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네트워크 외부효과를 고려한 두 단계 공급체인에서의 신기술 도입과 확산속도에 대한 연구 : 구매자-공급자간 관계 요인에 대한 모형*

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Adoption and Diffusion Speed of New Technology with Network Externality in a Two-level Supply Chain : An Approach to Relative Factors in Buyer-Supplier Relationships

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■ Abstract ■

This paper develops a model to predict the adoption and level of usage of network technology in a two-level supply chain with buyer-supplier relationships. A firm's adoption of a new technology depends not only on its own beliefs of the new technology's costs and benefits, but also on the adoption decisions of other firms in the supply chain. A model first analyzes an individual supplier's decision about a new technology adoption considering with multiple suppliers and buyers. Individual suppliers' decisions are aggregated with a population model to project how new technology diffuses across the supply chain and examine the pattern of diffusion process. This study found that as more firms adopt in initial periods, the total amount of information to the potential adopters in the population increases, and then the number of firms persuaded by the information increases as the process moves up the distribution of adoption process. We consider three factors influencing the diffusion speed of the new technology in a supply chain network : mean benefits, cost sharing, and information provision. This study examines how such factors affect the reduction of threshold levels, which implies that reductions in threshold levels have an aggregate effect by accelerating the rate of adoption. In particular, we explore relationship factors available in practice in a buyer-supplier relationship and numerically examines how these relationship factors contribute to increase the diffusion speed of the technology in a two-level supply chain

Keyword : Technology Adoption, Diffusion Speed, Buyer-Supplier Relationships, Two-Level Supply Chain, Network Externalities

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1. Introduction

Over the past decade, network technologies such as electronic data interchange (EDI), point of sales (POS), and radio frequency identification (RFID) have been promoted with the premised benefits in a supply chain. Such technologies cause significant changes internally in processes, work routine, and the patterns of interaction among units within an organization [1, 4] and among organizations as well. At the firm level, firms must consider several factors before making a decision to adopt a new technology: should they adopt a new technology now or wait and see how others do? Or should they wait for other more advanced technology? Will the benefits of the new technology outweigh the costs? The diffusion of new technologies or innovations in the network occurs through the learning process by information and knowledge transfer by the embedded organizations [35]. That is, potential adopters are largely influenced by prior adopters, and the level of influence by them may be different by various factors. These reasons may lead to different adoption timing. The motivation of this research starts from following research questions. How do firms make a decision to adopt new network technologies in a network with multiple levels? Why do they adopt it at different points of time across a population? What are the factors that stimulate the diffusion process of technologies in a supply chain network? What relationship climate factors in a buyer-supplier supply chain are important for predicting the adoption and usage of technology?

We first develops the decision model to explore inter-organizational relationship factors to predict the adoption and level of usage of network technology in a buyer-supplier relationship. The model focuses on a supplier's adoption decision in a buyer-supplier relationship. A supplier's adoption decision of a new technology depends not only on its own beliefs about the benefits of technology, but also on the adoption decisions of other suppliers and the size of adoptee buyers by network externality in the network. Potential adopters update their beliefs about the potential of the technology through the observed outcomes by information collected from others in the network. Through information flows, uncertainties about the benefits of technology will be reduced over time. In each period, potential adopters face adoption decisions with the updated estimate of adoption benefit. In particular, we considers two relationship factors available in practice between organizations to observe the effects of these factors on the adoption process in a buyer-supplier relationship; the dependence on the others (i.e. the supplier's dependence level on a buyer) and the openness for information sharing (i.e. the supplier's willingness to share information). Considering all the components of the estimated benefit above, we derive an individual firm's decision model to identify how a firm decides to adopt the technology in a two-level supply chain. Then, by aggregating dynamics from individual firms' decisions, we see how the technology diffuses across firms and examine under what conditions are important to make to speed up the diffusion process. The population model actually allows considering the effect of several strategies observed in practice and can yield some valuable managerial insights contribute to increase the adoption rates of the technology in a two-level supply chain.

In the second part of the paper, we seek several factors influencing the diffusion speed of the new technology in a supply chain network. More importantly, we consider about how to push up the time that the process begins to accelerate and which factors are able to make to speed up the diffusion process in the initial periods. This part also introduces relationship factors available in practice in a buyer-supplier relationship: the dependence on a trading partner (i.e. the supplier's dependence level on a buyer) and the openness for information sharing (i.e. the supplier's willingness to share information with other suppliers). By these two relationship factors, four different types of firms can be categorized in a set of firms of a major trading partner. These findings will provide some valuable insights for firms who wish to improve the adoption rates of their trading partners in a two-level supply chain. For example, a firm can use either persuasive or coercive approaches to encourage their trading partners' adoptions, depending on their types.

This study follows a stream of technology adoption literature and makes a contribution to it. Many prior studies showed that the adoption decision rules are determined through information-updating process for uncertain benefits of the new technology. In particular, this study has similar research questions and modeling approach as the literature by Chatterjee and Eliashberg [8]. They presented a model to explicitly capture the effect of uncertainty on the firm's utility and to aggregate individual firms' decisions to produce a diffusion curve. They also introduce a stochastic element to the diffusion process. Recently, Whang [36] considered adoption timing of RFID technology in a supply chain as another related paper. He proposed that the cost split will be able to speed up the retailer's adoption under some conditions. That is, he showed that the equal-cost-split arrangement always induces the upstream (suppliers) to adopt RFID earlier than under no cost sharing. But this study differs from theirs in that none of these papers consider the effect of other adoptee firms on the availability of information signals. The impact of firms' adoption decisions on the others is new to this literature. For the inter- organizational technology, none of existing literature consider relationship factors to predict the adoption of the technology by a set of suppliers and buyers in a multiple level of supply chain. In this study, some relationship factors available in practice are considered by a set of suppliers of a major trading partner (buyer) in a buyer-supplier relationship and are examined how these relationship factors contribute to increase the diffusion speed of the technology in a two-level supply chain.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the individual adoption model and develops it into a population model. Section 4 analyzes factors influencing the diffusion speed of new technology in a two-level supply chain. Section 5 explores relationship factors leading to technology adoption in a buyer-supplier relationship. We conclude and discuss future research direction in §6.

2. Literature Review

There has been considerable research for the adoption and diffusion of technologies and innovations. First, many prior studies showed that the adoption decision rules are determined through information-updating process for uncertain benefits of the new technology. The Bayesian modeling approach to conceptualizing the information integration process has been employed in many models for technology adoption. Oren and Schwartz [27] shows that consumers are Bayesian in that they update their uncertainties by combining the observed outcomes with their prior uncertainty according to the beta-Bernoulli updating. They also consider that the consumer will adopt once passing a risk-aversion threshold. McCardle [26] and Ulu and Smith [34] presented that the firm updates its distribution of the firm's belief after collecting a piece of information, accounting for the new information in a Bayesian fashion. McCardle [26] considered the technology adoption decision for a firm using a dynamic model where, in each period, information can be purchased to update the estimate of adoption benefit. Ulu and Smith [34] recently extended that model to consider general probability distributions for benefit and general information signals. Chatterjee and Eliashberg [15] adopt a similar approach to Oren and Schwartz [27]. They applied normal-normal Bayesian updating for the potential adopter's perception of the innovation as one unit of new information is received. They also introduce a stochastic element to the diffusion process. Eliashberg and Chatterjee [8] has more comprehensive treatment of the stochastic models in diffusion. In our model, the firm's benefits from an adoption depend upon one's belief updated by the total amount of information from previous adopters. This study enrich the above literature by revealing the possibility of the effects of network externality in the multi-level supply chain and considering relationship factors available in practice in buyer-supplier relationships.

Second, a firm's adoption of an innovation in network may give rise to positive externalities. The concept of network externality has been applied in the economics literature earlier. Katz and Shapiro [18] and Farrell and Saloner [10] consider the choice of standards and technology using the game-theoretic models. All of these studies developed decision problems through the welfare effects of aggregate behavior. As a recent literature for technology choice considering positive network effects, Kornish [20] have studied the decision problem facing a consumer with a choice between two competing technologies each subject to positive network benefits, using a decision-theoretic model and a stochastic process that captures the dynamics of market share of two competing technologies. This study considers that adoption can generate information flows across a same population group which may spill over to the rest of the industry, and the technology have network characteristics that give rise to positive externalities between two population groups. The firm's benefits directly depend on the number of adopters in other population group connected through the technology on supply chain network. In addition, the number of adopters in the same population by any given time will indirectly affect the adoption decision, even though some firms are not connected directly in the network.

Lastly, for the inter-organizational relationships between firms in the context of technology adoption literatures, many supply chain management literatures have applied the power source literature to the analysis of marketing channel relationships, finding that the different bases of power will affect inter-firm relationships in significant yet contrasting ways. Existing study has shown the factors affecting technology adoption in inter-organization and how the buyer-supplier related IT infrastructure influences the firm's performance [23, 24, 33]. For the dependence on the other firm, the review of the marketing power literature indicates the significant effects of power upon inter-firm relationships in a supply chain. Maloni and Benton [25] and Benton and Maloni [3] study the power influences in the supply chain from empirical tests. They provide that the power-affected buyer-supplier relationship was found to have a significant positive effect on both performance and satisfaction from the test. Hart and Saunders [15] have developed a theoretical framework, positing relative power and trust between trading partners as determinants of EDI adoption and usage. They empirically test that the greater the sales revenue from a retailer, the more dependent the supplier is on that retailer, so its power level is low. In this paper, a supplier's degree of power is defined as the measure of dependence on a buyer population. For the openness for information sharing, there are several literature in this field. The concept of the threshold model in this paper is similar to that of Rogers and Kincaid [32] who consider that an individual is more likely to adopt an innovation if more of the other individuals in his or her personal (local) network have adopted previously. Most of prior threshold models assume that all potential adopters receive the same amount of information by previous adopters in a population. In this case, firm's beliefs are primarily driven by the attributes of individuals. In the network, however, individuals' beliefs can be explained by the level of openness for information and knowledge sharing through the patterns of interactions between them. That is, firms with a high level of openness are more likely to receive the information and knowledge from others. Therefore firms' threshold values are distributed heterogeneously across the population, and then it enables to derive dynamics for the distribution of the threshold values to see the shape of adoption curve over time.

3. Models

This paper consider a two-level supply chain network with multiple suppliers and buyers, N buyers and M suppliers. Firms in each population may have different benefits and costs from an adoption based on their own network sizes. Individual firm's network size is defined by the number of trading partners in the other population group. That is, one buyer (supplier) may have more or less suppliers (buyers) than other buyers (suppliers) at any given time. This paper focuses on a supplier's adoption decision model in a buyer-supplier relationship. We assume that the buyer's adoption process exogenously given over period, which is commonly known.

3.1 The Benefits

As already defined it earlier, the firm's benefits from an adoption come from two sources: internal benefits and coordination benefits from trading partners. Firm's benefits are determined by the prior beliefs about the benefits from an adoption and her own network size by externalities. It is usually considered that larger network size yields greater returns by network externalities [9]. In the model derived below, the supplier's network size is defined by the combination of its dependence level and the number of linked users on a buyer population. Only network size at the date of adoption is considered and the size may change over time in such a way that the whole benefit distribution not only shifts but also changes shape of distribution.

The supplier j's network size is $Q_j(t)=w_j n_{jt}$ where w_j is the supplier j's fraction of his dependence level, which is defined as the ability of one firm to affect the intentions and actions of firms on the other side. We assume that it is known but differ across firms by (0, 1). n_{jt} is the number of buyers who have adopted by time t, which is exogenously determined, and linked with supplier j, $j=1,\cdots,M$. m and n_{jt} are commonly known to all firms at the beginning of each period. The supplier's benefits depend upon their network size and belief about benefits from an adoption. The supplier j's benefit function is given by

$$B_j(t)=\Pi_M(t,\ Q_j(t),\ \mu_{jt})=w_j n_{jt}\mu_{jt}, \tag{1}$$

where $j=1,\cdots,M$. μ_{jt} represents the supplier j's beliefs about unknown benefits at time period t. It will be updated by information coming from prior adopters over time. We assume that the supplier's benefits from adoption are normally distributed with unknown mean $\mu(>0)$ and variance σ^2, which is independent and identically distributed among suppliers and time periods.

3.2 The Costs

The cost structure simply consists of two components: fixed and variable costs. The fixed costs are incurred once at the time of installation on both buyers and suppliers. There is no variable cost for buyers and only supplier charges variable costs for tags continuously. c_f is the fixed costs and c_v is the variable costs of the supplier. It assumes that there is no any discount rate, so the installation costs are same through all periods across all suppliers. In general, discounting factor might significantly affect the benefit-cost structure, but it would distract from the main analysis because this study focuses on aggregate dynamics based on the individual firms' myopic adoption decision of the new technology at any given time.[1)] The supplier's costs are defined as

$$C_j(t)=c_f+n_{jt}c_v, \tag{2}$$

where n_{jt} is the number of buyers who have connected with a supplier j and already adopted by time t. The adoption costs, c_f, are also same at every time period across all suppliers, whereas they all may have different variable costs for tags by different size of network n_{jt}, which is the number of buyers.[2)] In this model, suppliers

1) Focusing upon the long-term decision about the technology adoption, we need to construct the decision model considering net present value, the discount rate and return on investment. But, in this paper, the discount rate is ignored because this study considers the individuals' myopic decision models at any given time and also focus on the impacts of the perception updated from information and the network size on firms' decisions.

2) In general, the number of items or pallets should be considered for the tag costs, but only the number of linked buyers is considered because we have already assumed that all buyers are identical in size. If this assumption is relaxed, the problem will get more complicated because all firms have different produced items at different times.

are not allowed to make change their decisions. In other words, suppliers adopt it once and they do not switch back to the existing legacy system within the time frame of the analysis. This is a reasonable assumption for many technology markets under consideration, particularly when the goods, in general, are durable and switching costs are relatively high.

3.3 The Adoption Decision

This section provides the adoption decision rule for the supplier, based on the benefit-cost structure provided from previous sections. At any given time t, a supplier will have seen prior outcomes from other priors in the same population. In a continuous time phase, the total prior outcomes by time t are measured by the cumulative amount of information generated by all prior adopters. Let $p(t)$ be the proportion of adopters at time t, and set that $p(0)=0$. I can simply define the cumulative amount of information generated by all prior adopters by time t which is given by the integral under the adoption curve, namely,

$$q(t)=\int_0^t p(s)ds.$$

To make a decision, suppose that each supplier has a prior belief about the benefits with unknown mean μ and unknown precision $\rho(=1/\sigma^2)$. For each value of ρ, the conditional distribution of μ is normally distributed with mean μ_{j0} and precision $\rho\tau_j$ [6]. Low values of μ_{j0} mean pessimism about the benefits from the adoption and low values of $\rho\tau_j$ reflect flexibility in beliefs. At starting point of the process, if supplier j's benefits are initially larger than the costs, he will want to adopt. Otherwise, he will not adopt now and update the beliefs about the benefits by information flow. The supplier updates the perception of the technology and the uncertainties about it are reduced over time. Assuming that the supplier is equally likely to see any outcome from buyers who have already adopted it, supplier's information flow can be modeled by followings. Let $\widetilde{M}_{jt}$ be the number of supplier j's observations by time t, and the expected number of supplier j's observations by time t is $E[\widetilde{M}_{jt}]=\alpha_j q(t)$, where the parameter α_j represents the degree of openness which is the willingness to share information in a same population (among suppliers). $q(t)=\int_0^t p(s)ds$ is the integral under the adoption curve, which means the cumulative information collected from all suppliers who have already adopted by time t. That is, they may have different weights of α_j and will have different amount of information by their different degree of openness for information and knowledge sharing.

Let I_{jt} be the realization of $\widetilde{M}_{jt}$ and $\overline{m}_{jt}$ be the mean of the benefits among I_{jt} observations at time t. I assume that $\overline{m}_{jt}$ is normally distributed with mean μ and variance σ^2/I_{jt}. By the normal-normal updating process, supplier j's posterior estimate of the mean benefits, μ_{jt}, can be defined as the weighted average of the prior and the observed mean [6], namely,

$$\mu_{jt}=(I_{jt}\overline{m}_{jt}+\tau_j\mu_{j0})/(I_{jt}+\tau_j).$$

Substituting this into (1), supplier j's benefits from an adoption at period t is $B_j(t)=w_j n_{jt}\cdot(I_{jt}\overline{m}_{jt}+\tau_j\mu_{j0})/(I_{jt}+\tau_j)$. If supplier j is myopic, then he will adopt when the benefits are at least equal to the fixed adoption costs c_f, which is,

$$w_j n_{jt}(I_{jt}\overline{m}_{jt}+\tau_j\mu_{j0}) \geq (c_f+n_{jt}c_v)(I_{jt}+\tau_j).$$

Suppose that $\overline{m}_{jt}$ follows the standard normal distribution with mean 0 and standard deviation 1, so it can be rewritten as $\overline{m}_{jt}=\mu+(\sigma z_{jt}/\sqrt{I_{jt}})$. As defined earlier, $I_{jt}=\alpha_j q(t)$, thus rearranging this inequality,

$$q(t) \geq \frac{((c_f+n_{jt}c_v)-w_j n_{jt}\mu_{jo})\tau_j}{\alpha_j(w_j n_{jt}\mu-(c_f+n_{jt}c_v))} - \frac{w_j n_{jt}\sigma\sqrt{q(t)}}{\sqrt{\alpha_j}(w_j n_{jt}\mu-(c_f+n_{jt}c_v))}z_{jt}.$$

Supplier j's threshold level of the amount of information at time t is defined as r_{jt}, which is the first term of the right-hand side of this inequality ,

$$r_{jt}=\frac{(C_{jt}-w_j n_{jt}\mu_{j0})\tau_j}{\alpha_j(w_j n_{jt}\mu-C_{jt})}. \tag{3}$$

From (3), supplier j adopts as $q(t)$ passes the marginal information level r_{jt}. By observation, firms with high threshold values are pessimistic about the technology: low initial belief for the technology or small number of previous adopters (large values of $(c_f+n_{jt}c_v)-w_j n_{jt}\mu_{j0}$), high level of uncertainty (large τ_j), low marginal profits (low values of $w_j n_{jt}\mu-(c_f+n_{jt}c_v)$), and relatively uninformed knowledge from previous adopters (low α_j).

3.4 Aggregating Dynamics

The myopic decision rule has been developed from the updating process of individual's belief about the benefits of technology adoption. Suppliers adopt when they see certain positive outcomes from the information of previous adopters. That is, a supplier adopts once getting its marginal information levels for adopting the technology, which is called by the threshold levels. Based on the individual's adoption decision in the previous section, this section is to embed it into a population model to see how the technology diffuses.

Let $\phi(q)$ be the probability that the firm is ready to adopt the technology, given that the cumulated information collected by the prior adopters is q. In other words, this is the probability that a firm's information thresholds have been exceeded by the total amount of information from the previous adopters, $\phi(q)=\Pr(r \leq q)$. Suppose that supplier j's number of observations, I_{jt}, is Poisson distributed with mean $\alpha_j q$ and $\overline{M}_{jt}$ is normally distributed with mean μ and variance σ^2/d, given any observation $I_{jt}=d(>0)$. The probability that supplier $j(=1,\cdots,M)$'s threshold level exceeds the amount of information by time t is

$$\phi_j(q(t))=\sum_{d=1}^{\infty}\Pr(I_{jt}=d|\alpha_j q)\cdot\phi(\bullet) \tag{4}$$

where ϕ is the standard normal cumulative distribution function.[3)]

We next see the aggregate dynamics of this process. The dynamics can be determined by the distribution of the threshold levels in the population. Let $F(q)$ be the cumulative distribution function of the information threshold levels in a population.

3) $\phi(\bullet)=$
$$\phi\left(\frac{(w_j n_{jt}\mu-(c_f+n_{jt}c))\sqrt{d}}{\sigma}-\frac{((c_f+n_{jt}c_v)-w_j n_{jt}\mu_{j0})\tau_j}{\sigma\sqrt{d}}\right).$$

$$F(q(t)) = \sum_j \phi_j(q(t)),\ t > 0,\ \forall j \in \{1, \cdots, M\} \quad (5)$$

where $\phi(q(t))$ is defined as (4). $F(q)$ is a monotone non-decreasing function and $\lim_{q \to \infty} F(q)$ may be less than 1 since the firm with the infinite threshold value should be allowed. Let $p(t)$ is the proportion of adopters at time period t and we assume that $p(0)=0$, i.e. the process starts in period zero when no one has adopted yet. In the initial period, however, some firms may adopt the technology, which are called by innovators or early adopters. They are in the fraction $F(0)$ of the population, $F(0) > 0$, and F has a continuous density f. The reason is that the process is initially driven by the innovators who represent a positive fraction $F(0)$ of the population by assumption. From (11), the dynamics for the distribution of the firms' threshold values can be derived. The proportion of the population whose thresholds have been exceeded is $F(q)$.

Recall $q(t) = \int_0^t p(s)ds$, which is the cumulative amount of information collected from prior adopters in a same population by time t. $p(t)$ is the proportion of adopters in each population at time t, and $F(q(t)) - p(t)$ is the proportion of potential adopters at time t, that is, the proportion of remaining firms whose thresholds have been exceeded but have not adopted yet at time t. Let λ be the instantaneous rate at which these remaining firms adopt. λ is constant to make it simple. The instantaneous rate can be relaxed in some limit later. Then the adoption process by the differential equation in the discrete time setting is given by $p(t+1) - p(t) = \lambda[F(q(t)) - p(t)]$. In the continuous time setting, this can be rewrite as, simply,

$$\dot{p}(t) = \lambda[F(q(t)) - p(t)],\ t > 0,\ \lambda > 0 \quad (6)$$

It shows that firms use the accumulated information from all prior adopters in a same population group. This equation is always positive, which means that the rate of adoption increases for all time.

3.5 The Diffusion Curve

This section examines the pattern of diffusion based on the aggregate dynamics. As Chatterjee and Eliashberg [8] addressed about it, the shape of the curve can be determined by convexity or concavity properties and by the number and location of inflection points. It is a well-known fact that, in general, when the number of users of a new technology or product is accumulated over time, the curve is typically an S-shaped. That is, adoption proceeds slowly at first, and accelerates as it spreads throughout the potential adopting population, and then slows down as the relevant population becomes saturated. To track the diffusion curve over time in our model, equation (6) first should be differentiated with respect to t again. The second order (acceleration) equation is $\ddot{p}(t) = \lambda[p(t)f(q(t)) - \dot{p}(t)]$.

From this acceleration equation, we can first examine the shape of the curve of the initial period. When t is 0, the first term in brackets is zero and the second term is not zero because $p(0)=0$ and $\dot{p}(0) = \lambda F(0) > 0$ since $F(0) > 0$. Thus, the equation (7) is less than 0. When t goes to 0, that is, in the initial period, we can state it as $\lim_{t \to 0} \ddot{p}(t) = -\lambda^2 F(0)$. The above will be less than zero since $F(0) > 0$ and $p(t)$ is close to 0 as $t \to 0$. We can see that the adoption curve has weak growth in the early phases and there is a weak acceleration in the initial period until arriving at some point that the process begins to accelerate. Intuitively, the reason is that there are only few early adopters in the initial period, so the amount of information collected from them is not enough. Moreover, early adopters are not willing to share the information about the technology until the outcomes from adoption will be validated later.

Next consider about the inflection point of the adoption curve. Let Y_1 and Y_2 denote the fraction of the early adopters and potential adopters in each population. Then $G(t) = Y_1 + Y_2 F(q(t))$ is the penetration curve for proportion of the population in which firms are ready to adopt by time t. The first term, Y_1, is the proportion of firms who will adopt the technology immediately, $F(0)$. These firms have no any information from prior adopters when they make a decision to adopt.

These firms determine the initial fraction of adopters at the starting point of diffusion process. The second term represents the proportion of the population in which thresholds have been exceeded by the cumulative information in any period t. That is, they are ready to adopt it. In here, only the second term determines the shape of an adoption curve because the adoption curve totally depends upon the change of F over time. As addressed earlier, the number and locations of inflection points are determinants for the shape of the adoption curve. The inflection point actually occurs when a curve increases or decreases most rapidly. The first-order inflection point occurs when the rate of adoption is fastest and the second derivative equals to zero. The second-order inflection point occurs when the rate of the rate of adoption is fastest and the third derivative equals zero [35]. $G(t)$ can be differentiated as $\dot{G}(t) = Y_2 p(t) f(q(t)),\ t > 0$.

Proposition 1 : *The rate of adoption is fastest at time $\hat{t}$ such that $\ddot{G}(\hat{t}) = 0$ is satisfied (See proof in Appendix.). That is,*

$$\dot{p}(\hat{t}) = -p^2(\hat{t}) \frac{f'(q(\hat{t}))}{f(q(\hat{t}))}.$$

It is actually difficult to find the second-order inflection points in our diffusion model. Therefore we need a reasonable approximation to examine the whole shape of a diffusion curve, using two different patterns of information flows. Following Chatterjee and Eliashberg [5], two patterns of information flows over time can be examined in the model-monotone decreasing rate and monotone increasing rate of information.[4)]

Monotonically decreasing information rates

If the information rate is monotonically decreasing, $\dot{p}_k(t)$ is less than zero. Differentiating $\dot{G}(t)$ with respect to t again, then,

$$\ddot{G}(t) = Y_2 \dot{p}(t) f(q(t)) + Y_2 p^2(t) f'(q(t)),\ t > 0. \quad (7)$$

At any time $t > \hat{t}$, this double-differentiated equation is less than 0 since $\dot{p}(t) < 0$ and $f'(q(t)) < 0$. Therefore, the diffusion curve should be concave for $\hat{t} < t < \infty$(See [Figure 1(i)]).

Let t_1 denote any possible point of time that the rate of information flow changes rapidly after a slow diffusion in the initial period. For $t_1 < t \leq \hat{t}$, if we have the following condition

$$\dot{p}(t) < -p^2(t) \frac{f'(q(t))}{f(q(t))}.$$

4) To illustrate the shape of the diffusion curve, Chatterjee and Eliashberg [5] consider three patterns of information flows over time-constant, monotonically decreasing, and monotonically increasing. In our work, however, the constant rate is ignored.

Thus, the penetration curve will be concave in this range. As shown in [Figure 1(i)], there is no inflection point in the whole process because the initial period $(0<t\le t_1)$, the second period $(t_1<t\le \hat{t})$, and the third period $(\hat{t}<t<\infty)$ are concave for all $t>0$.

Monotonically increasing information rates

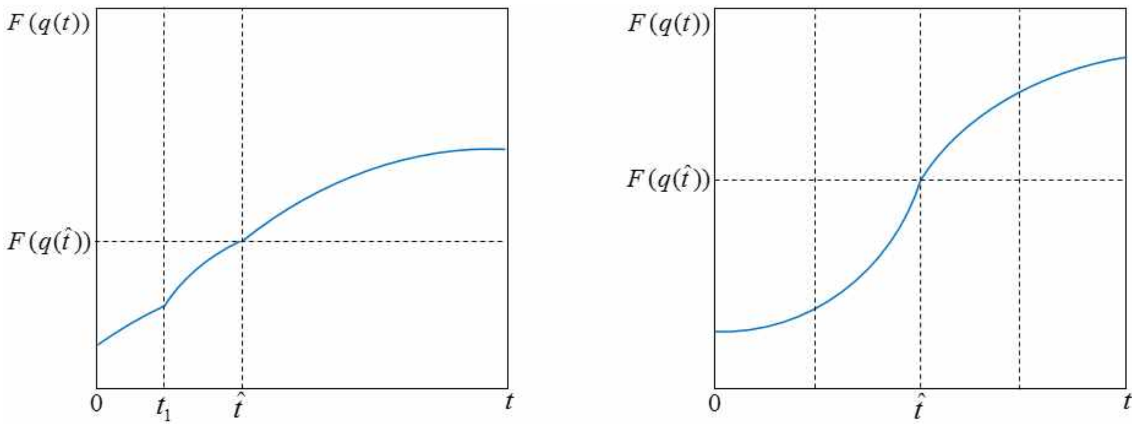
If the information rate is monotonically increasing, $\dot{p}(t)$ is larger than zero. From equation (8), at any time $t\le \hat{t}$, $\ddot{G}(t)>0$ since $\dot{p}(t)>0$ and $f'(q(t))>0$. Therefore, the adoption curve should be convex for $t\le \hat{t}$ (See [Figure 1(ii)]). After a slow diffusion in the initial period, the speed of adoption of technology will rapidly increase by the increasing rate of information. In this case, there exists a second-order inflection point satisfying the condition, $\ddot{G}(t)=0$, in the range $t\le \hat{t}$. For $\hat{t}<t<\infty$, it is not easy to find the inflection points.

Instead, we can intuitively think of the case that $f(q(t))\to 0$ as $q(t)\to\infty$. For this argument, we analyze that the rate of information flow will decrease and then go to zero as t goes to infinity. That is,

$$\lim_{t\to\infty}\ddot{G}(t)=\dot{p}(t)f(q(t))+p^2(t)f'(q(t))<0,$$

because of $f(q(t))\to 0$ and $f'(q(t))\to 0$ as $t\to\infty$. Thus, the adoption curve should be concave in this range. In this case, there should exist one first-order inflection point and at least two second-order inflection points since the initial part of the diffusion curve is concave, the second part is convex, and the third is concave. This draws the traditional S-curve in the diffusion model as shown in [Figure 1(ii)].

In the initial phases, the diffusion of the technology is initially driven by very few adopters who are convinced that this technology is good enough (pure inertia) or who are persuaded by mandates from powerful firms in the market. Once this situation is overcome by increase of the cumulative information from all previous adopters at some time point, there is rapid acceleration by the information collected from prior adopters, combined with the increasing number of firms who are persuaded by the new information. Therefore, in order to increase the adoption rates in a supply chain rapidly, more firms should adopt in the initial periods. In other words,



(i) Monotonically Decreasing Information Rates (ii) Monotonically Increasing Information Rates

[Figure 1] Diffusion Curves in Two Different Types of Information Rates as more firms adopt initially, the total amount of information to the potential adopters in the population increases, and then the number of firms persuaded by the information increases as the process moves up the distribution.

4. The Factors Influencing the Diffusion Speed

In this section, we consider about how to push up the time that the process begins to accelerate and which factors are able to make to speed up the diffusion process in the initial periods. This section introduces three factors influencing the diffusion speed of the new technology in a supply chain network : mean benefits, cost sharing, and information provision. We examine how such factors affect the reduction of threshold levels, which implies that reductions in threshold levels have an aggregate effect by accelerating the rate of adoption.

4.1 The Mean Benefits

The mean benefits from the adoption are considered as a key factor to shift the individual firms' threshold values.

Proposition 2 : *The adoption occurs more rapidly in firms with higher mean benefits from adoption than in those with lower mean benefits. (See proof in Appendix.)*

If it is assumed that q is the same threshold level for all two populations, the distributions satisfy $F_H(q) \geq F_L(q)$ for all q, that is, F_L first-order stochastically dominates F_H. Under these conditions, adoption occurs more rapidly in firms with higher mean benefits than in those with lower mean benefits. In other words, $p_H(t) \geq p_L(t)$ is satisfied for all time t. This result can be also used in the case of firms with different mean benefits each other if we relax the assumption that all firms in same population group are identical in the mean benefits from same size and capacity.

4.2 Cost Sharing with Partners

One of big hurdles of technology adoption may be the cost issue. The adoption costs include the cost of infrastructure as fixed costs and some operating costs as variable costs. For example, we consider the case of the adoption of radio frequency identification (RFID) in supply chains. Industry experience with its adoption shows that some firms, especially on the supply side in the retail industry, have had some significant cost issues as well as uncertainty in estimating the benefits of adoption. Typically, fixed costs for hardware and infrastructure tend not to vary significantly with the amount of product that passes through the supply chain while tag costs as variable costs might be very significant for suppliers who should pay for placing tags on products [11]. Actually, fixed costs and tag costs have come down over time, and then it will encourage more non-adopted firms no doubt. But they are still high compared to previous technology like barcoding. In practice, the suppliers may want to negotiate for sharing the variable cost with buyers for a return on investment (ROI). Under these cost issues, several literature has already studied about the optimal policies for RFID investment. Gaukler et al. [13] show that in the absence of mandating entities, there exists a unique optimal way of sharing the cost of RFID tags between a manufacturer and a retailer. Sharing the tag costs is optimal in the sense that the total supply chain profits are maximized. Whang [36] found that the equal-cost-split arrangement always induces the upstream (suppliers) to adopt RFID earlier than under no cost sharing and the cost split will be able to speed up the retailer's adoption under some conditions.

Proposition 3 : *The adoption occurs more rapidly in firms under the cost sharing with trading partners.*

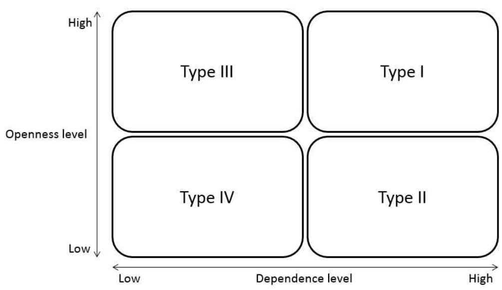
In §3, it has been assumed that buyers have only the fixed adoption costs and suppliers have the variable costs for tags as well as the fixed adoption costs. There has no discount rate for both cost components. To examine the effect of cost factors, suppose that all retailers equally share the tag costs with suppliers who have already adopted. From (2), the costs of supplier j who are facing the decision are simply defined as $C_{jt}' = c_f + (n_{jt}c_v)/2$, where n_{jt} is the number of buyers who have connected with supplier j and already adopted by time t, and c_v is the variable cost per a buyer. That is, all buyers linked with supplier j takes the equal-cost-split for tags. Supplier j's threshold value at time t will be

$$r_{jt,cs} = \frac{(C_{jt}' - w_j n_{jt} \mu_{j0})\tau_j}{\alpha_j (w_{MJ} n_{jt} \mu_M - C_{jt}')}.$$

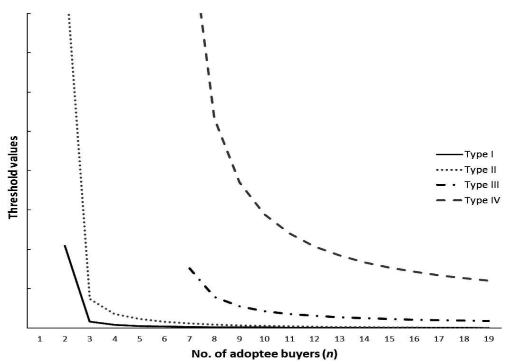
Comparing with the threshold of equation (3), the threshold decreases with the adjustment of cost sharing. In a total supply chain's perspective, one can expect more speedy diffusion in total network due to significant decreases of suppliers' threshold values for the adoption. This result corresponds with Gaukler et al. [13] such that in the absence of mandating entities, there exists a unique optimal way of sharing the cost of RFID tags between a supplier and a buyer. They also found that sharing the its tag cost is optimal in the sense that the total supply chain profits are maximized.

4.3 Information Provision

Lastly, we consider that firm's threshold values will decrease and the speed of diffusion proc-



(i)



(ii)

[Figure 2] (i) Four different types of suppliers classified by the degrees of dependence on buyer (w) and openness for information sharing with other suppliers (α) and (ii) the change of threshold levels of suppliers' types over the size of adoptee buyers $c_f = 10,\ c_v = 1, \mu_0 = 2,\ \mu_r = 15,\ \tau = 10,\ w = \{low: 0.2,\ high: 0.8\},\ \alpha = \{low: 0.2,\ high: 0.9\}$

ess will increase as providing more information about the technology adoption.

Proposition 4 : *The adoption rates increases as the firm's threshold level decreases by providing more information about the technology, and this will lead to speedy diffusion process.*

It has been already defined that I_{jt} are the amount of information from previous adopters in the supplier population, $I_{jt}=\alpha_j q(t)$. The additional information is also provided by buyers who have already adopted. Let $\hat{I}_{jt}$ be the new amount of information for the supplier, which is expressed as $\hat{I}_{jt}=\alpha_j q(t)+G(n_{jt})$ where $G(\cdot)$ is non-decreasing function with $G'(\cdot)>0$. The updated supplier j's threshold value of the amount of information at time t are

$$r_{jt,bi}=\frac{((c_f+n_{jt}c_v)-w_j n_{jt}\mu_{j0})\tau_j}{\alpha_j(w_j n_{jt}\mu-(c_f+n_{jt}c_v))}-\frac{G(n_{jt})}{\alpha_j}.$$

We see that the supplier's threshold level decreases by additional information, $G(n_{jt})/\alpha_j$. That is, each potential adopter has more information about the technology, and then has lower threshold levels required adopting it than before. This will lead to an increase of adoption rates. From equation above, the supplier's threshold level decreases over all possible number of buyers once considering a case that additional information coming from buyers increase linearly by the number of adopters in the buyer population. Therefore we expect that the diffusion speed with enhanced information will get faster. This implies that potential adopters need to collect more confidential information about technology implementation from previous adopters in the same population. In addition, a firm who wish to speed up their trading partners' adoption rates should need share more trustable information with them.

5. Buyer-Supplier Relationship Factors Leading to Technology Adoption

This section considers relationship factors to predict the adoption of network technology by a set of suppliers of a major buyer as a trading partner. Specifically, the model will examine that a buyer should use with its suppliers to encourage adoption and use of technology when the buyer can actively or passively encourage trading partner adoption. It will include factors available in practice between organizations in a buyer-supplier relationship; the dependence on the other (i.e. the supplier's dependence level on a buyer) and the openness for information sharing (i.e. the supplier's willingness to share information with other suppliers). This section shows how these relationship factors affect the change of suppliers' threshold levels.

We introduces four different types of the suppliers (See [Figure 2(i)]). These types are classified by two parameters, w_j and α_j, $j=1,\cdots,M$. As defined earlier, w_j represents the degree of dependence of supplier j over the buyer. Following Emerson's [9] definition of dependence, supplier's dependence is based on the percent of sales revenue from a particular buyer and the ability of the buyer to reselect another supplier. That is, the greater the sales revenue from a buyer, the more dependent the supplier is on that buyer [15], so its dependence is low. But in the model, it was already assumed that all suppliers' sizes are identical. So supplier j with a high w_j has less dependence on buyers and more ability to reselect other buyer. α_j represents the degree of openness of supplier j for the information (or know-how) coming from other adoptee suppliers. Supplier j with a high α_j is more likely to receive the information from other suppliers who have already adopted. Such supplier is willing to listen to new ideas and share the know-how about an adoption. However, suppliers under extremely competitive industry are less willing to share the information with their competitors, because they want to keep their superior position in the linked trading network, while suppliers under more localized and less competitive industry structure are frequently share the information among them.

Type I suppliers are relatively more dependent on buyers and have more openness for information sharing among other suppliers. As shown in [Figure 2(ii)], this type has very low threshold levels needed to adopt. In other words, their resistance levels for the technology adoption are relatively low and they are under weak competition. Type Ⅱ suppliers are relatively more dependent on buyers and has less openness for information sharing among others. Therefore suppliers with this type has no enough information about the technology, and so they have too high threshold levels than those of other types. Type Ⅲ suppliers are relatively less dependent on buyers and have more openness for information sharing among other suppliers. Such suppliers might be willing to share the information with other suppliers and are less affected by the buyer's persuasion for the technology adoption. Type Ⅳ suppliers are relatively less dependent on buyers and have less openness degree for information sharing. There is almost no information or know-how sharing of adoption among other suppliers, which implies that they are under strong competition.

We next examine how these relationship factors influence the supplier's adoption in the buyer-supplier supply chain, From equation (3) in §3.3, we numerically test the change of suppliers' threshold levels, depending on the supplier's type and the size of adoptee buyers. For relationship factors by each supplier's type, we set the parameters of dependence(w) and openness(α) in the buyer-supplier relationship in which type I has more dependence and more openness($w=0.8$, $\alpha=0.8$), type Ⅱ has more dependence and less openness($w=0.8$, $\alpha=0.2$), type Ⅲ has more dependence and more openness($w=0.2$, $\alpha=0.8$), and type Ⅳ has more dependence and more openness ($w=0.2$, $\alpha=0.2$).

In [Figure 2(ii)], the numerical results show the change curves of the supplier's threshold values of each type over the size of adoptee buyers. As shown in this figure, it requires more buyers to adopt the technology to encourage of use of technology for these four types of suppliers, which implies network externality. Regardless of types of supplier, this will enable them to significantly reduce their threshold levels, which means the low marginal level needed to adopt the technology. Observing the change of threshold values in more detail, the degree of dependence on a buyer determines different marginal numbers of adoptee buyers as the staring points of threshold levels. Type I supplier and type Ⅱ supplier need a small number of adoptee buyers while type Ⅲ supplier and type Ⅳ supplier need a large number of adoptee buyers to observe the initial value to track the change of threshold values. It implies that if the supplier has a low dependence level on buyers, it may have more resistance level on the pressure of adoption from buyers and it causes delay the adoption. By network externality, it requires more buyers to adopt the technology to encourage of use of technology for these types of suppliers. On the other dimension, the degree of openness for information sharing determines whether the starting point of threshold curves is high or low. The curves of type I supplier and type Ⅲ supplier have low level of initial threshold while those of type Ⅱ supplier and Ⅳ supplier are so high. It implies that if the supplier has a low degree of openness for information sharing, which means that the supplier has a high level of threshold, it is less willing to share their information and know-how with others and cannot acquire more information about the technology earlier. Then it causes delay of adoption.

6. Conclusion

This study develops a model to predict the adoption and level of usage of network technology with network externality in a two-level supply chain network. Based on the suggested benefit-cost structure, this paper has studied the individual supplier's adoption model to make a decision to adopt the technology or delay. By aggregating dynamics for the distribution of the threshold values, we see how the technology adoption in the network diffuses over time. From the shape of adoption curve, we are able to know the critical point of time that the diffusion process begins to accelerate. From the results, as more firms adopt in the initial period, the total amount of information to the potential adopters in the population increases, and then the number of firms persuaded by the information increases as the process moves up the distribution. We found that several factors influencing the diffusion speed, the mean benefits, cost sharing and better information, are useful to increase adoption rates at the initial period in the supply chain network. With the analytical results, this paper have also provided some important implications to the powerful buyers who wish to improve supply chain collaboration with their trading partners through the inter-organizational network technology. This study considers two relationship factors available in practice in a buyer-supplier relationship by a set of suppliers of a major buyer as a trading partner : the dependence on the other (i.e. the supplier's dependence level on a buyer) and the openness for information sharing (i.e. the supplier's willingness to share information with other suppliers). We have considered four different types of suppliers by these two relationship factors. By numerical tests, regardless of types of suppliers, the results show the change curves of the threshold values of each type of suppliers over the size of adoptee buyers. It implies that it requires more buyers to adopt the technology to encourage of use of technology for these four types of suppliers, which calls network externality. Using the suggested classification of suppliers, we will be able to provide some valuable insights for powerful buyers who wish to improve the adoption rates of their trading partners (suppliers). The buyer need to use persuasive and coercive approaches to encourage their partners to adopt the technology, depending on the supplier's type. For example, the persuasive approach might be more effective for suppliers with less dependence on buyers (type Ⅲ and Ⅳ) rather than force these firms, while the coercive approach might be better for suppliers with high dependence (type I and II). This part should be examined more in future.

One of the limit in this paper is that the model have only considered about the case of the suppliers' adoption process and not developed the model precisely for this case. But one can develop specific models under different scenarios as future researches : (1) the buyer is dominant; (2) the supplier is dominant; and (3) the supply chain system is under centralized control. One can study a game theoretic approach to find an equilibrium strategy for the adoption decision under each scenario above. More explicit structure to explain benefits and costs from adoption will be required, and then one can develop the technology adoption model precisely. Second, considering other internal and external factors, such as firm size, competition levels, relationships between trading partners, and contracts with the partners, was actually difficult in this paper. More rigorous models should be developed for analyzing how these various factors influence the adoption rates and diffusion speed across firms. To test insights generated from the analytical model, the empirical study should be also examined. Lastly, the analytical model assumes that each supplier in the population is connected to all the buyers; in reality the specific nature of the network of linkages between supplier and buyers will influence the adoption curve. In addition, the strength of connections between suppliers will influence the quantity and quality of information transmission between suppliers. One can collect data about technology adoptions and gathered information to build a network of linkages between these firms. Drawing upon the idea of an adoption threshold from the analytical model, one can fit an empirical threshold model to this data and then use the empirical model to test several hypotheses proposed by the analytical model presented in this paper.

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<Appendix>

Proof of proposition 1 : The rate of adoption is fastest at time $\hat{t}$ such that $\ddot{G}(\hat{t})=0$ is satisfied. From $\dot{G}_K(t)=p_K(t)f_k(q(t)),\ t>0$. Differentiating this, $\ddot{G}(\hat{t})=Y_2\dot{p}(\hat{t})f(q(\hat{t}))+Y_2p^2(\hat{t})f'(q(\hat{t}))=0$.

Then, $\dot{p}(\hat{t})=\ -p^2(\hat{t})f'(q(\hat{t}))/f(q(\hat{t}))$. The second-order is $\ddot{p}(t_0)=\lambda[p(t_0)f(q(t_0))-\dot{p}(t_0)]$.

Thus, $\Theta(t_0)=\ddot{p}(t_0)/\dot{p}(t_0)=\lambda(p(t_0)f(q(t_0))/\dot{p}(t_0)-1)=\lambda(F'(q(t_0))/\dot{p}(t_0)-1)$.

At time t_0, $\Theta(t_0)=0$ and $\Theta'(t_0)>0$.

Therefore $F'(q(t_0))=\dot{p}(t_0)$.

$$\Theta'(t_0)=\lambda\cdot[F''(q(t_0))\dot{p}(t_0)-F'(q(t_0))\ddot{p}(t_0)]/\dot{p}^2(t_0).$$

To show $\Theta'(t_0)>0$, we only need the part of numerator in a square bracket because $\lambda/\dot{p}^2(t_0)>0$. Therefore, $F''(q(t_0))\dot{p}(t_0)-F'(q(t_0))\ddot{p}(t_0)>0$. From $F'(q(t_0))=\dot{p}(t_0)$, thus, we see that $F''(q(t_0))>\ddot{p}(t_0)$.

Proof of proposition 2 : We prove it using contradiction. By contradiction, we assume that $F_H(0)\geq F_L(0)$ and $p_H(0)=p_L(0)=0$. From the dynamic equations, $\dot{p}_K(t)=\lambda_K\left[F_K\left(\int_0^t p_K(s)ds\right)-p_K(t)\right]$, $K=H, L$. Let $p_H(t)$ and $p_L(t)$ be the solutions. We know that $\dot{p}_H(0)\geq\dot{p}_L(0)$, because $F_H(0)\geq F_L(0)$ by assumption.

Suppose that at any given time $\tilde{t}>0$, $p_H(\tilde{t})<p_L(\tilde{t})$.

There is a time $\tilde{t}$ such that $p_H(\tau)\geq p_L(\tau)$ for all $\tau\leq\tilde{t}$ and $\dot{p}_H(\tilde{t})<\dot{p}_L(\tilde{t})$.

Therefore $\int_0^{\tilde{t}} p_H(s)ds\geq\int_0^{\tilde{t}} p_L(s)ds$ and $F_H\left(\int_0^{\tilde{t}} p_H(s)ds\right)\geq F_L\left(\int_0^{\tilde{t}} p_L(s)ds\right)$.

From the dynamic equations, hence, $\dot{p}_H(\tilde{t})\geq\dot{p}_L(\tilde{t})$ should be satisfied. This is a contradiction!