

COMMON FIXED POINT THEOREMS FOR HYBRID MAPS IN NON-ARCHIMEDEAN FUZZY METRIC SPACES

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ABSTRACT. In this paper, we have established some common fixed point theorems for two pairs of occasionally weakly compatible hybrid maps satisfying a strict contractive condition in a non-archimedean fuzzy metric space. Our result extend, generalized and fuzzify several fixed point theorems on metric space.

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1. Introduction

Fixed point of functions and operators are important in many classical mathematical areas ranging from analysis to dynamical systems to geometry etc. So many articles on fixed point theorem for single valued map have been written under different contractive conditions. B. Fisher [5] initiated the study of fixed point for hybrid maps and thereafter many authors [3, 1] etc. tried to develop this concept for hybrid maps under different contractive conditions. But, the concepts of weak compatibility and occasionally weak compatibility were frequently used to prove existence theorems in fixed and common fixed point for hybrid maps satisfying certain conditions in different spaces. The study of common fixed point on occasionally weakly compatible maps is new and also interesting. Works along these lines have recently been initiated by Jungck and Rhoades [8] in 2006 and by Abbas and Rhoades [2] in 2007.

Fuzzy set theory, a generalization of crisp set theory, was first introduced by Zadeh [15] in 1965 to describe situations in which data are imprecise or vague or uncertain. Consequently, the last three decades remained productive for various authors [6, 12, 4] etc. have extensively developed the theory of fuzzy sets due to

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a wide range of application in the field of population dynamics , chaos control , computer programming , medicine , etc. Kramosil and Michalek [11] introduced the concept of fuzzy metric spaces (briefly , FM-spaces) in 1975, which opened an avenue for further development of analysis in such spaces.

In this paper, our target is to establish some common fixed point theorems for two pairs of occasionally weakly compatible hybrid maps satisfying a strict contractive condition in a non-archimedean fuzzy metric space. Our result extend, generalized and fuzzify several fixed point theorems on metric space.

2. Preliminaries

We quote some definitions and statements of a few theorems which will be needed in the sequel.

Definition 2.1 ([13]). A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t -norm if $*$ satisfies the following conditions :

- (i) $*$ is commutative and associative;
- (ii) $*$ is continuous;
- (iii) $a * 1 = a \quad \forall a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0, 1]$.

Result 2.1 ([10]). (a) For any $r_1, r_2 \in (0, 1)$ with $r_1 > r_2$, there exists $r_3 \in (0, 1)$ such that $r_1 * r_3 > r_2$;

(b) For any $r_5 \in (0, 1)$, there exist $r_6 \in (0, 1)$ such that $r_6 * r_6 \geq r_5$.

Definition 2.2 ([7]). The 3-tuple $(X, \mu, *)$ is called a **fuzzy metric space** if X is an arbitrary non-empty set, $*$ is a continuous t-norm and μ is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions :

- (i) $\mu(x, y, t) > 0$;
 - (ii) $\mu(x, y, t) = 1$ if and only if $x = y$;
 - (iii) $\mu(x, y, t) = \mu(y, x, t)$;
 - (iv) $\mu(x, y, s) * \mu(y, z, t) \leq \mu(x, z, s + t)$;
 - (v) $\mu(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous
- for all $x, y, z \in X$ and $t, s > 0$.

Note that $\mu(x, y, t)$ can be thought of as the degree of nearness between x and y with respect to t .

Example 2.1. Let $X = [0, \infty)$, $a * b = ab$ for every $a, b \in [0, 1]$ and d be the usual metric defined on X . Define $\mu(x, y, t) = e^{-\frac{d(x, y)}{t}}$ for all $x, y \in X$. Then clearly $(X, \mu, *)$ is a fuzzy metric space.

Example 2.2. Let (X, d) be a metric space, and let $a * b = ab$ or $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$. Let $\mu(x, y, t) = \frac{t}{t + d(x, y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, \mu, *)$ is a fuzzy metric space and this fuzzy metric μ induced by d is called the standard fuzzy metric [6].

Note 2.1. George and Veeramani [6] proved that every fuzzy metric space is a metrizable topological space. In this paper, also they have proved, if (X, d) is a metric space, then the topology generated by d coincides with the topology generated by the fuzzy metric μ of example (2.2). As a result, we can say that an ordinary metric space is a special case of fuzzy metric space.

Note 2.2. Consider the following condition :

$$(iv') \quad \mu(x, y, s) * \mu(y, z, t) \leq \mu(x, z, \max\{s, t\});$$

If the condition (iv) in the definition (2.2) is replaced by the condition (iv'), the fuzzy metric space $(X, \mu, *)$ is called a non-archimedean fuzzy metric space .

Definition 2.3 ([14]). Let $(X, \mu, *)$ be a fuzzy metric space.

A sequence $\{x_n\}_n$ in X is said to converge to $x \in X$ if and only if

$$\lim_{n \rightarrow \infty} \mu(x_n, x, t) = 1 \text{ for each } t > 0.$$

A subset P of X is said to be closed if for any sequence $\{x_n\}$ in P converges to $x \in P$, that is,

$$\lim_{n \rightarrow \infty} \mu(x_n, x, t) = 1 \implies x \in P \quad \forall t > 0.$$

A subset P of X is said to be bounded if and only if there exists $t > 0, r \in (0, 1)$ such that

$$\mu(x, y, t) > 1 - r \quad \forall x, y \in X.$$

Remark 2.1. In fuzzy metric space X , for all $x, y \in X$, $\mu(x, y, \cdot)$ is non-decreasing with respect to the variable t . In fact, in a non-archimedean fuzzy metric space, $\mu(x, y, t) \geq \mu(x, z, t) * \mu(z, y, t)$ for $x, y, z \in X, t > 0$. Every non-archimedean fuzzy metric space is also a fuzzy metric space.

Through out this paper X will represent a non-archimedean fuzzy metric space $(X, \mu, *)$ and $CB(X)$, the set of all non-empty closed and bounded sub-set of X . We recall a few usual notations : for $x \in X, A \subseteq X$ and for every $t > 0$,

$$\mu(x, A, t) = \max\{\mu(x, y, t) : y \in A\}$$

Let H be the associated Hausdorff fuzzy metric on $CB(X)$: for every A, B in $CB(X)$,

$$H(A, B, t) = \min \left\{ \min_{x \in A} \mu(x, B, t), \min_{y \in B} \mu(A, y, t) \right\}$$

and let $\delta(A, B, t)$ be the function defined by

$$\delta(A, B, t) = \min \{ \mu(a, b, t) : a \in A, b \in B \}$$

If A consists of a single point a , we write $\delta(A, B, t) = \delta(a, B, t)$. If B also consists of a single point b , we write $\delta(A, B, t) = \delta(A, b, t)$. It follows immediately from the definition that

$$\delta(A, B, t) = \delta(B, A, t) \geq 0,$$

$$\delta(A, B, t) \geq \delta(A, C, t) * \delta(C, B, t),$$

$$\delta(A, B, t) = 1 \iff A = B = \{a\},$$

for all A, B, C in $CB(X)$.

Definition 2.4. A sequence $\{A_n\}$ of nonempty subsets of X is said to be **convergent** to a subset A of X if the following holds :

(i) for each point $a \in A$, there is a sequence $\{a_n\}$ in X such that $a_n \in A_n$ for $n = 1, 2, \dots$, and $\{a_n\}$ converges to a in $(X, \mu, *)$.

(ii) given $\epsilon > 0$, there exists a positive integer m such that $A_n \subseteq A_\epsilon$ for $n > m$, where A_ϵ denotes the set of all points x in X for which there exists a point a in A , depending on x , such that $\mu(x, a, t) > \epsilon$ for all $t > 0$. A is then said to be the limit of the sequence $\{A_n\}$.

Through this section, we suppose that $f : X \rightarrow X$, $F : X \rightarrow CB(X)$.

Definition 2.5. A point $x \in X$ is called a **coincidence point** (resp. **fixed point**) of the hybrid pair (f, F) if $fx \in Fx$ (resp. $x = fx \in Fx$)

Definition 2.6. The hybrid pair (f, F) is said to be **compatible** if $fFx \in CB(X)$ for all $x \in X$ and

$$\lim_{n \rightarrow \infty} H(fFx_n, Ffx_n, t) = 1$$

whenever $\{x_n\}$ is a sequence in X such that $Fx_n \rightarrow M \in CB(X)$ and $fx_n \rightarrow x \in M$.

Definition 2.7 ([9]). The hybrid pair (f, F) is said to be **weakly compatible** if they commute at coincidence points. ie., if $fFx = Ffx$ whenever $fx \in Fx$.

Definition 2.8 ([2]). The hybrid pair (f, F) is said to be **occasionally weakly compatible (owc)** if there exists some point $x \in X$ such that $fx \in Fx$ and $fFx \subseteq Ffx$.

Example 2.3. Let $X = [1, \infty)$ with the usual metric. Define $f : X \rightarrow X$ and $F : X \rightarrow CB(X)$ by, for all $x \in X$,

$$fx = x + 1, Fx = [1, x + 1]$$

$$fx = x + 1 \in Fx \text{ and } fFx = [2, x + 2] \subset Ffx = [1, x + 2]$$

Hence, f and F are occasionally weakly compatible but non weakly compatible.

Definition 2.9. Let $F : X \rightarrow 2^X$ be a set-valued map on X . $x \in X$ is a fixed point of F if $x \in Fx$.

3. General fixed point theorems

Theorem 3.1. *Let $f, g : X \rightarrow X$ be mappings and $F, G : X \rightarrow CB(X)$ be set-valued maps such that the pairs $\{f, F\}$ and $\{g, G\}$ are **owc**. Let $\varphi : R^5 \rightarrow R$ be a real map satisfying the following conditions :*

$$(\varphi_1) : \varphi \text{ is increasing in variables } t_4 \text{ and } t_5$$

$$(\varphi_2) : \varphi(t, 1, 1, t, t) > 1 \quad \forall t \in [0, 1)$$

If, for all x and $y \in X$ for which

$$(\star) \quad \varphi(\mu(fx, gy, t), \mu(fx, Fx, t), \mu(gy, Gy, t), \mu(fx, Gy, t), \mu(gy, Fx, t)) < 1$$

then f, g, F and G have a unique common fixed point .

Proof. (i) We begin to show the existence of a common fixed point .

Since the pairs $\{f, F\}$ and $\{g, G\}$ are **owc** then , there exist u, v in X such that $fu \in Fu$, $gv \in Gv$, $fFu \subseteq Ffu$ and $gGv \subseteq Ggv$.

First , we show that $gv = fu$. The condition (\star) implies that

$$\varphi(\mu(fu, gv, t), \mu(fu, Fu, t), \mu(gv, Gv, t), \mu(fu, Gv, t), \mu(gv, Fu, t)) < 1$$

$$\implies \varphi(\mu(fu, gv, t), 1, 1, \mu(fu, Gv, t), \mu(gv, Fu, t)) < 1$$

By (φ_1) we have

$$\implies \varphi(\mu(fu, gv, t), 1, 1, \mu(fu, gv, t), \mu(fu, gv, t)) < 1$$

which from (φ_2) gives $\mu(fu, gv, t) = 1$. So $fu = gv$.

Next , we prove that $f^2u = fu$. Then condition (\star) implies that

$$\varphi(\mu(f^2u, gv, t), \mu(f^2u, Ffu, t), \mu(gv, Gv, t), \mu(f^2u, Gv, t), \mu(gv, Ffu, t)) < 1$$

$$\implies \varphi(\mu(f^2u, fu, t), 1, 1, \mu(f^2u, Gv, t), \mu(fu, Ffu, t)) < 1$$

By (φ_1) we have

$$\implies \varphi(\mu(f^2u, fu, t), 1, 1, \mu(f^2u, fu, t), \mu(fu, f^2u, t)) < 1$$

which , from (φ_2) , gives $f^2u = fu$.

Since (f, F) and (g, G) have the same role , we have $gv = g^2v$. Therefore,

$$ffu = fu = gv = ggv = gfu$$

and

$$fu = f^2u \in fFu \subseteq Ffu$$

So $fu \in Ffu$ and $fu = gfu \in Gfu$. Then fu is common fixed point of f, g, F and G .

(ii) Now , we show uniqueness of the common fixed point .

Put $fu = w$ and let w' be another common fixed point of the four maps, then by (\star) , we get

$$\begin{aligned} & \varphi(\mu(fw, gw', t), \mu(fw, Fw, t), \mu(gw', Gw', t), \mu(fw, Gw', t), \\ & \qquad \qquad \qquad \mu(gw', Fw, t)) < 1 \\ \implies & \varphi(\mu(fw, gw', t), 1, 1, \mu(fw, Gw', t), \mu(gw', Fw, t)) < 1 \end{aligned}$$

By (φ_1) , we get

$$\implies \varphi(\mu(fw, gw', t), 1, 1, \mu(fw, gw', t), \mu(fw, gw', t)) < 1$$

So, by (φ_2) , $\mu(fw, gw', t) = 1$ and thus

$$\mu(fw, gw', t) = \mu(w, w', t) = 1 \implies w = w'$$

This completes the proof. \square

Theorem 3.2. Let $f, g : X \rightarrow X$ be maps and $F, G : X \rightarrow CB(X)$ be set-valued maps such that the pairs $\{f, F\}$ and $\{g, G\}$ are **owc**. Let $\varphi : R^6 \rightarrow R$ be a real map satisfying the following conditions :

(φ_1) : φ is increasing in variables t_5 and t_6 ,

(φ_2) : for every $t', \varphi(t', t, 1, 1, t, t) > 1 \quad \forall t \in [0, 1)$

If for all x and $y \in X$ for which

$$\begin{aligned} (\star) \quad & \varphi(H(Fx, Gy, t), \mu(fx, gy, t), \mu(fx, Fx, t), \mu(gy, Gy, t), \\ & \qquad \qquad \qquad \mu(fx, Gy, t), \mu(gy, Fx, t)) < 1 \end{aligned}$$

then f, g, F and G have a unique common fixed point.

Proof. (i) We being to show the existence of a common fixed point .

Since the pairs $\{f, F\}$ and $\{g, G\}$ are **owc** then, there exist u, v in X such that $fu \in Fu, gv \in Gv, fFu \subseteq Ffu$ and $gGv \subseteq Ggv$.

First, we show that $gv = fu$. Then condition (\star) implies that

$$\begin{aligned} & \varphi(H(Fu, Gv, t), \mu(fu, gv, t), \mu(fu, Fu, t), \mu(gv, Gv, t), \\ & \qquad \qquad \qquad \mu(fu, Gv, t), \mu(gv, Fu, t)) < 1 \\ \implies & \varphi(H(Fu, Gv, t), \mu(fu, gv, t), 1, 1, \mu(fu, Gv, t), \\ & \qquad \qquad \qquad \mu(gv, Fu, t)) < 1 \end{aligned}$$

By (φ_1) we have

$$\begin{aligned} \implies & \varphi(H(Fu, Gv, t), \mu(fu, gv, t), 1, 1, \mu(fu, gv, t), \\ & \qquad \qquad \qquad \mu(fu, gv, t)) < 1 \end{aligned}$$

which, from (φ_2) , gives $\mu(fu, gv, t) = 1$. So $fu = gv$.

Next, we show that $f^2u = fu$. Then condition (\star) implies that

$$\begin{aligned} & \varphi(H(Ffu, Gv, t), \mu(f^2u, gv, t), \mu(f^2u, Ffu, t), \mu(gv, Gv, t), \\ & \qquad \qquad \qquad \mu(f^2u, Gv, t), \mu(gv, Ffu, t)) < 1 \\ \implies & \varphi(H(Ffu, Gv, t), \mu(f^2u, fu, t), 1, 1, \mu(f^2u, Gv, t), \end{aligned}$$

$$\mu(fu, Ffu, t) < 1$$

By (φ_1) we have

$$\begin{aligned} \Rightarrow \varphi(H(Ffu, Gv, t), \mu(f^2u, fu, t), 1, 1, \mu(f^2u, fu, t), \\ \mu(f^2u, fu, t)) < 1 \end{aligned}$$

which, from (φ_2) , gives $\mu(f^2u, fu, t) = 1$. We have $f^2u = fu$.

Since $\{f, F\}$ and $\{g, G\}$ have the same role, we have $gv = g^2v$. Therefore,

$$ffu = fu = gv = ggv = gfu$$

and

$$fu = f^2u \in fFu \subset Ffu$$

So $fu \in Ffu$ and $fu = gfu \in Gfu$. Then fu is common fixed point of f, g, F and G .

(ii) Now, we show uniqueness of the common fixed point.

Put $fu = w$ and let w' be another common fixed point of the four maps, then by (\star) , we get

$$\begin{aligned} \varphi(H(Fw, Gw', t), \mu(fw, gw', t), \mu(fw, Fw, t), \mu(gw', Gw', t), \\ \mu(fw, Gw', t), \mu(gw', Fw, t)) < 1 \\ \Rightarrow \varphi(H(Fw, Gw', t), \mu(fw, gw', t), 1, 1, \mu(fw, Gw', t), \\ \mu(gw', Fw, t)) < 1 \end{aligned}$$

By (φ_1) we get

$$\begin{aligned} \Rightarrow \varphi(H(Fw, Gw', t), \mu(fw, gw', t), 1, 1, \mu(fw, gw', t), \\ \mu(fw, gw', t)) < 1 \end{aligned}$$

So, by (φ_2) , $\mu(fw, gw', t) = 1$ and thus

$$\mu(fw, gw', t) = \mu(w, w', t) = 1 \quad \Rightarrow w = w'$$

□

Theorem 3.3. Let $f, g : X \rightarrow X$ be maps and $F, G : X \rightarrow CB(X)$ be set-valued maps such that the pairs $\{f, F\}$ and $\{g, G\}$ are **owc**. Let $\varphi : R^6 \rightarrow R$ be a real map satisfying the following conditions :

(φ_1) : φ is non-increasing in variables t_1 and non-decreasing in variables t_5 and t_6 ,

(φ_2) : for every $t', \varphi(t, t, 1, 1, t, t) > 1 \quad \forall t \in [0, 1)$.

If for all x and $y \in X$ for which

$$\begin{aligned} (\star) \quad \varphi(\delta(Fx, Gy, t), \mu(fx, gy, t), \mu(fx, Fx, t), \mu(gy, Gy, t), \\ \mu(fx, Gy, t), \mu(gy, Fx, t)) < 1 \end{aligned}$$

then f, g, F and G have a unique common fixed point.

Proof. The proof is similar to that of the theorem (3.2). \square

4. Other type common fixed point theorems

Theorem 4.1. Let $f, g : X \rightarrow X$ be maps and $F, G : X \rightarrow CB(X)$ be set-valued maps such that the pairs $\{f, F\}$ and $\{g, G\}$ are **owc**. Let $\psi : R \rightarrow R$ be a non-decreasing map such that, for every $0 \leq l < 1$, $\psi(l) > l$ and satisfying the following condition :

$$(\star) \quad \delta^p(Fx, Gy, t) \geq \psi[a\mu^p(fx, gy, t) + (1-a)\mu^{\frac{p}{2}}(gy, Fx, t) \\ \mu^{\frac{p}{2}}(fx, Gy, t)]$$

for all x and $y \in X$, where $0 < a \leq 1$ and $p \geq 1$. Then f, g, F and G have a unique common fixed point.

Proof. Since f, F and g, G are **owc**, as in proof theorem 3.1, there exist $u, v \in X$ such that $fu \in Fu, gv \in Gv, fFu \subseteq Ffu, gGv \subseteq Ggv$. (i) As in proof of theorem (3.1), we begin to show the existence of a common fixed point. We have,

$$\delta^p(Fu, Gv, t) \geq \psi[a\mu^p(fu, gv, t) + (1-a)\mu^{\frac{p}{2}}(gv, Fu, t) \\ \mu^{\frac{p}{2}}(fu, Gv, t)]$$

and by the properties of δ and ψ , we get

$$\mu^p(fu, gv, t) \geq \delta^p(Fu, Gv, t) \geq \psi(\mu^p(fu, gv, t))$$

So, if $0 \leq \mu(fu, gv, t) < 1$, $\psi(\mu(fu, gv, t)) > \mu(fu, gv, t)$, which implies that

$$\mu^p(fu, gv, t) \geq \delta^p(Fu, Gv, t) \geq \psi(\mu^p(fu, gv, t)) > \mu^p(fu, gv, t)$$

which is a contradiction, thus we have $\mu(fu, gv, t) = 1$ hence $fu = gv$. Again, if $0 \leq \mu(f^2u, gv, t) < 1$ then by (\star) , we have

$$\delta^p(Ffu, Gv, t) \geq \psi[a\mu^p(f^2u, gv, t) + (1-a)\mu^{\frac{p}{2}}(gv, Ffu, t) \\ \mu^{\frac{p}{2}}(f^2u, Gv, t)]$$

and hence

$$\mu^p(f^2u, fu, t) \geq \delta^p(Ffu, Gv, t) \geq \psi(\mu^p(f^2u, fu, t))$$

which implies

$$\mu^p(f^2u, fu, t) \geq \delta^p(Ffu, Gv, t) \geq \psi(\mu^p(f^2u, fu, t)) > \mu^p(f^2u, fu, t)$$

what it is impossible. Then we have $\mu(f^2u, fu, t) = 1$ hence $f^2u = fu$. Similarly, we can prove that $g^2v = gv$.

Let $fu = w$ then, $fw = w = gw, w \in Fw$ and $w \in Gw$, this completes the proof of the existence.

(ii) For the uniqueness, let w' be a second common fixed point of f, g, F and G . If $0 \leq \mu(w, w', t) < 1$ then

$$\mu(w, w', t) = \mu(fw, gw', t) \geq \delta(Fw, Gw', t)$$

and, by assumption (\star) , we obtain

$$\delta^p(Fw, Gw', t) \geq \psi[a\mu^p(fw, gw', t) + (1-a)\mu^{\frac{p}{2}}(fw, Gw', t) \mu^{\frac{p}{2}}(gw', Fw, t)]$$

and thus

$$\mu^p(w, w', t) = \mu^p(fw, gw', t) \geq \delta^p(Fw, Gw', t) \geq \psi[\mu^p(w, w', t) > \mu^p(w, w', t)]$$

which is a contradiction. So, we have $\mu(w, w', t) = 1$, that is, $w = w'$. \square

Theorem 4.2. Let $f, g : X \rightarrow X$ be maps and $F, G : X \rightarrow CB(X)$ be set-valued maps such that there exist two elements u and v in X for which $fu \in Fu, fFu \subseteq Ffu$ and $gv \in Gv, gGv \subseteq Ggv$. Let $\psi : \mathbb{R} \rightarrow \mathbb{R}$ be a non-decreasing map such that, for every $0 \leq l < 1, \psi(l) > l$ and satisfying the following condition :

$$(\star) \quad H^p(Fx, Gy, t) \geq \psi[a\mu^p(fx, gy, t) + (1-a)\mu^{\frac{p}{2}}(gy, Fx, t) \mu^{\frac{p}{2}}(fx, Gy, t)]$$

for all x and $y \in X$, where $0 < a \leq 1$ and $p \geq 1$.

If $fu = gv$, then fu is a common fixed point of f, g, F and G , and $Fu = Gv$.

Proof. We see that

$$\begin{aligned} \mu(Fu, gv, t) &\geq H(Fu, Gv, t), \quad \mu(fu, Gv, t) \geq H(Fu, Gv, t) \\ \mu(Ffu, gv, t) &\geq H(Ffu, Gv, t) \quad \text{and} \quad \mu(f^2u, Gv, t) \geq H(Ffu, Gv, t) \end{aligned}$$

From the nondecreasing property of ψ , we obtain

$$\begin{aligned} H^p(Ffu, Gv, t) &\geq \psi[a\mu^p(f^2u, gv, t) + (1-a)\mu^{\frac{p}{2}}(gv, Ffu, t) \mu^{\frac{p}{2}}(f^2u, Gv, t)] \\ &\geq \psi[a\mu^p(f^2u, gv, t) + (1-a)H^p(Ffu, Gv, t)], \end{aligned}$$

$$H^p(Fu, Gv, t) \geq \psi[a\mu^p(fu, gv, t) + (1-a)H^p(Fu, Gv, t)],$$

$$H^p(Fu, Ggv, t) \geq \psi[a\mu^p(fu, g^2v, t) + (1-a)H^p(Fu, Ggv, t)]$$

Now we suppose that $fu = gv$. From the first inequality, we see that

$$\begin{aligned} H^p(Ffu, Gv, t) &\geq \psi[a\mu^p(f^2u, gv, t) + (1-a)H^p(Ffu, Gv, t)] \\ &\geq \psi[H^p(Ffu, Gv, t)] \end{aligned}$$

If $0 \leq H^p(Ffu, Gv, t) < 1$ then we see that

$$H^p(Ffu, Gv, t) \geq \psi[H^p(Ffu, Gv, t)] > H^p(Ffu, Gv, t),$$

which is a contradiction and this contradiction shows that $H^p(Ffu, Gv, t) = 1$, which implies that $Ffu = Gv$. Similarly, also we have $Fu = Gv$ and $Fu = Ggv$. This completes the proof. \square

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