for two classes. For the case of more than two classes, adjustment for optimal booking limit is a need.

7.2 Overbooking

The demand cancellation is used to calculate the acceptable order to anticipate cancellation or no-show customers. Based on Eq. (5), q is a mean of the fitted distribution. Tables 1 and 2 show the result of Stat::Fit software with Anderson Darling and Kolmogorov Smirnov as parameters. The fitted distribution are normal for hotel A and triangular for hotel B. Based on Eq. (5), the optimal overbooking hotel A is 83 and is 35 for the hotel B. It may be concluded that hotel A is permitted to accept customers up to 83 to anticipate probability of cancellation or no shows.

7.3 Duopoly Deterministic Occupancy Rate Model

Pricing-based RM model is developed using some data which are occupancy rate, peak season factor, and discount price. The occupancy rate can be obtained by dividing demand with hotel's capacity while peak season can be determined by plotting monthly demand as shown in Figures 2 and 3.

Based on Figures 2 and 3, both hotels have peak season. Hotel A has peak seasons in January, May, June, and December. Hotel B has peak seasons in January, June, September, and December. Hence, Eq. (8) is used to develop occupancy rate models for both hotels.

Occupancy rate models for hotel A and B are developed using regression method with price and peak season factor as independent variables and occupancy rate as dependent variable. Eqs. (22) and (23) below are the results.

$$d_A = 1.618 - 1.472\times10^{-5}\, p_{D1} + 5.580\times10^{-7}\, p_{D2} + 0.124x_1 \quad (22)$$

$$d_B = 1.921 - 7.576\times10^{-6}\, p_{D2} - 5.585\times10^{-6}\, p_{D1} + 0.126x_2 \quad (23)$$

Table 1. Theoretical distribution fitting for customer show of hotel A

Distribution	Rank
Lognormal (-468; 6.15; 3.27E-004)	100
Normal (0.805; 0.153)	76.9
Triangular (0.331; 1.04; 0.93)	40.8
Uniform (0.4; 1)	0.529

Table 2. Theoretical distribution fitting for customer show of hotel B

Distribution	Rank
Triangular (0.571; 1; 1)	95
Lognormal (-291; 5.68; 3.6E-004)	82.5
Normal (0.865; 0.106)	75.3
Uniform (0.63; 1)	6.57

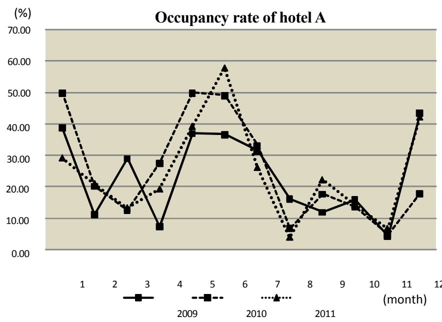

Figure 2. Occupancy rate of hotel A in 2009 to 2011.

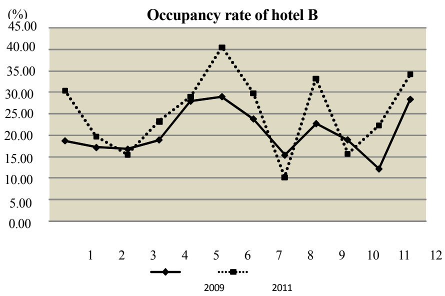

Figure 3. Occupancy rate of hotel B in 2010 and 2011.

7.4 Duopoly Stochastic Occupancy Rate Model

The condition, $p_{DA} \geq p_{DB}$, had never occurred during 2011. So, the first condition of duopoly stochastic occupancy rate model (Eq. (11)) is not considered because the case used in this research has lack of data. Nonetheless, if other cases have the second condition depicted in Eq. (11), modeling process should follow the same steps as the first condition.

There is no fitted theoretical distribution for price of hotel A. Hence, for simplification purposes, we use empirical distribution for both prices with price as independent variable and cumulative percentage of frequency as dependent variable. Eqs. (24) and (25) are cumulative density function for hotel A and B.

$$F_A = -3.688 + 4.629 \times 10^{-5} P_A,\ \ R^2 = 0.857 \tag{24}$$

$$F_B = -1.806 + 1.762 \times 10^{-5} P_B,\ \ R^2 = 0.970 \tag{25}$$

7.5 Model Evaluation

It is acknowledged that there are differences between the proposed model and the existing system for both quantity-based RM and pricing-based RM model. However, in order to apply the proposed model, how significant the differences between the existing system and the proposed model should be analyzed. If there are significant differences, then the proposed model is not feasible to be applied to the system.

Because the data is not normally distributed, Mann-Whitney test is used. Using α = 0.95, Table 3 shows that there is no significant difference between model and the real system for stochastic occupancy rate model of hotel A and hotel B. However, based on Table 3, the duopoly deterministic of occupancy rate model and the real data are statistically different.

Although demand (occupancy rate) models have been validated, in order to validate the quantity-based RM, statistical study on revenue model is performed. The reason why it should be done is stated on Section 4. Based on the case study, it can be concluded that all revenue models except deterministic demand for hotel B are not significantly different from the existing system as shown in Table 4. The result of the Mann-Whitney test also suggests that stochastic models result more accurate models.

Particularly for hotel B, booking limit should be adjusted because hotel B has multilevel prices. Thus the applied booking limit should be suggested using following scenarios depicted in Table 5. From Table 5, it can be seen that 60% is chosen as its p-value is approximately same as p-value of occupancy rate model. It means hotel B uses 60% of optimal booking limit as actual booking limit. However, there is no suggestion for hotel A because it has two classes in which this condition confirms with the assumption in the model.

Table 3. Result of Mann-Whitney test for occupancy rate model

	Mann-Whitney	Wilcoxon	Z	Sig.	Decision
Hotel A					
Deterministic	5.939E3	1.272E4	-1.545	0.122	Do not reject
Stochastic	6.541E3	1.333E4	-0.366	0.714	Do not reject
Hotel B					
Deterministic	5.040E3	1.183E4	-3.303	0.001	Reject
Stochastic	6.692E3	1.348E4	-0.071	0.943	Do not reject

Table 4. Result of Mann-Whitney test for expected discounted revenue model

	Mann-Whitney	Wilcoxon	Z	Sig.	Decision
Hotel A					
Deterministic	5.900E3	1.269E4	-1.622	0.105	Do not reject
Stochastic	6.658E3	1.544E4	-0.358	0.720	Do not reject
Hotel B					
Deterministic	4.938E3	1.1725E4	-3.501	0.000	Reject
Stochastic	6.802E3	1.382E4	-0.082	0.935	Do not reject

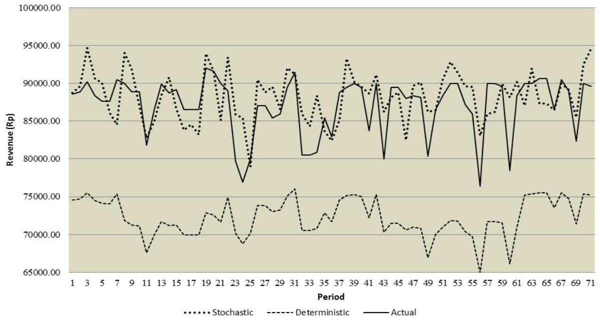

Figure 4. Actual and optimal prices for hotel A.

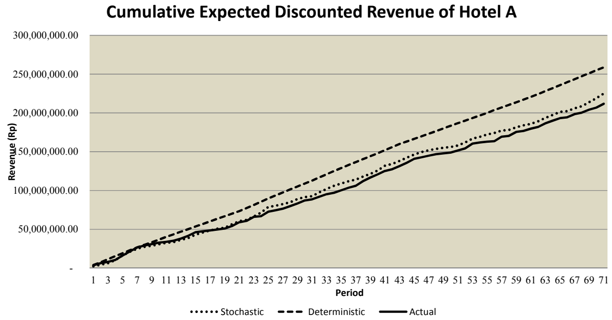

Figure 6. Cumulative revenue of hotel A.

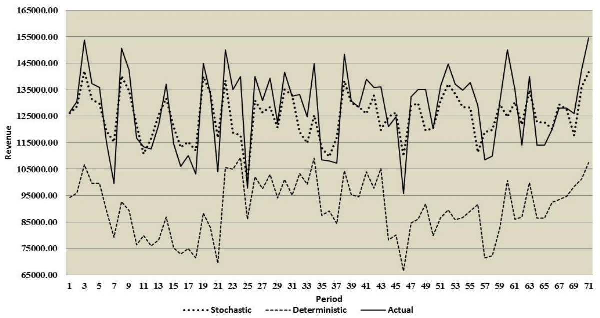

Figure 5. Actual and optimal prices for hotel B.

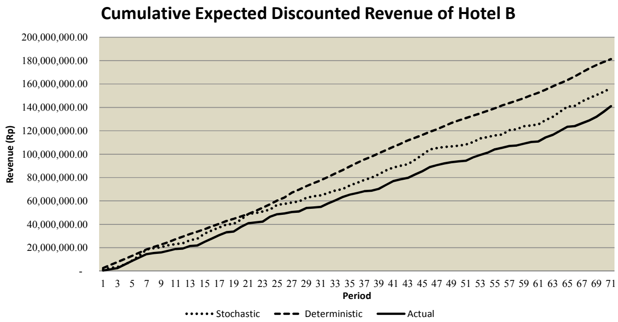

Figure 7. Cumulative revenue of hotel B.

Table 5. Adjustment of booking limit for hotel B

No.	Optimal booking limit (%)	p-value
1	0.1	0.272
2	0.2	0.424
3	0.3	0.626
4	0.4	0.779
5	0.5	0.751
6	0.6	0.935
7	0.7	0.842
8	0.8	0.740
9	0.9	0.651
10	1.0	0.580

7.6 Performance of Optimal Pricing Model

Models have represented actual system which is evaluated using statistical tool so further analysis can be conducted. Using Eqs. (16) and (17), optimal prices in deterministic and stochastic condition can be determined for both hotels depicted in Table 6.

Table 6. Optimal prices for both hotels

	$P^*_{i,STO} \ge P_{i,AKT}$	$P^*_{i,STO} < P_{i,AKT}$	$\overline{\overline{P^*_{i,STO}}}$
Hotel A			
Deterministic	0 period	71 periods	Decline
Stochastic	43 periods	28 periods	Increase
Hotel B			
Deterministic	0 period	71 periods	Decline
Stochastic	43 periods	28 periods	Decline

Figures 4 and 5 and Table 6 show that deterministic models suggest both hotels to lower their actual prices. On the other hand, the result shows that the deterministic optimal pricing model will push the prices to reach 100% occupancy. However, there are 23 points that are impossible because the expected demand exceeds market sizes. If the hotel lowers the prices according to deterministic optimal prices, the hotel will not increase the expected discounted revenue because the occupancy rate will not reach the deterministic model predictions and are always worth less than their market sizes. Based on this result, we use stochastic process for further analysis.

The purpose of optimal pricing modeling is to examine how price changes can increase the expected discounted revenue but still enable to keep the market share. Losing customers is indicated by market share changes. Based on Table 4, actual price of hotel A is not significantly different from its optimal in the stochastic condition. It can be concluded that hotel A has reach its optimal condition. Hotel B should be lower its prices because its actual prices have not been optimal prices condition. By price changes, hotel B can keep its customers. This conclusion is based on paired t-test of actual and optimal market share shown in Table 7.

In addition, we also have to consider the expected discounted revenue as primary goal of business, whether the optimal pricing model can increase revenue. Figure 6 shows expected discounted revenue for hotel A. Because hotel A has been in the optimal condition, there is no significant difference of actual and optimal discounted revenue. On the other hand, if hotel B reduces its price to reach the optimal point, hotel B can raise its revenue as can be seen in Figure 7.

7.7 Nash Equilibrium

We consider that the two models apply optimal pricing model. Equilibrium price occurs when both hotels do their best responses. So, equilibrium price can be determined when $P_i^* = P_j^* = P^*$ is set. The explanation of Nash equilibrium has been explained in the Section 5. In this case, Eqs. (18) and (19) are used to calculate the equilibrium point for both hotels.

Table 7. The differences of actual and optimal condition

	Z or t	Sig.	Decision
Hotel A			
Price	-1.173	0.241	Do not reject H_0
Market share	-0.424	0.673	Do not reject H_0
Hotel B			
Price	2.776	0.007	Reject H_0
Market share	0.244	0.808	Do not reject H_0

H_0 is null hypothesis.

Table 8. Deterministic equilibrium point

	Peak season	Non-peak season
Hotel A		
Peak season	$\frac{P_A^{t-1}}{2}+25,669.3$	$\frac{P_A^{t-1}}{2}+16,861.34$
Non-peak season	$\frac{P_A^{t-1}}{2}+30,073.28$	$\frac{P_A^{t-1}}{2}+21,265.32$
Hotel B		
Peak season	$\frac{P_B^{t-1}}{2}+9,476.95$	$\frac{P_B^{t-1}}{2}-661.15$
Non-peak season	$\frac{P_B^{t-1}}{2}+14,546$	$\frac{P_B^{t-1}}{2}+4,407.90$

Table 9. The application of deterministic equilibrium point

	Peak season	Non-peak season
Hotel A		
Peak season	75,669.30	66,861.34
Non-peak season	80,073.28	71,265.32
Hotel B		
Peak season	88,217.95	78,088.85
Non-peak season	93,296.00	83,157.90

Table 10. The application of stochastic equilibrium point

	Stochastic equilibrium point
Hotel A	100,202.71
Hotel B	156,967.46

Firstly, regarding to Eq. (18), equilibrium point depends on peak season and the price of both hotels in the previous time (t – 1). Accordingly, assumed that hotel A and B are in the same peak season, Table 8 shows the deterministic equilibrium point for both hotels. If in the end of this period (t – 1), price of hotel A is 100,000 and price of hotel B is 157,500, then Tables 9 and 10 shows the equilibrium point for both hotels for deterministic and stochastic condition.

8. CONCLUSION

In this article we developed the mathematical models for hotel revenue management under competition. The models consider not only pricing strategy but also capacity allocation strategy including overbooking and booking limit. Furthermore, the pricing strategy includes soft customer cost, i.e. cost incurred due to customer unsatisfaction. The concepts of revenue management and game theory are applied. It is assumed that the competition is under duopoly environment. Demand models are developed as function of price and considered both deterministic and stochastic conditions. The case studies show that applying the optimal price enables to increase the expected revenueas well as to maintain the market share due to the price changes. Additionally, in general stochastic demand model outperforms the deterministic demand model.

REFERENCES

Bertsimas, D. and De Boer, S. (2005), Simulation-based booking limits for airline revenue management, *Operations Research*, **53**(1), 90-106.

Bitran, G. B. and Gilbert, S. M. (1996), Managing hotel reservation with uncertain arrivals, *Operations Research*, **44**(1), 35-49.

Chen, X., Zhou, S. X., and Chen, Y. (2010), Integration of inventory and pricing decisions with costly price adjustments, *Operations Research*, **59**(5), 1144-1158.

Dolgui, A. and Porth, J. (2010), Pricing strategy and models, *Annual Reviews in Control,* **34**(1), 101-110.

Dai, Y., Chao, X., Fang, S. C., and Nuttle, H. L. W. (2004), Pricing in revenue management for multiple firms competing for customers, *International Journal Production Economics*, **98**(1), 1-16.

Göthesson, L. and Riman, S. (2004), *Revenue manage-*

ment within Swedish hotels, M.S. thesis, School of Economics and Commercial Law, Göteborg University, Göteborg, Sweden.

Netessine, S. and Shumsky, R. A. (2004), Revenue management games: horizontal and vertical competition, *Management Science*, **51**(5), 813-831.

Netessine, S., Shumky, R. A., and Sadagopan, N. (2002), Introduction to the theory and practice of yield management, *INFORMS Transactions on Education*, **3**(1), 34-44.

Rapoport, A. (1971), Three- and four-person games, *Comparative Group Studies*, **2**(2), 191-226.

Talluri, K. T. and Van Ryzin, G. J. (2004), *The Theory and Practice of Revenue Management*, Kluwer Academic Publishers, Boston, MA.

Turocy, T. L. and Von Stengel, B. (2001), Game theory, *CDAM Research Report LSE-CDAM-2001-09*, Mathematics Department, London School of Economics, London, UK.

Wolman, A. L. (2000), The frequency and costs of individual price adjustment, *Federal Reserve Bank of Richmond Economic Quarterly*, **86**(4), 1-22.

Appendix 1. Proof of Eq. (16)

$$f_{\det} = \Delta d_i \left(P_i^t\right)$$

$$f_{\det} = \left(d_i^{t-1} - d_i^t\right)\left(P_i^t\right)$$

$$f_{\det} = \left[\left(\alpha_i - \beta_i P_i^{t-1} + \gamma_i P_j^{t-1} + \theta x_i^{t-1}\right) - \left(\alpha_i - \beta_i P_i^t + \gamma_i P_j^t + \theta_i x_i^t\right)\right] P_i^t$$

$$f_{\det} = \left[\beta_i\left(P_i^t - P_i^{t-1}\right) + \gamma_i\left(P_j^{t-1} - P_j^t\right) + \theta_i\left(x_i^{t-1} - x_i^t\right)\right] P_i^t$$

$$f_{\det} = \beta_i\left(P_i^t\right)^2 - \left[\beta_i P_i^{t-1} + \gamma_i\left(P_j^t - P_j^{t-1}\right) + \theta_i\left(x_i^t - x_i^{t-1}\right)\right] P_i^t$$

$$\min L \equiv \beta_i\left(P_i^t\right)^2 - \left(\beta_i P_i^{t-1} + \gamma_i\left(P_j^t - P_j^{t-1}\right) + \theta_i\left(x_i^t - x_i^{t-1}\right)\right) P_i^t + \lambda\left(\alpha_i - \beta_i P_i^t + \gamma_i \gamma P_j^t + \theta_i x_i^t - 1\right)$$

$$\frac{dL}{dP_i^t} = 2\beta_i P_i^t - \left(\beta_i P_i^{t-1} + \gamma_i\left(P_j^t - P_j^{t-1}\right) + \theta_i\left(x_i^t - x_i^{t-1}\right)\right) - \beta_i \lambda = 0 \tag{1}$$

$$\frac{dL}{dP_j^t} = -\gamma_i P_i^t + \gamma_i \lambda = 0 \tag{2}$$

$$\frac{dL}{x_i^t} = -\theta_i P_i^t + \theta_i \lambda = 0 \tag{3}$$

$$\frac{dL}{d\lambda} = \alpha - \beta_i P_i^t + \gamma_i P_j^t + \theta_i x_i^t - 1 = 0 \tag{4}$$

From Eqs. (2) and (3), we obtain Eq. (5):

$$P_i^t = \lambda \tag{5}$$

By replacing Eqs. (4) and (5), we obtain Eq. (6)

$$\lambda = \frac{\alpha_i + \gamma_i P_j^t + \theta_i x_i^t - 1}{\beta_i}$$

By replacing Eqs. (6) and (1), we obtain equation below:

$$2\beta_i P_i^t - \left(\beta_i P_i^{t-1} + \gamma_i\left(P_j^t - P_j^{t-1}\right) + \theta_i\left(x_i^t - x_i^{t-1}\right)\right) - \beta_i\left(\frac{\alpha_i + \gamma_i P_j^t + \theta_i x_i^t - 1}{\beta_i}\right) = 0$$

$$2\beta_i P_i^t - \beta_i P_i^{t-1} - \gamma_i P_j^t + \gamma_i P_j^{t-1} + \theta_i x_i^t - \theta_i x_i^{t-1} - \alpha_i - \gamma_i P_j^t - \theta_i x_i^t + 1 = 0$$

$$2\beta_i P_i^t - \beta P_i^{t-1} - 2\gamma_i P_j^t + \gamma_i P_j^{t-1} - 2\theta_i x_i^t + \theta_i x_i^{t-1} - \alpha_i + 1 = 0$$

$$2\beta_i P_i^t = \beta_i P_i^{t-1} + \gamma_i\left(2P_j^t - P_j^{t-1}\right) + \theta_i\left(2x_i^t - x_i^{t-1}\right) + \alpha_i - 1$$

$$P_{i,\det}^* = \frac{P_i^{t-1}}{2} + \frac{\gamma_i}{2\beta_i}\left(2P_j^t - P_j^{t-1}\right) + \frac{\theta_i}{2\beta_i}\left(2x_i^t - x_i^{t-1}\right) + \frac{\alpha_i - 1}{2\beta_i}$$

Appendix 2. Proof of Eq. (17)

$$f_{Sto} = \Delta d_{i,sto}\left(P_i^t\right)$$

$$f_{Sto} = \left(d_{i,sto}^{t-1} - d_{i,sto}^t\right)\left(P_i^t\right)$$

$$f_{Sto} = \frac{NC\left[\left(1-\alpha_{i,sto}+\beta_{i,sto}P_i^{t-1}+\alpha_{j,sto}+\beta_{j,sto}P_j^{t-1}\right)-\left(1-\alpha_{i,sto}+\beta_{i,sto}P_i^t+\alpha_{j,sto}+\beta_{j,sto}P_j^t\right)\right]}{2}\left(P_i^t\right)$$

$$f_{Sto} = \frac{NC}{2}\left[\beta_{i,sto}P_{i,sto}^t - \beta_{i,sto}P_i^{t-1} + \beta_{j,sto}\left(P_j^{t-1}-P_j^t\right)\right]\left(P_i^t\right)$$

$$\min f_{Sto} = \frac{NC}{2}\times\left[\beta_{i,sto}\left(P_i^t\right)^2 - \beta_{i,sto}P_i^{t-1}P_i^t + \beta_{j,sto}\left(P_j^{t-1}-P_j^t\right)P_i^t\right] + \lambda\left[-\alpha_{i,sto}+\beta_{i,sto}P_i^t+\alpha_{j,sto}+\beta_{j,sto}P_j^t\right]$$

$$\frac{dL}{dP_i^t} = 2\beta_{i,sto}P_i^t - \beta_{i,sto}P_i^{t-1} + \beta_{j,sto}\left(P_j^{t-1}-P_j^t\right) - \lambda\beta_{i,sto} = 0 \tag{1}$$

$$\frac{dL}{dP_j^t} = -\beta_{j,sto}P_i^t + \lambda\beta_{j,sto} = 0 \tag{2}$$

$$\frac{dL}{d\lambda} = -\alpha_i + \beta_{i,sto}P_i^t + \alpha_{j,sto} + \beta_{j,sto}P_j^t = 0 \tag{3}$$

By replacing Eqs. (2) and (3), we get λ^*, and P_i^* can be formulated as follows:

$$P_{i,sto}^* = \frac{P_i^{t-1}}{2} + \frac{\beta_{j,sto}}{2\beta_{i,sto}}\left(2P_j^t - P_j^{t-1}\right) + \frac{-\alpha_{i,sto}+\alpha_{j,sto}}{2\beta_{i,sto}}$$

Appendix 3. Proof of Eq. (18)

Optimal pricing model for hotel i:

$$P_{i,\det}^* = \frac{P_i^{t-1}}{2} + \frac{\gamma_i}{2\beta_i}\left(2P_j^t - P_j^{t-1}\right) + \frac{\theta_i}{2\beta_i}\left(2x_i^t - x_i^{t-1}\right) + \frac{\alpha_i - 1}{2\beta_i} \tag{1}$$

Optimal pricing model for hotel j:

$$P_{j,\det}^* = \frac{P_j^{t-1}}{2} + \frac{\gamma_j}{2\beta_j}\left(2P_i^t - P_i^{t-1}\right) + \frac{\theta_j}{2\beta_j}\left(2x_j^t - x_j^{t-1}\right) + \frac{\alpha_j - 1}{2\beta_j} \tag{2}$$

Equilibrium point will occur when we substitute Eqs. (1) and (2) so both hotels get their equilibrium point simultaneously.

$$P_{i,\det}^* = \frac{P_i^{t-1}}{2} + \frac{\gamma_i}{2\beta_i}\left[2\left(\frac{P_j^{t-1}}{2} + \frac{\gamma_j}{2\beta_j}\left(2P_i^t - P_i^{t-1}\right) + \frac{\theta_j}{2\beta_j}\left(2x_j^t - x_j^{t-1}\right) + \frac{\alpha_j - 1}{2\beta_j}\right) - P_j^{t-1}\right] + \frac{\theta_i}{2\beta_i}\left(2x_i^t - x_i^{t-1}\right) + \frac{\alpha_i - 1}{2\beta_i}$$

$$P_{i,\det}^{*}=\frac{P_i^{t-1}}{2}+\frac{\gamma_i}{2\beta_i}\left[2\left(\frac{P_j^{t-1}}{2}+\frac{\gamma_j}{2\beta_j}\left(2P_i^t-P_i^{t-1}\right)+\frac{\theta_j}{2\beta_j}\left(2x_j^t-x_j^{t-1}\right)+\frac{\alpha_j-1}{2\beta_j}\right)-P_j^{t-1}\right]+\frac{\theta_i}{2\beta_i}\left(2x_i^t-x_i^{t-1}\right)+\frac{\alpha_i-1}{2\beta_i}$$

$$\left(1-\frac{\gamma_i\gamma_j}{\beta_i\beta_j}\right)P_{i,\det}^{*}=\frac{1}{2}\left(1-\frac{\gamma_i\gamma_j}{\beta_i\beta_j}\right)P_i^{t-1}+\frac{\gamma_i\theta_j}{\beta_i\beta_j}\left(2x_j^t-x_j^{t-1}\right)+\frac{\theta_i}{2\beta_i}\left(2x_i^t-x_i^{t-1}\right)+\frac{\gamma_i\left(\alpha_j-1\right)}{2\beta_i\beta_j}+\frac{\alpha_i-1}{2\beta_i}$$

$$P_i^{*}=\frac{P_i^{t-1}}{2}+\frac{\gamma_i\left(\alpha_j-1\right)+\beta_j(\alpha_i-1)}{2(\beta_i\beta_j-\gamma_i\gamma_j)}+\frac{\gamma_i\theta_j\left(2x_j^t-x_j^{t-1}\right)+\beta_j\theta_i\left(2x_i^t-x_i^{t-1}\right)}{2(\beta_i\beta_j-\gamma_i\gamma_j)}$$

Appendix 4. Proof of Eq. (19)

Optimal pricing model for hotel i:

$$P_{i,sto}^{*}=\frac{P_i^{t-1}}{2}+\frac{\beta_{j,sto}}{2\beta_{i,sto}}\left(2P_j^t-P_j^{t-1}\right)+\frac{-\alpha_{i,sto}+\alpha_{j,sto}}{2\beta_{i,sto}} \tag{1}$$

Optimal pricing model for hotel j:

$$P_{j,sto}^{*}=\frac{P_j^{t-1}}{2}+\frac{\beta_{i,sto}}{2\beta_{j,sto}}\left(2P_i^t-P_i^{t-1}\right)+\frac{-\alpha_{j,sto}+\alpha_{i,sto}}{2\beta_{j,sto}} \tag{2}$$

Do the same step as before, assumed that competitor's price is fixed, we can obtain equilibrium point below:

$$P_{i,sto}^{*}=\frac{P_i^{t-1}}{2}+\frac{\beta_{j,sto}}{2\beta_{i,sto}}\left(\frac{P_j^{t-1}}{2}+\frac{\beta_{i,sto}}{2\beta_{j,sto}}P_i^t+\frac{-\alpha_{j,sto}+\alpha_{i,sto}}{2\beta_{j,sto}}\right)+\frac{-\alpha_{i,sto}+\alpha_{j,sto}}{2\beta_{i,sto}}$$

$$P_{i,sto}^{*}=\frac{2P_i^{t-1}}{3}+\frac{\beta_{j,sto}}{3\beta_{i,sto}}P_j^{t-1}+\frac{-\alpha_{i,sto}+\alpha_{j,sto}}{3\beta_{i,sto}}$$

Appendix 5. Revenue prediction

Hotel A

No.	Data		Deterministic model		Stochastic model	
	Occupancy rate	Revenue	Occupancy rate	Revenue	Occupancy rate	Revenue
1	0.34	2,300,000	0.33	2,200,000	0.47	3,100,000
2	0.09	600,000	0.33	2,200,000	0.13	900,000
3	0.61	3,757,500	0.52	3,100,417	0.37	2,214,583
4	0.42	2,522,500	0.51	3,111,500	0.37	2,222,500
5	0.28	1,762,500	0.50	3,066,071	0.36	2,164,286
6	0.42	2,475,000	0.52	3,182,143	0.69	4,154,464
7	0.06	400,000	0.33	2,200,000	0.10	700,000
8	0.25	1,500,000	0.56	3,293,333	0.31	1,820,000
9	0.94	5,522,000	0.54	3,155,428	1.04	5,987,150
10	0.94	5,522,000	0.53	3,155,428	0.76	4,494,889
11	0.06	400,000	0.32	2,200,000	0.13	800,000
12	0.81	4,887,500	0.48	2,986,806	0.44	2,715,278
13	0.45	2,720,000	0.38	2,340,000	0.45	2,700,000
14	0.30	1,777,500	0.40	2,399,625	0.27	1,599,750
15	0.31	1,877,500	0.39	2,399,625	0.29	1,777,500
16	0.18	1,000,000	0.51	2,863,636	0.33	1,800,000
17	0.24	1,425,000	0.43	2,509,615	0.16	951,923
18	0.31	2,100,000	0.20	1,400,000	0.05	400,000
19	0.09	600,000	0.20	1,400,000	0.17	1,200,000
20	0.12	800,000	0.20	1,400,000	0.06	400,000
21	0.21	1,400,000	0.20	1,400,000	0.37	2,500,000
22	0.31	2,100,000	0.20	1,400,000	0.04	300,000
23	0.12	800,000	0.20	1,400,000	0.39	2,600,000
24	0.45	2,727,500	0.37	2,337,593	0.44	2,607,315
25	0.61	3,650,000	0.40	2,396,250	0.42	2,485,000
26	0.27	1,800,000	0.20	1,400,000	0.16	1,100,000
27	0.78	4,635,000	0.39	2,317,500	0.65	3,832,789
28	0.19	1,125,000	0.43	2,509,615	0.41	2,336,538
29	0.19	1,175,000	0.43	2,509,615	0.43	2,509,615
30	0.19	1,175,000	0.43	2,509,615	0.34	1,990,385
31	0.19	1,212,500	0.34	2,117,045	0.19	1,196,591
32	0.51	3,282,500	0.35	2,198,571	0.75	4,588,750
33	0.16	1,100,000	0.20	1,400,000	0.45	3,000,000
34	0.79	4,770,000	0.37	2,250,000	0.49	2,970,000
35	0.31	1,870,000	0.51	3,116,667	0.31	1,870,000
36	0.96	5,100,000	0.67	3,665,625	0.91	5,145,313
37	0.19	1,000,000	0.72	3,815,385	0.99	5,515,385
38	1.00	5,600,000	0.66	3,600,000	0.85	4,740,000
39	0.34	2,002,000	0.55	3,220,609	0.31	1,827,913
40	0.34	2,002,000	0.55	3,220,609	0.37	2,176,087
41	0.57	3,245,000	0.58	3,330,395	0.46	2,647,237
42	0.6	3,465,000	0.56	3,265,000	0.74	4,310,131
43	0.66	3,947,500	0.51	3,042,209	0.47	2,773,779
44	0.24	1,462,500	0.47	2,925,000	0.18	1,096,875
45	0.60	3,240,000	0.66	3,623,077	1.00	5,764,615
46	0.60	3,260,000	0.66	3,623,684	0.88	4,965,263
47	0.34	1,860,000	0.65	3,558,261	0.86	4,781,739
48	0.55	3,160,500	0.57	3,331,338	0.48	2,733,405
49	0.60	3,318,000	0.61	3,483,900	0.49	2,737,350
50	0.42	2,486,000	0.51	3,107,500	0.29	1,775,714
51	0.78	4,680,000	0.49	2,970,000	0.74	4,485,134
52	0.78	4,680,000	0.50	3,060,000	0.59	3,510,000
53	0.67	4,037,500	0.50	3,042,614	0.72	4,295,455
54	0.79	4,436,500	0.60	3,432,010	1.16	5,488,723
55	0.37	2,250,000	0.50	3,060,000	0.41	2,430,000

No.	Data		Deterministic model		Stochastic model	
	Occupancy rate	Revenue	Occupancy rate	Revenue	Occupancy rate	Revenue
56	0.75	4,000,000	0.67	3,600,000	0.80	4,440,000
57	0.72	4,337,500	0.38	2,326,705	0.66	3,937,500
58	0.82	5,037,500	0.38	2,326,705	0.65	3,937,500
59	0.93	5,557,500	0.38	2,330,565	0.85	5,192,218
60	0.57	3,385,000	0.40	2,397,162	0.57	3,373,784
61	0.22	1,425,000	0.31	2,062,500	0.23	1,406,250
62	0.33	1,935,000	0.40	2,462,727	0.41	2,374,773
63	0.19	1,300,000	0.20	1,400,000	0.19	1,200,000
64	0.36	2,237,500	0.40	2,386,607	0.41	2,386,607
65	0.30	1,857,500	0.41	2,467,500	0.31	1,850,625
66	0.16	962,500	0.54	2,973,214	0.30	1,607,143
67	0.07	500,000	0.20	1,400,000	0.14	900,000
68	0.19	1,125,000	0.43	2,509,615	0.18	1,038,462
69	0.43	2,585,000	0.40	2,473,333	0.34	1,943,333
70	0.19	1,300,000	0.32	2,200,000	0.12	800,000
71	0.12	800,000	0.20	1,400,000	0.08	500,000
72	0.18	1,100,000	0.21	1,400,000	0.16	1,100,000
73	0.12	700,000	0.20	1,400,000	0.05	300,000
74	0.34	2,300,000	0.20	1,400,000	0.03	200,000
75	0.39	2,600,000	0.20	1,400,000	0.38	2,500,000
76	0.97	5,802,500	0.47	2,714,783	1.47	4,132,174
77	0.21	1,400,000	0.21	1,400,000	0.43	2,900,000
78	0.48	2,900,000	0.55	2,960,000	0.79	4,340,000
79	0.36	2,217,500	0.40	2,392,031	0.41	2,480,625
80	0.24	1,600,000	0.20	1,400,000	0.14	1,000,000
81	0.09	600,000	0.20	1,400,000	0.14	900,000
82	0.46	2,850,000	0.38	2,340,000	0.72	4,320,000
83	0.19	1,300,000	0.20	1,400,000	0.06	400,000
84	0.3	2,000,000	0.20	1,400,000	0.18	1,200,000
85	0.21	1,400,000	0.21	1,400,000	0.14	1,000,000
86	0.06	400,000	0.20	1,400,000	0.05	400,000
87	0.99	6,000,000	0.38	2,340,000	0.86	5,310,000
88	0.25	1,495,000	0.42	2,528,438	0.42	2,441,250
89	0.16	945,000	0.45	2,577,273	0.57	3,264,545
90	0.16	840,000	0.61	3,130,909	0.39	1,985,455
91	0.15	1,000,000	0.20	1,400,000	0.01	-
92	0.93	5,600,000	0.37	2,250,000	0.42	2,520,000
93	0.07	500,000	0.20	1,400,000	0.06	400,000
94	0.09	600,000	0.20	1,400,000	0.04	300,000
95	0.07	500,000	0.20	1,400,000	0.07	500,000
96	0.21	1,290,000	0.37	2,250,000	0.11	630,000
97	0.06	400,000	0.20	1,400,000	0.07	500,000
98	0.82	4,982,500	0.38	2,329,364	0.59	3,583,636
99	0.24	1,320,000	0.58	3,060,000	0.52	2,746,154
100	0.07	500,000	0.20	1,400,000	0.05	300,000
101	0.46	2,775,000	0.40	2,386,607	0.33	1,944,643
102	0.06	400,000	0.20	1,400,000	0.06	400,000
103	0.06	400,000	0.21	1,400,000	0.10	700,000
104	0.07	500,000	0.32	2,200,000	0.03	200,000
105	0.37	2,250,000	0.49	3,060,000	0.50	2,970,000
106	0.76	4,810,000	0.50	3,060,000	0.76	4,610,000
107	0.55	3,362,500	0.48	2,990,625	0.54	3,262,500
108	0.54	3,262,500	0.48	2,990,625	0.54	3,262,500
109	0.19	1,125,000	0.55	3,288,461	0.16	951,923
110	0.66	3,982,500	0.49	2,986,875	0.58	3,529,943
111	0.34	2,047,500	0.51	3,115,761	0.46	2,759,674
112	0.67	3,705,000	0.63	3,458,000	0.99	5,852,000
113	0.45	2,700,000	0.50	3,060,000	0.89	5,510,000
114	0.84	5,017,500	0.51	3,046,339	1.34	5,315,646
115	0.75	4,520,000	0.50	3,060,000	0.63	3,780,000
116	0.64	3,892,500	0.49	2,987,268	0.56	3,439,884

Hotel B

No.	Data		Deterministic model		Stochastic model	
	Occupancy rate	Revenue	Occupancy rate	Revenue	Occupancy rate	Revenue
1	0.80	4,120,500	0.50	2,373,000	0.67	3,413,000
2	0.17	795,000	0.44	2,205,000	0.13	630,000
3	0.13	506,500	0.67	2,906,125	0.37	1,392,875
4	0.27	1,043,750	0.65	2,798,594	0.32	1,304,688
5	0.23	1,076,250	0.53	2,467,500	0.15	615,000
6	0.73	3,019,500	0.62	2,709,000	0.46	1,921,500
7	0.20	863,500	0.50	2,329,833	0.16	719,583
8	0.13	703,500	0.53	2,556,750	0.07	315,000
9	0.77	3,126,000	0.63	2,869,370	0.67	2,869,370
10	0.83	2,883,000	0.74	3,074,160	1.01	4,176,660
11	0.83	2,883,000	0.65	2,601,660	0.76	3,074,160
12	0.97	2,888,000	0.80	3,085,035	1.34	4,030,035
13	0.20	904,000	0.42	1,958,667	0.20	904,000
14	0.13	570,000	0.47	1,995,000	0.16	712,500
15	0.40	1,400,000	0.60	2,304,167	0.42	1,516,667
16	0.43	1,525,000	0.66	2,680,000	0.28	906,667
17	0.13	450,000	0.63	2,610,000	0.21	675,000
18	0.17	558,500	0.54	2,127,900	0.43	1,497,900
19	1.00	3,520,000	0.51	1,997,833	0.92	3,730,333
20	0.13	505,500	0.47	1,800,375	0.19	758,250
21	0.90	3,700,000	0.41	1,781,482	0.74	3,178,519
22	1.00	3,057,500	0.59	2,168,000	1.27	4,058,000
23	1.00	1,005,000	0.41	1,798,333	0.73	3,373,333
24	0.57	2,120,000	0.56	2,333,750	0.58	2,333,750
25	0.10	411,500	0.49	2,077,833	0.29	1,234,500
26	0.17	731,500	0.40	1,680,000	0.28	1,120,000
27	0.90	3,090,500	0.60	2,476,056	1.03	4,208,556
28	0.90	2,862,500	0.67	2,532,222	0.68	2,689,722
29	0.87	2,862,500	0.65	2,581,154	0.63	2,423,654
30	0.8	2,477,000	0.68	2,656,000	0.65	2,341,000
31	0.13	580,000	0.43	1,885,000	0.13	580,000
32	1.00	4,007,500	0.49	2,051,583	0.76	3,311,583
33	1.00	4,287,500	0.38	1,715,000	0.71	3,103,333
34	1.00	3,120,000	0.65	2,508,000	1.30	4,083,000
35	0.13	600,000	0.55	2,572,500	0.13	600,000
36	0.20	810,000	0.70	3,015,000	0.25	945,000
37	1.00	4,200,000	0.69	3,062,500	0.20	840,000
38	0.73	2,150,000	0.88	3,535,227	0.88	3,377,727
39	0.13	560,000	0.62	2,747,500	0.16	700,000
40	0.30	1,318,000	0.66	3,871,000	0.27	1,048,000
41	0.10	418,000	0.63	2,895,667	0.21	836,000
42	0.80	2,980,000	0.72	3,076,087	0.66	2,746,087
43	0.10	425,000	0.59	2,613,333	0.29	1,275,000
44	0.13	530,000	0.62	2,667,500	0.19	795,000
45	0.73	2,929,000	0.70	3,148,273	0.33	1,331,364
46	0.70	2,620,000	0.74	3,196,905	0.42	1,621,905
47	0.67	2,900,000	0.64	2,975,000	0.15	725,000
48	0.60	2,020,000	0.79	3,402,353	0.67	2,702,353
49	0.43	1,406,000	0.81	3,345,346	0.54	1,927,846
50	0.47	1,501,000	0.77	3,176,571	0.60	2,231,571
51	0.10	445,000	0.56	2,549,167	0.36	1,631,667
52	0.37	1,610,000	0.64	3,074,167	0.56	2,509,167
53	0.77	3,275,000	0.66	2,793,500	0.72	3,114,500
54	0.80	3,362,500	0.65	2,962,283	0.43	1,807,283
55	0.37	1,522,500	0.62	2,754,750	0.33	1,357,500
56	0.27	1,117,500	0.69	3,088,929	0.22	816,429
57	0.80	3,092,500	0.57	2,458,500	0.86	3,798,500
58	0.57	2,797,000	0.55	2,542,875	0.74	3,589,750

No.	Data		Deterministic model		Stochastic model	
	Occupancy rate	Revenue	Occupancy rate	Revenue	Occupancy rate	Revenue
59	0.20	1,030,000	0.38	1,890,000	0.28	1,260,000
60	0.17	1,185,000	0.39	1,890,000	0.17	787,500
61	0.13	690,000	0.35	1,732,500	0.12	630,000
62	1.00	3,415,000	0.71	2,961,460	0.92	4,002,888
63	0.40	2,070,000	0.42	1,755,000	0.40	1,620,000
64	0.37	1,657,500	0.52	2,217,900	0.32	1,323,000
65	0.23	1,400,000	0.51	2,524,000	0.22	945,000
66	0.23	1,230,000	0.57	2,651,250	0.09	405,000
67	0.83	2,925,000	0.51	1,993,500	0.76	3,096,000
68	0.17	601,000	0.59	2,350,100	0.18	601,000
69	0.13	546,000	0.50	2,089,500	0.22	955,500
70	0.20	800,000	0.56	2,363,333	0.27	1,066,667
71	0.27	1,190,000	0.49	1,911,667	0.31	1,095,000
72	0.13	775,000	0.31	1,575,000	0.15	787,500
73	0.40	2,165,000	0.55	2,377,000	0.47	1,793,500
74	0.77	5,245,000	0.60	2,870,909	1.08	5,830,000
75	0.60	3,220,000	0.38	1,710,000	0.61	2,783,333
76	0.73	4,392,500	0.42	2,047,500	0.23	1,102,500
77	0.63	3,902,500	0.31	1,575,000	0.41	1,890,000
78	0.33	1,946,250	0.45	2,242,125	0.02	157,500
79	0.17	1,026,250	0.39	1,890,000	0.12	630,000
80	0.13	573,500	0.38	1,720,500	0.23	1,003,625
81	0.43	1,682,000	0.45	1,839,500	0.38	1,423,231
82	0.67	2,894,000	0.45	2,025,800	0.41	1,736,400
83	0.13	486,500	0.49	1,896,125	0.26	973,000
84	0.27	1,076,500	0.42	1,749,313	0.39	1,614,750
85	0.13	586,500	0.36	1,612,875	0.20	879,750
86	0.10	410,000	0.41	1,776,667	0.11	410,000
87	0.53	2,192,000	0.49	2,075,500	0.66	2,863,000
88	0.43	1,752,000	0.52	2,224,500	0.26	1,078,154
89	0.70	2,893,000	0.51	2,243,667	0.29	1,239,857
90	0.37	1,420,000	0.62	2,623,182	0.14	516,364
91	0.10	306,000	0.59	2,169,000	0.24	714,000
92	0.40	1,477,000	0.63	2,368,000	0.91	3,588,000
93	0.23	849,750	0.49	1,893,107	0.24	849,750
94	0.10	375,000	0.47	1,940,000	0.15	500,000
95	0.17	660,000	0.44	1,873,500	0.17	660,000
96	0.13	440,000	0.62	2,422,500	0.23	770,000
97	0.17	663,500	0.43	1,882,600	0.16	663,500
98	0.33	1,541,000	0.52	2,189,143	0.56	2,353,357
99	0.33	1,515,000	0.50	2,430,000	0.05	150,000
100	0.10	395,000	0.44	1,869,167	0.12	526,667
101	0.10	405,000	0.51	2,227,500	0.23	945,000
102	0.10	435,000	0.37	1,740,000	0.10	435,000
103	0.17	747,000	0.35	1,633,500	0.13	594,000
104	0.23	855,000	0.67	2,643,000	0.27	888,000
105	1.00	3,420,000	0.73	2,943,000	0.87	3,573,000
106	0.53	2,285,000	0.60	2,745,000	0.53	2,375,000
107	1.00	3,420,000	0.72	2,943,000	1.01	4,203,000
108	1.00	3,420,000	0.72	2,943,000	1.00	4,203,000
109	0.13	600,000	0.72	3,240,000	0.16	600,000
110	0.57	2,390,000	0.65	3,523,000	0.65	3,288,000
111	0.57	2,390,000	0.66	3,523,000	0.45	1,878,000
112	0.87	3,277,500	0.72	3,056,250	0.55	2,111,250
113	1.00	4,280,000	0.58	2,627,333	0.56	2,469,833
114	1.00	4,640,000	0.53	2,480,333	0.50	2,322,833
115	0.10	670,000	0.51	2,783,333	0.22	945,000
116	0.13	975,000	0.51	2,865,000	0.21	945,000