J. Appl. Math. & Informatics Vol. 31(2013), No. 3 - 4, pp. 523 - 531
Website: http://www.kcam.biz

A NOTE ON THE q-EULER NUMBERS AND POLYNOMIALS WITH WEAK WEIGHT α AND q-BERNSTEIN POLYNOMIALS†

H.Y. LEE*, N.S. JUNG, J.Y. KANG

Abstract. In this paper we construct a new type of q-Bernstein polynomials related to q-Euler numbers and polynomials with weak weight α ; $E_{n,q}^{(\alpha)}$, $E_{n,q}^{(\alpha)}(x)$ respectively. Some interesting results and relationships are obtained.

AMS Mathematics Subject Classification : 11B68, 11S40, 11S80.
Key words and phrases : q-Bernstein polynomials, q-Euler numbers and polynomials,Weak weight α,Euler numbers and polynomials, q-Euler numbers and polynomials.

1. Introduction

The q-Euler numbers and polynomials with weak weight α is introduced by H.Y. Lee, N.S. Jung, C.S. Ryoo. The main motivation of this paper is the paper [3,4,6-10] by Kim, in which he introduced and studied relations of the q-Euler numbers and polynomials with weight α and q-Bernstein polynomials. The Euler numbers and polynomials possess many interesting properties and rising in many areas of mathematics and physics. Recently, many mathematicians have studied in the area of the q-Euler numbers and polynomials (see [8,9,11,13,16,17,18]). In this paper, we construct a new type of q-Euler numbers $E_{n,q}^{(\alpha)}$ and polynomials $E_{n,q}^{(\alpha)}(x)$. We introduce the q-Euler numbers and polynomials with weak weight α and observe relations of the q-Euler numbers and polynomials with weak weight α and q-Bernstein polynomials. The p-adic q-integral are originally constructed by Kim [15]. In various parts, we use the p-adic q-integral. Throughout this paper we use the following notations. By $\mathbb{Z}_p$ we denote the ring of p-adic rational integers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$, $\mathbb{N}$ denotes the set of natural

Received February 14, 2013. Revised April 6, 2013. Accepted April 11, 2013. *Corresponding author. †This work was supported by the research grant of the Hannam University.

numbers, $\mathbb{Z}$ denotes the ring of rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{C}$ denotes the set of complex numbers, and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-\nu_p(p)} = p^{-1}$. When one talks of q-extension, q is considered in many ways such as an indeterminate, a complex number $q \in \mathbb{C}$, or p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$ one normally assume that $|q| < 1$. If $q \in \mathbb{C}_p$, we normally assume that $|q-1|_p < p^{-\frac{1}{p-1}}$ so that $q^x = \exp(x \log q)$ for $|x|_p \le 1$. Throughout this paper we use the notation:

$$[x]_q = \frac{1-q^x}{1-q}, \quad [x]_{-q} = \frac{1-(-q)^x}{1+q} \text{ (cf. [2,3,6,7,10,11,12,14,15]) .}$$

$\lim_{q \to 1}[x]_q = x$ for any x with $|x|_p \le 1$ in the present p-adic case. To investigate relation of the twisted q-Euler numbers and polynomials weak weight α and the q-Bernstein polynomials, we will use useful property for $[x]_q$ as following;

$$\begin{aligned} [x]_q &= 1 - [1-x]_q \\ [1-x]_q &= 1 - [x]_q \\ [1-x]_{q^{-1}} &= -q[1-x]_q \end{aligned} \tag{1.1}$$

For

$$g \in UD(\mathbb{Z}_p) = \{g | g : \mathbb{Z}_p \to \mathbb{C}_p \text{ is uniformly differentiable function}\},$$

the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-q}(g) = \int_{\mathbb{Z}_p} g(x) d\mu_{-q}(x) = \lim_{N \to \infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} g(x)(-q)^x \text{ (cf. [3-6]) .} \tag{1.2}$$

Let

$$T_p = \cup_{m \ge 0} C_{p^m} = \lim_{m \to \infty} C_{p^m},$$

where $C_{p^m} = \{w | w^{p^m} = 1\}$ is the cyclic group of order p^m. For $w \in T_p$, we denote by $\phi_w : \mathbb{Z}_p \to \mathbb{C}_p$ the locally constant function $x \longmapsto w^x$.

From (1.2), we obtain

$$q^n I_{-q}(g_n) + (-1)^{n-1} I_{-q}(g) = [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l g(l), \tag{1.3}$$

where $g_n(x) = g(x+n)$ (cf. [10]).

If we take $g_1(x) = g(x+1)$ in (1.3), then we easily see that

$$q I_{-q}(g_1) + I_{-q}(g) = [2]_q g(0). \tag{1.4}$$

The q-Euler numbers and polynomials with weak weight α are defined as follows;

For $\alpha \in \mathbb{Z}$ and $q \in \mathbb{C}_p$ with $|1-q|_p \le 1$, q-Euler numbers $E_{n,q}^{(\alpha)}$ are defined by

$$E_{n,q}^{(\alpha)} = \int_{\mathbb{Z}_p} [x]_q^n d\mu_{-q^\alpha}(x). \tag{1.5}$$

$$E_{n,q}^{(\alpha)}(x) = \int_{\mathbb{Z}_p} [x+y]_q^n d\mu_{-q^\alpha}(y). \tag{1.6}$$

with the usual convention of replacing $\left(E_q^{(\alpha)}(x)\right)^n$ by $E_{n,q}^{(\alpha)}(x)$. In the special case, $x = 0$, $E_{n,q}^{(\alpha)}(0) = E_{n,q}^{(\alpha)}$ are called the n-th q-Euler numbers with weak weight α.

In [18], C.S. Ryoo, H.Y. Lee, N.S. Jung introduced (h,q)-Euler numbers and polynomials; $E_{n,q}^{(h)}, E_{n,q}^{(h)}(x)$. We can find a little difference between (h,q)-Euler numbers and polynomials and q-Euler numbers and polynomials with weak weight α.

Our aim in this paper is to investigate relations of q-Euler numbers $E_{n,q}^{(\alpha)}$ and polynomials $E_{n,q}^{(\alpha)}(x)$ with weak weight α and q-Bernstein polynomials. First, we investigate some properties which are related to q-Euler numbers $E_{n,q}^{(\alpha)}$ and polynomials $E_{n,q}^{(\alpha)}(x)$ with weak weight α. The next, We derive the relations of q-Bernstein polynomials with q-Euler numbers $E_{n,q}^{(\alpha)}$ and polynomials $E_{n,q}^{(\alpha)}(x)$ with weak weight α at negative integers.

2. Main results

From (1.5),(1.6), we can derive the following recurrence formula for the q-Euler numbers and polynomials with weight α :

$$\begin{aligned} E_{n,q}^{(\alpha)} &= [2]_{q^\alpha} \left(\frac{1}{1-q}\right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l \frac{1}{1+q^{\alpha+l}} \\ &= [2]_{q^\alpha} \sum_{m=0}^{\infty} (-1)^m q^{\alpha m} [m]_q^n. \end{aligned} \tag{2.1}$$

$$\begin{aligned} E_{n,q}^{(\alpha)}(x) &= [2]_{q^\alpha} \left(\frac{1}{1-q}\right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{xl} \frac{1}{1+q^{\alpha+l}} \\ &= \sum_{l=0}^{n} \binom{n}{l} [x]_q^{n-l} q^{xl} E_{l,q}^{(\alpha)} \\ &= \left([x]_q + q^x E_q^{(\alpha)}\right)^n. \end{aligned} \tag{2.2}$$

By (2.1),(2.2), we have properties as below;

For $n \in \mathbb{Z}_+$, we have

$$q^\alpha E_{n,q}^{(\alpha)}(1) + E_{n,q}^{(\alpha)} = \begin{cases} [2]_{q^\alpha}, & \text{if } n = 0, \\ 0, & \text{if } n > 0. \end{cases} \tag{2.3}$$

For $n \in \mathbb{Z}_+$, we have

$$q^\alpha (qE_q^{(\alpha)} + 1)^n + E_{n,q}^{(\alpha)} = \begin{cases} [2]_{q^\alpha}, & \text{if } n = 0, \\ 0, & \text{if } n > 0, \end{cases} \tag{2.4}$$

with the usual convention of replacing $(E_q^{(\alpha)})^n$ by $E_{n,q}^{(\alpha)}$.

Theorem 2.1. *For $n \in \mathbb{Z}_+$*

$$E_{n,q}^{(\alpha)}(2) = q^{-\alpha}[2]_{q^\alpha} + q^{-2\alpha} E_{n,q}^{(\alpha)}.$$

Proof. By (1.3) we easily see that

$$[2]_{q^\alpha} \sum_{l=0}^{n-1} (-1)^{n-1-l} q^{\alpha l} [l]_q^m = q^{\alpha n} E_{m,q}^{(\alpha)}(n) + (-1)^{n-1} E_{m,q}^{(\alpha)}.$$

Take $n = 2$, then we have Theorem 2.1. $\square$

Theorem 2.2. *For $n, k \in \mathbb{Z}_+$, with $n > k$, we have*

$$\begin{aligned} \int_{\mathbb{Z}_p} B_{k,n}(x,q) d\mu_{-q^\alpha}(x) &= \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} \left([2]_{q^\alpha} + q^{2\alpha} E_{n-l,q^{-1}}^{(\alpha)} \right) \\ &= \begin{cases} q^\alpha [2]_{q^\alpha} + q^{2\alpha} E_{n,q^{-1}}^{(\alpha)}, & \text{if } k = 0, \\ q^{2\alpha} \binom{n}{k} \sum_{l=0}^{k} \binom{k}{l} (-1)^{k-l} E_{n-l,q^{-1}}^{(\alpha)}, & \text{if } k > 0, \end{cases} \end{aligned}$$

Proof. By definition of q-Euler polynomials with weak weight α , we get the following;

$$\int_{\mathbb{Z}_p} [x+2]_q^n d\mu_{-q^\alpha}(x) = E_{n,q}^{(\alpha)}(2).$$

By using p-adic q-integral and (1.1), we obtain a property as follows;

$$\begin{aligned} \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^n d\mu_{-q^\alpha}(x) &= \int_{\mathbb{Z}_p} (-q)^n [1-x]_q^n d\mu_{-q^\alpha}(x) \\ &= (-q)^n E_{n,q}^{(\alpha)}(-1) \\ &= (-q)^n (-1)^n q^{-n} E_{n,q^{-1}}^{(\alpha)}(2) \\ &= E_{n,q^{-1}}^{(\alpha)}(2) \\ &= q^\alpha [2]_{q^{-\alpha}} + q^{2\alpha} E_{n,q^{-1}}^{(\alpha)}. \end{aligned} \tag{2.5}$$

For $x \in \mathbb{Z}_p$, the p-adic q-Bernstein polynomials of degree n are given by

$$B_{k,n}(x,q) = \binom{n}{k} [x]_q^k [1-x]_{q^{-1}}^{n-k} \qquad \text{where } n, k \in \mathbb{Z}_+. \tag{2.6}$$

By (2.6), we get the symmetry of q-Bernstein polynomials as follows;

$$B_{k,n}(x,q) = B_{n-k,n}(1-x,q^{-1}). \tag{2.7}$$

Thus by (2.5) and (2.7)

$$\begin{aligned}
\int_{\mathbb{Z}_p} B_{k,n}(x,q)d\mu_{-q^\alpha}(x) &= \int_{\mathbb{Z}_p} B_{n-k,n}(1-x,q^{-\alpha})d\mu_{-q^\alpha}(x) \\
&= \int_{\mathbb{Z}_p} \binom{n}{k}[x]_q^k[1-x]_{q^{-1}}^{n-k}d\mu_{-q^\alpha}(x) \\
&= \int_{\mathbb{Z}_p} \binom{n}{k}(1-[1-x]_{q^{-1}})^k[1-x]_{q^{-1}}^{n-k}d\mu_{-q^\alpha}(x) \\
&= \int_{\mathbb{Z}_p} \binom{n}{k}\left(\sum_{l=0}^{k}\binom{k}{l}(-1)^{k-l}[1-x]_{q^{-1}}^{k-l}\right)[1-x]_{q^{-1}}^{n-k}d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k-l}\int_{\mathbb{Z}_p}[1-x]_{q^{-1}}^{n-l}d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k-l}\left(q^\alpha[2]_{q^\alpha}+q^{2\alpha}E_{n-l,q^{-1}}^{(\alpha)}\right).
\end{aligned} \tag{2.8}$$

□

Theorem 2.3. *Let $n,k \in \mathbb{Z}_+$ with $n>k$. Then we have*

$$\sum_{l=0}^{n-k}\binom{n-k}{l}E_{k+l,q}^{(\alpha)} = \begin{cases} q^{2\alpha}E_{n,q^{-1}}^{(\alpha)} + q^\alpha[2]_{q^\alpha}, & \text{if } k=0 \\ q^{2\alpha}\sum_{l=0}^{k}\binom{k}{l}(-1)^{k-l}E_{n-l,q^{-1}}^{(\alpha)}, & \text{if } k>0. \end{cases}$$

Proof. Let us take the fermionic q-integral on $\mathbb{Z}_p$ for the q-Bernstein polynomials of degree n as follows;

$$\begin{aligned}
\int_{\mathbb{Z}_p} B_{k,n}(x,q)d\mu_{-q^\alpha}(x) &= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_q^k[1-x]_{q^{-1}}^{n-k}d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_q^k\left(1-[x]_q\right)^{n-k}d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\int_{\mathbb{Z}_p}[x]_q^k\left(\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l[x]_q^l\right)d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{k+l}d\mu_{-q^\alpha}(x) \\
&= \binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^lE_{k+l,q}^{(\alpha)}.
\end{aligned} \tag{2.9}$$

□

Theorem 2.4. *Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$. Then we have*

$$\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)B_{k,n_2}(x,q)d\mu_{-q^\alpha}(x)$$
$$= \begin{cases} q^{2\alpha}E^{(\alpha)}_{n_1+n_2-l,q^{-1}} + q^\alpha[2]_{q^\alpha}, & \text{if } k=0 \\ q^{2\alpha}\binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}E^{(\alpha)}_{n_1+n_2-l,q^{-1}}, & \text{if } k>0 \end{cases}$$

Proof. Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$, then we get

$$\begin{aligned} &\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)B_{k,n_2}(x,q)d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\int_{\mathbb{Z}_p}[x]_q^{2k}[1-x]_{q^{-1}}^{n_1+n_2-2k}d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}\int_{\mathbb{Z}_p}[1-x]_{q^{-1}}^{n_1+n_2-l}d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}\left([2]_{q^\alpha} + q^{2\alpha}E^{(\alpha)}_{n_1+n_2-l,q^{-1}}\right). \end{aligned} \tag{2.11}$$

□

Theorem 2.5. *Let $n_1, n_2, k \in \mathbb{Z}_+$ with $n_1 + n_2 > 2k$, then we get*

$$\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l E^{(\alpha)}_{2k+l,q}$$
$$= \begin{cases} q^{2\alpha}E^{(\alpha)}_{n_1+n_2,q^{-1}} + q^\alpha[2]_{q^\alpha}, & \text{if } k=0 \\ q^{2\alpha}\sum_{l=0}^{2k}\binom{2k}{l}(-1)^{2k-l}E^{(\alpha)}_{n_1+n_2-l,q^{-1}}, & \text{if } k>0 \end{cases}$$

Proof. From the binomial theorem, we can derive the following equation.

$$\begin{aligned} &\int_{\mathbb{Z}_p} B_{k,n_1}(x,q)B_{k,n_2}(x,q)d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\int_{\mathbb{Z}_p}[x]_q^{2k}[1-x]_{q^{-1}}^{n_1+n_2-2k}d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l\int_{\mathbb{Z}_p}[x]_q^{2k+l}d\mu_{-q^\alpha}(x) \\ &= \binom{n_1}{k}\binom{n_2}{k}\sum_{l=0}^{n_1+n_2-2k}\binom{n_1+n_2-2k}{l}(-1)^l E^{(\alpha)}_{2k+l,q}. \end{aligned} \tag{2.11}$$

□

Theorem 2.6. *For $n_1, n_2, n_2, \cdots, n_s, k \in \mathbb{Z}_+$ with $n_1 + n_2 + \cdots + n_s > sk$, we have*

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \prod_{i=1}^{s} B_{k,n_i}(x,q) d\mu_{-q^\alpha}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left(q^\alpha [2]_{q^\alpha} + q^{2\alpha} E_{m-l,q^{-1}}^{(\alpha)} \right) \\
&= \begin{cases} q^{2\alpha} E_{m-l,q^{-1}}^{(\alpha)} + q^\alpha [2]_{q^\alpha}, & \text{if } k = 0 \\ q^{2\alpha} \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} E_{m-l,q^{-1}}^{(\alpha)}, & \text{if } k > 0 \end{cases}
\end{aligned}$$

where $n_1 + \cdots + n_s = m$.

Proof. For $n_1, n_2, \cdots, n_s, k \in \mathbb{Z}_+$, $n_1 + n_2 + \cdots + n_s > sk$, and let $\sum_{i=1}^{s} n_i = m$, then by the symmetry of q-Bernstein polynomials, we see that

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \prod_{i=1}^{s} B_{k,n_i}^{(\alpha)}(x,q) d\mu_{-q^\alpha}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \int_{\mathbb{Z}_p} [1-x]_{q^{-1}}^{m-l} d\mu_{-q^\alpha}(x) \qquad (2.12) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left([2]_{q^\alpha} + q^{2\alpha} E_{m-l,q^{-1}}^{(\alpha)} \right).
\end{aligned}$$

□

Corollary 2.7. *Let $m \in \mathbb{N}$. For $n_1, n_2, \ldots, n_s, k \in \mathbb{Z}_+$ with $n_1 + \cdots + n_s > sk$, we have*

$$\begin{aligned}
&\sum_{l=0}^{sk} \binom{sk}{l} (-1)^{sk-l} \left(q^\alpha [2]_{q^\alpha} + q^{2\alpha} E_{m-l,q^{-1}}^{(\alpha)} \right) \\
&= \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} E_{sk+l,q}^{(\alpha)},
\end{aligned}$$

where $n_1 + \cdots + n_s = m$.

Proof.

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \prod_{i=1}^{s} B_{k,n_i}(x,q) d\mu_{-q^\alpha}(x) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \int_{\mathbb{Z}_p} [x]_q^{sk} \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} (-1)^l [x]_q^l d\mu_{-q^\alpha}(x) \qquad (2.13) \\
&= \prod_{i=1}^{s} \binom{n_i}{k} \sum_{l=0}^{m-sk} (-1)^l \binom{m-sk}{l} E_{sk+l,q}^{(\alpha)},
\end{aligned}$$

where $n_1, n_2, \cdots, n_s, k \in \mathbb{Z}_+$ with $m = n_1 + n_2 + \cdots + n_s > sk$. □

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H.Y. Lee is currently at Hannam Unuiversity since 2010. His reseach interests are polynomials in Z_p and it's identities.

Department of Mathematics, Hannam University, Daejeon 307-791, Korea.
e-mail: normaliz@hnu.kr

N.S. Jung is currently at Hannam Unuiversity since 2010. Her reseach interests are polynomials in Z_p and it's identities.

Department of Mathematics,Hannam University, Daejeon 307-791, Korea.
e-mail: jns4235@nate.com

J.Y. Kang is currently at Hannam Unuiversity since 2011. Her reseach interests are polynomials in Z_p and it's identities.

Department of Mathematics, Hannam University, Daejeon 307-791, Korea.
e-mail: rkdwjddnr2002@yahoo.co.kr