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SENSITIVITY ANALYSIS OF ATMOSPHERIC DISPERSION MODEL-RIMPUFF USING THE HARTLEY-LIKE MEASURE†

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Abstract. In this article, sensitivity analysis of atmospheric dispersion model RIMPUFF is considered. Uncertain parameters are taken to be triangular fuzzy numbers, and sensitivity analysis is carried out by using the Hartley-like measure. Codes for evaluating membership function using the Vertex method and the Hartley-like measure are prepared using Matlab.

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1. Introduction

Atmospheric diffusion model is a mathematical expression relating the emission of hazardous material into the atmosphere to the downwind ambient concentration of the hazardous material. The main aim of the modelling process is to estimate the concentration of a pollutant at a particular receptor point by calculating from the basic information about the source of the pollutant and meteorological conditions. Atmospheric dispersion is a phenomenon based on uncertainties, and in general, the concentration of pollutants observed at a given time and location downwind of a source cannot be predicted [1]. The concentration of air pollutant at a given place is a function of a number of variables, such as rate of emission, distance of receptor from source and atmospheric conditions. The most important atmospheric conditions are wind speed, wind direction and vertical temperature structure of local atmosphere. Air pollution dispersion models are subject to scientific uncertainty, but the way this is handled for air quality management policy is different depending on scale of modelling and impact under consideration [2].

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Information about dispersion model input parameters can be gained through measurement, calibration, expert judgement etc. However, the value of these parameters may be subject to uncertainty due to lack of measurement point and over-calibration or inaccurate expert judgement. Inherent uncertainty of the input parameters is one of the main causes of uncertainty in model output. Parameter uncertainty is present because not always a single value of a parameter can completely characterize a modelling domain [3]. In the cases when not much data are available, or design values can be only estimated by an expert, the fuzzy set theory is useful as it assigns each value a degree of credibility [4]. The use of fuzzy numbers is proposed as a suitable technique for handling atmospheric dispersion criteria and tackling decisions made under uncertainty. Fuzzy analysis based on fuzzy set introduced by Zadeh [4] is widely used to represent such uncertain knowledge.

Sensitivity analysis aims to quantify relative importance of input variables in determining the value of an assigned output variable. It can be used as an aid to identifying the important uncertainties for the purpose of prioritizing additional data collection or research [5]. A Sensitivity study examines the way a particular model responds to variations in values of input variables or internal parameters [6]. Parameter sensitivity refers to the case where the output function values are largely affected by variations in the values of one or more parameters.

2. Fuzzy Number and Fuzzy Arithmetic

The notion of fuzzy set was introduced by Zadeh [4], since then its application has been evident in different field of study. The notion of fuzzy number arises from experiences of everyday life when many phenomena which can be quantified are not characterised in terms of absolutely precise numbers. Fuzzy numbers are convex and normalised fuzzy sets which are defined on the set of real numbers. Membership function of fuzzy number assigns degree of 1 to the most probable value, also called mean value and lower degrees to other numbers which reflect their proximity to the most probable value according to the used membership function. Thus, the membership function decreases from 1 to 0 on both sides of the most probable value. α-level set or α-cut of a fuzzy number is an interval defined for a specific value of the membership function.

Vertex Method [7] simplifies manipulations of arithmetic of fuzzy numbers. This method is based on a combination of the α-cut concept and standard interval arithmetic. In this method, rather than discretizing the variable domain, the membership domain is discretized. The discretization of the membership domain is accomplished by dividing the membership domain into a series of equally spaced α-cuts. Arithmetic is performed on each of these α-cuts on the basis of standard interval arithmetic and for each α-cut the upper and the lower value of the fuzzy variable is selected.

3. Hartley-like Measure

Hartley measure is well established measure of uncertainty in the classical set theory [8, 9]. This type of measure quantifies the most fundamental type of uncertainty, one expressed in terms of a finite set of possible alternatives. The type of uncertainty quantified by the Hartley measure is well captured by the term non-specificity. This theory was generalised to fuzzy set by Higashi and Klir [10, 11]. The generalised measure H for any non-empty fuzzy set A defined on a finite universal set X has the form

$$H(A) = \frac{1}{h(A)} \int_0^{h(A)} \log_2 |A_\alpha| \, d\alpha$$

where $|A_\alpha|$ denotes the cardinality of the α-cuts of the fuzzy set A and $h(A)$ height of A. For fuzzy intervals or numbers on the real line, the Hartley-like measure is defined as

$$HL(A) = \int_0^1 \log_2 \left(1 + \lambda\left(A_\alpha\right)\right) d\alpha$$

where $\lambda\left(A_\alpha\right)$ is the Lebesgue measure of A_α [10]. Mathematically, for a triangular fuzzy number $A < a_L, a_m, a_R >$ given by the membership function

$$A(x) = \begin{cases} \frac{x-a_L}{a_m-a_L}, & \text{if } a_L \le x \le a_m \\ \frac{x-a_R}{a_m-a_R}, & \text{if } a_m \le x \le a_R \\ 0, & \text{otherwise} \end{cases}$$

the Hartley-like measure is given by the expression below, which is valid for any type of triangular fuzzy number.

$$HL(A) = \frac{1}{(a_R - a_L)\ln(2)} \left([1 + (a_R - a_L)] \ln[1 + (a_R - a_L)] - (a_R - a_L)\right)$$

4. Sensitivity Analysis

The main aim of sensitivity analysis is to estimate the rate of change of output of a model with respect to the changes in the model inputs. Sensitivity analysis is the study of how uncertainty in the output of a model can be appropriated to different sources of uncertainty in the model input. It will in turn instruct modellers as to the relative importance of the inputs in determining the output. Sensitivity analysis provides useful risk insights, but alternative approaches are also needed to understand which parameters show up as important and why they show up as important.

Traditional and most used method of sensitivity analysis is derivative method which provides local sensitivity analysis. The mathematical definition of the sensitivity analysis a model Y_j versus input X_i can be obtained from the derivative $\frac{\partial Y_j}{\partial X_i}$. But due to implicit correlation of the parameters under investigation, a global sensitivity analysis is essential. Global sensitivity analysis method is based on stepwise regression and this rigorous computational procedure is shortened by carrying out correlation matrix method. However, all these requires specific probability distribution of all the parameters of interest. In practice, probability distribution of the parameters are not always possible due to their lack of measurement and spare behaviour in their prediction. Accordingly, imprecise method of sensitivity analysis is searched. The Hartley-like measure is the method of sensitivity analysis with imprecise parameter, because non-specificity is quantified using the Hartley-like measure.

Parameter sensitivity analysis by Hartley-like measure is a method from imprecise probability theory. When the information about the parameters consist of a central value and a coefficient or range of variation then the Hartley-like measure method of sensitivity analysis is employed [12]. The basic strategy for arriving at a sensitivity analysis or assessment is by successively fixing the input parameters and drawing the effect on the variability of the output.

Suppose for neutral air, in a case of purely mechanical turbulence, classical logarithmic wind profile [13] is

$$\overline{u} = \frac{u_*}{k_a} \ln \frac{z}{z_0}, z > z_0 \tag{1}$$

Here k_a is *von Karman constant*, z_0 is value of puff height z at which average wind speed $\overline{u}$ vanishes is termed as *surface roughness length* and u_* is *friction velocity*. The sensitivity analysis of average wind speed with respect to the parameters k_a, z_0 and u_* is carried out by modelling the parameters as triangular fuzzy numbers by assigning degree 1 to the most possible value. These input parameters are modelled from Table 1 as shown below [16]. The value of the von Karman constant varies from 0.35-0.43 and most likely value 0.40 is used for the case in which measurement have not been made [16]. Fuzziness of input parameters friction velocity u_* (m/sec) is being modelled as fuzzy number $< 0.23, 0.4, 0.65 >$ from the Table 1, so, the surface roughness length z_0 (cm) is defined as fuzzy number $< 0.03, 1, 9 >$. The output membership function of the average wind speed in m/sec are depicted in Figure 1, outer membership function is the membership function of average wind speed without fixing any parameter, while the shaded membership function is the membership function of the average wind speed fixing input parameters to the most probable value successively. Hartley-like measure of the fuzzy output under successive fixing the input parameters are depicted in Table 2, the parameter friction velocity u_* is found to be the most sensitive parameter followed by the surface roughness length z_0 and then the von Karman constant k_a.

5. RIMPUFF Model

The RIMPUFF is a Lagrangian mesoscale atmospheric dispersion puff model designed for calculating the concentration and doses resulting from dispersion of airborne materials. The Riso Mesoscale Puff model, RIMPUFF, Mikkelsen [14], Thykier-Nielsen [15], models the plume of released material as a number of individual puffs. The RIMPUFF model calculates the concentration at each

TABLE 1. Values of z_0 and u_* for use in Average Wind Speed Profile

Type of Surface	z_0 in cm	u_* in m/sec
Smooth mud flats, ice	0.001	0.16
Smooth snow	0.005	0.19
Smooth sea	0.02	0.22
Level desert	0.03	0.23
Snow surface, lawn grass to 1.0 cm high	0.1	0.26
Lawn, grass to 5 cm	1-2	0.38-0.43
Lawn, grass to 60 cm	4-9	0.51-0.65
Fully grown root crops	14	0.75
Pasture land	20	0.87
Suburban housing	60	1.66
Forest, cities	100	2.89

TABLE 2. Hartley-like Measure of average wind speed ($\overline{u}$) successively fixing friction velocity u_*, von Karman constant k_a and surface roughness length z_0.

Membership Function	HL-measure
u_* Fixed	0.6947
k_a Fixed	0.9434
z_0 Fixed	0.8174

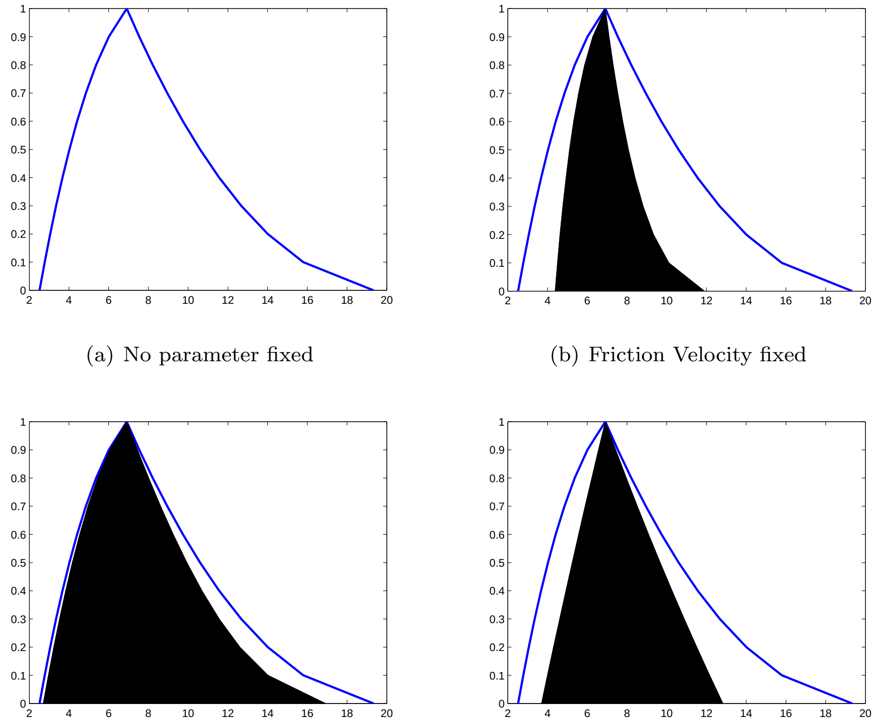
grid point by summing the contributions from surrounding puffs at each advection step. The grid concentrations or doses can either accumulate or simply be updated with the latest instantaneous value calculated for average time. The model output consists of time integrated air concentration and depositions in grid points at times specified in the input data. Updated grid concentrations $\chi(x_g, y_g, z_g)$ are evaluated at each grid point (x_g, y_g, z_g) summing up all the contributions from the puffs in the grid. The concentration in a grid point (x_g, y_g, z_g) from puff number (i) is given in Bq/m^3:

$$\chi(x_g, y_g, z_g) = \frac{Q(i)}{(2\pi)^{\frac{3}{2}}(\sigma_{xy}(i))^2\sigma_z(i)} \exp\left[-\frac{1}{2}\left(\left(\frac{x_g - x_c(i)}{\sigma_{xy}(i)}\right)^2 + \left(\frac{y_g - y_c(i)}{\sigma_{xy}(i)}\right)^2\right)\right]$$
$$\exp\left[-\frac{1}{2}\left(\frac{z_g - z_c(i)}{\sigma_z(i)}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{2z_{inv} - z_c(i)}{\sigma_z(i)}\right)^2\right]$$

where,

$Q(i)$	Puff inventory in puff number (i).
$x_c(i), y_c(i), z_c(i)$	Centre co-ordinate of puff number (i).
z_{inv}	Height of the Inversion lid.
$\sigma_{xy}(i), \sigma_z(i)$	Puff dispersion parameters in horizontal and vertical directions respectively and $\sigma_{xy}(i), \sigma_z(i) > 0$.

The Pasquill-Gifford parametrisation of plume spread σ_y and σ_z could be replaced by so-called *Similarity-scaling* of atmospheric turbulence and diffusion. The basic concept is to base the calculations of plume spread on the physical



(a) No parameter fixed

(b) Friction Velocity fixed

(c) von Karman constant fixed

(d) Roughness length fixed

Figure 1. Membership functions of average wind speed ($\overline{u}$) successively fixing friction velocity u_*, von Karman constant k_a and surface roughness length z_0.

parameters that governs the atmospheric boundary layer turbulence-this is parameters for heat flux w_*, shear stress u_*, the inversion height z_{inv} and then from them derived Monin-Obukhov length L. With these the Pasquills stability classes are replaced by continuous non-dimensional parameter "z/L", where z is height above the ground or release height [10]. The standard deviations σ_x in downwind directions used as a mathematical tool and for simplicity the value $\sigma_x = \sigma_y$ is used and this common value is marked as σ_{xy} [15]. The spread parameter σ_{xy} and σ_z can be written as follows [16]

$$\sigma_{xy} = \sigma_\theta x f_y\left(\frac{x}{\overline{u}T_y}\right), \quad \sigma_z = \sigma_e x f_z\left(\frac{x}{\overline{u}T_z}\right)$$

where,

$$\sigma_\theta = \frac{\sigma_v}{\overline{u}},\ \sigma_e = \frac{\sigma_w}{\overline{u}},\ f_y\left(\frac{t}{T_y}\right) = \sqrt{2}\frac{T_y}{t}\left[\frac{t}{T_y} - 1 + exp\left(-\frac{t}{T_y}\right)\right]^{1/2}$$

$t = x/\overline{u}$ is diffusion time.

The σ_{xy} and σ_z are completely determined by observation of σ_θ and σ_e, the standard deviations of wind direction fluctuations in horizontal and vertical directions, respectively. But, a universal function for f_z is very difficult to determine since few data are available on vertical concentration distribution [16]. Parameters standard deviations of the wind direction fluctuations in horizontal and vertical direction are implemented based on the equation by Carruthers [17]:

$$\sigma_u^2 = 0.3w_*^2 + 6.25T_{W_N}^2(z)u_*^2, \quad \sigma_v^2 = 0.3w_*^2 + 4.0T_{W_N}^2(z)u_*^2$$

$$\sigma_w^2 = \left\{0.4T_{W_C}^2 + \left(1.3T_{W_N}(z)\frac{u_*}{w_*}\right)^2\right\} w_*^2$$

where,

$$T_{W_C}(z) = 2.1\left(\frac{z}{h}\right)^{1/3}\left(1 - 0.8\frac{z}{h}\right), \quad T_{W_N} = \left(1 - 0.8\frac{z}{h}\right)$$

z	Puff height (m)
h	Boundary layer height (m)
w_*	Heat flux (W/m^2)
$\overline{u}$	Average wind velocity (m/sec)
u_*	Fiction velocity or shear stress (m/sec)
T_y, T_z	Averaging time (sec)

5.1. Input Parameters and their Fuzziness. The parameters with inherent uncertainty are taken to be triangular fuzzy numbers. These inputs to the model are modelled as triangular fuzzy numbers assigning degree 1 to the most probable value. Average wind speed, roughness length, shear stress or fiction velocity, boundary layer height and heat flux etc. are such parameters. The average wind speed $\overline{u}$ (m/sec) is being obtained from Equation 1, since the parameters friction velocity u_*, roughness length z_0 and Von Karmann constant are fuzzy, average wind so obtained is also fuzzy, however for simplification it is taken to be triangular fuzzy number defined as $< 2, 5, 17 >$. Boundary layer height h (m) is obtained as fuzzy number from the equation $h = 0.2\frac{u_*}{2\Omega sin\phi}$ and found to be triangular fuzzy number $< 673.5, 1171.3, 1903.46 >$. Heat flux w_* (W/m^2) is obtained from $w_* = 0.4(R_s - 100)$ and fuzzy number found is $< 178.84, 186.12, 195.46 >$. The concentration is evaluated at a particular grid point at puff height $z = 10$ m and downwind distance $x = 100$ m away from the source at an averaging time 1 *hour*.

5.2. Results and Discussions. Sensitivity analysis of the horizontal dispersion coefficient reveals that the average wind speed $\overline{u}$ is the most influential parameter followed by heat flux w_*, the parameter fiction velocity u_* and boundary layer height h has non influence on the horizontal dispersion coefficient. The membership function of horizontal dispersion coefficient (m) is shown in Figure 2, shaded regions are the membership function of the horizontal dispersion coefficient in meters obtained by fixing the fuzzy parameter to the most probable

TABLE 3. Hartley-like Measure of Horizontal Dispersion Co-efficient (σ_{xy}) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

Membership Function	HL-measure
$\bar{u}$ Fixed	0.5815
u_* Fixed	1.0000
w_* Fixed	0.9939
h Fixed	1.0000

TABLE 4. Hartley-like Measure of Vertical Dispersion Co-efficient (σ_z) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

Membership Function	HL-measure
$\bar{u}$ Fixed	0.7469
u_* Fixed	1.0000
w_* Fixed	0.9940
h Fixed	0.9783

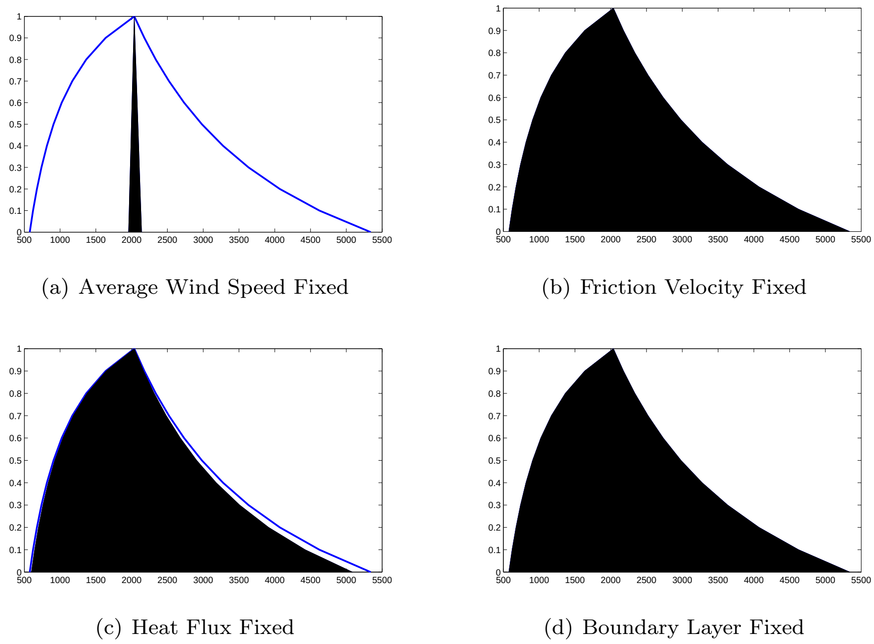
value of degree 1 successively. The Hartley-like measures are depicted in the Table 3.

Sensitivity analysis of the vertical dispersion coefficient is depicted in Figure 3 and Table 4, average wind speed $\overline{u}$ is most influential parameter followed by boundary layer height h and then heat flux w_*, the parameter friction velocity u_* has no influence on the vertical dispersion coefficient.In Figure 3 the outer membership functions are the membership function of the vertical dispersion coefficient (m) and the inner shaded membership functions are the membership function obtained by fixing the fuzzy parameters to the most probable value successively. Sensitivity analysis can be studied from the figures by observing the membership functions.

Sensitivity analysis of the concentration (Bq/m^3) emitted from the source can be studied from Figure 4 and Table 5. The outer contour of Figure 4 is the membership function of the concentration obtained without fixing any fuzzy parameters, whereas the inner contour is the membership function of the concentration obtained by fixing the fuzzy parameters to the most probable value of degree 1. The most influential parameter is found to be the average wind speed $\overline{u}$ followed by the boundary layer height h and then the heat flux w_*. The parameter friction velocity u_* does not have any significant influence on the concentration obtained from the RIMPUFF model.

6. Conclusion

Sensitivity analysis of the atmospheric dispersion model yield that the uncertain model parameter average wind speed is the most sensitive parameters which



(a) Average Wind Speed Fixed

(b) Friction Velocity Fixed

(c) Heat Flux Fixed

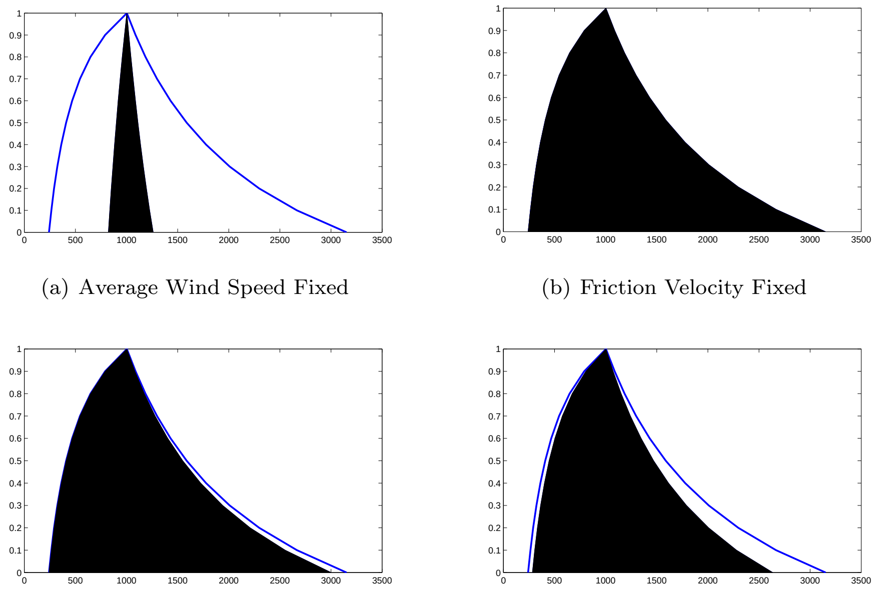
(d) Boundary Layer Fixed

FIGURE 2. Membership function of Horizontal Dispersion Co-efficient (σ_{xy}) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

TABLE 5. Hartley-like Measure of Concentration (χ) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

Membership Function	HL-measure
$\bar{u}$ Fixed	0.8714
u_* Fixed	1.0000
w_* Fixed	0.9958
h Fixed	0.9953

influences the downwind concentration. The sensitivity analysis has been carried on the stand point that the input parameters are uncertain, these parameters are treated as triangular fuzzy number. Hartley-like measure has been evaluated on each membership function of the output with/without fixing. Sensitivity analysis of the atmosphere dispersion model yield that the uncertain model parameter average wind speed is the most sensitive parameters which influences the downwind concentration. The other parameters such as the boundary layer height and the heat flux or the shear stress has less influence on the downwind



(a) Average Wind Speed Fixed

(b) Friction Velocity Fixed

(c) Heat Flux Fixed

(d) Boundary Layer Fixed

FIGURE 3. Membership function of Vertical Dispersion Co-efficient (σ_{xy}) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

concentration of the RIMPUFF model, whereas the uncertain parameter fiction velocity has no significant influence on the downwind concentration of the model.

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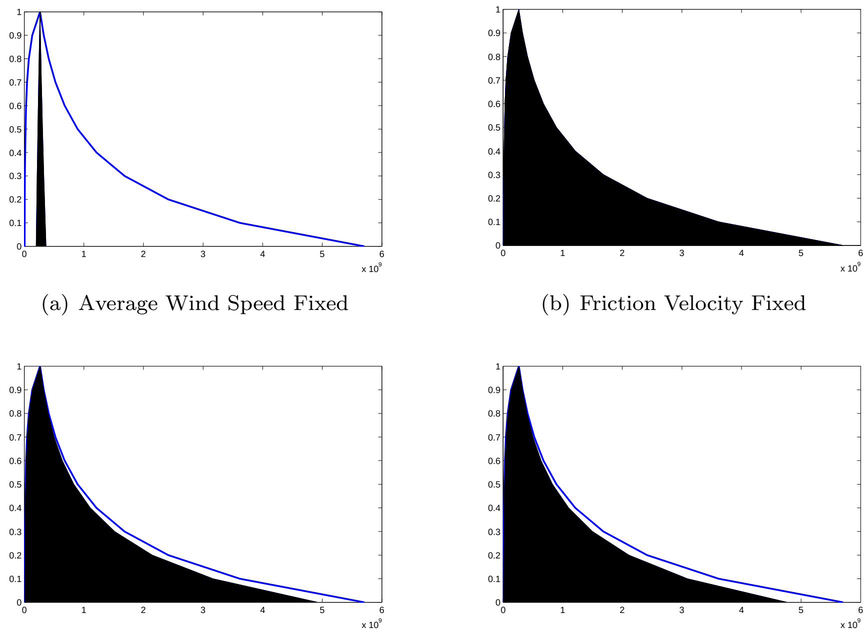

(a) Average Wind Speed Fixed

(b) Friction Velocity Fixed

(c) Heat Flux Fixed

(d) Boundary Layer Fixed

FIGURE 4. Membership function of Concentration (χ) successively fixing average wind speed $\overline{u}$, friction velocity u_*, heat flux w_* and boundary layer height h.

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